To test the sensitivity of these order-of-magnitude estimations, we may set, instead of (B14), $(4\pi\mu)^{-1}\kappa_0=1$; this changes the values for κ/κ_0 and $Z_\phi^{1/2}$ slightly, from (B15) and (B16) to $(\kappa/\kappa_0)\cong 1.56$ and $Z_\phi^{1/2}\cong 0.84$.

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*This research was supported in part by the U. S. Atomic Energy Commission.

[1]S. D. Drell and T. D. Lee, Phys. Rev. D 5, 1738 (1972). The fields $P(x)$ and $X(x)$ used in this reference correspond, respectively, to $\psi(x)$ and $\pi(x)$ in the present paper. See also the various other references mentioned therein.

[2]Formally, one may consider the Lagrangian of this well-defined theory to be of the form $\mathfrak{L}_0-\eta\pi^4$, where $\mathfrak{L}_0$ is given by (7) and η is a function of κ_0 satisfying $\lim_{\kappa_0\to 0}\kappa_0^n\,\eta\to 0$ for all negative powers n. As an example, one may choose, say, $\eta=\exp(-b/\kappa_0^2)$ where b is a positive constant. The corresponding Hamiltonian now clearly has a lower bound. By choosing the appropriate renormalization counterterms, one can easily arrange in this theory to have an (asymptotic) expansion in powers of κ_0 which is determined by $\mathfrak{L}_0$ only.

[3]K. Johnson, Nucl. Phys. 25, 435 (1961).

[4]R. P. Feynman, Phys. Rev. Letters 23, 1415 (1969); in *High Energy Collisions*, Third International Conference held at the State University of New York, Stony Brook, 1969, edited by C. N. Yang *et al*. (Gordon and Breach, New York, 1969); J. D. Bjorken and E. A. Paschos, Phys. Rev. 185, 1975 (1969).

[5]J. S. Ball and F. Zachariasen, Phys. Rev. 170, 1541 (1968); D. Amati, L. Caneschi, and R. Jengo, Nuovo Cimento 58, 783 (1968); D. Amati, R. Jengo, H. R. Rubinstein, G. Veneziano, and M. A. Virasoro, Phys. Letters 27B, 38 (1968); M. Ciafaloni and P. Menotti, Phys. Rev. 173, 1575 (1968); M. Ciafaloni, *ibid*. 176, 1898 (1968).

[6]Compare also P. V. Landshoff, J. C. Polkinghorne, and R. D. Short, Nucl. Phys. B28, 225 (1971), and S. Brodsky, F. E. Close, and J. F. Gunion, Phys. Rev. D 5, 1384 (1972), for a similar expression especially for the spin-0 particle.

[7]S. L. Adler, Phys. Rev. 143, 1144 (1966); see also J. D. Bjorken, *ibid*. 148, 1467 (1966).

[8]Dr. A. Mueller has kindly pointed out to me that, in the scaling region, the final meson multiplicity remains finite in this model provided the scaling variable $x\neq 0$.

[9]Without a specific Lagrangian, it would be difficult to make a sharp distinction between a composite and an elementary-particle state. See the discussions given by W. Zimmermann, Nuovo Cimento 10, 597 (1958).

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Light-Cone Limit Sequences in Electroproduction*

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Calculating to a high order of approximation in the $\lambda\phi^3$ model, we investigate the possible connection between the deep Regge and deep scaling limits. A link between these limits is provided by fixed poles, even when canonical dimension is not conserved.

I. INTRODUCTION

This is the first of two papers in which we investigate light-cone limit sequences for inclusive processes which include a large mass. In this paper we formulate and investigate the problem for electroproduction. In the sequel, the considerations in this paper are extended to processes which involve multiparticle matrix elements of current products. A preliminary account of some of this work has been published elsewhere.[1]

The experimental data on inelastic electron-proton scattering[2] tend to support Bjorken's hypothesis that, in the deep-inelastic region, the associated invariant inclusive structure functions exhibit scale invariance.[3] This has led to considerable speculation regarding the structure of hadrons and their currents.[4-12] Of particular interest here is the observation that the kinematical region in which Bjorken scaling may obtain is canonically related to the light-cone region in configuration space, and that scaling places stringent constraints on the strength of singularities permitted in the matrix element of the current commutator, in the neighborhood of the light cone.[9-12] Thus, generalizing Wilson's hypothesis on the short-distance behavior of operator products,[13] electromagnetic scale invariance has been implemented quite elegantly with mass-independent operator-product expansions along the light cone.[14,15]

The essential observation which justifies the framework of operator-product expansions and makes it powerful is that light-cone dominance in the Bjorken scaling limit is equivalent to leading-singularity dominance. There are other kinematic routes to the light cone, however, and the possibility exists that the singularities which dominate the scaling limit may become important in these. Many authors have suggested that Regge behavior may be prominent in the deep-scaling limit,[9,10,12,16-20] and that the deep-Regge limit may become leading-singularity-dominated.[9,10,12,14,20] It is this question with which we are concerned here.

We investigate here, in the laboratory of the $\lambda\phi^3$ model, whether the deep-Regge limit is equivalent to the deep-scaling limit. The process we consider then is: electron + scalar hadron → electron + anything. The inclusive cross section is given in the one-photon-exchange approximation by[21]

$$\frac{d\sigma}{d\Omega dE'}=\frac{\alpha^2}{4E^2\sin^4(\frac{1}{2}\theta)}\times[W_2(\nu,q^2)\cos^2(\tfrac{1}{2}\theta)+2W_1(\nu,q^2)\sin^2(\tfrac{1}{2}\theta)],$$

where E, E', and θ are, respectively, the initial and final electron laboratory energies and scattering angle, q^2 is the spacelike mass squared of the exchanged photon, $\nu=q\cdot p=m(E-E')$, and p is the momentum of the hadron. The structure functions are given by

$$W_{\mu\nu}=(2\pi)^6 2p_0\sum_n\langle p|J_\mu|n\rangle\langle n|J_\nu|p\rangle\delta^4(p+q-n),$$

$$W_{\mu\nu}=\frac{W_2}{m^2}\left(p_\mu-\frac{\nu}{q^2}q_\mu\right)\left(p_\nu-\frac{\nu}{q^2}q_\nu\right)+W_1\left(-g_{\mu\nu}+\frac{q_\mu q_\nu}{q^2}\right).$$

If we consider the amplitude for the forward current–scalar-hadron scattering,

$$T_{\mu\nu}=-i(2\pi)^3 2p_0\int d^4x\, e^{iqx}\langle p|T(J_\mu(x)J_\nu(0))|p\rangle$$
$$=\frac{T_2}{m^2}\left(p_\mu-\frac{\nu}{q^2}q_\mu\right)\left(p_\nu-\frac{\nu}{q^2}q_\nu\right)+T_1\left(-g_{\mu\nu}+\frac{q_\mu q_\nu}{q^2}\right),$$

then the relation between T_i and W_i is shown by unitarity to be

$$W_i=\frac{1}{\pi}\,\mathrm{Im}T_i(\nu,q^2).$$

We follow Brandt's terminology[10] and refer to the Bjorken scaling limit as the A limit, defined as

$$\lim_A=\lim_{\nu,\,-q^2\to\infty;\ \omega=-2\nu/q^2\ \text{fixed}},$$

and the Regge limit as the R limit, defined as

$$\lim_R=\lim_{\nu\to\infty;\ q^2<0\ \text{fixed}}$$

Limits taken in sequel to these, the deep-scaling and deep-Regge limits, letting the value of the fixed parameter grow indefinitely, are

$$\lim_{A'}=\lim_{\omega\to\infty}:\lim_A,$$
$$\lim_{R'}=\lim_{q^2\to-\infty}:\lim_R.$$

Bjorken scaling means

$$\lim_A(\nu/m)W_2(\nu,q^2)=F_2(\omega),$$
$$\lim_A mW_1(\nu,q^2)=F_1(\omega),$$

with $0\leqslant F_i(\omega)<\infty$. In the R limit, the behavior is expected to be

$$\lim_R(\nu/m)W_2(\nu,q^2)=\beta_2(q^2,\alpha)(-2\nu)^{\alpha-1},$$
$$\lim_R mW_1(\nu,q^2)=\beta_1(q^2,\alpha)(-2\nu)^{\alpha},$$

where α is the $t=0$ intercept of the leading contributing Regge trajectory.

The explicit connection between the $W_i(\nu,q^2)$ and their configuration-space representation has been discussed thoroughly in the literature and we refer there for details.[10,12] The approach to the light cone in both the R' and A limit is as $0<x^2\lesssim 1/q^2\to 0$. The A limit projects the most singular component in x^2 of $W_i(p\cdot x,x^2)$. In the R' limit, however, we project first the leading $p\cdot x$ behavior, and then, taking $q^2\to-\infty$, the leading singularity in x^2 of this component. If one assumes the leading singularity in x^2 of $W_i(p\cdot x,x^2)$ carries also the leading $p\cdot x$ behavior, then these limits will be the same. In q space this requires that

$$\lim_{A'}(\nu/m)W_2(\nu,q^2)=\beta_2\omega^{\alpha-1},$$
$$\lim_{A'}mW_1(\nu,q^2)=\beta_1\omega^{\alpha},$$
$$\lim_{R'}(\nu/m)W_2(\nu,q^2)=\beta_2(q^2)^{-\alpha-1}(-2\nu)^{\alpha-1},$$
$$\lim_{R'}mW_1(\nu,q^2)=\beta_1(q^2)^{-\alpha}(-2\nu)^{\alpha},$$

where we have assumed canonical dimensionality to obtain scaling.[22,23] β_1 and β_2 are constant factors and, for boson currents, $\beta_1\to 0$. For fermion currents we have[24] $\beta_1\to\beta_2$.

The question of the relevance of Regge behavior in the A' limit for inclusive electroproduction has been investigated in the $\lambda\phi^3$ model to leading orders in the ladder approximation.[19] In this model

$$\lim_{A'}(-2\nu)\,W_2(\nu,q^2)=C_2\omega^{\alpha-1},$$

$$\lim_{A'} mW_1(\nu, q^2) = C_1 \frac{\ln(q^2/\mu^2)}{q^2}\, \omega^\alpha .$$

In Sec. II we investigate the connection between the R' and A' limits. We work always in q space, and make repeated use of Mellin-transform techniques. In order to obtain realistic q^2 dependence in the Regge residues, $\beta_i(q^2, \alpha)$, it is necessary to go beyond the leading-order approximations. We find, in ladder theory, that when all leading contributions are accounted for, the R' limit is identical with the A' limit, inclusive of the constant coefficients C_1 and C_2. The essential link between the two limits is found to be the fixed poles which occur in the model. The noncanonical $\ln(q^2/\mu^2)$ term, which has been linked[19] with the breakdown in the model of the Bjorken, Johnson, and Low theorem, is shown to be a direct consequence in the R' limit of a fixed J-plane dipole at $J = -1$.

We conclude in Sec. III with a further discussion of our results.

II. THE A' AND R' LIMITS IN ELECTROPRODUCTION

The amplitudes we retain are illustrated in Fig. 1. To leading order in q^2 in the A' and R' limits, we do not need to separate the current insertions by more than one parton line. Contact terms are dropped altogether here, as they do not contribute to the W_i. We compute the R' and A' limits to all leading orders in ladder theory, and then make direct comparisons.

Integral representations for the L-rung ladder contribution to the T_i are well known[25] and are given by

$$T_2^L = 4m^2e^2\left(-\frac{\lambda}{16\pi^2}\right)^L \int_0^\infty d\alpha_0 \left(\prod_{i=1}^L d\alpha_i d\beta_i d\gamma_i\right) \times \frac{(F_1^L)^2 e^{D_L/\Delta_L}}{\alpha_0{}^2 \Delta_L{}^4},$$

$$T_1^L = 2e^2\left(-\frac{\lambda}{16\pi^2}\right)^L \int_0^\infty d\alpha_0 \left(\prod_{i=1}^L d\alpha_i d\beta_i d\gamma_i\right) \times \frac{\Delta_L(\xi - \alpha_0) e^{D_L/\Delta_L}}{\Delta_L{}^3},$$

where e is the weak coupling, and Δ_L and D_L are the usual determinant and discriminant functions which are obtained for scalar-scalar ladders.[26] $\Delta_L(\xi - \alpha_0)$ denotes the Δ function for the diagram with the α_0 rung removed. Δ_L satisfies the relation

$$\Delta_L = \alpha_0 \Delta_L(\xi - \alpha_0) + \Delta_L^0 , \quad \Delta_L^0 = \Delta_L|_{\alpha_0 = 0} .$$

The explicit form of D_L is given by

$$D_L = 2F_1^L \nu + (F_1^L + F_3^L + F_6^L) q^2 + (F_1^L + F_4^L + F_5^L) m^2 - \mu^2 \Delta_L \sum_i \xi_i ,$$

where the ξ_i make up the set $\{\xi\}$ of Feynman parameters α_i, β_i, and γ_i. It is sufficient here to note

$$F_1^L + F_3^L + F_6^L = \alpha_0 \Delta_L^0 ,$$

and

$$F_1^L = \prod_{i=0}^L \alpha_i .$$

We compute now the R limit, using the Mellin-transform technique[26] to obtain the leading behaviors in asymptotic domains of the invariants. Applying a Mellin transform to T_2^L and T_1^L with respect to (-2ν), to avoid the cuts coming from multiparticle thresholds, we obtain

$$\tilde{T}_2^L = \Gamma(-\beta) \int_0^\infty d\alpha_0 d\xi^0 \frac{(F_1^L)^{\beta+2} e^{-R_L}}{\alpha_0{}^2 \Delta_L{}^{\beta+4}} ,$$

$$\tilde{T}_1^L(\beta, q^2) = \Gamma(-\beta) \int_0^\infty d\alpha_0 d\xi^0 \frac{(F_1^L)^\beta \Delta_L(\xi - \alpha_0) e^{-R_L}}{\Delta_L{}^{\beta+3}} ,$$

where we have defined

$$D_L = 2\nu F_1^L - \Delta_L R_L , \quad R_L = \frac{\alpha_0 \Delta_L^0}{\Delta_L} q^2 - K_L ,$$

$$d\xi = d\alpha_0 d\xi^0 = d\alpha_0 \prod_{i=1}^L d\alpha_i d\beta_i d\gamma_i ,$$

and β is the Mellin variable conjugate to -2ν. Here and hereafter, we will drop inessential factors such as $-\lambda/16\pi^2$, etc. The leading poles in the left half of the β plane, at $\beta = -3$ in $\tilde{T}_2^L$, and $\beta = -1$ in $\tilde{T}_1^L$, may be exposed by integrating by parts in the rung parameters α_i. This gives terms of the form $\alpha_i{}^{\beta+3}/\beta+3$ in $\tilde{T}_2^L$, and $\alpha_i{}^{\beta+1}/(\beta+1)$ in $\tilde{T}_1^L$. In order to compute the sum in all ladders of all terms of order ν^{-3} in T_2 and ν^{-1} in T_1, including all factors of $\ln\nu$, we employ the following device.[27] Write

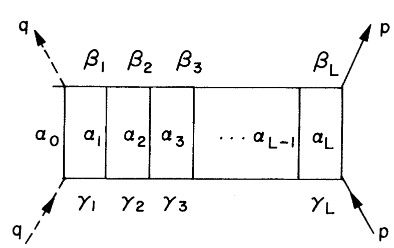

FIG. 1. The ladder amplitude. The Feynman parameters α_i, β_i, and γ_i comprise the set $\{\xi\}$.

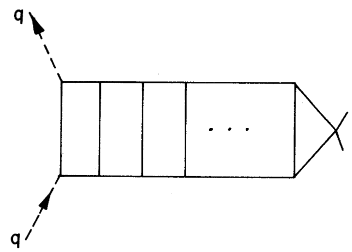

FIG. 2. One of the components of the vertex function $C(\beta, 0, q^2)$.

$$\frac{{\alpha_i}^{\beta+3}}{\beta+3}=\frac{{\alpha_i}^{\beta+3}-1}{\beta+3}+\frac{1}{\beta+3}\,,$$

$$\frac{{\alpha_i}^{\beta+1}}{\beta+1}=\frac{{\alpha_i}^{\beta+1}-1}{\beta+1}+\frac{1}{\beta+1}\,.$$

The singular part is now contained in only the second terms on the right. In the subsequent expansions of F_1^L, only contributions with at least one factor of these second terms will contribute to the leading behavior. The possible insertions of a given power of these singular terms into an L-rung ladder contribute partial multinomial expressions which are completed in the sum over all L, and recognized as a power of order near L of the trajectory function $F(\beta, t)$ obtained by Polkinghorne.[28]

Now we must discuss an essential point of this part of the analysis, namely, α_0 does not contribute to the Regge part of the high-energy behavior, but contributes only fixed poles.[29] This is essentially because the first rung, α_0, is proportional to the weak coupling, e^2, rather than λ^2. As for T_1, we show in the Appendix that there is a series of fixed J-plane dipoles starting at $J=-1$. Thus, for T_1, not only should we separate the α_0 term which contributes to simple, multiplicative fixed-pole behavior, we must separate off a second fixed pole at the same point which gives a fixed dipole. The result of summing over all L, distinguishing fixed-pole contributions from contributions to moving trajectories, is

$$\tilde{T}_2(\beta, q^2)=C_2(\beta, t, q^2)\frac{1}{\beta+3-F(\beta+2, t)}\,\mathcal{G}(\beta+2, t)\Gamma(-\beta)\bigg|_{t=0}\,,$$

$$\tilde{T}_1(\beta, q^2)=C_1(\beta, t, q^2)\frac{1}{\beta+1-F(\beta, t)}\,\mathcal{G}(\beta, t)\Gamma(-\beta)\bigg|_{t=0}\,,$$

where $\mathcal{G}(\beta, t)$ and $F(\beta, t)$ are the same residue and trajectory functions which occur in scalar-scalar scattering[28] and

$$C_i(\beta, 0, q^2)=\sum_{L=0}^{\infty} C_i^L(\beta, 0, q^2)\,,$$

with

$$C_2^L(\beta, 0, q^2)=e^2\int_0^{\infty} d\alpha_0 d\xi^0 {\alpha_0}^{\beta}\prod_{j=1}^{L}\frac{{\alpha_j}^{\beta+3}-1}{\beta+3}\,\partial_{\alpha_j}({\Delta_L}^{-\beta-4}e^{-K_L})\,,$$

$$C_1^L(\beta, 0, q^2)=e^2\int_0^{\infty}\frac{d\xi^0 d\rho}{d\beta_1 d\alpha_1 d\gamma_1}\int_0^{\infty} d\tilde{\alpha}_0 d\tilde{\alpha}_1 d\tilde{\beta}_1(\rho\tilde{\alpha}_0)^{\beta}\frac{{\tilde{\alpha}_1}^{+1}}{\beta+1}\,\partial_{\tilde{\alpha}_1}\prod_{j=2}^{L}\frac{{\alpha_j}^{\beta+1}-1}{\beta+1}\,\partial_{\alpha_j}[{\bar{\Delta}_L}^{-\beta-3}\tilde{\Delta}_L(\xi-\alpha_0)e^{-\tilde{K}_L}]\,,$$

using the notation of the Appendix. All the functions in this representation for the C's are to be calculated for the contracted-vertex graph illustrated in Fig. 2. A similar graph is appropriate[28] for the computation of $\mathcal{G}$.

The R' limit is now obtained by applying the same Mellin-transform technique. It is not necessary to first invert the $\tilde{T}_i$ and make explicit the R limit, since there is a unique correspondence between β and the pole singularities. Letting the Mellin variable conjugate to $-q^2$ be τ, we apply the Mellin transform to $C_i^L(\beta, 0, q^2)$ to obtain

$$\tilde{C}_2^L(\beta, 0, \tau)=e^2\Gamma(-\tau)\int_0^{\infty} d\xi^0 d\alpha_0 {\alpha_0}^{\beta+\tau}\prod_{j=1}^{L}\frac{{\alpha_j}^{\beta+3}-1}{\beta+3}\,\partial_{\alpha_j}\left[{\Delta_L}^{-\beta-4}\left(\frac{\Delta_L^0}{\Delta_L}\right)^{\tau}e^{-K_L}\right]\,,$$

$$\tilde{C}_1^L(\beta, 0, \tau)=e^2\Gamma(-\tau)\int_0^{\infty}\frac{d\xi^0 d\rho}{d\beta_1 d\gamma_1 d\alpha_1}\int_0^{\infty} d\tilde{\alpha}_0 d\tilde{\alpha}_1 d\tilde{\beta}_1(\rho\tilde{\alpha}_0)^{\beta+\tau}\frac{{\tilde{\alpha}_1}^{\beta+1}-1}{\beta+1}\,\partial_{\tilde{\alpha}_1}$$
$$\times\prod_{j=2}^{L}\frac{{\alpha_j}^{\beta+1}-1}{\beta+1}\,\partial_{\alpha_j}\left[{\bar{\Delta}_L}^{-\beta-3}\tilde{\Delta}_L(\xi-\alpha_0)\left(\frac{\Delta_L^0}{\Delta_L}\right)^{\tau}e^{-\tilde{K}_L}\right].$$

In $\tilde{C}_2^L$, integrating by parts in α_0 exposes the leading pole at $\tau=-\beta-1$. In C_1^L we find a double pole at $\tau=-\beta-1$. Displacing the inversion contours across these singularities, and retaining only the residues, we obtain

$$\lim_{-q^2\to\infty} C_2^L(\beta,0,q^2)=\Gamma(\beta+1)C_2^L(\beta,0)(-q^2)^{-\beta-1},$$

$$\lim_{-q^2\to\infty} C_1^L(\beta,0,q^2)=\Gamma(\beta+1)C_1(\beta,0)(-q^2)^{-\beta-1}\ln(q^2/\mu^2),$$

where

$$C_2^L(\beta,0)=e^2\int_0^\infty d\xi^0\prod_{j=1}^{L}\frac{\alpha_j{}^{\beta+3}-1}{\beta+3}\,\partial_{\alpha_j}(\Delta_L{}^{-\beta-4}e^{-K_L})_{\alpha_0=0},$$

$$C_1^L(\beta,0)=e^2\int_0^\infty\frac{d\xi^0}{d\alpha_1 d\beta_1 d\gamma_1}\int d\tilde{\beta}_1 d\tilde{\alpha}_1\frac{\alpha_1{}^{\beta+1}-1}{\beta+1}\,\partial_{\tilde{\alpha}_1}\prod_{j=2}^{L}\frac{\alpha_j{}^{\beta+1}-1}{\beta+1}\,\partial_{\alpha_j}[\bar{\Delta}_L{}^{-\beta-3}\tilde{\Delta}_L(\xi-\alpha_0)e^{-\tilde{K}_L}]_{\tilde{\alpha}_0=\rho=0}.$$

The noncanonical $\ln(q^2/\mu^2)$ factor is thus seen to be a direct consequence of the fixed dipole at $J=-1$.

It remains now to calculate the A' limit in comparable approximation. The details are given largely in Ref. 19. The A limit introduces a factor $-\pi(\sin\pi\beta)^{-1}$, which is just $\Gamma(-\beta)\Gamma(1+\beta)$. Using again the device of subtracting the singular portions in the rung parameters, the calculation is generalized to all leading orders as above, and the Mellin transform with respect to ω in the A' limit becomes

$$\tilde{T}_2(\beta,0)=C_2(\beta,t)\frac{q^{-2}}{\beta+3-\Gamma(\beta+2,t)}\,\mathcal{G}(\beta+2,t)\Gamma(-\beta)\Gamma(1+\beta)\bigg|_{t=0},$$

$$\tilde{T}_1(\beta,0)=C_1(\beta,t)\frac{q^{-2}\ln(q^2/\mu^2)}{\beta+1-\Gamma(\beta,t)}\,\mathcal{G}(\beta,t)\Gamma(-\beta)\Gamma(1+\beta)\bigg|_{t=0},$$

where, as the notation suggests, $C_2(\beta,0)$ and $C_1(\beta,0)$ are the same constants, term by term in the series

$$C_i=\sum_{L=1}^{\infty}C_i^L(\beta,0),$$

as obtained in the R' limit. Inverting, we obtain finally[30]

$$\lim_{A'(R')}(-2\nu)T_2(\nu,q^2)=C_2(\alpha-2,0)\mathcal{G}(\alpha,0)\Gamma(-1+\alpha)\Gamma(-\alpha+2)\omega^{\alpha-1}+\text{fixed-pole terms},$$

$$\lim_{A'(R')} mT_1(\nu,q^2)=C_1(\alpha,0)\mathcal{G}(\alpha,0)\Gamma(1+\alpha)\Gamma(-\alpha)\frac{\ln(q^2/\mu^2)}{q^2}+\text{fixed-pole terms}.$$

The scaling behavior obtained in the A' limit thus manifests itself fully in the R' limit, and the constant factors are equal. More remarkable, the noncanonical factor $\ln(q^2/\mu^2)$ appears also in the R' limit, by virtue of the fixed J-plane dipole.

III. DISCUSSION

The foregoing analysis suggests that perhaps a crucial link between the A' and R' limit is afforded by the fixed pole. It has been suggested that the fixed pole may provide the mechanism for Bjorken scaling.[7] It is certainly true in the present model that scaling comes in finite order of perturbation theory from the fixed pole of the α_0 rung. However, if we regard Regge behavior as being built up from parton exchanges, here taken in the ladder approximation, there does not have to be a fixed pole in the A' limit in the net amplitude. To obtain further insight into the connection in perturbation theory between fixed poles and interchangeable A' and R' limits, we may recall scalar-photon Compton scattering in ladder theory. Abarbanel, Goldberger, and Treiman found[16]

$$\lim_{A'} W(q^2,\nu)\sim\frac{C(\alpha)}{q^2}\,\omega^{\alpha(0)}.$$

If we compute first the R limit, in leading order all ladder rungs contribute to the trajectory function $F(\beta,t)$ and the Regge residue $C(\beta,q^2)$ is independent of q^2. We may include all leading logarithmic terms, by using Halliday's device as we did for W_1 and W_2 in Sec. II. We obtain again a series for $C(\beta,q^2)$, and in each term after the first we will have q^2 dependence. A typical term has the form

$$C_L(\beta,q^2)=\int d\xi\prod_{i=1}^{L}\frac{\alpha_i{}^{\beta+1}-1}{\beta+1}\,\partial_{\alpha_i}\left[\frac{e^{R_L(q^2)}}{\Delta_L{}^{\beta+2}}\right],$$

and we note the absence of the fixed pole. For such a term the leading behavior as $q^2\to\infty$ is a constant, and it appears that summing the series would not alter this. Therefore, to all leading orders in ν, the A' and R' limits behave differently. Unless one introduces a weak coupling, any configuration of particle exchanges will recur in all possible ways, and presumably will contribute finally to the Regge trajectory function, $F(\beta, t)$, giving again a constant leading behavior in q^2 in $C(\beta, q^2)$.

Thus the free-particle propagator in the α_0 rung may contribute to the trajectory function, or remain as a factor which shows up as a fixed pole in the J plane. There does not seem to be a necessary general connection between fixed poles and interchangeable limits, however. Some additional constraints conceivably could introduce fixed-pole killing factors. In the absence of such extra constraints it seems likely, assuming partonlike exchanges build up Regge trajectories as in the $\lambda\phi^3$ model, that interchangeability may obtain or not, depending on whether or not fixed poles also appear at the first nonsense point in the current-hadron scattering amplitude.

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APPENDIX

In this appendix we examine the J-plane structure of the spin-nonflip amplitude T_1. It is well known that, in the present model, T_2 has fixed simple poles in the J plane which enter multiplicatively with the moving poles.[25] We wish now to demonstrate that T_1 contains multiplicative fixed dipoles, which must be distinguished from contributions to moving-trajectory behavior. That is, as we have discussed, it is possible artificially to introduce multiplicative fixed poles of any order, but in T_1 they enter naturally. We start from the Mellin transform with respect to (-2ν) of T_1^L:

$$\tilde{T}_1^L(\beta, q^2) = e^2\Gamma(-\beta)\int_0^\infty d\alpha_0 d\xi^0 \frac{\Delta_L(\xi-\alpha_0)}{\Delta_L^{\beta+3}}\left(\prod_{i=0}^{L}\alpha_i\right)^{\beta} e^{-R_L}\,.$$

After the scaling transformation

$$\alpha_0 = \rho\tilde{\alpha}_0\,,\quad \beta_1 = \rho\tilde{\beta}_1\,,$$
$$\alpha_1 = \rho\tilde{\alpha}_1\,,\quad \gamma_1 = \rho(1-\tilde{\alpha}_0-\tilde{\alpha}_1-\tilde{\beta}_1)\,,$$

$\tilde{T}_1^L$ becomes

$$\tilde{T}_1^L(\beta, q^2) = e^2\Gamma(-\beta)\int_0^\infty \frac{d\xi^0 d\rho}{d\alpha_1 d\beta_1 d\gamma_1}\int_0^1 d\tilde{\alpha}_0 d\tilde{\alpha}_1 d\tilde{\beta}_1 (\rho\tilde{\alpha}_0)^\beta \tilde{\alpha}_1{}^\beta \prod_{i=2}^{L}\alpha_i{}^\beta \frac{\Delta_L(\xi-\alpha_0)}{\Delta_L{}^{\beta+3}}\, e^{-R_L}\,,$$

where

$$\tilde{\Delta}_L = \tilde{\Delta}_L(\tilde{\alpha}_0, \tilde{\alpha}_1, \tilde{\beta}_1, \rho, \alpha_2, \ldots)$$
$$= \Delta_L(\xi)\,,$$
$$\Delta_L = \rho^{-1}\tilde{\Delta}_L\,,$$

etc. Exposing the poles in $T_1^L(\beta)$ by integrating the α_i by parts gives

$$\tilde{T}_1^L(\beta, q^2) = \left(\frac{-1}{\beta+1}\right)^{L+2} e^2\Gamma(-\beta)\int_0^\infty \frac{d\xi^0 d\rho}{d\alpha_1 d\alpha_1 d\gamma_1}\int_0^\infty d\tilde{\alpha}_0 d\tilde{\alpha}_1 d\tilde{\beta}_1 (\rho\tilde{\alpha}_0)^{\beta+1}\tilde{\alpha}_1{}^{\beta+1}\partial_{\tilde{\alpha}_1}\prod_{i=2}^{L}\alpha_i{}^{\beta+1}\partial_{\alpha_i}\partial_{\alpha_0}\partial_\rho\left[\frac{\Delta_L(\xi-\alpha_0)}{\tilde{\Delta}_L{}^{\beta+3}}e^{-R_L}\right],$$

or, more briefly,

$$\tilde{T}_1^L(\beta, q^2) = (-1)^{L+2}\Gamma(-\beta)\mathcal{G}_L(\beta, q^2)(\beta+1)^{-L-2}\,.$$

To obtain the large-ν behavior we must invert this expression:

$$T_1^L(\nu, q^2) = \frac{(-1)^{L+2}}{2\pi i}\int_{-\epsilon-i\infty}^{-\epsilon+i\infty}\frac{\Gamma(-\beta)\mathcal{G}_L(\beta, q^2)(-2\nu)^\beta}{(\beta+1)^{L+2}}\,d\beta\,,$$

and, keeping only the pole terms at $\beta = -1$,

$$T_1^L(\nu,q^2)\approx\frac{(-1)^{L+2}}{(L+1)!}\,\partial_\beta{}^{L+1}[\mathcal{G}_L(\beta,q^2)\Gamma(-\beta)(-2\nu)^\beta]\Big|_{\beta=-1}$$

$$\approx\frac{K_2{}^{L+1}\partial_\beta{}^{L+1}}{(L+1)!}[K_e(q^2,t=0,\beta)(-2\nu)^\beta\Gamma(0)]\Big|_{\beta=-1}\,,$$

where

$$K_\lambda(t)=\int_0^\infty d\beta d\gamma\,\frac{\exp\{[\beta\gamma t-\mu^2(\beta+\gamma)]/(\beta+\gamma)\}}{\beta+\gamma}\,,$$

$$K_e(q^2,t,\beta)=e^2\int_0^\infty d\rho\int^1 d\tilde{\alpha}_0 d\tilde{\beta}_1(\rho\tilde{\alpha}_0)^{\beta+1}\partial_{\tilde{\alpha}_0}\partial_\rho\exp[\rho\tilde{\beta}_1(1-\tilde{\beta}_1-\tilde{\alpha}_0)t+\rho\tilde{\alpha}_0(1-\tilde{\alpha}_0)q^2-\rho\mu^2]\,.$$

Because $K_e(q^2,\beta)$ does not contribute to the Regge behavior, it remains as a factor and becomes a part of the Regge residue. To obtain the actual moving behavior we sum over all L:

$$T_1=\sum_{L=1}^\infty\frac{K_\lambda{}^{L+1}(0)}{(L+1)!}\,\partial_\beta{}^{L+1}[K_e(q^2,0,\beta)\Gamma(-\beta)(-2\nu)^\beta]\Big|_{\beta=-1}$$

$$=\sum_{L=0}^\infty\frac{K_\lambda{}^L(0)}{K^2}\,\frac{\partial_\beta{}^L}{L!}\,[K_e(q^2,0,\beta)\Gamma(-\beta)(-2\nu)^\beta]-\partial_\beta[K_e\Gamma(-\beta)(-2\nu)^\beta]\Big|_{\beta=-1}\,.$$

Here we immediately recognize the single and double fixed poles in addition to the moving singularity. The Regge contribution becomes

$$T_1^{\text{Regge}}=\frac{e^{K_\lambda(0)\partial_\beta}}{K_\lambda{}^2(0)}\,[K_e(q^2,0,\beta)\Gamma(-\beta)(-2\nu)^\beta]\Big|_{\beta=-1}$$

$$=\frac{K_e(q^2,0,\alpha)\Gamma(-\alpha)(-2\nu)^\alpha}{K_\lambda{}^2(0)}\,,$$

with $\alpha(t)=-1+K(t)$ as usual. Integrating back by parts in ρ and α_0 we have

$$T_1^{\text{Regge}}=C_1^0(\alpha(0),0,q^2)\Gamma(-\alpha(0)(-2\nu)^{\alpha(0)})\,,$$

which has a singular residue at $\alpha(0)=-1$.

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[1]H. C. Baker, E. Y. C. Lu, and E. Shrauner, Phys. Letters 38B, 110 (1972).

[2]E. D. Bloom *et al.*, MIT-SLAC Report No. SLAC-PUB-796, 1970 (unpublished), presented at the Fifteenth International Conference on High Energy Physics, Kiev, U.S.S.R., 1970.

[3]J. D. Bjorken, Phys. Rev. 179, 1547 (1969).

[4]R. D. Feynman, Phys. Rev. Letters 23, 1415 (1969).

[5]J. D. Bjorken and E. A. Paschos, Phys. Rev. 185, 1975 (1969).

[6]S. D. Drell, Donald Levy, and T.-M. Yan, Phys. Rev. Letters 22, 744 (1969); Phys. Rev. 187, 2159 (1969); Phys. Rev. D 1, 1035 (1970); 1, 1617 (1970).

[7]P. V. Landshoff, J. C. Polkinghorne, and R. D. Short, Nucl. Phys. B28, 226 (1971); P. V. Landshoff and J. C. Polkinghorne, *ibid.*, B28, 240 (1971).

[8]C. H. Llewellyn Smith, Nucl. Phys. B17, 277 (1970).

[9]Richard A. Brandt, Phys. Rev. Letters 22, 1149 (1969); *ibid.* 23, 1260 (1969).

[10]Richard A. Brandt, Phys. Rev. D 1, 2808 (1970).

[11]B. L. Ioffe, Phys. Letters 30B, 123 (1969).

[12]R. Jackiw, R. Van Royen, and G. B. West, Phys. Rev. D 2, 2473 (1970).

[13]Kenneth G. Wilson, Phys. Rev. 179, 1499 (1969).

[14]Y. Frishman, Phys. Rev. Letters 25, 966 (1970); Ann. Phys. (N.Y.) 66, 373 (1971).

[15]Richard A. Brandt and Giuliano Preparata, Nucl. Phys. B27, 541 (1971).

[16]H. D. I. Abarbanel, M. L. Goldberger, and S. B. Treiman, Phys. Rev. Letters 22, 500 (1969).

[17]H. Harari, Phys. Rev. Letters 22, 1078 (1969); *ibid.* 24, 1260 (1970).

[18]Y. Frishman, Phys. Rev. Letters 25, 966 (1970); Ann. Phys. (N.Y.) 66, 373 (1971).

[19]Guido Altarelli and Hector R. Rubinstein, Phys. Rev. 187, 2111 (1969).

[20]Richard A. Brandt, Phys. Rev. D 4, 444 (1971).

[21]S. D. Drell and J. D. Walecka, Ann. Phys. (N.Y.) 28, 18 (1968).

[22]P. Carruthers, Phys. Reports 1, 1 (1969).

[23]H. Fritzsch and M. Gell-Mann, in *Broken Scale Invariance and the Light Cone,* 1971 Coral Gables Conference on Fundamental Interactions at High Energy, edited

by M. Dal Cin, G. Y. Iverson, and A. Perlmutter (Gordon and Breach, New York, 1971), Vol. 2.
[24]C. G. Callan, Jr. and D. J. Gross, Phys. Rev. Letters 22, 156 (1969).
[25]I. G. Halliday, Nuovo Cimento 51, 970 (1967).
[26]R. Eden, P. Landshoff, D. Olive, and J. Polkinghorne, *The Analytic S-Matrix* (Cambridge Univ. Press, Cambridge, England, 1966), Chap. III.
[27]I. G. Halliday, Nucl. Phys. B33, 285 (1971). Our original analysis was done by inverting before summing in order to obtain $(q^2)^{-\alpha-1}$ behavior, much as in the Appendix, for both electroproduction and the multiparticle amplitudes.
[28]J. C. Polkinghorne, J. Math. Phys. 5, 431 (1964).
[29]J. B. Bronzan, I. S. Gerstein, B. W. Lee, and F. E. Low, Phys. Rev. 157, 1448 (1967).
[30]In the present model the Regge trajectory function α is always negative. We assume that our results may be continued to $\alpha > 0$.

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Compositeness and Light-Cone Singularities

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We offer two arguments that compositeness, with regular binding potentials, tends to "soften" short-distance singularities, and suggest that compositeness may reconcile Bjorken scaling with renormalizable strong interactions.

Since Lagrangians with massive particles are obviously not scale-invariant, one must formulate specific dynamical assumptions as to why scale invariance is nevertheless relevant in physics. One such set of hypotheses is clearly formulated by Wilson[1] as follows:

(i) It is assumed that there exists a scale-invariant "skeleton theory" in which the hadrons are massless, with a nontrivial interaction invariant under SU(3)×SU(3). In this theory operator-product expansions are valid for small distances:

$$A(\tfrac{1}{2}x)B(-\tfrac{1}{2}x) \underset{x\to 0}{\sim} \sum_{i=1}^{N} C_i(x)O_i(0),$$

with

$$C_i(x) \underset{x\to 0}{\sim} x^{d_i-d_A-d_B},$$

where the d_i's denote the (not necessarily canonical) scaling dimensions of the respective fields.

(ii) It is further assumed that the more realistic situation corresponds to the addition of some scale-noninvariant interactions, which are of the mass-term or superrenormalizable type. Consequently these are "soft" in the sense of giving corrections to $C_i(x)$ smaller by one power of x as $x \to 0$. Hence, unless a particular $C_i(x)$ was identically zero in the skeleton theory due to internal-symmetry selection rules, the strength of its strongest singularity as $x \to 0$ remains the same as in the skeleton theory.

Such a formulation provides a clear rationale for the relevance of scale invariance at small distances. However, as emphasized by Wilson, the scale dimensions d_i in (i) are expected to be noncanonical as a result of renormalization effects. On the other hand, the experimental results on deep-inelastic electron-nucleon scattering, up to this time, seem to indicate the existence of fields other than the stress-energy tensor and currents with canonical dimensions.[2] Also, the above formulation is somewhat different in philosophy from the conventional bootstrap idea,[3] in the sense that if the bootstrap solution really exists for only a unique set of values of masses and coupling constants, then the "skeleton" world must not satisfy all the bootstrap constraints.

In this paper we wish to suggest that if the hadrons are composites of one another, the strong interaction as a whole may be somewhat "soft" in the sense of giving rise to short-distance singularities which are often weaker than naively expected from perturbation calculations. We propose that this softness of the strong interaction as a whole, rather than the softness of the scale-noninvariant part, is responsible for the visibility of some remnants of canonical dimensionality in certain processes such as deep-inelastic electroproduction. Thus compositeness may be the reason why nature seems to "read books on free field theory, as far