$g_A = -1.23 \pm 0.01$.[6] Secondly, there may be situations in which one has a (1, 8) tensor along with some knowledge of its coupling constants between octet baryons. In this event, comparison with Eq. (9) should be of some use in determining how much the tensor in question "leaks" out of the baryon octet and decuplet. Evidently this particular aspect has some relevance to the hadronic part of the weak Hamiltonian – a matter which will be discussed in a future publication.

The author is happy to acknowledge several stimulating conversations with Professor Michael Scadron.

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Chiral Symmetry and Structure of the Nucleon

George F. Chapline, Jr.
1252 Walpert Street, Hayward, California 94541

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Starting from the nonlinear σ model it is shown that for large momentum transfer the proton charge form factor is approximately equal to $(1+Q^2/Q_0^2)^{-2}$, where the constant Q_0 is related by a simple transcendental equation to the pion-nucleon coupling constant f and to a constant f_π related to the charged-pion decay rate. Using the experimental values of f and f_π we obtain $Q_0 = 800$ MeV. The charge radius of the proton is calculated to be 0.85 F. Further, it is shown that the "hard core" radius for nucleon-nucleon interactions should be approximately equal to $(2/Q_0) = 0.5$ F. An interpretation of the most prominent pion-nucleon resonances is also offered.

I. INTRODUCTION

The fact that the axial-vector coupling constant for nucleon β decay does not differ too much from the vector coupling constant has led to the hypothesis that the strong interactions are approximately invariant under transformations which change parity but do maintain handedness, or chirality. This hypothesis has, in turn, led to a number of interesting relations between low-energy processes.[1] As yet, however, there have been very few applications of chiral symmetry to intermediate- and high-energy process.[2] It has been shown[3] that the consequences of chiral symmetry for low-energy processes can be obtained by applying lowest-order perturbation theory to certain nonlinear Lagrangians. The question of whether these nonlinear Lagrangians can be applied to intermediate- and high-energy phenomena has remained unanswered, due largely to the lack of understanding of how to make calculations when one cannot use perturbation theory. In this paper we will make use of some simple-minded approximations to show that a nonlinear chiral Lagrangian may be useful even in situations where perturbation theory does not apply. In particular, we will show that the nonlinear σ model of Gell-Mann and Lévy[4] can be used to explain the behavior of the electron-proton elastic scattering form factor. We will also suggest on the basis of the explanation given for the form factor that a simple interpretation can be given for the most prominent pion-nucleon resonances.

In the nonlinear σ model the Lagrangian density for the pion field has the form

$$\mathcal{L}_\pi = \tfrac{1}{2}(\partial_\mu \vec{\phi})^2 + \tfrac{1}{2}(\partial_\mu \sigma)^2 + \mu^2 f_\pi \sigma , \tag{1}$$

where μ is the pion mass. The pion field $\vec{\phi}$ is related to σ by

$$\vec{\phi}^2 + \sigma^2 = C^2 , \tag{2}$$

where C is a constant. The divergence of the β-

decay axial-vector current, P_μ, in this model is

$$\partial_\mu P_\mu = \sqrt{2}\, f_\pi \mu^2 \phi_{\pi^-} \,. \tag{3}$$

This relation allows one to relate f_π to the charged-pion decay rate.[5] In the unrenormalized σ model one has $C = f_\pi$. However, it has been claimed[6] that because of the presence of the axial-vector A_1 meson the pion field is renormalized so that Eq. (3) still holds but $C = Zf_\pi$ where $Z = m_{A_1}/m_\rho$. According to Weinberg[7] $m_{A_1} = \sqrt{2}\, m_\rho$ so we would have $C = \sqrt{2}\, f_\pi$.

Since the Lagrangian must be a real function, Eq. (2) implies that the magnitude of the pion field satisfies $|\vec{\phi}|^2 \leq C^2$. From the physical point of view we can regard this condition as resulting from pion-pion interactions. That is, because of pion-pion interactions the density of pions in any region of space cannot exceed a certain value.
If we assume that the charge distribution of a proton is due to virtual pions and that the density of pions is bounded then it follows immediately that the form factor for elastic electron-proton scattering must asymptotically decrease faster than the square of the momentum transfer. In fact, it can be shown[8] that if the charge density is bounded then the form factor must decrease at least as fast as the fourth power of the momentum transfer. Experimentally,[9] the form factor is observed to decrease approximately as the fourth power of the momentum transfer for large momentum transfers – a fact that has been regarded as somewhat of a mystery. In the next section we will show that if one makes certain plausable assumptions, then our condition $|\vec{\phi}^2| \leq C^2$ leads to a simple explanation for the observed proton form factor.

II. PROTON CHARGE DISTRIBUTION

If we suppose that the virtual pions surrounding a proton are emitted from a fixed point source and do not interact with one another, then the pseudoscalar pion field ϕ_α would be given by

$$\vec{\phi} = \frac{f}{4\pi} \vec{\sigma} \cdot \vec{\nabla} \frac{e^{-\mu r}}{\mu r} \vec{\tau} \,, \tag{4}$$

where $f \approx 1.0$ is the pseudovector coupling constant and σ and τ are the Pauli matrices for spin and isospin. Let us now turn on the pion-pion interactions so that $|\vec{\phi}|^2 \leq C^2$. We see that the r^{-1} factor in Eq. (4) must somehow be cut off as $r \to 0$. A mathematically simple way to describe this behavior would be to set $\phi_\alpha = \vec{\sigma} \cdot \vec{\nabla} f \tau_\alpha$ where

$$f(r) = \begin{cases} c_1 e^{-\alpha r}/\alpha a\,, & r < a \\ c_2 e^{-\mu r}/\mu r\,, & r > a\,. \end{cases} \tag{5}$$

Although the choice of the specific form for $f(r)$ given in Eq. (5) would appear to be completely *ad hoc*, the most important restriction involved in the choice of this specific form is that the second derivative of $r|\vec{\phi}|^2$ with respect to r does not vanish as $r \to 0$. That is, any reasonable choice for the function $f(r)$ which has this property and satisfies the condition $|\vec{\phi}^2| \leq C$ would give about the same behavior for the form factor.[8] A possible justification for assuming that the second derivative of $r|\vec{\phi}|^2$ does not vanish as $r \to 0$ is mentioned at the end of this section in connection with a discussion of the quantum corrections to our theory.

If we impose the physically reasonable requirements that the charge density and its gradient be continuous, then

$$C_1 = e^{(\alpha - \mu)a}\left(1 + \frac{1}{\mu a}\right) C_2 \tag{6}$$

and

$$\alpha = \mu + \frac{1}{a}\left(1 + \frac{1}{1 + \mu a}\right) \,.$$

We see that the function $f(r)$ depends on two constants, C_2 and a. Since the pion field must approach the form given in Eq. (4) as r becomes large we have that $C_2 = f/4\pi$. The constant a can be determined from the condition that as $r \to 0$ the square norm of the pion field, $\phi_\alpha^* \phi_\alpha$, will approach its maximum value, i.e., C^2. This condition gives $C_1 = aC$. Using this relation together with Eq. (6) we find that a satisfies the equation

$$a = \frac{f}{4\pi C}\left(1 + \frac{1}{\mu a}\right) \exp\left(1 + \frac{1}{1 + \mu a}\right) \,. \tag{7}$$

Substituting $C = \sqrt{2}\, f_\pi$ and using the experimental values of f and f_π gives $a = 0.85\mu^{-1}$. Thus we find that pion-pion interactions affect the distribution of virtual pions when the distance to the center of the proton is less than about 1 F.

The charge density due to virtual pions is proportional to $(\nabla f)^2$. The form factor for the charge distribution can thus be found by taking the Fourier transform of $[\nabla f(r)]^2$. We find, therefore, that the form factor for the charge distribution will be proportional to the function

$$F(Q^2) = e^{-2\mu a}\left(1 + \frac{1}{\mu a}\right)^2 \int_0^1 e^{-2\alpha a x} \frac{\sin(Qax)}{(Qa)}\, x dx + e^{-2\alpha a} \int_1^\infty e^{-2\mu a x}\left(1 + \frac{1}{\mu a x}\right)^2 \frac{\sin(Qax)}{(Qa)} \frac{dx}{x} \,. \tag{8}$$

The first term on the right-hand side can be evaluated explicitly. The second term can be evaluated for large Q^2 by integrating by parts. One finds that for large Q^2 the function $F(Q^2)$ is given by

$$F(Q^2) \underset{Q^2\to\infty}{\sim} \frac{2}{(1+Q^2/4\alpha^2)^2}\,\frac{e^{-2\mu a}}{(2\alpha a)^3} \times\left[1+\frac{e^{-2\alpha a}}{2\alpha a}\left(3-\frac{4Q^2}{Q^2+4\alpha^2}\right)\cos Qa\right]. \quad (9)$$

Putting in the numerical values of a and α implied by Eqs. (6) and (7) one finds that the numerical value of the second term in the brackets is small compared to 1. Thus for large Q^2 we find that the form factor is approximately equal to $(1+Q^2/4\alpha^2)^{-2}$. That the form factor for large Q^2 can be approximately described by such an expression is, of course, a well-known experimental fact.[9] The value of 2α implied by Eqs. (6) and (7) turns out to be 800 MeV, in remarkably good agreement with the experimental value, 840 MeV. In Fig. 1 we compare the experimental measurements[9] of the magnetic form factor with the values of $F(Q^2)/F(0)$ calculated directly from Eq. (8). It can be seen that the over-all agreement is fairly good, the difference being mainly due to the fact that our calculated value of 2α is slightly smaller than the experimental value. It is interesting that Eq. (8) predicts that for large Q^2 the form factor should oscillate about a value slightly below the value predicted by the simple dipole formula. This behavior is also observed experimentally (cf. Fig. 2 of Ref. 8); however, the deviations from the dipole formula predicted by Eq. (8) are smaller than are observed experimentally.

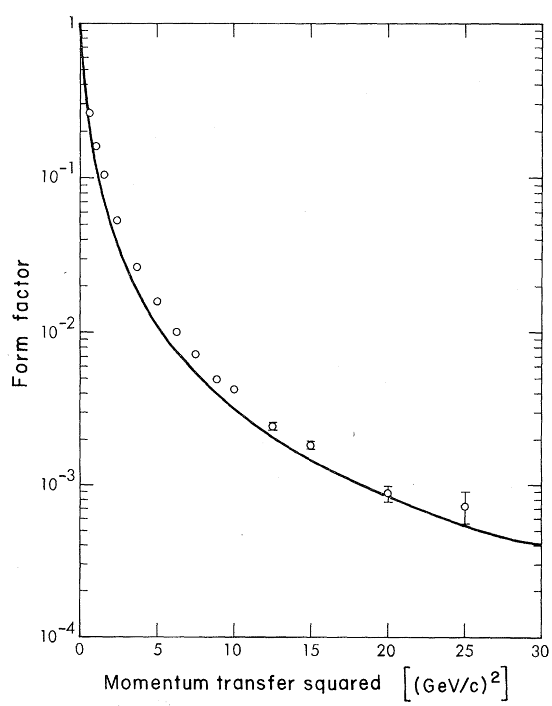

FIG. 1. The curve shows the function $F(Q^2)/F(0)$ where $F(Q^2)$ is given by Eq. (8). The points show measurements of the magnetic form factor as given in Ref. 8.

If we had chosen the renormalization constant $Z=1$ instead of $Z=\sqrt{2}$ we would have obtained $2\alpha = 700$ MeV instead of 800 MeV. Thus we can conclude that the qualitative nature of our results does not depend on the exact value of Z (provided, of course, it is not too different from 1). It is also interesting to note that if we had used the scalar-meson theory instead of the pseudoscalar-meson theory [i.e., omit the $\sigma\cdot\nabla$ in Eq. (4)] we would have obtained $2\alpha = 1500$ MeV. Thus the pseudoscalar-meson theory works much better than the scalar-meson theory.

The root-mean-square charge radius of the proton, $\langle\langle r^2\rangle\rangle^{1/2}$, will in our theory be approximately equal to $\sqrt{3}/\alpha$. Using the value of α implied by Eqs. (6) and (7) we obtain

$$\langle\langle r^2\rangle\rangle^{1/2} = \left(1+c+\frac{c^2}{1+c}\right)^{-1}\frac{\sqrt{3}}{\mu}, \quad (10)$$

where the numerical constant c is the solution of the transcendental equation

$$c=\frac{c_0}{1+c}\exp\left[-\left(1+\frac{c}{1+c}\right)\right]$$

and

$$c_0=\frac{4\pi}{f}\,\frac{\sqrt{2}\,f_\pi}{\mu}.$$

All the constants describing the strength of the strong interactions are contained in c_0. Although f_π refers to a weak decay we can explicitly exhibit its dependence on the strong interactions by making use of the Goldberger-Treiman[10] and Adler-Weisberger[1] relations. Together, these relations imply that $f_\pi \simeq (1/2f_0)\mu$, where $f_0 \approx 0.8$ is the vector coupling constant which describes the strength of the low-energy s-wave pion-nucleon interaction. Thus we have that

$$c_0 \simeq \frac{4\pi}{\sqrt{2}\,ff_0}, \quad (11)$$

which shows that the proton charge radius is related to the low-energy s-wave pion-nucleon interaction. If we use the experimental values of f_π and f we obtain $\langle\langle r^2\rangle\rangle^{1/2} = 0.85$ F.

We conclude this section with a comment on the quantum corrections to our theory. In calculating the form factor we have assumed that the quantity $|\vec{\phi}|^2$ is a classical field. In reality, of course, $|\vec{\phi}|^2$ is subject to quantum fluctuations. In fact, it has been shown by Yennie[11] that the quantum corrections to a nonlinear meson theory become increasingly important as the source strength is increased. Yennie gave formulas for the first-order quantum corrections to the energy and equation of motion when the nonlinear term in the Lagrangian had the form $\lambda\phi^4$. If we use Yennie's formulas together with the coefficient of $|\vec{\phi}|^4$ in an expansion of our Lagrangian in powers of $|\vec{\phi}|^2$ to estimate the quantum corrections to our classical theory, we find that the quantum corrections are on the order of a few percent. Thus the quantum corrections to our theory are not expected to be very important. The basic reason why the quantum corrections to a nonlinear meson theory increase with increasing source strength is that the classical field acts as a potential for the fluctuating field so that as the strength of this potential increases, the zero-point energy of the fluctuating field increases. This argument suggests that in our theory where $|\vec{\phi}|^2$ is constrained to be less than C^2, it would be energetically favorable for $|\vec{\phi}|^2$ to approach the limit C^2 as slowly as possible. In other words, we would expect that the derivative of the charge density to be finite as $r\to 0$. This, of course, would lead to the nonvanishing of the second derivative of $r|\vec{\phi}|^2$ as $r\to 0$, as was assumed in Eq. (5).

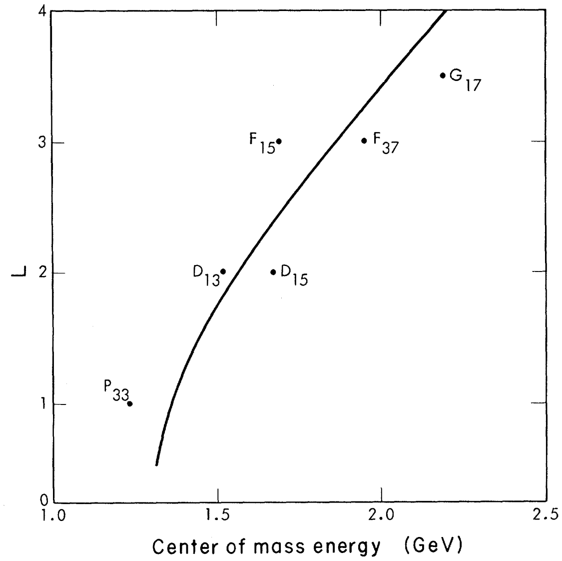

FIG. 2. The curve shows $L=qb-\frac{1}{2}$ plotted as a function of center-of-mass energy. The points are the positions of the most prominent pion-nucleon resonances.

III. PION-NUCLEON RESONANCES

According to our picture as one approaches the center of a proton the density of virtual pions does not become infinite but instead approaches a certain maximum value. Further we have found that the density of virtual pions does not begin to fall appreciably below the maximum value until one reaches a radius $r_0=(2\alpha)^{-1}$. Consequently, we expect that a pion introduced from the outside would have a difficult time entering the region $r<r_0$. Thus if we bring two protons close together we expect that a strong repulsion would set in when the separation of the protons is less than $2r_0$. Indeed, our calculated value for α gives $2r_0=0.49$ F, in good agreement with the experimental value of the distance at which the nucleon-nucleon force becomes strongly repulsive, as determined, for example, from nucleon-nucleon scattering.[12]

Although a pion incident on a proton from the outside would have a difficult time entering the region $r\lesssim r_0$, it could move around in the region $r>r_0$. Further, we showed in the last section that pion-pion interactions are important when r is less than about 1 F but that the pion can propagate freely at larger values of r. Therefore, as in the case of motion around an impenetrable sphere we would expect that a resonance in pion-proton scattering would occur when the center-of-mass momentum q of the pion satisfied

$$qb=L+\tfrac{1}{2}\,, \tag{12}$$

where b is some length on the order of 1 F. The integer L in this formula represents the orbital angular momentum of the resonance. In Fig. 2 we have plotted L as given by Eq. (12) as a function of the center-of-mass energy. In making this plot we have chosen $b=1$ F. On the same graph we have shown the positions of the most prominent pion-nucleon resonances. It can be seen that the positions of these resonances roughly follow the curve, which suggests that these resonances can be interpreted as being due to pions orbiting about an impenetrable region.

We should mention in this connection that Walecka[13] has shown that one can qualitatively understand the excitation of nucleon excited states in electron scattering if one assumes that the pion field in the excited states oscillates about an equilibrium field somewhat similar in form to that given in Eq. (5). In particular he assumes that the pion field in the ground state does not become infinite as one approaches the center of the nucleon but instead levels off at a finite value.

IV. CONCLUSION

There are obviously a number of questions which we have left unanswered. For example, what effect does nucleon recoil have on our calculation of the proton charge form factor? Can one explain the neutron charge distribution? Also can one explain inelastic electron proton scattering? Nevertheless, the good agreement with experimental results that we have obtained suggests that our ideas concerning the structure of a proton have some validity.

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Bose Distribution and Scaling Law*

T. F. Hoang
Argonne National Laboratory, Argonne, Illinois 60439
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Properties of a two-parameter Bose-type distribution have been investigated using data on π and K mesons of inclusive p-p collisions from 12 to 30 GeV/c. It is found that the temperature T is characteristic of the particle emission: $T_\pi > T_K$, and T for π^- is about 5 MeV higher than that of π^+. The scaling behavior is found to hold for both π and K for incident lab momentum above 20 GeV/c.

In a recent work on the inclusive $\bar{p}$-p reactions,[1] it has been found that the salient features of the double-differential cross sections in terms of the c.m. longitudinal, P_L, and the transverse, P_T, momentum, can be accounted for by the following Bose-type distribution:

$$\frac{d\sigma}{dP_T{}^2 dP_L} \sim \frac{1}{e^{\epsilon(\lambda)/T} - 1}, \tag{1}$$

where T is the temperature, the Boltzmann constant being set $k = 1$, and

$$\epsilon(\lambda) = (P_T{}^2 + \lambda^2 P_L{}^2 + m^2)^{1/2}, \tag{2}$$

m being the mass of the secondary meson. The dimensionless parameter λ is related to the scaling law,[2,3] according to which we should expect

$$\lambda \sim 1/P_{\max}, \tag{3}$$

$P_{\max}$ being the maximum of the c.m. momentum of the secondary meson.[4] It should be mentioned that another interpretation for λ is also possible.[5]

The purpose of this paper is to investigate the intrinsic properties of the two parameters T and λ. In particular, we propose to investigate the scaling behavior according to (3).

We recall that the temperature T, as in the Bose distribution, i.e., $\lambda = 1$, is determined entirely by the average of P_T, and that the parameter λ depends on the ratio of averages of P_T and P_L. Referring to formula (33) of Ref. 4, we write

$$\langle P_T \rangle / \langle P_L \rangle = \tfrac{1}{2}\pi\lambda. \tag{4}$$

In the present work, we shall apply the distribution to the currently available data on π and K production by p-p collisions. We begin with the inclusive reactions

$$p + p \to \pi^{\pm} + \text{anything}$$

from the following experiments[6]: Berkeley,[7] Brookhaven,[8] Scandinavian Collaboration,[9] and Day *et al.*[10] The parameters T and λ are estimated by fitting the experimental data with the distribution (1), except the Berkeley data. In this case, the