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PHYSICAL REVIEW D VOLUME 6, NUMBER 4 15 AUGUST 1972

Some General Consequences of Regge Theory for Pomeranchukon-Pole Couplings*

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(Received 13 April 1972)

Without reference to specific Regge models, and assuming only the validity of asymptotic Regge-pole expansions, we use unitarity and exact sum rules to draw conclusions concerning the couplings of the vacuum pole $\alpha_p(0)$. If $\alpha_p(0)<1$ we derive an upper bound on the triple-vacuum coupling. Assuming $\alpha_p(0)=1$ we prove either that the $(t=0)$ Pomeranchukon-particle-Reggeon coupling vanishes for all Reggeon masses, or the $(t=0)$ Pomeranchukon-Reggeon-Reggeon coupling vanishes. We also prove the vanishing of either the Pomeranchukon-Pomeranchukon-Reggeon vertex at zero mass, or the Mueller-like Pomeranchukon-Reggeon-particle-particle vertex, again all Reggeons at zero mass. Our model-independent technique is used to recover the Finkelstein-Kajantie result that the Pomeranchukon-Pomeranchukon-particle coupling vanishes for $\alpha_p(0)=1$; if $\alpha_p(0)<1$ an inequality is obtained for the coupling. The influence of Regge cuts is neglected.

I. INTRODUCTION

The problem of self-consistency of a dominant Regge pole in the vicinity of $J=1$ at $t=0$, which was studied in a classic paper by Finkelstein and Kajantie,[1] has received renewed interest during the past year. A new approach to this problem has followed the line of Mueller's analysis[2] of inclusive cross sections. By putting absolute bounds on integrals over inclusive cross sections one can, for example, show[3] that the so-called triple-Pomeranchukon vertex must vanish at $t_1=t_2=t_3=0$ if $\alpha_p(0)=1$. A rather elegant way of deriving this result[4] is based on sum rules expressing conservation of energy and momentum in terms of inclusive cross sections. As recently emphasized by one of us[5] these kinematical constraints become nontrivial dynamical conditions once unitarity (in the sense of Mueller) is used to express inclusive cross sections as discontinuities of elastic multiparticle amplitudes[6] and the properties of these latter are studied in terms of (multi-) Regge theory. In this paper we apply these sum rules to derive further rigorous inequalities on the parameters of dominant pole with $\alpha_p(0)\approx 1$. In particular a lower bound on $\delta = 1-\alpha_p(0)$ is derived in terms of a nonvanishing triple-Pomeranchukon coupling, and the connection with the result of FK is discussed. We also prove that, under the same conditions for which this triple coupling vanishes, a whole set of double-Regge vertices has to vanish as well. In Sec. II we derive the basic inequalities relating integrals over two-particle inclusive cross sections to one-particle inclusive cross sections and integrals over the latter to total cross sections. In Sec. III we give bounds on the triple-Pomeranchukon coupling in terms of the distance $1-\alpha_p(0)$ of the $t=0$ intercept of the Pomeranchukon trajectory from one. Section IV is devoted to a discussion of the further consequences of a dominant Pomeranchukon trajectory with $t=0$ intercept $\alpha(0)=1$; it is shown there that in this case the $(t=0)$ Pomeranchukon-particle-Reggeon coupling must vanish for arbitrary Reggeon mass, or the $(t=0)$ Pomeranchukon-Reggeon-Reggeon coupling must vanish for equal but otherwise arbitrary Reggeon masses. We also show, in this case, the vanishing of either the Pomeranchukon-Pomeranchukon-Reggeon vertex at zero mass, or the Mueller-like Pomeranchukon-Reggeon-particle-particle vertex, again all Reggeons at zero mass. We finally give a rigorous derivation of the Finkelstein-Kajantie[1] result for the double-Regge coupling of the Pomeranchukon to a hypothetical scalar particle.

We assume in this paper the dominance of Regge poles and we do not consider possible compensations coming from Regge cuts in the neighborhood of the poles.

II. SUM RULES

The sum rules which are relevant to our discussion have been discussed in considerable detail in Refs. 4 and 5. These are

$$(p_a+p_b)_\mu \sigma_{ab} = \sum_c \int \frac{d\sigma_{ab}}{dp_c} p_{c\mu} dp_c \tag{2.1}$$

and

$$(p_a+p_b-p_c)_\mu \frac{d\sigma_{ab}}{dp_c} = \sum_d \int \frac{d\sigma_{ab}}{dp_c dp_d} p_{d\mu} dp_d , \tag{2.2}$$

where σ_{ab} is the total cross section for particle a with four-momentum $p_{a\mu}$ and particle b with four-momentum $p_{b\mu}$; $d\sigma_{ab}/dp_c$ is the inclusive cross-section for $a+b\to c$ (with four-momentum $p_{c\mu}$) + anything; dp_c is the invariant three-dimensional volume element $dp_c = d^3p_c/2E_c$, with E_c the energy. The sums over c and d are over species, where the distinguishable members of an isomultiplet are considered as separate species. Finally, $d\sigma_{ab}/dp_c dp_d$ is the inclusive cross section $a+b \to c+d+$ anything.

We use the sum of the time component and the longitudinal component of Eq. (2.1) in the center-of-mass system to derive the inequality

$$\sigma_{ab} > \int \frac{E_c+p_{cL}}{E} \frac{d\sigma_{ab}}{dp_c} dp_c , \tag{2.3}$$

where c is now a single chosen species, E is the total center-of-mass energy $E=\sqrt{s}$, and p_{cL} is the longitudinal momentum of particle c. Since $E_c+p_{cL}>0$, we may restrict the integral in Eq. (2.3) to $p_{cL}>0$, thereby including one of the two fragmentation regions (by convention, $p_{cL}>0$ corresponds to c a fragment of a). Since we will wish to study the triple-Pomeranchukon vertex, we choose c and a to be of the same species.

It is convenient to consider the Feynman variable $x_c = 2p_{cL}/E$; for relativistic p_{cL}, and fixed transverse momentum $\vec{p}_{cT}$, we have $E_c \sim p_{cL}$ and $dp_c \sim d\vec{p}_{cT} dx_c/2x_c$. Equation (2.3) then becomes

$$\sigma_{ab} > \tfrac{1}{2} \int dx_c d\vec{p}_{cT} \frac{d\sigma_{ab}}{dp_c} . \tag{2.4}$$

If we further restrict the integral in (2.4) to values of $\vec{p}_{cT}^{\,2}$ that are held fixed as $E\to\infty$, the limits on x_c are independent of $\vec{p}_{cT}$: $0<x_c<1$.

We also note the relation between the momentum transfer squared $t=(p_a-p_c)^2$, x_c, and $\vec{p}_{cT}$:

$$t = -\frac{\vec{p}_{cT}^{\,2} + m^2(1-x_c)^2}{x_c} + O(1/s) , \tag{2.5}$$

where m is the common mass of a and c.

We turn now to Eq. (2.2), where again we use the sum of the time component and the longitudinal component[7] to derive the inequality

$$\frac{d\sigma_{ab}}{dp_c} > \int dp_d \frac{E_d+p_{dL}}{E-E_c-p_{cL}} \frac{d\sigma}{dp_c dp_d} \tag{2.6}$$

or, in the relativistic limit,

$$\frac{d\sigma_{ab}}{dp_c} > \tfrac{1}{2} \int d\vec{p}_{d\perp} \frac{dx_d}{1-x_c} \frac{d\sigma_{ab}}{dp_c dp_d} , \tag{2.7}$$

where $x_d = 2p_{dL}/E$ and the limits on x_d are $0<x_d<1-x_c$, where again we have limited the integration to positive longitudinal momenta.

We reserve further discussion of the kinematics of this process to Sec. IV.

III. TRIPLE-POMERANCHUKON VERTEX

We first explore the consequences of a dominant Pomeranchukon (or vacuum) trajectory $\alpha_v(t)$, with $\alpha_v'(0) \neq 0$ and $\alpha_v(0)$ near 1. The Mueller[2] technique enables us to express $d\sigma_{ab}/dp_c$ in terms of the Regge parameters of the six-point function, and thus gives content to the inequality Eq. (2.4). As $s=E^2 \to \infty$, the left-hand side, σ_{ab}, goes like

$$\sigma_{ab} \sim \gamma_{av}(0)\gamma_{bv}(0)s^{\alpha_v(0)-1}, \tag{3.1}$$

where γ_{av} and γ_{bv} are the particle-particle-Pomeranchukon couplings at $t=0$.

As $s\to\infty$, $M^2\to\infty$, and $s/M^2\to\infty$ [where M is the missing mass, $M^2=(p_a+p_b-p_c)^2$], the integrand goes like

$$\frac{d\sigma_{ab}}{dp_c} \sim \gamma_{bv}(0)\gamma_{ap}{}^2(t)\,\frac{(M^2)^{\alpha_v(0)}}{s}\left(\frac{s}{M^2}\right)^{2\alpha_p(t)}\Gamma_{ppv}(t,t,0), \tag{3.2}$$

where Γ_{ppv} is the triple-Pomeranchukon vertex.[8] We will eventually set $\alpha_p=\alpha_v$.[9] In terms of our variables x_c and $\vec{p}_{cT}$, t is given by Eq. (2.5), and s/M^2 by $1/(1-x_c)$. Since M^2 must be asymptotic, say $M^2>ks^\gamma$ with $0<\gamma<1$, and $1/1-x_c$ large, we must have

$$1-\epsilon<x_c<1-k/s^{1-\gamma}, \tag{3.3}$$

where we take ϵ to be very small (but independent of s). The requirement on ϵ is that secondary trajectories make a negligible contribution to Eq. (3.2); that is,

$$(1/\epsilon)^{\alpha_p(t)-\alpha_s(t)} \gg 1,$$

where $\alpha_s(t)$ is the next trajectory that couples to the $\bar{a}c$ system.

We now rewrite Eq. (2.4):

$$2>\int d\vec{p}_{cT}dx_c g(t)\left(\frac{1}{1-x_c}\right)^{2\alpha_p(t)-\alpha_v(0)}, \tag{3.4}$$

where

$$g(t)=\frac{\Gamma_{ppv}(t,t,0)\gamma_{ap}{}^2(t)}{\gamma_{av}(0)} \tag{3.5}$$

with $g(0)$ assumed different from zero.

(a) Suppose first that $\alpha_p(0)=1$. The integral given by Eq. (3.4) will diverge as $s\to\infty$, unless $\alpha_v(0)>1$. Thus $\alpha_p(0)=1$ implies $\alpha_v(0)>1$; since this result is both inconsistent and in violation of the Froissart bound, it implies $g(0)=0$, in analogy to the famous result of Finkelstein and Kajantie,[1] with the difference that the present argument shows the vanishing of the triple-Pomeranchukon vertex, rather than that of the Pomeranchukon-Pomeranchukon-particle vertex. In particular, the assertion $\alpha_p(0)=\alpha_v(0)=1$ leads to an inconsistency unless $g(0)=0$. This argument duplicates the one given by DeTar *et al.*[4] and essentially duplicates that of Abarbanel *et al.*[3]

(b) A more natural starting point is to impose the condition $\alpha_v(0)=\alpha_p(0)$ from the outset. Here, $\alpha_p(0)>1$ is immediately inconsistent, unless $g(t)$ vanishes identically, since the upper limit of the integral now gives the right-hand side of Eq. (3.4) a positive power dependence on s.

(c) A consistent starting point, within our pole-dominant framework, is $\alpha_p(0)<1$. In this case, the integral (3.4) converges as $s\to\infty$, and we obtain the inequality

$$2>\int d\vec{p}_{cT}\int_{1-\epsilon}^{1}dx_c g(t)\left(\frac{1}{1-x_c}\right)^{2\alpha_p(t)-\alpha_p(0)} \tag{3.6}$$

or, roughly, if we assume dominance of $x\sim1$ in (3.6),

$$2>\frac{\pi g(0)}{2\alpha_p'(0)}\left[-\mathrm{Ei}\left(-[1-\alpha_p(0)]\ln\frac{1}{\epsilon}\right)\right], \tag{3.7}$$

where

$$-\mathrm{Ei}(-z)=\int_z^\infty\frac{du\,e^{-u}}{u} \approx \ln\frac{1}{z}\quad\text{for } z\ll1. \tag{3.8}$$

Presumably, the bound (3.7) can be somewhat improved for $\alpha_p(0)$ not too close to one by relaxing the assumption of $x\sim1$ made above.

IV. POMERANCHUKON-PARTICLE-REGGE VERTICES

We study next some implications of the more complicated inequality Eq. (2.7). We consider the same triple-Regge limit for $d\sigma_{ab}/dp_c$ as we have been discussing earlier, to wit

$$\frac{d\sigma_{ab}}{dp_c}\sim\gamma_{bp}(0)\gamma_{ap}{}^2(t)\Gamma_{ppp}(t,t,0)\times\left(\frac{1}{1-x_c}\right)^{2\alpha_p(t)-\alpha_p(0)}. \tag{4.1}$$

In this limit, the two-particle inclusive cross section which contributes to the integral Eq. (2.7) can be calculated from the discontinuity of the

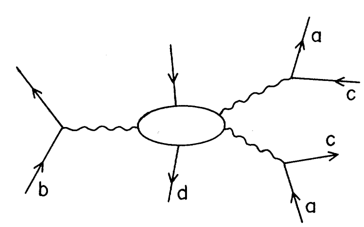

FIG. 1. Multi-Regge representation of the eight-point function in the triple-Regge region.

eight-point amplitude, whose multi-Regge representation (corresponding to the diagram in Fig. 1) is

$$\frac{d\sigma_{ab}}{dp_c dp_d} = \frac{\gamma_{bp}(0)\gamma_{ap}{}^2(t)(\bar{M}^2)^{\alpha_p(0)}}{s}\left(\frac{s}{M^2}\right)^{2\alpha_p(t)} \times B\left(t, \bar{t}, \frac{M^2}{\bar{M}^2}, \phi\right), \tag{4.2}$$

where, as before,

$$\begin{aligned} M^2 &= (p_a + p_b - p_c)^2, \\ \bar{M}^2 &= (p_a + p_b - p_c - p_d)^2, \\ t &= (p_a - p_c)^2, \end{aligned} \tag{4.3}$$

and

$$\bar{t} = (p_a - p_c - p_d)^2 .$$

Here B represents the (triple-Pomeranchukon)-particle-particle coupling, and ϕ is a Toller-like angle defined by a series of Lorentz transformations taking us along the following chain of Lorentz frames. We start with (see Fig. 2):

(i) Frame F_b; this frame is one in which p_b is at rest $p_b = (m_b, 0, 0, 0)$ and $Q_d = (Q_{dt}, 0, 0, Q_{dz})$.

(ii) The frame F_d is defined as one in which $Q_d = p_a - p_c - p_d$ has the form $Q_d = (0, 0, 0, \sqrt{-\bar{t}})$. To get from F_b to F_d one applies a z boost $B_z(\xi_b)$ to the four-vectors in F_b.

(iii) Frame $\bar{F}_d$ is one in which $Q_d = (0, 0, 0, \sqrt{-\bar{t}})$ and p_d has only nonzero time and z components: $p_d = (E_d, 0, 0, p_{dz})$, where E_d and p_{dz} are determined by the t's and the masses. $\bar{F}_d$ is obtained from F_d by an O(2, 1) transformation which leaves Q_d invariant. Let us parametrize this transformation by

$$R_z(\phi_{d2})B_x(\xi_d)R_z(\phi_{d1}) .$$

(iv) Frame F_{ac} has

$$Q_{ac} = p_a - p_c = (0, 0, 0, \sqrt{-t}),$$
$$p_d = (E_d', 0, 0, p_{dz}') .$$

One gets to F_{ac} from $\bar{F}_d$ by means of a z boost $B_z(\eta)$ where η is determined by the t's and the masses.

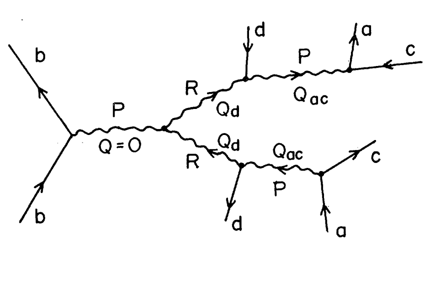

FIG. 2. Diagram illustrating the Toller parametrization of the eight-point function.

(v) Finally, frame $\bar{F}_{ac}$ has

$$Q_{ac} = (0, 0, 0, \sqrt{-t}),$$
$$p_a = (E_a, 0, 0, p_{az}) .$$

Frame $\bar{F}_{ac}$ is obtained from F_{ac} by an O(2, 1) transformation leaving Q_{ac} fixed:

$$R_z(\phi_{acz})B_x(\xi_{ac})R_z(\phi_{ac1}) .$$

The four-vectors in $\bar{F}_d$ can be written

$$\begin{aligned} p_b &= R_z(\phi_{d2})B_x(\xi_d)R_z(\phi_{d1})B_z(\xi_b)(m_b, 0, 0, 0), \\ p_d &= (E_d, 0, 0, p_{dz}), \\ p_a &= B_z(-\eta)R_z(-\phi_{ac1})B_x(-\xi_{ac})R_z(-\phi_{ac2}) \\ &\quad \times (E_a, 0, 0, p_{az}), \\ p_c &= B_z(-\eta)R_z(-\phi_{ac1})B_x(-\xi_{ac})R_z(-\phi_{acz}) \\ &\quad \times (E_a, 0, 0, p_{az} - \sqrt{-t}) . \end{aligned} \tag{4.4}$$

It is clear by transforming the above vectors by $R_z(-\phi_{d2})$ that the invariants are functions only of the group variables ξ_d, ξ_{ac}, ξ_b, and $\phi = \phi_{d2} + \phi_{ac1}$.

The limits $\bar{M}^2$, $M^2/\bar{M}^2$, and s/M^2 large correspond, respectively, to the boosts ξ_b, ξ_d, and ξ_{ac} large. We have so far only considered ξ_{ac} and ξ_b large, maintaining ξ_d fixed.

We now insert the formulas (4.2) for $d\sigma_{ab}/dp_c dp_d$ and (3.2) for $d\sigma_{ab}/dp_c$ into our inequality, Eq. (2.7). We find, after some obvious cancellations,

$$\Gamma_{ppp}(t, t, 0) \geq \tfrac{1}{2}\int d\vec{p}_{dT}\int_{>0}^{1-x_c} \frac{dx_d}{1-x_c}\left(\frac{\bar{M}^2}{M^2}\right)^{\alpha_p(0)} \times B\left(t, \bar{t}, \frac{M^2}{\bar{M}^2}, \phi\right), \tag{4.5}$$

where, we remind the reader, $1 - x_c \ll 1$.

In order to exploit the previously proved vanishing of $\Gamma_{ppp}(0, 0, 0)$, we change variables to $y = x_d/(1 - x_c)$, and note that

$$\bar{M}^2 = s(1 - x_c - x_d) + \text{finite terms},$$
$$M^2 = s(1 - x_c) + \text{ finite terms},$$

so

$$\bar{M}^2/M^2 \cong 1 - y,$$

where y now runs from any positive lower limit to 1. We note that y is the Feynman variable for the process $p_b + Q_{ac} \to p_d$ + anything. In terms of the new variable y, Eq. (4.5) becomes

$$\Gamma_{ppp}(t, 0) \geq \tfrac{1}{2}\int d\vec{p}_{dT}\int_{0+}^{1-} dy(1-y)^{\alpha_p(0)} \times B\left(t, \bar{t}, \frac{1}{1-y}, \phi\right). \tag{4.6}$$

We consider first the case $\alpha_p(0) = 1$, and let $t \to 0$. We then must have $\Gamma_{ppp}(0, 0, 0) = 0$, and, since our integration is over a finite range of y and $d\vec{p}_{d\perp}$, the positive quantity (cross section)

$$B\left(0, \bar{t}, \frac{1}{1-y}, \phi\right) = 0. \tag{4.7}$$

We remark that, as $t \to 0$, the kinematics of the forward direction dictates that $B(t, \bar{t}, 1/(1-y), \phi)$ become independent of ϕ. In particular, in the limit $y \to 1$, we get the double-Regge limit of the three-Reggeon–two-particle vertex of Fig. 1, as shown in Fig. 2. The corresponding formula replacing (4.7) is

$$\left(\frac{1}{1-y}\right)^{2\alpha_R(\bar{t})} |g_{PRd}(0, \bar{t}, \phi)|^2 \Gamma_{RRP}(\bar{t}, \bar{t}, 0), \tag{4.8}$$

where $\alpha_R(\bar{t})$ is, in the first instance, the leading trajectory with appropriate quantum numbers to couple to particle d and $g_{PRd}(0, \bar{t}, \phi)$ is independent of ϕ. Of course, since our inequality holds exactly in y and not only asymptotically, and since d is an arbitrary particle, all couplings g_{PRd} must vanish. An alternative conclusion is that $\Gamma_{RRP}(\bar{t}, \bar{t}, 0)$ vanishes. The strongest alternative is that $\gamma_{bp}(0) = 0$ for all b, thus completely decoupling the vacuum trajectory from total cross sections.

We make two observations:

(i) One might be tempted to infer the complete decoupling of the vacuum trajectory at $t = 0$ from Eq. (4.8); that is, the vanishing of $\gamma_{pd}(0)$, obtained by continuing g or Γ to the pole at $\bar{t} = M_d^2$. Model calculations seem to show that the special nature of the point $t = 0$ makes this continuation a surprisingly subtle matter. This point is currently under study by Brower and Weis.[10]

(ii) The result of Finkelstein and Kajantie already cited[1] shows that the vertex shown in Fig. 3 must vanish for fixed internal variables. Clearly, our result corresponds to a statement about the asymptotic limit of the diagram, or $\bar{M}_{d\bar{d}} \to \infty$. One may also ask, with respect to this latter diagram whether continuation to the particle pole shown in Fig. 4 does not imply decoupling of the vacuum trajectory. Here, again, model calculations[10] show that considerable caution is needed.

Evidently the vanishing of $B(0, \bar{t}, 1/(1-y), \phi)$ is a very strong constraint; our result has only skimmed the surface, since *all* contributions from

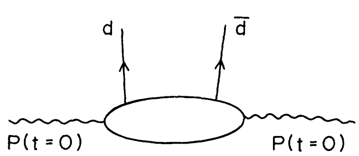

FIG. 3. Double-P coupling to a system with vacuum quantum numbers.

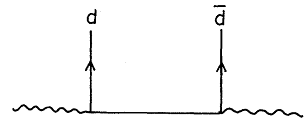

FIG. 4. Particle-pole contribution to double-P coupling to a system with vacuum quantum numbers.

identifiable regions must vanish. For example, by considering the limit of B in the pionization region of particle d (i.e., y near zero and $\bar{t}$ large), one finds the product

$$\Gamma_{ppR}(0, 0, 0)\Gamma_{pRdd}(0, 0) = 0$$

for all trajectories R. Here $\Gamma_{pRdd}(0, 0)$ is the Mueller-like vertex shown in Fig. 5.

We consider next the case $\alpha_p(0) \neq 1$, and derive an inequality for the ppd double-Regge coupling constant, g_{ppd}. We return to Eq. (4.6) in the limit $t = 0$:

$$\Gamma_{ppp}(0) \geq \tfrac{1}{2} \int d\vec{p}_{dT} \int_{0+}^{1-} dy (1-y)^{\alpha_p(0)} \times B\left(0, \bar{t}, \frac{1}{1-y}, \phi\right). \tag{4.9}$$

Now, near $y = 1$, we have for d a particle with vacuum quantum numbers

$$B \sim \left(\frac{1}{1-y}\right)^{2\alpha_p(\bar{t})} |g_{ppd}(0, \bar{t}, \phi)|^2 \Gamma_{ppp}(\bar{t}, \bar{t}, 0). \tag{4.10}$$

Near $\bar{t} = 0$, we may cancel the triple vertex from the two sides of Eq. (4.9), and find

$$1 \geq \frac{|g|^2}{2} \int d\vec{p}_{dT} \int^{1-} dy (1-y)^{\alpha_p(0) - 2\alpha_p(-p_{dT}^2)}, \tag{4.11}$$

where

$$g = g_{ppd}(0, 0).$$

Proceeding as in Sec. III, we have

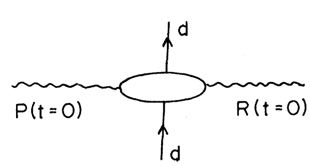

FIG. 5. Nonleading Mueller pionization vertex.

$$|g|^2 \leq \frac{\text{const}}{\ln\{[1-\alpha_p(0)]\ln(1/\epsilon)\}^{-1}} \tag{4.12}$$

in very close analogy to the result of Finkelstein and Kajantie.[1]

ACKNOWLEDGMENTS

It is a pleasure to thank Dr. R. C. Brower and Dr. J. Weis for many interesting and useful conversations during the course of this work.

*This work is supported in part through funds provided by the Atomic Energy Commission under Contract No. AT(11-1)-3069.

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[9]The reader may wonder why we have distinguished between α_p and α_v; this is for the sole purpose of comparing to the work of Finkelstein and Kajantie (Ref. 1) where "input" trajectories and "output" power are compared.

[10]R. C. Brower and J. Weis (private communication).

PHYSICAL REVIEW D VOLUME 6, NUMBER 4 15 AUGUST 1972

Calculations of the Pomeranchuk Residue at $t=0$ Using Intermediate-Energy Bootstrap Dynamics Based on Average Nonlinear Duality*

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(Received 20 March 1972)

By imposing nonlinear forms of average duality at intermediate energies it is possible to obtain simple inhomogeneous relations between Regge residues. This reopens the possibility of using duality to make genuine bootstrap calculations. We consider two different forms of nonlinear duality, both of which are applied to $\pi\pi$ scattering, in combination with the usual linear duality. The first (Type A) asserts that, if σ^c is the cross section for a particular reaction c, $\sigma^c_{\text{resonance}}=\sigma^c_{\text{Regge}}$, when averaged appropriately over one or more resonances. Applying it at the g resonance, we obtain a total g width of 123 MeV, in agreement with experiment. The second (Type B) uses the optical theorem and states that $A_{\text{Regge}}=\sum_c K\sigma^c_{\text{Regge}}$ on the average, where A is the forward $\pi\pi$ absorptive part and K is a kinematic factor. The sum is over all reactions c making up the total cross section, which we take to be $\pi\pi\to\pi\pi$, $\pi\pi\to\rho\rho$, $\pi\pi\to\rho\epsilon$, and $\pi\pi\to\epsilon\epsilon$ below the 3ρ threshold. The last three are treated in a model-independent way, assuming only semilocal linear duality and the dominance of $I=1$ exchanges, such as the π and A_2. Calculations are then made in which Pomeranchuk exchange is included in A_{Regge} and different energy intervals are selected for the averaging procedure. For example, a semilocal calculation around the g resonance gives a ρ Regge residue corresponding to a ρ-meson width of 133 MeV, and a Pomeranchuk residue corresponding to an asymptotic $\sigma_{\text{tot}}=13.5$ mb; the only input parameters are the resonance masses, which can be fixed by using the partial conservation of axial-vector current.

I. INTRODUCTION

There has recently been a certain revival of interest in the use of duality for making bootstrap calculations.[1-4] At first sight this may not appear to be very promising. For example, in the familiar linear Dolen-Horn-Schmid average absorptive-part duality condition[5]

$$\int A_{\text{resonance}} = \int A_{\text{Regge}}\,, \tag{1}$$

both sides of the equation are proportional to a Regge residue function (excluding the Pomeranchukon). The over-all scale of such functions cannot therefore be determined from such conditions alone, and so we can have at best only a partial bootstrap. This is explicitly evident in the dual Veneziano model,[6] for which the normalization is completely arbitrary. Fortunately, this objection does not apply to nonlinear forms of duality, which lead to inhomogeneous conditions on residues and reopen the