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Exotic Fragmenting States in Inclusive Reactions*

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The general inclusive process $a+b\to c+X$ is considered in the b fragmentation region. It is proved that if the fragmenting state $(b\bar{c})$ is exotic, the presence or absence of energy-dependent terms in the invariant cross section f cannot depend on whether or not $(ab\bar{c})$ is exotic. If f is a smooth function of the kinematic variables, there must be a nonzero but limited region of the variables, in which the energy-dependent terms for exotic $(b\bar{c})$ are small compared to their average size when $(b\bar{c})$ is not exotic. In this region, present experimental evidence is in agreement with the hypothesis that the energy-dependent terms are small when $(ab\bar{c})$ is exotic.

I. INTRODUCTION

We consider the inclusive reaction $a+b\to c+X$, where a, b, and c are specific hadrons and X is any set of particles. Mueller has shown that the invariant cross section is proportional to the imaginary part of the elastic $a+b+\bar{c}$ scattering amplitude in the appropriate unphysical region.[1] In the fragmentation region of the particle b, the Mueller-Regge diagram of Fig. 1 dominates, where α denotes Regge trajectories. Since the $(b\bar{c})$ state may be exotic, inclusive reactions involve a phenomenon that does not occur in two-hadron → two-hadron reactions, namely, the coupling of trajectories to exotic states. Nonexotic states are meson states with Y (hypercharge) and I_z obtainable from quark-antiquark pairs, and baryon states with the Y and I_z of three-quark composites. Understanding this new phenomenon may be important to our understanding of strong interactions. In this paper, we are concerned particularly with the couplings of Regge trajectories to exotic $(b\bar{c})$ states.

First, we define the important dynamical quantities considered. The invariant cross section is $f=Ed\sigma/d^3p$, where E and p are the energy and momentum of the produced particle c. In the fragmentation region of particle b at high energy, the Mueller analysis implies that f may be written in the form[1,2]

$$f(s,p_\parallel^b,p_\perp^2)=A(p_\parallel^b,p_\perp^2)+s^{\alpha_{VT}-1}B(p_\parallel^b,p_\perp^2),\qquad(1)$$

where s is the square of the total energy in the center-of-mass system, $p_\parallel^b$ is the momentum of c in the direction of a in the b rest system, $p_\perp$ is the transverse momentum of c, and α_{VT} is the intercept of the Regge trajectories of the $Y=I_z=0$ vector and tensor mesons. An alternate longitudinal variable is $x=2p_\parallel^{\text{c.m.}}/s^{1/2}$, where $p_\parallel^{\text{c.m.}}$ is the momentum of c in the direction of the projectile in the center-of-mass system. Clearly, $|x|<1$. If b is the projectile, x is given by

$$x=[E_c^b(1-\epsilon_+^2)^{1/2}(1-\epsilon_-^2)^{1/2}-p_\parallel^b(1-\epsilon_+\epsilon_-)]/m_b,\qquad(2)$$

where $\epsilon_\pm=(m_a\pm m_b)/s^{1/2}$, m_i is the mass of i, and E_c^b is the energy of c in the b rest system. If b is the target, one replaces x by $(-x)$ in this equation. The fragmentation regions are those for which x is not close to zero.

In Sec. II, we prove a theorem concerning the energy-dependent terms [B terms in Eq. (1)] when $(b\bar{c})$ is exotic. In Sec. III we consider the question of when the B terms are negligible. Exotic $(b\bar{c})$ states are crucial to this consideration.

II. EXOTIC-STATE THEOREM

We discuss and prove the following theorem: *If the Regge trajectories are all nonexotic, $(b\bar{c})$ is exotic, and the energy-dependent B term of Eq. (1) is nonzero for any $(ab\bar{c})$, then B is nonzero for some exotic $(ab\bar{c})$*. There is one special exception, noted below.

This theorem is related to a recent controversy concerning the energy-dependent terms. Chan, Hsue, Quigg, and Wang have postulated that these terms vanish in both fragmentation regions whenever the state $(ab\bar{c})$ is exotic.[2] We call this the "exotic $ab\bar{c}$ hypothesis." This hypothesis has been criticized by Ellis, Finkelstein, Frampton, and Jacob, and by Logan; these authors use factorization arguments to show that the exotic $ab\bar{c}$ hypothesis implies that the B terms must vanish in some amplitudes for which $(ab\bar{c})$ is not exotic.[3] These factorization arguments involve exotic $(b\bar{c})$ states, although this is not stressed in Ref. 3. The results of Ref. 3 are special cases of our theorem.

As shown below, the theorem follows from either of the following two sets of assumptions: (i) exact

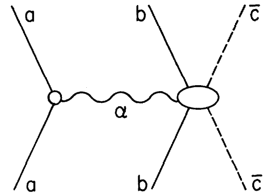

FIG. 1. Mueller-Regge diagram for fragmentation region of b.

SU(3) symmetry, with any number of singlet and octet, mesonic Regge trajectories allowed; (ii) no SU(3) symmetry, with certain restrictions on the types and couplings of the trajectories.[4]

(i) Exact SU(3) case. We allow the particles a and b to be any members of the baryon and pseudoscalar meson octets. If N_B, the baryon number of $(ab\bar{c})$, is 2 or 3, the theorem is trivial, so we may restrict N_B to the values 1, 0, and -1. First, we consider the cases $N_B=0$ and -1. Then, the $(b\bar{c})$ state is of baryon number 0 or -1, and is exotic if the $(b\bar{c})$ representation R is $\underline{10}$, $\underline{27}$, or $\underline{35}^*$. (The representation $\underline{35}^*$ is possible if $\bar{c}$ is an antibaryon resonance state of the representation $\underline{10}^*$.) If $R=\underline{10}$, one can see from the $(\underline{8}+\underline{8}\to\underline{10}+\underline{10}^*;\ \underline{8}+\underline{10}\to\underline{8}+\underline{10})$ crossing matrix that if only t-channel singlets and octets make contributions to the B terms, and if the sum of these contributions is zero in the exotic s channels $\underline{10}$, $\underline{27}$, and $\underline{35}$, then all t-channel contributions are identically zero.[5]

If R is $\underline{27}$ or $\underline{35}^*$, the relevant crossing matrix is not available, so we make an alternate proof. We will assume that some B term is nonzero, and show that this leads to a nonzero B for some exotic $(ab\bar{c})$. First we prove the following lemma: If any B is nonzero, then for every state of the $(b\bar{c})$ representation R, there is some member of the a octet such that $B\neq 0$. If a direct-channel (a-$b\bar{c}$ channel) representation r contributes to the B terms, the contribution is proportional to the sum $\sum_s \alpha^2(s-a-b\bar{c})$, where α is the SU(3) Clebsch-Gordan coefficient, and the sum is over all states s of r. It is a property of Clebsch-Gordan coefficients that for every $(b\bar{c})$ state, there is an s and a such that $\alpha(s-a-b\bar{c})\neq 0$. Since all contributions are of the same sign, the lemma follows.

The Regge trajectories are limited to $Y=I_z=0$ members of singlets and octets. It follows from the lemma, SU(3) Clebsch-Gordan coefficients, and consideration of the trajectory-a couplings that for any $(b\bar{c})$ state, there must be a nonzero B for at least one of the $I=0$, $I=1$, or $Y=1$ states of the a octet.[6] We take $(b\bar{c})$ to be the $Y=0$, $I=2$ quintet of R. It is a property of s-t channel, SU(2) crossing matrices that the isosinglet-exchange contribution is the same for all s-channel isospins, while the isotriplet-exchange contribution is a monotonic changing function of s-channel isospin. Therefore, B is nonzero for some exotic (a-$b\bar{c}$) involving the $(b\bar{c})$ isoquintet. For example, if B is nonzero when a is the $Y=1$ isodoublet of the octet, then $B\neq 0$ for at least one of the two exotic (a-$b\bar{c}$) isospins $\frac{3}{2}$ and $\frac{5}{2}$. The theorem is proved for $N_B=0$ and -1.

If $N_B=1$, the arguments are similar, but there is one exception, occurring when a is a baryon and $(b\bar{c})$ is a state of baryon number 0 and SU(3) representation $\underline{10}$. In this case, however, the $\underline{10}^*$ states obtained by applying the particle-antiparticle conjugation operator to $(b\bar{c})$ do satisfy the theorem.

(ii) Case with no SU(3) symmetry. In this case, we assume that the only trajectories contributing to the B terms are those of ω, ρ, f, and A_2 mesons. We first let particle a be a meson, and consider a specific $(b\bar{c})$ state of exotic (Y, I_z).[7] The set of a-octet states that can lead to nonexotic $(ab\bar{c})$ consists at most of two adjacent outer states on the Y-I_z diagram. Hence, the exotic $(ab\bar{c})$ condition of the theorem implies that $B=0$ for at least two members of the pion triplet, compelling the ρ and f trajectories to be decoupled from the $(b\bar{c})$. The exotic $(ab\bar{c})$ condition also requires that $B=0$ for both members of at least one of the K and $\bar{K}$ doublets, thus compelling the ω and A_2 also to be decoupled from the $(b\bar{c})$ state. Thus all B terms vanish, proving the theorem.

In the case when particle a is a baryon, an extra assumption is needed. A simple, sufficient assumption (which is stronger than needed) is that the f and A_2 trajectory couplings to the baryon a are proportional to those of the ω and ρ. (This follows from the duality condition that the imaginary part of the vector and tensor trajectories cancel for all two-body baryon-baryon amplitudes.) The contributions of isospin-0 and isospin-1 trajectories are then proportional to $g(\omega)+\kappa g(f)$ and $g(\rho)+\kappa g(A_2)$, where $g(i)$ is the coupling constant of the i trajectory to the $(b\bar{c})$ state in question, and κ is the product of the above-mentioned proportionality constant and a signature factor. We label these two contributions G_0 and G_1. The exotic $(ab\bar{c})$ condition of the theorem implies that B must vanish for at least two members of the Σ multiplet. If (as is generally believed) the ω and ρ trajectories are coupled to Σ's, it follows that $G_0=G_1=0$, completing the proof.

III. ENERGY-DEPENDENT TERMS WHEN $(b\bar{c})$ IS EXOTIC

We turn to the question of whether or not the B term of Eq. (1) is expected to be small when $(b\bar{c})$ is exotic. For definiteness we take b to be the projectile. In the limit when s is large compared to m_a^2, m_b^2, and $(E_c^b)^2$, it can be shown that the ratio

(M^2/s) approaches the limit

$$(M^2/s) \to (1-x), \qquad (3)$$

where M is the mass of the state X and x is given in Eq. (2). At the outer limit of the b fragmentation region, $x \to 1$, and M^2 remains constant as $s \to \infty$. This is the conventional Regge limit for $a+b \to c+X$, conceived as a two-hadron → two-hadron process, at small $t=(p_b - p_c)^2$. In conventional Regge theory, the cross section for such a process is small when the t channel $(b\bar{c})$ is exotic. We assume that the invariant cross section f is a smooth function of x even at the limits of x. Then, there must be a nonzero region of small $(1-x)$ where the A and B terms both are small when $(b\bar{c})$ is exotic. We define the "extreme fragmentation region" as the region of x and $p_\perp^2$ in which the B for exotic $(b\bar{c})$ are small relative to the average B for nonexotic $(b\bar{c})$.

On the other hand, in the double-Regge region, where x is small and the squared invariant momentum transfers t and $u=(p_a - p_c)^2$ are large, neither the A nor the B term of Eq. (1) can be small for all exotic $(b\bar{c})$. This follows from the Regge-pole analysis of Ref. 1 and of DeTar *et al.*[8]; in this region the states b and $\bar{c}$ are connected by the Pomeranchukon trajectory, whose couplings do not depend on whether or not $(b\bar{c})$ is exotic. Thus, the extreme fragmentation region cannot include all of positive x. When b is the target, similar considerations apply to negative x.

Our smoothness condition is a particular type of duality hypothesis. Since $|x|$ cannot be exactly equal to one, it follows from Eq. (3) that M^2 does increase as $s \to \infty$ for any fixed x and $p_\perp^2$. Our condition is that when $|x|$ is large (when M^2 increases slowly with s), the conventional two-particle Regge representation is valid, as well as the Mueller representation of Eq. (1). The condition will be useful if it is valid over an appreciable range of x and $p_\perp^2$.

One important consequence of these considerations is that the hypothesis of Ref. 2, that $B \sim 0$ when $(ab\bar{c})$ is exotic, is possible only in the extreme fragmentation region. In this region, the arguments of Ref. 3 do not apply, and the exotic $ab\bar{c}$ hypothesis is a conceivable extension of duality. Outside the extreme fragmentation region, one expects appreciable B terms for exotic $ab\bar{c}$ *whether or not* $(b\bar{c})$ *is exotic*. This follows because $(b\bar{c})$ states of specific hadrons have appreciable components of exotic SU(3) representations even when (Y, I_z) is not exotic. Therefore, if the exotic $ab\bar{c}$ hypothesis proves to be accurate in a given region of $p_\parallel^b$ and $p_\perp^2$ for almost all processes, this region must be part of the extreme fragmentation region.

One may attempt to determine the range of the extreme fragmentation region without knowing whether or not the exotic $ab\bar{c}$ hypothesis is correct, by comparing processes of exotic $(b\bar{c})$ to processes for which neither $(b\bar{c})$ nor $(ab\bar{c})$ is exotic. Some data of this kind have been published, involving the $\pi^\pm + p \to \pi^\mp + X$ and $\pi^\pm + p \to \pi^\pm + X$ reactions in the meson fragmentation region.[9] These data do show clearly that the A terms of f decrease rapidly as x approaches unity in the $\pi^\pm \to \pi^\mp$ (exotic $b\bar{c}$) reactions. However, the data are too sparse for an accurate estimate concerning the B terms.

We next consider some evidence concerning the exotic $ab\bar{c}$ hypothesis. Crennell, Gordon, Ioffredo, and Lai have shown recently that data from the reactions $\pi^+ + p \to c + X$ (where c is a π^+, π^-, and K_S^0) at 6 and 22 GeV/c support the hypothesis in both fragmentation regions, for an appreciable range of $p_\parallel^b$.[10] (If b is a meson, the hypothesis is well satisfied when $p_\parallel^b < 1.0$ GeV/c.) The extreme fragmentation region appears to extend this far in $p_\parallel^b$. The energy-dependent term in the $\pi^+ + p \to \pi^- + X$ reaction is appreciable when $p_\parallel^b > 2.5$ GeV/c in the meson fragmentation region, supporting our contention that the extreme fragmentation region is limited.

Berger, Oh, and Smith have pointed out that the exotic $ab\bar{c}$ hypothesis does not work for an integral over all x and $p_\perp^2$ in the process $p+p \to K_1^0 + X$; the total K_1^0 inclusive cross section increases by $\sim 50\%$ in the range of beam momentum 13–28 GeV/c.[11] However, it is seen from Figs. 2(c) and 2(d) of this reference that when $p_\parallel^b < 1.0$ GeV/c, the energy dependence is small, and perhaps zero. Therefore, these data support the principal conclusions obtained from Ref. 10.

Another significant experimental analysis involving exotic $(b\bar{c})$ states is that of M. Foster *et al.*[12] These authors show that in the projectile fragmentation region ($p_\parallel^b < 0.5$ GeV/c), the invariant cross section f for the process $K^- + p \to \pi^+ + X$ at 9 GeV/c is almost a factor of two larger than the f for $K^+ + p \to \pi^- + X$ at 12.7 GeV/c, at corresponding values of $p_\perp^2$. Since the Pomeranchukon-exchange contributions to the $(b\bar{c})$ states $(K^-\pi^-)$ and $(K^+\pi^+)$ are expected to be equal as well as energy independent, this is evidence that the energy-dependent terms are significant. However, this result does not imply that this range of $p_\parallel^b$ and $p_\perp^2$ is outside the extreme fragmentation region. We have defined the extreme fragmentation region by the condition that the B terms for exotic $(b\bar{c})$ are small compared to their average size for nonexotic $(b\bar{c})$. The result of Foster *et al.* may mean only that in the $K^+ \to \pi^-$ and $K^- \to \pi^+$ processes, the A term of f decreases faster than the B term as x approaches unity. More data on these processes are needed.

IV. CONCLUSIONS

Our principal conclusions, in addition to the theorem, are that the extreme fragmentation region is of appreciable extent in $p_\parallel^b$ and $p_\perp^2$, and that outside this region, nonzero energy-dependent terms in f are expected, whether or not $(b\bar{c})$ is of exotic (Y, I_z). There has been much confusion in the literature concerning the exotic $(ab\bar{c})$ hypothesis, caused by the fact that most authors have not distinguished between the regions of small and large $|x|$. In the extreme fragmentation region, the present experimental evidence is consistent with the exotic $(ab\bar{c})$ hypothesis. More experiments are needed in order to establish the size of this region in x and $p_\perp^2$.

Note added. After completing this paper, I was informed that the theorem of Sec. II has been proved, under slightly different assumptions, by M. Kugler, V. Rittenberg, and H. J. Lipkin, in *Particles and Fields – 1971,* proceedings of the 1971 Rochester Meeting of the Division of Particles and Fields of the American Physical Society, edited by A. C. Melissinos and P. F. Slattery (A.I.P., New York, 1971), p. 106.

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Backward-Scattering Extrapolations and Baryon Couplings

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Extrapolation of an asymptotic cross section to a baryon pole cannot uniquely determine the coupling constant. The only asymptotic quantities capable of doing this are the R and A parameters. The notorious discrepancy between extrapolation of $d\sigma(\pi^-p)/du$ and the known Δ width is shown to be purely illusory.

It is perhaps well known in backward meson-nucleon scattering that a parity nondegenerate baryon pole decouples from the extrapolated $d\sigma/du$ in the leading spin term. A direct consequence of this, which seems to have been ignored so far, is that an asymptotic model for $d\sigma/du$ is basically incapable of providing a unique extrapolation for the baryon-pole coupling. For instance, it has become a belief that pure pole[1] or weak-cut[2] Regge models for backward π^-p cross section extrapolate to too low a value for the $\Delta(1236)$ coupling. Strong cuts[3] or a wrong-signature sense zero[4,5] have been suggested as possible remedies of this supposed discrepancy. Moreover it has been suggested that the baryon couplings in a pure pole model can be extrapolated from the $d\sigma/du$ data in a parameter-