General structure of the graviton self-energy

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(Received 27 January 1999; published 11 May 1999)

The graviton self-energy at finite temperature depends on 14 structure functions. We show that, in the absence of tadpoles, the gauge invariance of the effective action imposes three non-linear relations among these functions. The consequences of such constraints, which must be satisfied by the thermal graviton self-energy to all orders, are explicitly verified in general linear gauges to one loop order.
[S0556-2821(99)06112-3]

PACS number(s): 11.15.−q

The non-linear relation imposed by gauge invariance on the thermal self-energy of gluons has been recently discussed by Weldon in an interesting paper [1]. He proved that in QCD, the Slavnov-Taylor identities [2,3] require a non-linear constraint among the structure functions which occur at finite temperature. In this Brief Report, we show that a similar behavior occurs in the gauge theory of gravity. In this case, local gauge invariance leads to three non-linear relations which restrict the form of the thermal self-energy of gravitons.

The Einstein theory of gravity is described by the Lagrangian density [4–6]

$$\mathcal{L}=\frac{2}{\kappa^2}\sqrt{-g}R, \tag{1}$$

where $\kappa^2=32\pi G$, G is the Newton constant and R is the Ricci scalar. The graviton field $h_{\mu\nu}$ can be defined in terms of the metric tensor as

$$g_{\mu\nu}=\eta_{\mu\nu}+\kappa h_{\mu\nu}. \tag{2}$$

Using this parametrization, the Einstein action will be invariant under the gauge transformation [7]

$$\begin{aligned}\delta h_{\mu\nu}&=[\delta^{\lambda}_{\mu}\partial_\nu+\delta^{\lambda}_{\nu}\partial_\mu+\kappa(h^{\lambda}_{\mu}\partial_\nu+h^{\lambda}_{\nu}\partial_\mu+\partial^\lambda h_{\mu\nu})]\xi_\lambda\\ &\equiv G^{(0)\lambda}_{\mu\nu}\xi_\lambda,\end{aligned} \tag{3}$$

where ξ_λ is an infinitesimal gauge parameter.

In this gauge theory, the corresponding identities which occur at finite temperature differ from those at $T=0$ because of the appearance of one-particle graviton functions (tadpoles). Their thermal contribution is purely leading, being proportional to $T^{2(n+1)}$ at n-loop order. Hence, we may assume that the tadpoles are important, in the Ward identities, only for the leading thermal contributions to the graviton self-energy. [To one-loop order, for example, the tadpoles can be neglected for the purpose of studying the sub-leading T^2, $\log(T)$ and $T=0$ contributions.]

At finite temperature, the graviton self-energy may depend on the four-velocity u_α of the plasma, so that it can be a linear combination of the 14 independent tensors given in Table I. This contains three traceless tensors $T^A_{\alpha\beta,\mu\nu}$, $T^B_{\alpha\beta,\mu\nu}$ and $T^C_{\alpha\beta,\mu\nu}$, which are transverse with respect to the wave 4-vector k_μ. They are also, respectively, completely transverse, partially transverse and longitudinal with respect to the spatial component $\vec{k}$. The explicit form of these tensors is given by Eq. (5.16) of Ref. [8] [see also Eq. (4.7) of Ref. [11]]. They depend individually on the plasma four-velocity, but their sum is a Lorentz covariant tensor which is independent of u_α:

$$(T^A+T^B+T^C)_{\alpha\beta,\mu\nu}=\frac{1}{2}(P_{\alpha\mu}P_{\beta\nu}+P_{\alpha\nu}P_{\beta\mu})-\frac{1}{3}P_{\alpha\beta}P_{\mu\nu}, \tag{4}$$

where $P_{\alpha\beta}=\eta_{\alpha\beta}-k_\alpha k_\beta/k^2$.

In terms of this basis, which is convenient for our purpose, the graviton self-energy can be parametrized as

$$\begin{aligned}\Pi_{\alpha\beta,\mu\nu}&=\Pi_A T^A_{\alpha\beta,\mu\nu}+\Pi_B T^B_{\alpha\beta,\mu\nu}+\Pi_C T^C_{\alpha\beta,\mu\nu}\\ &\quad+\sum_{i=4}^{14}\Pi_i T^i_{\alpha\beta,\mu\nu}.\end{aligned} \tag{5}$$

In the hard thermal loop approximation, which represents a consistent high-temperature expansion [9,10], the one loop graviton self-energy has leading contributions proportional to T^4, and the corresponding functions $\Pi^{(l)}_j$ are gauge invari-

TABLE I. A basis of 14 independent tensors.

$T^{1,2,3}_{\alpha\beta,\mu\nu}=T^{A,B,C}_{\alpha\beta,\mu\nu}$
$T^{4}_{\alpha\beta,\mu\nu}=\eta_{\alpha\beta}\eta_{\mu\nu}$
$T^{5}_{\alpha\beta,\mu\nu}=u_\mu u_\nu\eta_{\alpha\beta}+u_\alpha u_\beta\eta_{\mu\nu}$
$T^{6}_{\alpha\beta,\mu\nu}=[u_\beta(k_\nu\eta_{\alpha\mu}+k_\mu\eta_{\alpha\nu})+k_\beta(u_\nu\eta_{\alpha\mu}+u_\mu\eta_{\alpha\nu})+u_\alpha(k_\nu\eta_{\beta\mu}+k_\mu\eta_{\beta\nu})+k_\alpha(u_\nu\eta_{\beta\mu}+u_\mu\eta_{\beta\nu})]/k\cdot u$
$T^{7}_{\alpha\beta,\mu\nu}=[k_\nu u_\alpha u_\beta u_\mu+k_\mu u_\alpha u_\beta u_\nu+k_\beta u_\alpha u_\mu u_\nu+k_\alpha u_\beta u_\mu u_\nu]/k\cdot u$
$T^{8}_{\alpha\beta,\mu\nu}=[k_\beta k_\nu\eta_{\alpha\mu}+k_\beta k_\mu\eta_{\alpha\nu}+k_\alpha k_\nu\eta_{\beta\mu}+k_\alpha k_\mu\eta_{\beta\nu}]/k^2$
$T^{9}_{\alpha\beta,\mu\nu}=[k_\mu k_\nu u_\alpha u_\beta+k_\alpha k_\beta u_\mu u_\nu]/k^2$
$T^{10}_{\alpha\beta,\mu\nu}=(k_\beta u_\alpha+k_\alpha u_\beta)(k_\nu u_\mu+k_\mu u_\nu)/(k\cdot u)^2$
$T^{11}_{\alpha\beta,\mu\nu}=[k_\beta k_\mu k_\nu u_\alpha+k_\alpha k_\mu k_\nu u_\beta+k_\alpha k_\beta k_\nu u_\mu+k_\alpha k_\beta k_\mu u_\nu]/(k^2 k\cdot u)$
$T^{12}_{\alpha\beta,\mu\nu}=(k_\alpha k_\beta k_\mu k_\nu)/k^4$
$T^{13}_{\alpha\beta,\mu\nu}=(k_\mu k_\nu\eta_{\alpha\beta}+k_\alpha k_\beta\eta_{\mu\nu})/k^2$
$T^{14}_{\alpha\beta,\mu\nu}=[(k_\nu u_\mu+k_\mu u_\nu)\eta_{\alpha\beta}+(k_\beta u_\alpha+k_\alpha u_\beta)\eta_{\mu\nu}]/(k\cdot u)$

ant [8]. However, when one goes beyond this approximation, by including contributions which are sub-leading in powers of T, this feature no longer occurs and the functions $\Pi_j^{(s)}$ become gauge dependent [11].

In order to investigate the structure of the exact graviton self-energy which includes also higher loops effects, we will make use of the Becchi-Rouet-Stora identities [12] which reflect the underlying gauge invariance of the theory [13,14]. A discussion of the consequences of these identities on the structure of the thermal self-energy is given in Appendix A. To explain these, we will denote by $\check{\Gamma}_{\alpha\beta,\mu\nu}$ the quadratic part of the graviton effective action, which is the sum of the free kinetic energy, without the gauge fixing term, and the one-particle-irreducible graviton self-energy:

$$\check{\Gamma}_{\alpha\beta,\mu\nu}=K^{(0)}_{\alpha\beta,\mu\nu}+\Pi_{\alpha\beta,\mu\nu}. \tag{6}$$

The free-graviton kinetic energy is given, in momentum space, by

$$\begin{aligned}K^{(0)}_{\alpha\beta,\mu\nu}(k)=&k^2(\eta_{\alpha\mu}\eta_{\beta\nu}+\eta_{\alpha\nu}\eta_{\beta\mu}-\eta_{\alpha\beta}\eta_{\mu\nu})\\&+k_\mu k_\nu\eta_{\alpha\beta}+k_\alpha k_\beta\eta_{\mu\nu}-(k_\alpha k_\mu\eta_{\beta\nu}+k_\alpha k_\nu\eta_{\beta\mu}\\&+k_\beta k_\mu\eta_{\alpha\nu}+k_\beta k_\nu\eta_{\alpha\mu}).\end{aligned} \tag{7}$$

The contributions to $\Pi_{\alpha\beta,\mu\nu}$ are calculated according to the usual Feynman rules, using a gauge fixing term which is quadratic in the graviton field. (Examples of such gauge-fixing terms are provided by the covariant [11] and axial gauges [13].) Hence, the gauge dependence of $\check{\Gamma}_{\alpha\beta,\mu\nu}$ comes only from the self-energy functions. In consequence of the Becchi-Rouet-Stora (BRS) identities, it turns out that the leading contributions to the longitudinal part of $\check{\Gamma}_{\alpha\beta,\mu\nu}$ are proportional to the tadpole terms [see Eq. (A5)]. Furthermore, in general linear gauges, the sub-leading contributions to $\check{\Gamma}_{\alpha\beta,\mu\nu}$ satisfy the following four constraints [see Eq. (A6)]:

$$\check{\Gamma}^{(s)\mu\nu}_{\alpha\beta}(k)G^{\lambda}_{\mu\nu}(k)|_{h=0}=0 \quad \text{for} \quad \lambda=0,1,2,3, \tag{8}$$

where the tensor $G^{\lambda}_{\mu\nu}$ is given, to lowest order, by Eq. (3).

Since $\check{\Gamma}^{(s)\mu\nu}_{\alpha\beta}$ is symmetric under permutations of indices $\alpha\leftrightarrow\beta$ and $\mu\leftrightarrow\nu$, it can viewed as a 10×10 matrix which must have 10 eigenvalues. Six of these are determined dynamically by the equations of motion of the gravitational field. The other four eigenvalues must be zero in consequence of the gauge invariance as expressed by Eq. (8). In order to obtain zero eigenvalues, it is necessary that the determinant of $\check{\Gamma}^{(s)\mu\nu}_{\alpha\beta}$ should vanish. These requirements lead to three distinct non-linear constraints among the exact functions $\Pi_j^{(s)}$, which determine the structure of the graviton self-energy at finite temperature. The first of these non-linear relations, which are derived in Appendix B, can be written as follows:

$$2\Pi_6^{(s)}+\Pi_8^{(s)}=2\left(\frac{k^2}{(k\cdot u)^2}-1\right)\frac{(\Pi_6^{(s)})^2}{k^2+\Pi_B^{(s)}}. \tag{9}$$

The other general non-linear relations are given in Eqs. (B16) and (B17), which involve quite lengthy expressions. We shall present here, for simplicity, only the corresponding results obtained in covariant gauges, at zero temperature. In this case, the general non-linear relations simplify because in Eq. (5), $\Pi_i=0$ for $i\neq 4,8,12,13$. Furthermore, we have now $\Pi_A=\Pi_B=\Pi_C$, since at zero temperature the only traceless-transverse tensor available is given by the sum $T_A+T_B+T_C$ in Eq. (4). Then, the condition (B16) implies that $\Pi_8=0$, and the non-linear constraint (B17) reduces to the following relation:

$$\Pi_4+\Pi_{12}+2\Pi_{13}=\frac{3}{2k^2}[\Pi_4\Pi_{12}-(\Pi_{13})^2]. \tag{10}$$

The above non-linear relations are rather interesting, since the structure functions Π_i are all gauge-dependent. In perturbation theory, these functions begin at least to order κ^2. Consequently, the linear combinations on the left-hand sides of Eqs. (9) and (10) must begin at order κ^4. We have verified these results to one loop order in general axial and covariant gauges, respectively, where the above linear combinations are found to vanish.

We are grateful to CNPq and Fapesp, Brazil, for a grant. J.F. would like to thank Professor J.C. Taylor for a helpful correspondence.

APPENDIX A: BRS IDENTITY

In order to derive the BRS identity for the graviton self-energy at finite temperature, we start from the effective action

$$\begin{aligned}\check{\Gamma}=&h_{\mu\nu}(0)\Gamma^{\mu\nu}+\int d^4x d^4y J^{\mu\nu}(x)G^{\lambda}_{\mu\nu}(x-y)C_\lambda(y)\\&+\frac{1}{2}\int d^4x d^4y h_{\alpha\beta}(x)\check{\Gamma}^{\alpha\beta,\mu\nu}(x-y)h_{\mu\nu}(y),\end{aligned} \tag{A1}$$

where $\Gamma^{\mu\nu}$ denotes the tadpole. $J^{\mu\nu}(x)$ represents the source term for the BRS transformation of the graviton field and $C_\lambda(y)$ is the vector ghost field. The tensor $G^{\lambda}_{\mu\nu}(x-y)$, which is given to lowest order by Eq. (3), can be expressed diagrammatically in a loop expansion, as shown in Refs. [11,13,14].

The relevant BRS invariance for the thermal graviton self-energy can now be written, in general linear gauges, as

$$\int d^4x\frac{\delta\check{\Gamma}}{\delta h_{\mu\nu}(x)}\frac{\delta\check{\Gamma}}{\delta J^{\mu\nu}(x)}=0. \tag{A2}$$

Differentiating Eq. (A2) with respect to $C_\lambda(y)$ and setting all fields and sources to zero, yields in momentum space the following condition for the tadpole:

$$\Gamma^{\mu\nu}G^{\lambda}_{\mu\nu}(k=0)|_{h=0}=0. \tag{A3}$$

On the other hand, differentiating Eq. (A2) with respect to $C_\lambda(y)$ and $h_{\alpha\beta}(z)$, and setting the source to zero, leads at $h=0$ to the identity

$$\check{\Gamma}^{\alpha\beta,\mu\nu}(k)G^{\lambda}_{\mu\nu}(k)+\Gamma^{\mu\nu}G^{\lambda,\alpha\beta}_{\mu\nu}(k)=0, \tag{A4}$$

where $G^{\lambda,\alpha\beta}_{\mu\nu}$ denotes the derivative of $G^{\lambda}_{\mu\nu}$ with respect to $h_{\alpha\beta}$. If we consider the tadpoles only for the purpose of calculating the leading contributions to the self-energy, Eq. (A4) can be written at $h=0$ as follows:

$$[\check{\Gamma}^{\alpha\beta,\mu\nu}(k)G^{\lambda}_{\mu\nu}(k)]^{(l)}=-\Gamma^{\mu\nu}G^{\lambda,\alpha\beta}_{\mu\nu}(k), \tag{A5}$$

$$[\check{\Gamma}^{\alpha\beta,\mu\nu}(k)G^{\lambda}_{\mu\nu}(k)]^{(s)}=0. \tag{A6}$$

Equation (A6) is equivalent to Eq. (8), since $G^{\lambda}_{\mu\nu}(k)$ does not contain leading thermal contributions. The above relations have been verified explicitly in [11], to one loop order. In this case, for example, Eq. (A6) becomes (at $h=0$)

$$\Pi^{(s)\alpha\beta,\mu\nu}(k)G^{(0)\lambda}_{\mu\nu}(k)+K^{(0)\alpha\beta,\mu\nu}(k)G^{(1)\lambda}_{\mu\nu}(k)=0. \tag{A7}$$

An important consequence of Eq. (A7), which follows from the fact that $K^{(0)}_{\alpha\beta,\mu\nu}(k)$ in Eq. (7) is transverse with respect to k_α, is the identity

$$k_\mu k_\alpha \Pi^{(s)\alpha\beta,\mu\nu}(k)=0. \tag{A8}$$

This is analogous to the identity $k_\mu k_\nu \Pi^{\mu\nu}(k)=0$, which holds in QCD for the exact gluon self-energy.

APPENDIX B: DERIVATION OF THE NON-LINEAR CONSTRAINTS

In this appendix, we will derive the non-linear relations involving the elements of $\check{\Gamma}^{(s)\mu\nu}_{\alpha\beta}$, which result in consequence of the vanishing of its determinant. [In what follows, for simplicity of notation, we shall drop the superscripts (s).] To this end, it is convenient to introduce a set of 10 polarization tensors $\epsilon^{l}_{\mu\nu}$ $(l=1,\ldots,10)$, which constitute a basis for the eigentensors of $\check{\Gamma}^{\mu\nu}_{\alpha\beta}$. Two of these tensors describe the physical graviton field, which has spin 2 and helicities ± 2. In terms of the transverse vectors $e^{1,2}_{\mu}$, which satisfy $k^\mu e^{1,2}_{\mu}=u^\mu e^{1,2}_{\mu}=0$, we can define the physical polarization tensors $\epsilon^{1,2}_{\mu\nu}$ as

$$\epsilon^{1,2}_{\mu\nu}=\frac{1}{2}[(e^1_\mu e^1_\nu-e^2_\mu e^2_\nu)\pm(e^1_\mu e^2_\nu+e^1_\nu e^2_\mu)], \tag{B1}$$

which are traceless and transverse with respect to k_μ and u_μ. These tensors are actually eigentensors of $\check{\Gamma}^{\alpha\beta}_{\mu\nu}$ with the same eigenvalue:

$$\check{\Gamma}^{\mu\nu}_{\alpha\beta}\epsilon^{1,2}_{\mu\nu}=(k^2+\Pi_A)\epsilon^{1,2}_{\alpha\beta} \tag{B2}$$

which reflects the fact that the purely spatially transverse mode A describes a physical gravitational wave propagating in the plasma.

Other two (unphysical) polarization tensors, defined as

$$\epsilon^{3,4}_{\mu\nu}=k_\mu e^{1,2}_\nu+k_\nu e^{1,2}_\mu-\frac{k^2}{k\cdot u}(u_\mu e^{1,2}_\nu+u_\nu e^{1,2}_\mu), \tag{B3}$$

are eigentensors of $K^{(0)}_{\alpha\beta,\mu\nu}$ with the same eigenvalue k^2. The following two unphysical polarization tensors can be written as

$$\epsilon^{5,6}_{\mu\nu}=k_\mu e^{1,2}_\nu+k_\nu e^{1,2}_\mu. \tag{B4}$$

These are eigentensors of $K^{(0)}_{\alpha\beta,\mu\nu}$ with zero eigenvalues. In a coordinate system where $\vec{k}$ is directed along the z-axis, they correspond to the elements of the tensor $G^{(0)1,2}_{\mu\nu}(h=0)$ defined in Eq. (3).

The next two unphysical polarization tensors can be represented as

$$\epsilon^{7}_{\mu\nu}=\left(\frac{k_\mu}{k\cdot u}-u_\mu\right)k_\nu+\left(\frac{k_\nu}{k\cdot u}-u_\nu\right)k_\mu, \tag{B5}$$

$$\epsilon^{8}_{\mu\nu}=u_\mu k_\nu+u_\nu k_\mu. \tag{B6}$$

These are also eigentensors of $K^{(0)}_{\alpha\beta,\mu\nu}$ with zero eigenvalues and correspond, in the above coordinate system, to the elements of the tensor $G^{(0)3,4}_{\mu\nu}(h=0)$. The remaining two polarization tensors, which are given by

$$\epsilon^{9}_{\mu\nu}=\eta_{\mu\nu}-\frac{k_\mu k_\nu}{k^2} \tag{B7}$$

and

$$\epsilon^{10}_{\mu\nu}=\epsilon^{9}_{\mu\nu}+\frac{3k\cdot u}{2(k\cdot u)^2+k^2}\times\left(k_\mu u_\nu+k_\nu u_\mu-\frac{k^2}{k\cdot u}u_\mu u_\nu-k\cdot u\,\eta_{\mu\nu}\right), \tag{B8}$$

are eigentensors of $K^{(0)}_{\alpha\beta,\mu\nu}$, with eigenvalues $-2k^2$ and k^2, respectively.

Using Eqs. (B3) and (B4), and the properties of the tensors $T^{i}_{\alpha\beta,\mu\nu}$, we get the system

$$\check{\Gamma}^{\mu\nu}_{\alpha\beta}\epsilon^{3,4}_{\mu\nu}=(k^2+\Pi_B)\epsilon^{3,4}_{\mu\nu}+2\left(1-\frac{k^2}{(k\cdot u)^2}\right)\Pi_6\epsilon^{5,6}_{\mu\nu}, \tag{B9}$$

$$\check{\Gamma}^{\mu\nu}_{\alpha\beta}\epsilon^{5,6}_{\mu\nu}=-2\Pi_6\epsilon^{3,4}_{\mu\nu}+(4\Pi_6+2\Pi_8)\epsilon^{5,6}_{\mu\nu}. \tag{B10}$$

In order to obtain non-trivial eigentensors of $\check{\Gamma}^{\mu\nu}_{\alpha\beta}$ from the above set of equations, we must impose the condition that the determinant involving the corresponding eigenvalues must vanish. This leads to a quadratic equation which determines a set of two eigenvalues. [Because of the form of Eqs.

(B9) and (B10), the other two eigenvalues will be degenerate with those in the first set.] The product of the eigenvalues must therefore vanish, yielding in this way the non-linear relation given by Eq. (9). The system (B9) and (B10) determines altogether four eigentensors of $\check{\Gamma}^{\mu\nu}_{\alpha\beta}$, two of which have zero eigenvalues.

Taking into account Eq. (B2), we have thus far a total of six eigentensors of $\check{\Gamma}^{\mu\nu}_{\alpha\beta}$, so that we must still find four more eigentensors. These must be each linear combinations of the tensors $\epsilon^{l}_{\mu\nu}$ $(l=7,\ldots,10)$. The requirement of non-trivial eigentensors leads to a quartic equation for the remaining four eigenvalues, which can be written in the form of a vanishing determinant:

$$\begin{vmatrix} L_{11}-\lambda & L_{12} & L_{13} & L_{14} \\ L_{21} & L_{22}-\lambda & L_{23} & L_{24} \\ L_{31} & L_{32} & L_{33}-2k^2-\lambda & L_{34} \\ L_{41} & L_{42} & L_{43} & \Pi_C+k^2-\lambda \end{vmatrix}=0 \tag{B11}$$

where L_{mn} are linear combinations of the structure functions Π_i $(i=4,\ldots,14)$. We have, for example,

$$L_{11}=\frac{1}{r(r+1)}[r(r-1)\Pi_4-2(r-1)\Pi_5+8r\Pi_6+4r^2\Pi_8 -2(r-1)\Pi_9+r(r-1)\Pi_{12}+2r(r-1)\Pi_{13}], \tag{B12}$$

$$L_{12}=\frac{2(r-1)}{(r+1)^2}[r\Pi_4+(r-1)\Pi_5+2r(r-1)\Pi_6-(r+1)\Pi_7 +2r\Pi_8+(r-1)\Pi_9+r(r+1)\Pi_{11}+r\Pi_{12}+2r\Pi_{13} +r(r+1)\Pi_{14}], \tag{B13}$$

$$L_{21}=\frac{2}{r+1}[\Pi_4+2\Pi_5+2(1+3r)\Pi_6+2(r+1)\Pi_7 +(r+3)\Pi_8+2\Pi_9+(r+1)^2\Pi_{10}+2(r+1)\Pi_{11} +\Pi_{12}+2\Pi_{13}+2(r+1)\Pi_{14}], \tag{B14}$$

$$L_{22}=\frac{1}{r^2}[r\Pi_4+(r-1)\Pi_5+2r(r-1)\Pi_6-(r+1)\Pi_7+2r\Pi_8 +(r-1)\Pi_9+r(r+1)\Pi_{11}+r\Pi_{12}+2r\Pi_{13} +r(r+1)\Pi_{14}], \tag{B15}$$

where $r\equiv k^2/(k\cdot u)^2$.

As a consequence of the BRS condition given by Eq. (8), we must have a total of four eigentensors with zero eigenvalues. Since Eq. (B9) and (B10) have already determined two such eigentensors, it follows that Eq. (B11) must have two vanishing roots. This yields two more non-linear relations among the structure functions, which can be written symbolically in the form

$$k^4(L_{11}+L_{22})=k^2L\otimes L+L\otimes L\otimes L \tag{B16}$$

and

$$k^4(L_{11}L_{22}-L_{12}L_{21})=k^2L\otimes L\otimes L+L\otimes L\otimes L\otimes L, \tag{B17}$$

where L denotes some matrix element of Eq. (B11).

[1] H. A. Weldon, Ann. Phys. (N.Y.) **271**, 141 (1999).
[2] A. A. Slavnov, Theor. Math. Phys. **10**, 99 (1972).
[3] J. C. Taylor, Nucl. Phys. **B33**, 436 (1971).
[4] D. J. Gross, M. J. Perry, and L. G. Yaffe, Phys. Rev. D **25**, 330 (1982).
[5] Y. Kikuchi, T. Moriya, and H. Tsukahara, Phys. Rev. D **20**, 2220 (1984).
[6] P. S. Gribosky, J. F. Donoghue, and B. R. Holstein, Ann. Phys. (N.Y.) **190**, 149 (1989).
[7] T. Matsuki, Phys. Rev. D **19**, 2879 (1979).
[8] A. Rebhan, Nucl. Phys. **B351**, 706 (1991).
[9] E. Braaten and R. D. Pisarski, Nucl. Phys. **B337**, 569 (1990); **B339**, 310 (1990); Phys. Rev. D **45**, 1827 (1992).
[10] J. Frenkel and J. C. Taylor, Nucl. Phys. **B334**, 199 (1990); **B374**, 156 (1992).
[11] F. T. Brandt and J. Frenkel, Phys. Rev. D **58**, 085012 (1998).
[12] C. Becchi, A. Rouet, and R. Stora, Commun. Math. Phys. **42**, 127 (1975).
[13] D. M. Capper and A. MacLean, J. Phys. A **15**, 3187 (1982).
[14] R. Delbourgo and T. Matsuki, Phys. Rev. D **32**, 2579 (1985).