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$$F_\mu^{K^+\pi^+\pi^-} = F_\mu^+ + F_\mu^-, \quad F_\mu^{K^+\pi^0\pi^0} = F_\mu^+/\sqrt{2}, \quad F_\mu^{K^0\pi^-\pi^0} = -\sqrt{2}\, F_\mu^-.$$

These give a sum rule

$$\Gamma(K^+ \to \pi^+\pi^- l^+\nu) = 2\Gamma(K^+ \to \pi^0\pi^0 l^+\nu) + \tfrac{1}{2}\Gamma(K^0 \to \pi^-\pi^0 l^+\nu).$$

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# High-Energy Scattering of a Charged Vector Meson in a Static Field: Simple Exponentiation and $s$-Channel Helicity Conservation

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It is shown that (a) the validity of $s$-channel helicity conservation and (b) the occurrence of simple exponentiation depend on the coupling between the vector meson and the external field. Both (a) and (b) can be realized in a theory where the charged vector meson has an anomalous magnetic moment ($\kappa = 1$) and where the external charge distribution is not too singular.

## I. INTRODUCTION

Recently, problems in connection with exponentiation and $s$-channel helicity conservation have received considerable attention in the study of high-energy processes.

It has been shown that the simple-exponentiation form of Molière[1] not only holds for the scattering amplitude for a charged scalar meson or a Dirac particle in a static field,[2] but also for the multi-photon-exchange amplitude of electron-electron scattering[3] as well as for the amplitude of pion-pion scattering.[4] On the one hand, simple-exponentiation forms have been successfully used in several models (e.g., the Glauber model,[5] the droplet model[6]) to describe various hadronic processes, but on the other hand, it has been shown that: Simple exponentiation breaks down when Dirac particles *with* anomalous magnetic moment[7] or charged vector mesons *without* anomalous magnetic moment[8] are scattered by a static potential. Now, we know that[9] to study the theoretical aspects of exponentiation, potential theory is a particularly fertile ground to gain physical insight. First, the formalism of high-energy potential scattering is simple and transparent. This is in marked contrast with the lengthy and involved field-theoretical calculations. Secondly, because of the large amount of effort required in any reliable field-theoretic calculation, only a small number of such calculations can be carried out. Finally, in many field-theoretic calculations, some assumptions are made about the region of integration from where the important contributions come. Results from high-energy potential scattering can be very useful in deciding which approximation may be used in field theories. Hence, in connection with the problem of applicability of simple-exponentiation forms to hadronic processes, the result obtained by Cheng and Wu[7,8] are extremely interesting and it seems worthwhile to investigate, in the framework of potential scattering, the following problem in further detail: Can the scattering amplitude have a simple-exponentiation form when the scattered particle has internal structure? If yes, what are the conditions under which this can occur? Because of simplicity and the reason given below, the vector meson seems to be the most interesting candidate for this study.

$s$-channel helicity conservation (SCHC) has been observed in $\rho$ photoproduction experiments[10] and in pion-nucleon analyses.[11] Ever since the discovery of this selection rule, there has been a considerable number of experimental and theoretical investigations on this subject.[12] Since potential scattering theory has proved itself to be one of the most powerful and most elegant approaches to gain physical insight in high-energy scattering processes,[9] it is of interest to see what this theory has to say about SCHC. Most recently, it has been shown that the above-mentioned high-energy selection rule does not hold (1) for Dirac particles *with* anomalous magnetic moment[7] and (2) for vector mesons *without* anomalous magnetic moment[8] when these particles are scattered by a static field. By considering the Dirac-particle case[7] as an example, one might think that the inclusion of an anomalous magnetic moment for the vector meson would make the violation of SCHC even worse. Now, in this connection, the vector-meson case is by no means irrelevant. Quite on the contrary, because of the impressive evidences of SCHC found in $\rho$ photoproduction[13] and because of the fact that the photon, in many qualitative respects, behaves at high energy like a spin-1 hadron,[14] the SCHC problem for scattering of a vector meson is of particular interest.

Last but not least, the very recent effort made at the National Accelerator Laboratory in the search of vector $W$ bosons[15] provides additional interest in the study of the structure of spin-one particles.

In this paper, an attempt is made to study the problems mentioned above. We start with the case in which the vector meson has an arbitrary magnetic moment.[16] The Born terms calculated in Sec. II show that $s$-channel helicity conservation and the occurrence of simple exponentiation depend on the value $\kappa$ assigned to the anomalous part of the magnetic moment. It is seen that the special case $\kappa=1$ plays a distinctive role. In Secs. III and IV, the high-energy approximation used by Cheng and Wu[8] is applied to this ($\kappa=1$) case. It is found that in contrast to the Proca theory[17,8] ($\kappa=0$), the solution of the differential equations can be readily given in closed form for all potentials provided that the corresponding charge distribution is not too singular. The main results are: (a) $s$-channel helicity is conserved. (b) The usual simple-exponentiation form of Molière is found for the helicity ±1 amplitudes while the helicity 0 amplitude obeys a slightly modified simple-exponentiation form.

## II. BORN TERM ($\kappa$ ARBITRARY)

We consider a vector meson with charge $e$, mass $m$, and magnetic moment $\vec{M}=[e(1+\kappa)/2m]\,\vec{s}$ where $\vec{s}$ denotes the spin, and $\kappa$ is a constant which characterizes the "anomalous" part of $\vec{M}$. The wave function $U_\mu(x)$, $\mu=1,2,3,4$ of this vector meson in a static field $V(\vec{x})$ satisfies the equation[16]

$$D_\mu G_{\mu\nu} - m^2 U_\nu - ie\kappa F_{\nu\mu} U_\mu = 0\,, \quad \nu=1,2,3,4\,, \tag{2.1}$$

where[18]

$$G_{\mu\nu} = D_\mu U_\nu - D_\nu U_\mu\,, \tag{2.2}$$

$$D_\mu = \partial_\mu - ieA_\mu = \frac{\partial}{\partial x_\mu} - ieA_\mu\,, \tag{2.3}$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu\,, \tag{2.4}$$

$$A_\mu = i\delta_{\mu 4} V(\vec{x})\,. \tag{2.5}$$

Making use of (2.2), (2.3), and (2.4), we rewrite (2.1) in the following form:

$$(\partial_\mu\partial_\mu - m^2)U_\nu - \partial_\nu\partial_\mu U_\mu = J_\nu\,, \quad \nu=1,2,3,4\,, \tag{2.6}$$

where

$$J_\nu = ie[(\kappa+1)F_{\nu\mu}U_\mu + (\partial_\mu A_\mu + A_\mu\partial_\mu)U_\nu - (\partial_\nu A_\mu + A_\nu\partial_\mu)U_\mu] + e^2(A_\mu A_\mu U_\nu - A_\nu A_\mu U_\mu)\,. \tag{2.7}$$

For the static-field case (2.5) this four-vector is reduced to

$$J_0 = -ie[(\kappa+1)(\partial_j V)U_j + V\partial_j U_j]\,,$$
$$J_k = -ie[\kappa(\partial_k V)U_0 - V(2\partial_0 U_k + \partial_k U_0)] - e^2V^2U_k\,, \quad k=1,2,3\,. \tag{2.8}$$

In terms of $J_\nu$, the scattering amplitude can be written as

$$M_{fi}=\bar{b}_\nu\int d^3x\, e^{-ip_\mu^{(f)}x_\mu}J_\nu, \qquad (2.9)$$

where $p_\mu^{(f)}$ is the four-momentum and $b_\nu$ the polarization vector of the outgoing vector meson. The Born terms[19] are obtained by inserting

$$U_\nu^{(i)}=e^{ip_\mu^{(i)}x_\mu}a_\nu, \quad \nu=1,2,3,4 \qquad (2.10)$$

into $J_\nu$ and collecting all terms linear in $e$. In (2.10) $p_\mu^{(i)}$ is the four-momentum, and $a_\nu$ the polarization vector of the incoming vector meson.

Let us now choose a coordinate system in which the incoming momentum $\vec{p}$ is taken to be in the $z$ direction, and the momentum transfer $\vec{\Delta}$ in the $x$ direction. In the high-energy limit, $E\sim p\gg m$, where $p=|\vec{p}|$, $E=(p^2+m^2)^{1/2}$, the incoming wave can be written as

$$U_\nu^{(i)}=e^{-iE(t-z)}a_\nu, \quad \nu=1,2,3,4. \qquad (2.11)$$

The three polarization vectors are

$$a_\nu^{(\parallel)}=\{1,0,0,0\},$$
$$a_\nu^{(\perp)}=\{0,1,0,0\}, \qquad (2.12)$$
$$a_\nu^{(0)}=\left\{0,0,\frac{E}{m},i\left(\frac{E}{m}-\frac{m}{2E}\right)\right\}.$$

Here, $a_\nu^{(\parallel)}$ and $a_\nu^{(\perp)}$ are transverse while $a_\nu^{(0)}$ is longitudinal. The index $(\parallel)$ means that $a_\nu$ is in the scattering plane while $(\perp)$ means that it is perpendicular to the scattering plane. Similarly, the three polarization vectors of the outgoing wave are

$$b_\nu^{(\parallel)}=\left\{1,0,-\frac{\Delta}{E},0\right\},$$
$$b_\nu^{(\perp)}=\{0,1,0,0\}, \qquad (2.13)$$
$$b_\nu^{(0)}=\left\{\frac{\Delta}{m},0,\frac{E}{m}-\frac{\Delta^2}{2Em},i\left(\frac{E}{m}-\frac{m}{2E}\right)\right\},$$

where $\Delta=|\vec{\Delta}|$.

From (2.8), (2.9), and (2.11) we obtain the desired Born terms in the high-energy limit:

$$M_{fi}^B=\int d^3x\, e^{-i\Delta x}\{EeV(2b_3^*a_3-b_0^*a_3-b_3^*a_0+2\vec{b}_\perp^*\cdot\vec{a}_\perp) + ie(\kappa+1)b_0^*[(\partial_zV)a_3+(\vec{\partial}_\perp V)\cdot\vec{a}_\perp]-ie\kappa[(\partial_zV)b_3^*+(\vec{\partial}_\perp V)\cdot\vec{b}_\perp^*]a_0\}. \qquad (2.14)$$

In the following, we shall explicitly calculate $M_{fi}^B$ from (2.12), (2.13), and (2.14):

### A. Transverse to Transverse

Since $a_0=a_3=b_0=0$, Eq. (2.14) reduces to

$$M_{fi}^B=\int d^3x\, e^{-i\Delta x}2EeV\vec{b}_\perp^*\cdot\vec{a}_\perp. \qquad (2.15)$$

Now

$$\vec{b}_\perp^{*(\parallel)}\cdot\vec{a}_\perp^{(\parallel)}=\vec{b}_\perp^{*(\perp)}\cdot\vec{a}_\perp^{(\perp)}=1,$$
$$\vec{b}_\perp^{*(\parallel)}\cdot\vec{a}_\perp^{(\perp)}=\vec{b}_\perp^{*(\perp)}\cdot\vec{a}_\perp^{(\parallel)}=0. \qquad (2.16)$$

Hence

$$M_{\parallel\parallel}^B=M_{\perp\perp}^B=2Ee\int d^3x\, e^{-i\Delta x}V(\vec{x}),$$
$$M_{\parallel\perp}^B=M_{\perp\parallel}^B=0. \qquad (2.17)$$

In terms of the helicity states, we have

$$M_{++}^B=M_{--}^B=2Ee\int d^3x\, e^{-i\Delta x}V(\vec{x}),$$
$$M_{+-}^B=M_{-+}^B=0. \qquad (2.18)$$

Here, $M_{+-}$ designates the scattering amplitude for the case in which the incoming vector meson is of helicity $-1$ and the outgoing vector meson is of helicity $+1$. The other notations should be obvious.

### B. Longitudinal to Transverse

From Eqs. (2.12), (2.13), and (2.14) we have

$$M_{fi}^B=\int d^3x\, e^{-i\Delta x}\{EeV(2b_3^*a_3-b_3^*a_0)-ie\kappa[(\partial_zV)b_3^*+(\vec{\partial}_\perp V)\cdot\vec{b}_\perp^*]a_0\}. \qquad (2.19)$$

Thus, if the polarization vector of the outgoing vector meson lies in the scattering plane, the scattering amplitude is given by

$$M^B_{\parallel 0} = -E\frac{e}{m}\int d^3x\, e^{-i\Delta x}[V(\vec{x}) + i\kappa\partial_x V(\vec{x})]$$

$$= (\kappa - 1)E\Delta\frac{e}{m}\int d^3x\, e^{-i\Delta x}V(\vec{x}), \qquad (2.20)$$

where the last step is a result of integration by parts and the assumption that $V(\vec{x}) \to 0$ for $|\vec{x}| \to \infty$. If the polarization vector of the outgoing vector meson is perpendicular to the scattering plane, the scattering amplitude $M_{\perp 0}$ vanishes.

We notice that, for the special case $\kappa = 1$ the amplitude $M^B_{\parallel 0}$ also vanishes. Hence in terms of the helicity amplitudes we have this special case:

$$M^B_{+0} = M^B_{-0} = 0, \qquad (2.21)$$

where $M^B_{+0}$ and $M^B_{-0}$ are the scattering amplitude in which the incoming vector meson is of helicity 0 and the outgoing vector meson is of helicity +1 and −1, respectively.

### C. Transverse to Longitudinal

In this case, we have

$$M^B_{fi} = E\frac{e}{m}\int d^3x\, e^{-i\Delta x}[2V(\vec{x})\Delta + i(\kappa+1)\partial_x V(\vec{x})], \qquad (2.22)$$

which gives

$$M^B_{0\parallel} = (1-\kappa)E\Delta\frac{e}{m}\int d^3x\, e^{-i\Delta x}V(\vec{x}),$$
$$M^B_{0\perp} = 0. \qquad (2.23)$$

Here we see again that $\kappa = 1$ plays a special role. Obviously for $\kappa = 1$, $M^B_{0\parallel}$ vanishes, and accordingly

$$M^B_{0+} = M^B_{0-} = 0, \qquad (2.24)$$

where $M^B_{0+}$ and $M^B_{0-}$ are the corresponding helicity amplitudes.

### D. Longitudinal to Longitudinal

The expression for this matrix element is

$$M^B_{00} = \int d^3x\, e^{-i\Delta x}\{EeV(2b_3^*a_3 - b_0^*a_3 - b_3^*a_0) + ie(\kappa+1)b_0^*(\partial_z V)a_3 - ie\kappa[(\partial_z V)b_3^* + (\vec{\partial}_\perp V)\cdot\vec{b}_\perp^*]a_0\}$$

$$= Ee\left[1 + \frac{\Delta^2}{2m^2}(1+2\kappa)\right]\int d^3x\, e^{-i\Delta x}V(\vec{x}). \qquad (2.25)$$

In this case, we see that the value of $\kappa$ does not influence the qualitative behavior of $M^B_{00}$. In the Proca case ($\kappa = 0$), this result is in agreement with that of Cheng and Wu[8] up to a term proportional to $(\Delta/m)^2$.

The main result of this section is: All helicity-flip Born terms vanish for $\kappa = 1$. This may be looked upon as a hint[20] for the occurrence of simple exponentiation and for the validity of $s$-channel helicity conservation for this particular case ($\kappa = 1$).

## III. WAVE FUNCTION ($\kappa = 1$)

In this section, we apply the method used by Cheng and Wu[8] to solve the wave Eq. (2.1) for the special case $\kappa = 1$.

This system of differential equations for $U_\mu$, $\mu = 1, 2, 3, 4$ can be put into a more convenient form: We first apply $D_\nu$ to (2.1), which gives

$$D_\nu D_\mu G_{\mu\nu} - m^2 D_\nu U_\nu - ie\kappa D_\nu F_{\nu\mu}U_\mu = 0. \qquad (3.1)$$

By making use of (2.2) and the commutation relations:

$$[D_\mu, D_\nu] = -ieF_{\mu\nu}, \qquad (3.2)$$

$$[D_\mu, D_\nu D_\nu] = -2ieF_{\mu\nu}D_\nu + ie(\partial_\nu F_{\nu\mu}) \qquad (3.3)$$

which are immediate consequences of (2.3) and (2.4), we can rewrite Eq. (2.1) as

$$(D_\mu D_\mu - m^2)U_\nu - D_\nu D_\mu U_\mu - ie(\kappa+1)F_{\nu\mu}U_\mu = 0,$$
$$\nu = 1, 2, 3, 4, \qquad (3.4)$$

and Eq. (3.1) as

$$D_\nu U_\nu = \frac{ie}{m^2}[(\kappa-1)F_{\mu\nu}D_\nu U_\mu - \kappa(\partial_\nu F_{\nu\mu})U_\mu]. \qquad (3.5)$$

Instead of (2.1) or (3.4), we shall use (3.5) together with the set of equations obtained by inserting (3.5) into (3.4). Thus, we have for the case $\kappa = 1$:

$$(D_\mu D_\mu - m^2)U_\nu + \frac{ie}{m^2}D_\nu(\partial_\rho F_{\rho\mu})U_\mu - ie\,2F_{\nu\mu}U_\mu = 0,$$
$$\nu = 1, 2, 3, 4 \qquad (3.6)$$
$$D_\nu U_\nu = -\frac{ie}{m^2}(\partial_\nu F_{\nu\mu})U_\mu.$$

For the static field case in which the external field is given by Eq. (2.5), the system of equations (3.6) becomes

$$[\partial_k\partial_k-(\partial_0+ieV)^2-m^2+J_0(\partial_0+ieV)]U_0+2ie(\partial_k V)U_k=0,$$

$$[\partial_k\partial_k-(\partial_0+ieV)^2-m^2]U_j+[2ie(\partial_j V)-(\partial_j J_0)-J_0\partial_j]U_0=0,\quad j=1,2,3 \tag{3.7}$$

$$(\partial_0+ieV-J_0)U_0+\partial_k U_k=0,$$

where $U_0=-i\,U_4$ and $J_0$ is defined by

$$J_0=\frac{ie}{m^2}(\partial_k\partial_k V). \tag{3.8}$$

In the following, we shall explicitly solve (3.7) in the high-energy limit. We use the same notation and the same choice of coordinate system as in Sec. II. The boundary conditions are

$$\lim_{z\to-\infty} U_\mu(t,\vec{x})=e^{-i(Et-pz)}a_\mu\,,\quad \mu=0,1,2,3 \tag{3.9}$$

where $p=|\vec{p}|$, $E=(p^2+m^2)^{1/2}$, and $a_\mu$ is the polarization vector of the incoming vector meson.

Following Cheng and Wu,[8] we make the ansatz

$$U_\mu(t,\vec{x})=e^{-iE(t-z)}W_\mu(\vec{x}),\quad \mu=0,1,2,3\,. \tag{3.10}$$

The $W_\mu$'s are supposed to be slowly varying functions of $\vec{x}$. In the high-energy limit $E\to\infty$, (3.9) and (3.10) give

$$\lim_{z\to-\infty} W_\mu(\vec{x})=a_\mu\,,\quad \mu=0,1,2,3 \tag{3.11}$$

where the incoming polarization vector $a_\mu$ is given in Eq. (2.12). The differential equation for $W_\mu$ can be obtained by inserting (3.10) in (3.7). One obtains

$$2E(\partial_z+ieV-\tfrac{1}{2}J_0)W_0-i[\vec{\partial}^2-m^2+e^2V^2+iJ_0eV+2ie(\partial_z V)]W_0-2e(\partial_z V)W_-+2e(\vec{\partial}_\perp V)\cdot\vec{W}_\perp=0, \tag{3.12a}$$

$$2E(\partial_z+ieV)W_--i[\vec{\partial}^2-m^2+e^2V^2-2ie(\partial_z V)]W_-+[J_0eV-i(\partial_z J_0)-iJ_0\partial_z]W_0+2e(\vec{\partial}_\perp V)\cdot\vec{W}_\perp=0, \tag{3.12b}$$

$$2E(\partial_z+ieV)\vec{W}_\perp-i(\vec{\partial}^2-m^2+e^2V^2)\vec{W}_\perp+[2e(\vec{\partial}_\perp V)+i(\vec{\partial}_\perp J_0)+iJ_0\vec{\partial}_\perp]W_0=0, \tag{3.12c}$$

$$-iEW_-+(\partial_z+ieV-J_0)W_0-\partial_z W_-+\vec{\partial}_\perp\cdot\vec{W}_\perp=0, \tag{3.12d}$$

where

$$W_-=W_0-W_3,\quad \vec{W}_\perp=(W_1,W_2),$$

$$\vec{\partial}=(\partial_1,\partial_2,\partial_3)=(\vec{\partial}_\perp,\partial_z).$$

In order to solve these equations in the limit $E\to\infty$, it is convenient to make the following asymptotic expansion for $W_\mu$:

$$W_\mu=\frac{E}{m}W_\mu^{(0)}+W_\mu^{(1)}+\frac{m}{E}W_\mu^{(2)}+\cdots,\quad \mu=0,-,1,2\,. \tag{3.13}$$

Substituting this expression into (3.12) and taking only the leading terms in $E$, we get

$$(\partial_z+ieV-\tfrac{1}{2}J_0)W_0^{(0)}=0, \tag{3.14}$$

$$W_-^{(0)}=0, \tag{3.15}$$

$$(\partial_z+ieV)\vec{W}_\perp^{(0)}=0\,. \tag{3.16}$$

Making use of the corresponding boundary conditions for $W_\mu^{(0)}$ ($\mu=0,-,1,2$) obtained from (3.11) and (3.13), we then have

$$W_0^{(0)}(z,\vec{x}_\perp)=\left(a_0\frac{m}{E}\right)\times\exp\left\{-i\int_{-\infty}^{z}dz'[eV(z',\vec{x}_\perp)-\tfrac{1}{2}\tilde{\rho}(z',\vec{x}_\perp)]\right\}, \tag{3.17}$$

$$\vec{W}_\perp^{(0)}(z,\vec{x}_\perp)=0 \tag{3.18}$$

provided that

$$\tilde{\rho}(z,\vec{x}_\perp)=-iJ_0(z,\vec{x}_\perp)=\frac{e}{m^2}(\partial_z{}^2+\vec{\partial}_\perp{}^2)V(z,\vec{x}_\perp) \tag{3.19}$$

is not too singular. We note that, in the high-energy limit, the factor $a_0m/E$ in (3.17) is a constant for all three cases given in (2.12).

To find $W_-^{(1)}$ and $\vec{W}_\perp^{(1)}$ we insert (3.13) into (3.12b), (3.12c) and consider the next higher order terms in $E$. The differential equations are

$$(\partial_z+ieV)W_-^{(1)}-\frac{i}{2m}[(\partial_z J_0)+\tfrac{1}{2}J_0{}^2]W_0^{(0)}=0 \tag{3.20}$$

and

$$(\partial_z+ieV)\vec{W}_\perp^{(1)}+\frac{1}{2m}[2e(\vec{\partial}_\perp V)+i(\vec{\partial}_\perp J_0)+iJ_0\vec{\partial}_\perp]W_0^{(0)}=0, \tag{3.21}$$

where in writing (3.20) we have employed (3.14). Making use of the boundary conditions

$$\lim_{z\to-\infty} W_-^{(1)}(z,\vec{x}_\perp)=a_0-a_3, \tag{3.22}$$

$$\lim_{z\to-\infty} \vec{W}_\perp^{(1)}(z,\vec{x}_\perp)=\vec{a}_\perp, \tag{3.23}$$

which follow immediately from (3.11) and (3.13), we obtain from (3.20), (3.21):

$$W_-^{(1)}(z,\vec{x}_\perp)=\exp\left[-i\int_{-\infty}^{z}dz'eV(z',\vec{x}_\perp)\right]\left\{a_- -\frac{a_0}{2E}\tilde{\rho}(z,\vec{x}_\perp)\exp\left[\frac{i}{2}\int_{-\infty}^{z}dz'\tilde{\rho}(z',\vec{x}_\perp)\right]\right\}, \tag{3.24}$$

$$\vec{W}_\perp^{(1)}(z,\vec{x}_\perp)=\exp\left[-i\int_{-\infty}^{z}dz'eV(z',\vec{x}_\perp)\right]\left\{\vec{a}_\perp-\frac{a_0}{2E}\int_{-\infty}^{z}dz'\exp\left[\frac{i}{2}\int_{-\infty}^{z'}dz''\tilde{\rho}(z'',\vec{x}_\perp)\right]\vec{g}_\perp(z',\vec{x}_\perp)\right\}. \tag{3.25}$$

In obtaining these expressions, the explicit expression for $W_0^{(0)}$ in (3.17) has been used. Here $a_-$ is $a_0-a_3$ and

$$\vec{g}_\perp(z,\vec{x}_\perp)=\vec{\partial}_\perp[2eV(z,\vec{x}_\perp)-\tilde{\rho}(z,\vec{x}_\perp)]+\tfrac{1}{2}i\tilde{\rho}(z,\vec{x}_\perp)\vec{\partial}_\perp\int_{-\infty}^{z}dz'[2eV(z',\vec{x}_\perp)-\tilde{\rho}(z',\vec{x}_\perp)]. \tag{3.26}$$

Since our goal is to calculate the scattering amplitude, we are primarily interested in $W_\mu(z,\vec{x}_\perp)$ for $z\to+\infty$. From (3.13), (3.15), (3.17), (3.18), (3.24), and (3.25) we have

$$\lim_{z\to+\infty}W_0(z,\vec{x}_\perp)=a_0\exp\left\{-i\int_{-\infty}^{+\infty}dz'[eV(z',\vec{x}_\perp)-\tfrac{1}{2}\tilde{\rho}(z',\vec{x}_\perp)]\right\}, \tag{3.27}$$

$$\lim_{z\to+\infty}W_-(z,\vec{x}_\perp)=a_-\exp\left[-i\int_{-\infty}^{+\infty}dz'eV(z',\vec{x}_\perp)\right], \tag{3.28}$$

$$\lim_{z\to+\infty}\vec{W}_\perp(z,\vec{x}_\perp)=\exp\left[-i\int_{-\infty}^{+\infty}dz'eV(z',\vec{x}_\perp)\right]\left\{\vec{a}_\perp-\frac{a_0}{2E}\int_{-\infty}^{+\infty}dz'\exp\left[\frac{i}{2}\int_{-\infty}^{z'}dz''\tilde{\rho}(z'',\vec{x}_\perp)\right]\vec{g}_\perp(z',\vec{x}_\perp)\right\}, \tag{3.29}$$

where we made use of the assumption that both $V(\vec{x})$ and $\tilde{\rho}(\vec{x})$ vanish for $|\vec{x}|\to\infty$. Here we include only the first nonvanishing term in the expansion (3.13). The wave functions to the next order can be found by solving the differential equation obtained by setting next $E$-power terms in (3.12). However, since for the calculation of scattering amplitudes we only need the asymptotic form in the limit $z\to+\infty$ and, in particular, only $\lim_{z\to+\infty}W_-^{(2)}(z,\vec{x}_\perp)$, we shall apply the formula derived by Cheng and Wu[8]:

$$W_-(z,\vec{x}_\perp)\sim-\frac{i}{E}\vec{\partial}_\perp\cdot\vec{W}_\perp(z,\vec{x}_\perp)+\frac{\vec{\partial}_\perp{}^2-m^2}{2E^2}W_0(z,\vec{x}_\perp)\quad\text{for}\quad z\to\infty. \tag{3.30}$$

This relation can be obtained by inserting (3.10) into (3.7) and considering the limit as $z\to\infty$, in which case the potential and the charge distribution can be neglected.

## IV. SCATTERING AMPLITUDE ($\kappa=1$)

The general expression for the scattering amplitude is given in Eq. (2.9). In the high-energy limit, this expression reduces to[8]

$$M_{fi}=2Ei\int d^2x_\perp e^{-i\Delta x}\lim_{z\to+\infty}[-b_3^*(W_--a_-)-b_-^*(W_0-a_0)+\vec{b}_\perp^*\cdot(\vec{W}_\perp-\vec{a}_\perp)], \tag{4.1}$$

where $b_-=b_0-b_3$, $W_-=W_0-W_3$, and the $W_\mu$'s ($\mu=0,1,2,3$) are related to the $U_\mu$'s through Eq. (3.10). In the following, we shall explicitly calculate the scattering amplitude for the case $\kappa=1$. In analogy to the Born terms given in Sec. II, we shall classify the matrix elements according to the polarization vectors of the incoming and outgoing vector meson.

### A. Transverse to Transverse

From Eqs. (2.12), (2.13) and (3.27), (3.28), (3.29) we see that $a_0=a_3=0$, both $b_3^*W_-$ and $b_-^*W_0$ are of order $1/E$, while $\vec{b}_\perp^*\cdot(\vec{W}_\perp-\vec{a}_\perp)$ is of order 1. Hence, we have

$$M_{fi}=i2E\int d^2x_\perp e^{-i\Delta x}\left\{\exp\left[-i\int_{-\infty}^{+\infty}dz\,eV(z,\vec{x}_\perp)\right]-1\right\}\vec{b}_\perp^*\cdot\vec{a}_\perp \tag{4.2}$$

which, together with (2.16), gives

$$M_{\parallel\parallel}=M_{\perp\perp}=i2E\int d^2x_\perp e^{-i\Delta x}\left\{\exp\left[-i\int_{-\infty}^{+\infty}dz\,eV(z,\vec{x}_\perp)\right]-1\right\},$$
$$M_{\parallel\perp}=M_{\perp\parallel}=0. \tag{4.3}$$

In terms of the helicity amplitudes, we have

$$M_{++}=M_{--}=i2E\int d^2x_\perp e^{-i\Delta x}\left\{\exp\left[-i\int_{-\infty}^{+\infty}dz\,eV(z,\vec{x}_\perp)\right]-1\right\},$$
$$M_{+-}=M_{-+}=0. \tag{4.4}$$

### B. Longitudinal to Transverse

Instead of inserting the relevant wave functions and polarization vectors into Eq. (4.1) and going through a rather lengthy calculation, we shall make use of the reciprocity theorem which relates the amplitudes $M_{\parallel 0}$, $M_{\perp 0}$, or $M_{\pm 0}$ to the amplitudes $M_{0\parallel}$, $M_{0\perp}$, or $M_{0\pm}$ of Sec. IV C. This is because, as can be easily verified, $U_\mu(t,x)$ and $U_\mu(-t,x)$, $\mu=1,2,3,4$ satisfy the same set of differential Eqs. (3.7) or (2.1) with the external field given by (2.5). Thus this theory is invariant under time reversal. The result will be given at the end of Sec. IV C.

### C. Transverse to Longitudinal

Substituting (2.13) into (4.1), we obtain

$$M_{fi}=i2E\int d^2x_\perp e^{-i\Delta x}\lim_{z\to\infty}\left[\frac{E}{m}W_-+\frac{m^2-\Delta^2}{2Em}W_0+\frac{\Delta}{m}(W_1-a_1)\right]. \tag{4.5}$$

By counting the powers of $E$ in the integrand, we see that $\lim_{z\to+\infty}W_-^{(2)}$ is also needed in this case. From (3.30), we get

$$\int d^2x_\perp e^{-i\Delta x}\lim_{z\to+\infty}\left(-\frac{E}{m}W_-\right)=\int d^2x_\perp e^{-i\Delta x}\lim_{z\to+\infty}\left[-\frac{\Delta}{m}(W_1-a_1)+\frac{\Delta^2+m^2}{2Em}W_0\right]. \tag{4.6}$$

Equations (4.5) and (4.6) lead immediately to

$$M_{fi}=i2m\int d^2x_\perp e^{-i\Delta x}\lim_{z\to+\infty}W_0. \tag{4.7}$$

Therefore we have

$$M_{0\parallel}=M_{0\perp}=0, \tag{4.8}$$

and in terms of helicity amplitudes

$$M_{0+}=M_{0-}=0. \tag{4.9}$$

Using the arguments given in Sec. IV B, we also have

$$M_{\parallel 0}=M_{\perp 0}=0, \tag{4.10}$$

$$M_{+0}=M_{-0}=0. \tag{4.11}$$

### D. Longitudinal to Longitudinal

From (2.13) and (4.1), we have

$$M_{fi}=i2E\int d^2x_\perp e^{-i\Delta x}\lim_{z\to+\infty}\left[-\frac{E}{m}W_-+\frac{m^2-\Delta^2}{2Em}W_0+\frac{\Delta}{m}W_1-1+\frac{\Delta^2}{2m^2}\right]. \tag{4.12}$$

From (3.30) we can derive an equality similar to that given in (4.6)

$$\int d^2x_\perp e^{-i\Delta x}\lim_{z\to+\infty}\left(-\frac{E}{m}W_-\right)=\int d^2x_\perp e^{-i\Delta x}\lim_{z\to+\infty}\left[-\frac{\Delta}{m}\left(W_1+\frac{\Delta}{2m}\right)+\frac{\Delta^2+m^2}{2Em}W_0\right]. \tag{4.13}$$

It follows from (4.12) and (4.13) that

$$M_{fi}=i2E\int d^2x_\perp e^{-i\Delta x}\lim_{z\to+\infty}\left(\frac{m}{E}W_0-1\right) \tag{4.14}$$

which, together with (2.12) and (3.27), gives

$$M_{00}=i2E\int d^2x_\perp e^{-i\Delta x}\left(\exp\left\{-i\int_{-\infty}^{+\infty}dz[eV(z,\vec{x}_\perp)-\tfrac{1}{2}\tilde{\rho}(z,\vec{x}_\perp)]\right\}-1\right). \tag{4.15}$$

The result of this section can be summarized as follows: For $\kappa=1$,

(i) The helicity amplitudes $M_{++}$, $M_{--}$ are equal and they have the usual simple-exponentiation form.

(ii) $M_{00}$ has a modified simple-exponentiation form. The occurrence of the additional term in the exponent is a consequence of minimal coupling and $\kappa\neq0$ in the wave equation (2.1).

(iii) All helicity-flip amplitudes vanish.

## V. DISCUSSION

We have shown that in the high-energy scattering of charged vector mesons in a static field: (a) the validity of $s$-channel helicity conservation and (b) the occurrence of simple exponentiation depend on the coupling between the vector meson and the external field. Both (a) and (b) can be realized in a theory where the vector meson has an anomalous magnetic moment, which is characterized by $\kappa=1$, and where the external charge distribution is not too singular.

As far as $s$-channel helicity conservation and occurrence of simple exponentiation is concerned, a comparison with the (trivial) spin-0 case and the (nontrivial) spin-$\frac{1}{2}$ case shows that charged vector mesons with two meson-magnetons ($\kappa=1$) behave "more normally" than those with "normal" magnetic moment (unit-magneton, $\kappa=0$).[21] Nevertheless, the result obtained in this paper should perhaps be looked upon as *an* example in which both SCHC and simple exponentiation can be realized through a suitable choice of the coupling between the particle and the external field. We speculate that more general conditions can be found under which SCHC as well as simple exponentiation would occur.

As a simple application, one may try to formulate a phenomenological model for the photoproduction of vector mesons by combining the vector-dominance idea[14] with the result obtained in the present paper. Investigations along this line are now in progress.

While this work was being written we received a preprint by S. Weinberg[22] where, in connection with the exponentiation problem, similar results were obtained. His method consists in summing the perturbation series in the eikonal approximation[23] under the following conditions:

(1) The external electromagnetic field $A_\mu(x)$ is a sufficiently smooth function of position so that the particle can absorb at most a finite four-momentum at each vertex. Indeed, $A_\mu(x)$ is chosen in such a way, that the exponential function in Eq. (5) of Ref. 22 may be approximated at each vertex by the zeroth- and first-order terms of its expansion in terms of the momentum transfer.

(2) One either demands on *a priori* grounds that the details of the electromagnetic interaction be arranged in such a way that the matrix element $S_{\beta\alpha}$ given in Eq. (1) of Ref. 22 approaches a finite value at the high-energy limit, or one simply assumes this asymptotic behavior.

This is to be compared with the method used, and the result obtained in the present paper: (A) Instead of summing the perturbation series, our starting point was the conventional[24] gauge-invariant wave function (2.1) which we have solved explicitly in the high-energy limit. (B) For the transverse-to-transverse amplitudes $M_{++}$ and $M_{--}$ the two different methods give the same answer. Here we see explicitly that the conditions for the occurrence of simple exponentiation is less restrictive than those given in Ref. 22. Indeed, at least one of the crucial assumptions made in Weinberg's paper [condition (2) mentioned above] is not necessary for the proof. (C) For the longitudinal to longitudinal amplitude $M_{00}$, Weinberg's result is different from ours. This discrepancy has its origin in the coupling terms in the wave Eq. (2.1) which are not included in the vertex functions of Ref. 22. That is to say, if we accept the conventional[24] wave equation for charged vector mesons, then we must conclude that relevent coupling terms have been neglected in the calculation of Ref. 22. (D) It is explicitly shown in this paper that those coupling terms mentioned in (C) do not contribute to the other amplitudes (e.g., $M_{++}$, $M_{+0}$, etc.).

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