Fourth-Order Charge Renormalization in Spin-Zero Electrodynamics

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The discrepancy between two recent calculations of the divergent part of Z_3 in scalar electrodynamics is resolved. It is shown that in both approaches certain improper operations have been performed on linearly divergent diagrams. The terms which are necessary to correct each of these two calculations are determined and the results thereby brought into agreement. It is found that while the corrections happen to cancel in Sinclair's approach, there is a nonvanishing contribution to the earlier result of the present authors.

I. INTRODUCTION

The hypothesis that the divergent contributions to the charge-renormalization constant Z_3 could cancel by means of an eigenvalue condition on the bare charge has led to a number of calculations of the divergent terms in both spinor and scalar electrodynamics. At the present time there exist three independent calculations[1-3] of the fourth-order terms in the scalar case with results which can be summarized by

$$(Z_3^{-1})_{\text{divergent}} = (\tfrac{1}{6}\alpha_0 + \lambda\alpha_0^2)\int \frac{dp^2}{p^2},$$

where $\alpha_0 \equiv e_0^2/8\pi^2$. The coefficient λ was determined to be unity by Białynicka-Birula and by Sinclair and $\frac{5}{4}$ by the present authors (hereafter referred to as KH). On the other hand it has been remarked[2] that inasmuch as Białynicka-Birula's calculation neglects certain polynomial terms in the external photon momentum which give a nonvanishing contribution to her result, it is fair to say that it is only the disagreement between Refs. 2 and 3 which is of concern at the present time.

In this paper the discrepancy between the two results is resolved, it being shown that in both calculations certain divergent integrals have been improperly translated. Although these integrals are the same in each case, the coefficients differ in the two approaches so that the correction terms are not identical. It is shown that while they cancel in Sinclair's calculation (for no obvious reason), there is a net shift of the KH result for λ by $-\frac{1}{4}$, thereby confirming the (apparently fortuitous) results of Sinclair and Białynicka-Birula.

In the following section a brief summary is given of Sinclair's application of Rosner's method[4] to the spin-zero case. A somewhat more explicit notation is used in order to demonstrate the existence of terms which must be added to his result if one pays close attention to the translation of divergent integrals. Section III displays the corresponding terms which must be added to the result of KH when such translations are carefully avoided. In the conclusion some general observations are made concerning the application of the Rosner method and the crucial differences between the calculation of Z_3^{-1} in spinor electrodynamics and scalar electrodynamics.

II. THE ROSNER METHOD

The distinctive feature of the Rosner method for the calculation of Z_3 is the explicit cancellation of the radiative corrections to Z_1 and Z_2. Consequently, it is convenient to write the second-order meson propagator and vertex function as

$$\Delta^{-1}(p) = (p^2 + \mu^2)(1 + B^{(2)}), \qquad (2.1)$$

$$\Gamma_\mu(p, p) = 2p_\mu(1 + B^{(2)}), \qquad (2.2)$$

where $B^{(2)}$ is of second order in e_0 and is finite in the Yennie gauge. To the same order one symbolically writes

$$\Gamma^{(2)}(p + \tfrac{1}{2}q, p - \tfrac{1}{2}q) \equiv \Gamma_p^{(2)} = 2p + K^{(2)}G_{p-k}^{(0)}\,2(p-k) + V_p^{(2)},$$

where K is the Bethe-Salpeter kernel[3] and a superscript notation has been used to indicate the order in e_0. The $V_p^{(2)}$ term is the sum of two diagrams, each of which involves a single two-photon vertex and G_p is defined by

$$G_p \equiv \Delta(p + \tfrac{1}{2}q)\Delta(p - \tfrac{1}{2}q).$$

In KH the expression for $\Pi^{\mu\nu}(q)$ has been given and its gauge invariance explicitly demonstrated. Using this result one can thus write the fourth-order result as

$$\Pi^{(4)} = ie_0^2\{(2pG_p\Gamma_p)^{(2)} + \tfrac{1}{2}[V_{p-k}G_{p-k}2(p-k)]^{(2)} + \tfrac{1}{2}(V_pG_p2p)^{(2)} - ie_0^2 2gG^{(0)}2gD\}, \tag{2.3}$$

where

$$G = \Delta(p+\tfrac{1}{2}q)\Delta(p+k-\tfrac{1}{2}q).$$

Use of (2.1) allows Eq. (2.3) to be rewritten as

$$\Pi^{(4)} = ie_0^2[-2B^{(2)}(2pG_p2p)^{(0)} + 2pG_p^{(0)}K^{(2)}G_{p-k}^{(0)}2(p-k) + 2pG_p^{(0)}V_p^{(2)} + \tfrac{1}{2}V_{p-k}^{(2)}G_{p-k}^{(0)}2(p-k) + \tfrac{1}{2}V_pG_p2p - ie_0^2 2gG^{(0)}2gD].$$

Since Z_3 is to be computed from

$$Z_3^{-1} - 1 = \frac{1}{24}\frac{\partial^2}{\partial q_\nu \partial q^\nu}\Pi_\mu{}^\mu(q^2)\bigg|_{q^2=0},$$

one denotes the combined operations of taking two derivatives with respect to q and setting $q^2=0$ by the customary double-prime notation thereby deriving

$$\Pi^{(4)\prime\prime} = ie_0^2[-B^{(2)}(2pG_p''2p)^{(0)} + 2pG_p^{(0)}K^{(2)\prime\prime}G_{p-k}^{(0)}2(p-k) + 2pG_p^{(0)}K^{(2)}G_{p-k}^{(0)\prime\prime}2(p-k) + \tfrac{1}{2}(V_{p-k}^{(2)}G_{p-k}^{(0)})''2(p-k) + \tfrac{1}{2}(V_p^{(2)}G_p^{(0)})''2p + 2pG_p^{(0)}V_p^{(2)\prime\prime} - ie_0^2 2gG^{(0)\prime\prime}2gD],$$

where use has been made of (2.1) and (2.2). A cancellation of the remaining $B^{(2)}$ term can also be effected and yields

$$\begin{aligned}\Pi^{(4)\prime\prime} = ie_0^2\{&[2pG_p^{(0)}K^{(2)\prime\prime}G_{p-k}2(p-k) + V_p^{(2)\prime\prime}G_p^{(0)}2p + 2pG_p^{(0)}V_p^{(2)\prime\prime} - ie_0^2 2gG''2gD]\\ &+[2pG_p^{(0)}K^{(2)}G_{p-k}^{(0)\prime\prime}2(p-k) - 2(p-k)G_{p-k}^{(0)}K^{(2)}G_p^{(0)\prime\prime}2p]\\ &+\tfrac{1}{2}[V_{p-k}^{(2)}G_{p-k}^{(0)\prime\prime}2(p-k) - V_p^{(2)}G_p^{(0)\prime\prime}2p] + \tfrac{1}{2}[V_{p-k}^{(2)\prime\prime}G_{p-k}^{(0)}2(p-k) - V_p^{(2)\prime\prime}G_p^{(0)}2p]\}.\end{aligned} \tag{2.4}$$

It is to be noted that the first square brackets reproduce the Sinclair result while the three remaining brackets can only be discarded if the translation $p \to p-k$ is admissible. Since it can readily be shown that the terms in the last bracket each possess no linear divergence and can consequently be discarded, the verification of Sinclair's result is seen to require the cancellation of the second and third terms in (2.4).

The contribution of the second bracket in (2.4) to Z_3^{-1} is

$$e_0^4\int\frac{dpdk}{(2\pi)^8}\,p(p-k)\Delta^2(p)\Delta^2(p-k)\{(p^2+2\mu^2)\Delta^2(p) - [(p-k)^2+2\mu^2]\Delta^2(p-k)\}.$$

It is straightforward to use the rules for translation of the argument of a linearly divergent integral and reduce the above to

$$-\tfrac{1}{8}\alpha_0^2\int dp^2\,\frac{p^4(p^2+2\mu^2)}{(p^2+\mu^2)^4}, \tag{2.5}$$

which has an ultraviolet divergence identical to

$$-\tfrac{1}{8}\alpha_0^2\int\frac{dp^2}{p^2}.$$

The third bracket in (2.4) contributes to Z_3^{-1} in the amount

$$\tfrac{1}{3}e_0^4\int\frac{dpdk}{(2\pi)^8}D^{\mu\nu}(k)(2p-k)_\mu\{(p-k)_\nu\Delta(p)\Delta^4(p-k)[(p-k)^2+2\mu^2] - p_\nu\Delta(p-k)\Delta^4(p)(p^2+2\mu^2)\}.$$

This can eventually be manipulated to the form

$$e_0^4\int\frac{dpdk}{(2\pi)^8}\{-(pk)\Delta^4(p)\Delta^2(p-k)(p^2+2\mu^2) - p(p-k)\Delta^2(p)\Delta^4(p-k)[(p-k)^2+2\mu^2]\},$$

which again allows the direct application of the rule for translating the argument of a linearly divergent integral. The result is

$$\tfrac{1}{8}\alpha_0^2 \int dp^2 \frac{p^4(p^2+2\mu^2)}{(p^2+\mu^2)^4},$$

which cancels (2.5), thereby verifying the Sinclair result.

It is important to note that at the present level at least, such a cancellation between the two different components of the vertex function cannot be made on general principles (i.e., application of the Ward identity) inasmuch as it is only one-half of the two-photon vertex part of the vertex function which is canceling the entire single-photon vertex part of the same function. Thus a careful and explicit treatment of surface terms is necessary to allow one to infer the cancellation of these additional contributions.[5]

III. THE KH CALCULATION

It is not difficult to show that any error in the authors' earlier calculation of Z_3^{-1} must occur in the computation of vertex corrections [$\Pi_B(0)$ and $\Pi_D(0)$ in the notation of KH]. In fact the identical operation (namely, the translation of $p \to p-k$) has been performed in that work and it is necessary to evaluate the corrections to Z_3^{-1} arising from discarded surface terms.

The additional contribution to $\Pi_D(0)$ [i.e., that part of $\Pi(0)$ arising from vertex parts containing only single-photon vertices] is

$$e_0^4 \int \frac{dp\,dk}{(2\pi)^8}\, p(k-p)\Delta^2(p)\Delta^2(p-k)\{(p^2+2\mu^2)\Delta^2(p) - [(p-k)^2+2\mu^2]\Delta^2(p-k)\}\,.$$

With the aid of the preceding section, this is readily found to contribute

$$\frac{\alpha_0^2}{8} \int dp^2 \frac{p^4(p^2+2\mu^2)}{(p^2+\mu^2)^4}\,. \tag{3.1}$$

One similarly finds the correction to $\Pi_B(0)$ [i.e., that part of $\Pi(0)$ which arises from vertex parts containing one two-photon vertex] to be

$$e_0^4 \int \frac{dp\,dk}{(2\pi)^8}\, D^{\mu\nu}(k)(2p-k)_\mu\{p_\nu\Delta(p-k)\Delta^4(p)(p^2+2\mu^2) - (p-k)_\nu\Delta(p)\Delta^4(p-k)[(p-k)^2+2\mu^2]\},$$

which is precisely -3 times the corresponding correction to the Sinclair calculation. This consequently gives an additional term

$$-\tfrac{3}{8}\alpha_0^2 \int dp^2 \frac{p^4(p^2+2\mu^2)}{(p^2+\mu^2)^4}\,,$$

a result which when combined with (3.1) yields the asserted correction of $-\frac{1}{4}$ to the value of λ previously computed in Ref. 2. It may be noted that these corrections imply the following forms for the complete vertex contributions to $\Pi(0)$:

$$\Pi_B(0) \sim \alpha_0^2 \int \frac{dp^2}{p^2}\left(-\tfrac{1}{2}\ln\left|\frac{p^2+\mu^2}{\mu^2}\right| - \tfrac{25}{24}\right), \quad \Pi_D(0) \sim \alpha_0^2 \int \frac{dp^2}{p^2}\left(\tfrac{1}{2}\ln\left|\frac{p^2+\mu^2}{\mu^2}\right| + \tfrac{7}{24}\right).$$

IV. CONCLUSION

While the principal result of this paper is presumably the resolution of the discrepancy in the fourth-order evaluation of Z_3^{-1} in scalar electrodynamics, there are some qualitative features of the calculation which are particularly deserving of emphasis. One recalls, for example, that in spinor electrodynamics the divergences in the propagator and vertex are respectively linear and logarithmic, whereas in the spin-zero case these become quadratic and linear. Since all recent calculations in this area of work seem to be rather careful in the treatment of self-energy parts, one sees that particular attention to detail is necessary in the scalar case because of the possible neglect of surface terms. Although formal and rough handling of vertex parts gives no difficulty in the spinor case (as logarithmically divergent integrals can be freely translated), it is essential in scalar electrodynamics to be extremely circumspect in one's handling of the relevant integrals. Because of this fact it is not sufficient merely to ensure the satisfaction of the Ward identity, but one must start from a polarization tensor whose gauge invariance has been explicitly demonstrated. Only in this way can the routing of momenta around each diagram be unambiguously defined. A failure to have these momenta correctly routed in each diagram will, in general (again because of linear divergences), give rise to errors in the calculation of Z_3^{-1}. Careful attention to such detail will be necessary in the sixth-order calculation.

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[5]Sinclair claims (his footnote 5) that the problems discussed in this section can be avoided by choosing the photon momentum to be $p-k$. This assertion is incorrect for two reasons. First the terms which are polynomial in q have been shown in Ref. 2 to vanish only for a particular gauge and a particular choice of integration variables. Secondly because of the delicate nature of the linear divergence it is essential that one start from a polarization tensor which is explicitly gauge-invariant no matter what integration variables are used. At present the only explicitly gauge-invariant fourth-order form is that of KH in which the photon momentum is taken to be k.

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Model of Weak and Electromagnetic Interactions

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We present here a model of weak and electromagnetic interactions of leptons and hadrons based on the spontaneously broken gauge symmetry O(3)×O(2). The advantages of the model are the following: (1) The universality of the β and μ decays emerges naturally; (2) there appears only positively charged heavy leptons, and no neutral heavy leptons which might affect the muon $(g-2)$ factor adversely; (3) a neutral current shows up only as a short-range weak parity violation in electromagnetism, and nowhere else.

We present here a model of weak and electromagnetic interactions[1] of leptons and hadrons based on the spontaneously broken gauge symmetry[2] O(3)×O(2). The model is in some sense intermediate between those of Weinberg[1] and of Glashow and Georgi.[3] The universality of the β and μ decays emerges naturally. The model contains positively charged heavy leptons, but no neutral heavy leptons. [A neutral heavy lepton in the Glashow-Georgi model may affect adversely the agreement between theory and experiment of the $(g-2)$ factor of the muon if the mass of the neutral lepton is too large and/or the mass of the weak vector boson is too small.[4]] The model contains a neutral current, but is shows up only as a minute short-range parity violation in electromagnetism, and nowhere else. The model can be embedded in a bigger group O(3)×O(3) (or even bigger ones) more or less naturally, but we shall not discuss it here. The model is anomaly-free[5] and renormalizable.[6]

We shall describe the model in terms of the electron first. We form a triplet

$$L_l = \frac{1-\gamma_5}{2}\begin{pmatrix} e^- \\ \nu^0 \\ E^+ \end{pmatrix},$$

with zero r charge, and two singlets e_r^-, E_r^+:

$$e_r^- = \frac{1+\gamma_5}{2}\,e^-, \quad E_r^+ = \frac{1+\gamma_5}{2}\,E^+,$$

with r charge $+1$ and -1, respectively. Let W_μ, $W_\mu^\dagger$, and A_μ^l be the O(3) gauge bosons and A_μ^r be the r charge O(2) one. Their couplings to the currents are given by