[6]N. M. Kocharian, Zh. Eksp. Teor. Fiz. 28, 160 (1955) [Sov. Phys. JETP 1, 128 (1955)].
[7]A. G. Meshkovskii and L. I. Sokolov, Zh. Eksp. Teor. Fiz. 31, 752 (1956) [Sov. Phys. JETP 4, 629 (1957)].
[8]O. C. Allkofer and E. Kraft, Nuovo Cimento 39, 1051 (1965).
[9]A. I. Alikhanov, A. V. Khrimayan, V. K. Kosmaohevsky, V. V. Avakyon, Yn. V. Gorodkov, K. Sh. Egiyan, and N. A. Nalbandyan, in *Proceedings of the Moscow Cosmic Ray Conference, 1959,* Vol. I, edited by N. M. Gerasimova (International Union of Pure and Applied Physics, Moscow, 1960).
[10]J. J. J. Kokkedee and L. Van Hove, Nuovo Cimento 42A, 711 (1966).
[11]G. Bellettini, G. Cocconi, A. N. Diddens, E. Lillithun, G. Matthiae, J. P. Scanlon, and M. A. Wetherell, Nucl. Phys. 79, 609 (1965); J. Engler, K. Horn, F. Monnig, P. Schludecker, W. Schmidt-Parzefall, H. Schopper, P. Sievers, H. Ullrich, R. Hartung, K. Runge, and Yu. Galaktionov, Phys. Letters 32B, 716 (1970); L. W. Jones, M. J. Longo, T. P. McCorriston, E. F. Parker, S. T. Powell, and M. N. Kreisler, Phys. Letters 36B, 509 (1971).
[12]A. Buhler-Broglin, G. Fortunato, T. Massam, and A. Zichichi, Nuovo Cimento 49A, 209 (1967).
[13]H. Kasha, R. C. Larsen, L. B. Leipuner, and R. K. Adair, Phys. Rev. Letters 20, 217 (1968).
[14]F. Ashton and J. King, J. Phys. A 4, L 31 (1971).

PHYSICAL REVIEW D VOLUME 6, NUMBER 5 1 SEPTEMBER 1972

Velocity Distribution of Secondary π Mesons from 22.6-GeV/c Proton-Nucleus Interactions in Emulsion

T. Roy and I. K. Daftari
Department of Physics, Jadavpur University, Calcutta-32, India
(Received 15 November 1971)

Emulsion plates exposed to a 24-GeV proton beam are scanned. A velocity distribution of the secondary π mesons produced in such nuclear interactions is obtained. A Gaussian-type function is observed to simulate the experimental data and the usual phenomena of isotropy in the center-of-mass frame. Our results indicate that the assumption of the constant velocity for all the secondary particles is only an approximation to the real world.

I. INTRODUCTION

An analysis of secondary π-meson production and determination of the primary energy of the cosmic-ray nucleon was first done by Duller and Walker[1] in a successful and elegant way. Their assumption regarding the isotropy of the secondary π-meson production has been verified amply by various workers. There is also another assumption in Duller and Walker's derivation with regard to momentum distribution of the secondary particles. Duller and Walker and others assumed that secondary particles come out with almost the same momentum. That this is not far from the truth comes from our investigations. We have analyzed the momentum distribution of the secondary particles and have seen that in the center-of-mass frame the distribution is almost Gaussian.

Our analysis has been solely confined to the emulsion plates exposed to the 22.6-GeV/c proton beam.

Various theoretical predictions have been made according to various theories, and the first theoretical proof of this meson production has been given by Fermi[2] who based his work on statistical theory. Takagi[3] and Kraushar and Marks[4] have suggested a two-center model for the production of mesons in high-energy nuclear collisions, which has been successfully followed by Lindenbaum and Sternheimer[5] and Holmquist.[6] In 1954 Duller and Walker[1] studied the angular distribution of penetrating showers through lead and carbon in a cloud chamber in the energy range 10–40 BeV. Aly and Fisher[7] have shown the isotropic nature of the secondary mesons in the c.m. system obtained at 6-GeV/c proton interactions with emulsion nuclei.

In the present experiment we have studied the velocity distribution of secondary π mesons in the c.m. system of 22.6-GeV/c proton–emulsion-nuclei interactions. It is seen that with the increase of the velocity of secondary mesons, the number of particles also increases up to a certain limit and then becomes almost constant. All the quantities such as velocities, ranges, and space angles of secondary particles have been measured in the laboratory and then converted into the c.m.

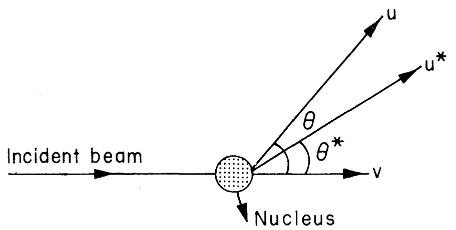

FIG. 1. Kinematics for an incident nucleon of mass m colliding with a nucleus of mass M at rest.

system. The theoretical idea of an error-function type of distribution is in agreement with our experimental analysis.

II. EXPRESSION OF THE MOMENTUM DISTRIBUTION

Assume that an incident nucleon of mass m collides with a nucleus of mass M at rest, which after collision gives rise to different particles and itself moves with some velocity as shown in Fig. 1. If we let u be the velocity of the scattered particles in the laboratory system, θ be the angle of the scattered particle with the incoming direction in the laboratory system, u^* be the velocity of the scattered particle in the c.m. system, θ^* be the angle of the outgoing particle with the incoming direction in the c.m. system, and v be the velocity of the center of mass (not obtained from momentum conservation, *a priori*), then as seen from Fig. 1 we have

$$u^* \cos\theta^* = \frac{u\cos\theta - v}{1 - uv\cos\theta}, \tag{1a}$$

$$u^* \sin\theta^* = \frac{u\sin\theta}{\gamma(1 - uv\cos\theta)}, \tag{1b}$$

where $\gamma = 1/(1-v^2)^{1/2}$ is the Lorentz factor. (We take $c=1$ throughout the paper.)

The relation between the distribution functions in the two frames of the outgoing mesons can be obtained from the constancy of the number of emitted particles and is seen to be

$$g_{\text{c.m.}}(u^*)du^*d\theta^* = g_l(u,\theta)du\,d\theta. \tag{2}$$

The above distribution can be obtained through the Jacobian determinant

$$\begin{vmatrix} \frac{\partial u^*}{\partial u} & \frac{\partial u^*}{\partial \theta} \\ \frac{\partial \theta^*}{\partial u} & \frac{\partial \theta^*}{\partial \theta} \end{vmatrix} du\,d\theta = du^* d\theta^*. \tag{3a}$$

Using Eqs. (1a) and (1b), the above Jacobian reduces to

$$J = \frac{u^{*2}\sin^3\theta^*}{u^2\sin^3\theta}. \tag{3b}$$

Thus, the distribution function (2) becomes

$$g_l(u,\theta) = g_{\text{c.m.}}(u^*)J = g_{\text{c.m.}}(u^*)\frac{u^{*2}\sin^3\theta^*}{u^2\sin^3\theta}, \tag{4}$$

where u^* and θ^* can be easily obtained from Eqs. (1a) and (1b) as given by

$$u^* = \frac{[u^2\sin^2\theta + \gamma^2(u\cos\theta - v)^2]^{1/2}}{\gamma(1 - uv\cos\theta)}, \tag{5a}$$

$$\theta^* = \tan^{-1}\frac{u\sin\theta}{\gamma(u\cos\theta - v)}. \tag{5b}$$

In the above two equations all the quantities (except v, the velocity of the center of mass) are known, and can be obtained by taking different values of v and checking for which value will give the best fit for isotropy in the c.m. system.

The velocity distribution (in the c.m. frame) we now assume to be

$$g(u^*) = A\,e^{-u^{*2}/\sigma^2}, \tag{6a}$$

where σ and A are two constants.

Thus, for all pions, the integrated distribution becomes

$$\begin{aligned} f(u^*) &= \int_0^{u^*} g(u^*)du^* \\ &= A\int_0^{u^*} e^{-u^{*2}/\sigma^2}du^* \end{aligned} \tag{6b}$$

or

$$f(u^*) = A\sigma\,\text{Ei}(u^*/\sigma), \tag{6c}$$

where Ei is the error function which is tabulated in mathematical tables (e.g., Jahnke and Emde).

III. EXPERIMENTAL PROCEDURE

Three plates 19.8×12.1 cm^2, 600 μ thick exposed to the scattered-out proton beam of momentum 22.6 GeV/c at the CERN proton synchrotron, were scanned by the usual "pick up and following the track" method. All events which were clear, having no blobs at the point of production, were selected for analysis. All the secondary tracks with grain density $g^* < 1.4$ were taken as pions. The ranges and space angles of all the mesons were measured in the laboratory system, and subsequently the kinematics was transformed to the c.m. system.

For studying the velocity distribution, we have divided each event observed in the photographic plate into three categories according to the star size:

(1) Nucleon–nucleon collision with $N_h \leq 2$;

(2) Nucleon–(C, N, O)-nucleus collision with $2 < N_h \leq 7$;

(3) Nucleon–(Ag, Br)-nucleus collision with $N_h > 7$.

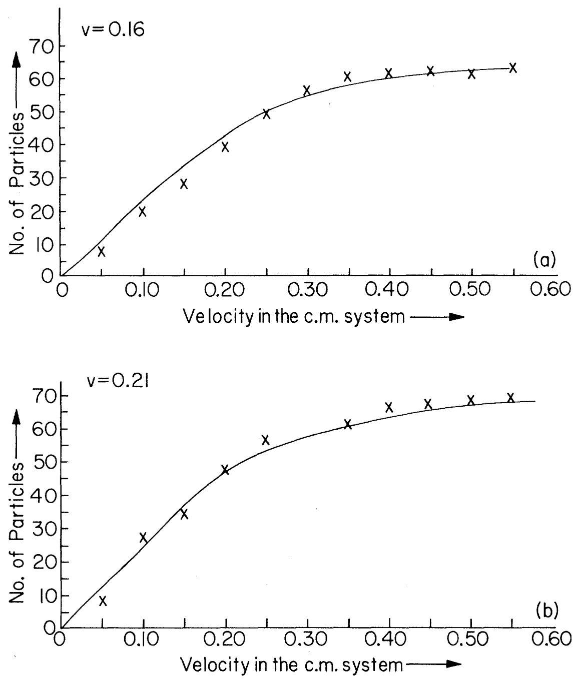

FIG. 2. Velocity distribution of secondary meson from (a) proton–(Ag, Br)-nucleus and (b) proton–(C, N, O)-nucleus interaction. The solid line represents the theoretical curve, x represents the experimental points.

Thus obtaining the relation (6c), we have seen that the velocity distribution in the c.m. system can be provided by the error function in the variable $f(u^*) = \sigma \mathrm{Ei}(\sigma y)$, where σ is the standard deviation. The experimental results are shown in Figs. 2(a) and 2(b) for different values of v and σ. From Figs. 2(a) and 2(b) it is seen that the secondary mesons follow a Gaussian distribution which is in agreement with the expression (6c). It is seen that $v = 0.16$ and $v = 0.21$ are the best fit for the proton–(Ag, Br)-nucleus and the proton–(C, N, O)-nucleus collision, respectively. The distribution $N = 63.2\ \mathrm{Ei}(3.5u^*)$ for (Ag, Br) collisions and $69.0\ \mathrm{Ei}(3.5u^*)$ for (C, N, O) collisions.

IV. DISCUSSION

In the present paper we are interested in the c.m. system velocity distribution of the secondary mesons. The distribution seems to be Gaussian. The assumption of the isotropic distribution of mesons of Duller and Walker[1] is found to be valid. The parameter v which fits for isotropic distribution also fits the momentum distribution. By assuming v and σ as known parameters, the numerical values of these are obtained by best fit. If the value of the velocity v is found by considering the head-on collision of a proton and (Ag, Br) or a proton and (C, N, O) nucleus, and by considering the mass and energy of the incoming proton and the known nucleus at rest by applying the conservation of energy and momentum, then it is seen that the value of V as 0.16 is obtained for (Ag, Br), and 0.21 is obtained for the (C, N, O) nucleus.

Besides this, it is also observed in the present experiment that the π^+/π^- ratio comes out to be unity and the α to proton ratio to be 0.41, which resembles the data given by Perkins[8] and Page.[9] The observed multiplicity of the secondary mesons is 3.7 ± 0.4, which quite resembles the values given by other authors.[10,11]

ACKNOWLEDGMENTS

The authors would like to thank Professor A. J. Herz of CERN, Geneva for supplying us with the exposed emulsion plates. The authors express their gratitude to Professor N. M. Duller of Texas A & M University for taking interest in the work and carefully reading the manuscript. The authors also thank Dr. B. K. Bandyopadhyay of Lawrence Radiation Laboratory for going through the manuscript and providing encouragement during the progress of the work.

[1]N. M. Duller and W. D. Walker, Phys. Rev. 93, 215 (1954).

[2]E. Fermi, Progr. Theoret. Phys. (Kyoto) 5, 570 (1950).

[3]S. Takagi, Progr. Theoret. Phys. (Kyoto) 7, 123 (1952).

[4]W. L. Kraushar and L. J. Marks, Phys. Rev. 93, 326 (1954).

[5]S. J. Lindenbaum and R. M. Sternheimer, Phys. Rev. 105, 1874 (1957).

[6]F. N. Holmquist, Ph.D. thesis, University of California, 1958 (unpublished).

[7]H. H. Aly and C. M. Fisher, Nuovo Cimento 17, 98 (1960).

[8]D. H. Perkins, Phil. Mag. 41, 138 (1950).

[9]N. Page, Proc. Phys. Soc. (London) A63, 250 (1950).

[10]G. Civijanouich, B. Dayton, P. Egli, B. Klaiber, W. Koch, M. Nikolic, R. Schneeberger, H. Winzeler, J. C. Combe, W. M. Gibson, W. O. Lock, M. Schneeberger, and G. Vanderhaeghe, Nuovo Cimento 20, 1012 (1961).

[11]Y. Bandinet-Robinet, M. Morand, Tsai-Chu, C. Castaguoli, G. Dascola, S. Mora, A. Barbaro-Galtieri, G. Baroni, and A. Manfredini, Nucl. Phys. A52, 32, (1962).