can the routing of momenta around each diagram be unambiguously defined. A failure to have these momenta correctly routed in each diagram will, in general (again because of linear divergences), give rise to errors in the calculation of Z_3^{-1}. Careful attention to such detail will be necessary in the sixth-order calculation.

*Research supported in part by the U. S. Atomic Energy Commission.

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[5]Sinclair claims (his footnote 5) that the problems discussed in this section can be avoided by choosing the photon momentum to be $p-k$. This assertion is incorrect for two reasons. First the terms which are polynomial in q have been shown in Ref. 2 to vanish only for a particular gauge and a particular choice of integration variables. Secondly because of the delicate nature of the linear divergence it is essential that one start from a polarization tensor which is explicitly gauge-invariant no matter what integration variables are used. At present the only explicitly gauge-invariant fourth-order form is that of KH in which the photon momentum is taken to be k.

PHYSICAL REVIEW D VOLUME 6, NUMBER 4 15 AUGUST 1972

Model of Weak and Electromagnetic Interactions

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(Received 4 May 1972)

We present here a model of weak and electromagnetic interactions of leptons and hadrons based on the spontaneously broken gauge symmetry O(3)×O(2). The advantages of the model are the following: (1) The universality of the β and μ decays emerges naturally; (2) there appears only positively charged heavy leptons, and no neutral heavy leptons which might affect the muon $(g-2)$ factor adversely; (3) a neutral current shows up only as a short-range weak parity violation in electromagnetism, and nowhere else.

We present here a model of weak and electromagnetic interactions[1] of leptons and hadrons based on the spontaneously broken gauge symmetry[2] O(3)×O(2). The model is in some sense intermediate between those of Weinberg[1] and of Glashow and Georgi.[3] The universality of the β and μ decays emerges naturally. The model contains positively charged heavy leptons, but no neutral heavy leptons. [A neutral heavy lepton in the Glashow-Georgi model may affect adversely the agreement between theory and experiment of the $(g-2)$ factor of the muon if the mass of the neutral lepton is too large and/or the mass of the weak vector boson is too small.[4]] The model contains a neutral current, but is shows up only as a minute short-range parity violation in electromagnetism, and nowhere else. The model can be embedded in a bigger group O(3)×O(3) (or even bigger ones) more or less naturally, but we shall not discuss it here. The model is anomaly-free[5] and renormalizable.[6]

We shall describe the model in terms of the electron first. We form a triplet

$$L_l=\frac{1-\gamma_5}{2}\begin{pmatrix}e^-\\ \nu^0\\ E^+\end{pmatrix},$$

with zero r charge, and two singlets e_r^-, E_r^+:

$$e_r^-=\frac{1+\gamma_5}{2}e^-,\quad E_r^+=\frac{1+\gamma_5}{2}E^+,$$

with r charge $+1$ and -1, respectively. Let W_μ, $W_\mu^\dagger$, and A_μ^l be the O(3) gauge bosons and A_μ^r be the r charge O(2) one. Their couplings to the currents are given by

$$g\left\{W_\mu\left[\bar{e}\gamma^\mu\left(\frac{1-\gamma_5}{2}\right)\nu+\bar{\nu}\gamma^\mu\left(\frac{1-\gamma_5}{2}\right)E^+\right]+\text{H.c.}\right.$$
$$\left.+A_\mu^l\left[\bar{e}\gamma^\mu\left(\frac{1-\gamma_5}{2}\right)e-\bar{E}\gamma^\mu\left(\frac{1-\gamma_5}{2}\right)E\right]\right\}$$
$$+g'A_\mu^r\left[\bar{e}\gamma^\mu\left(\frac{1+\gamma_5}{2}\right)e-\bar{E}\gamma^\mu\left(\frac{1+\gamma_5}{2}\right)E\right].$$

To induce the Higgs phenomenon,[2] we postulate a real scalar triplet ϕ with r charge 0, and a complex scalar triplet ξ with r charge $+1$ and its complex conjugate $\xi^\dagger$. We arrange their mutual interactions in such a way that their vacuum expectation values are given by[7]

$$\langle\phi\rangle_0=\begin{pmatrix}0\\ v\\ 0\end{pmatrix},$$

$$\langle\xi\rangle_0=\begin{pmatrix}0\\ 0\\ u\end{pmatrix},\quad\langle\tilde{\xi}\rangle_0=\begin{pmatrix}u\\ 0\\ 0\end{pmatrix},$$

where v and u are real numbers. Of the nine scalar mesons, three can be transformed away by a gauge transformation. There remain two neutral, two singly charged, and two doubly charged scalar particles.

The charged vector bosons $W^\pm$ become massive with the mass given by

$$m_W{}^2=g^2(u^2+v^2).$$

One combination of two neutral vector bosons, namely

$$Z_\mu=A_\mu^l\cos\theta-A_\mu^r\sin\theta,\quad\tan\theta=g'/g,$$

becomes massive, i.e.,

$$m_Z{}^2=2(g^2+g'^2)u^2$$

and couples to the parity-violating neutral current

$$(g^2+g'^2)^{1/2}Z_\mu\left[\frac{\cos2\theta}{2}(\bar{e}\gamma^\mu e-\bar{E}\gamma^\mu E)+\tfrac{1}{2}(\bar{e}\gamma^\mu\gamma^5 e-\bar{E}\gamma^\mu\gamma^5E)\right].$$

The other linear combination

$$A_\mu=A_\mu^l\sin\theta+A_\mu^r\cos\theta$$

is the photon. The electric charge e is given by

$$e=\frac{gg'}{(g^2+g'^2)^{1/2}}$$

and the Fermi constant $G_F/\sqrt{2}$ by

$$G_F/\sqrt{2}=[4(u^2+v^2)]^{-1}.$$

From these it follows that

$$m_W{}^2=\frac{\sqrt{2}\,e^2}{4G_F\sin^2\theta}\geq53\ \text{GeV}^2.$$

The lepton masses are generated by the couplings

$$-\frac{m_E}{2u}[(\bar{L}_l\cdot\xi)E_r^++\text{H.c.}],$$

$$-\frac{m_e}{2u}[(\bar{L}_l\cdot\tilde{\xi})e_r^-+\text{H.c.}].$$

To accommodate the muon, we need merely substitute μ^-, ν_μ, M^+ for e^-, ν, E^+ in the above discussions.

Integrally charged quarks $\mathcal{P}^+$, $\mathfrak{N}^0$, λ^0 may be incorporated into the scheme in the following way. We form two triplets of zero r charge:

$$\begin{pmatrix}\mathcal{P}\\ \mathfrak{N}'\\ q^-\end{pmatrix}_l,\quad\begin{pmatrix}p'\\ \lambda'\\ q'^-\end{pmatrix}_l$$

where q^-, p'^+, and q'^- are the fourth, fifth, and sixth quarks, and

$$\mathfrak{N}'=\mathfrak{N}\cos\theta_C+\lambda\sin\theta_C,$$

$$\lambda'=-\mathfrak{N}\sin\theta_C+\lambda\cos\theta_C,$$

with the Cabibbo angle θ_C. We treat the right-handed quarks as singlets of appropriate r charges. The quark masses, as well as the Cabibbo mixing can be generated by the couplings of ϕ and ξ to the quarks. In order to construct a fermion octet with correct electric charges, a seventh, electrically neutral fundamental fermion must be introduced in addition.[8] In this scheme, the magnitude of the amplitude $K_L\to\mu^+\mu^-$ is proportional to $G_F{}^2(\Delta m)^2$ where Δm is a typical quark mass difference, rather than to $G_F(G_FM_W{}^2)\sim G_F\alpha$.

To estimate the size of the parity-violating effects due to the Z meson, consider the μ-e force due to the Z-meson exchange. For small momentum transfer it is given by

$$-\frac{1}{8u^2}[\bar{e}\gamma_\mu(\cos2\theta+\gamma_5)e][\bar{\mu}\gamma^\mu(\cos2\theta+\gamma_5)\mu].$$

Since u is arbitrary, subject only to the constraint

$$\frac{G_F}{\sqrt{2}}\leq\frac{1}{4u^2},$$

the parity-violating effect can be made easily to be of the order of the typical weak effects. The contribution of the above interaction to the hyperfine splitting of the muonium is, assuming $u^2\simeq v^2$, about 10^2 cps, far below the present experimental detectability.

Note added in proof. A model essentially identical to the one reported here is described by J. Prentki and B. Zumino, Nucl. Phys. B (to be published).

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†Operated by Universities Research Association, Inc., under contract with the U. S. Atomic Energy Commission.

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[7]We write

$$\xi = \begin{pmatrix}\xi^{--}\\ \xi^{-}\\ \xi^{0}\end{pmatrix}, \qquad \tilde{\xi} = \begin{pmatrix}\bar{\xi}^{0}\\ \xi^{+}\\ \xi^{++}\end{pmatrix} = \begin{pmatrix}(\xi^{0})^*\\ (\xi^{-})^*\\ (\xi^{--})^*\end{pmatrix}.$$

[8]Alternatively we may form two triplets:

$$\begin{pmatrix}Q^{+}\\ \mathcal{P}^{0}\\ \mathfrak{N}^{-}\cos\theta + \lambda^{-}\sin\theta\end{pmatrix}, \begin{pmatrix}Q'^{+}\\ \mathcal{P}'^{0}\\ -\mathfrak{N}^{-}\sin\theta + \lambda^{-}\cos\theta\end{pmatrix},$$

where $\mathcal{P}'^{0}$, $\mathcal{P}^{0}$, $\mathfrak{N}^{-}$, λ^{-} are the fundamental quartet of integral quantum numbers of S. Glashow, J. Iliopoulos, and L. Maiani, Phys. Rev. D 2, 1285 (1970). This scheme does not require a seventh fermion. This scheme gives the opposite sign for the $\pi^0 \to 2\gamma$ amplitude from the usually accepted one.

PHYSICAL REVIEW D VOLUME 6, NUMBER 4 15 AUGUST 1972

Model for Lepton, Nucleon, and Pion Interactions

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(Received 31 March 1972)

An $SU(2)_L \times Y_L$ gauge theory is studied. The purely leptonic part is the Weinberg theory. The purely hadronic part is a form of the $SU(2)\times SU(2)$ σ model. It is shown, following the suggestion of Weinberg, that with a cross-coupling term the spontaneous breaking of the lepton symmetry induces a spontaneous breaking of the $SU(2)\times SU(2)$ hadron symmetry, resulting in a partially-conserved-axial-vector-current condition for axial-vector divergences and pions which are Goldstone-Nambu particles in an appropriate limit.

Weinberg has recently observed[1] that the usual picture of chiral symmetry breaking[2] in which the pion mass arises from an intrinsic breaking of $SU(2)\times SU(2)$ is inconsistent with renormalizability[3] of his weak- and electromagnetic-interaction theory[4] when hadrons are incorporated. He suggests therefore that the pion mass arises from the same symmetry-breaking mechanism responsible for the W and Z masses through a cross-coupling term[1]

$$\mathcal{L}_{WH} = f(\Sigma_L^\dagger \varphi + \varphi^\dagger \Sigma_L), \tag{1}$$

where

$$\varphi = \begin{pmatrix}\varphi^{\dagger}\\ \varphi^{0}\end{pmatrix}, \text{ left doublet, } Y_L = -\tfrac{1}{2},$$
$$\Sigma_L = \begin{pmatrix}-i\sqrt{2}\,\pi^{\dagger}\\ \sigma + i\pi^{0}\end{pmatrix}, \text{ left doublet, } Y_L = -\tfrac{1}{2}. \tag{2}$$

In this note we will show how this new mechanism for chiral symmetry breaking can be realized by joining the Weinberg lepton theory with the $SU(2)\times SU(2)$ σ model[5] for hadron interaction. The mechanism itself, however, is more general than either the Weinberg theory or the σ model; it depends basically on a lepton-symmetry-invariant cross-coupling term (whatever that symmetry may be) which mixes a scalar lepton multiplet φ with a hadron multiplet which transforms like the hadronic chiral-symmetry-breaking term responsible for the pseudoscalar masses.

With the $SU(2)_L\times Y_L$ assignments

$$L = \tfrac{1}{2}(1+\gamma_5)\begin{pmatrix}\nu\\ e\end{pmatrix}, \text{ left doublet, } Y_L = \tfrac{1}{2},$$
$$R = \tfrac{1}{2}(1-\gamma_5)e, \text{ left singlet, } Y_L = 1,$$
$$\vec{A}_\mu, \ SU(2)_L \text{ gauge triplet,} \tag{3}$$
$$B_\mu, \ Y_L \text{ gauge singlet,}$$

and the $SU(2)_L\times Y_L$ covariant derivative