Conformally coupled induced gravity with gradient torsion

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It is found that conformally coupled induced gravity with gradient torsion gives dilaton gravity in Riemann geometry. In the Einstein frame of dilaton gravity the conformal symmetry is hidden and a nonvanishing cosmological constant is not plausible due to the constraint of the conformal coupling.
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I. INTRODUCTION

Before the success of the Weinberg-Salam model, the weak interaction was characterized by the dimensional Fermi coupling constant $G_F=(300\ \text{GeV})^{-2}$, far below the electroweak scale. But later it turns out that the dimensional coupling constant is the low energy effective coupling which is determined by the dimensionless electroweak coupling constant and the vacuum expectation value of the Higgs scalar field through spontaneous symmetry breaking. The weakness of the weak interaction originates from the large vacuum expectation value of the Higgs field [1].

From this lesson, it is suspected that gravity may be also characterized by a dimensionless coupling constant ξ with the gravitational constant G_N given by the inverse square of the vacuum expectation value of a scalar field. The weakness of gravity can be associated with a symmetry breaking at a very high energy scale. It has been independently proposed by Zee [2], Smolin [3], and Adler [4] that the Einstein-Hilbert action can be replaced by the induced gravity action

$$S=\int d^4x\sqrt{-g}\left\{\frac{1}{2}\xi\phi^2R+\frac{1}{2}\partial_\mu\phi\partial^\mu\phi-V(\phi)\right\},\qquad(1)$$

where the coupling constant ξ is dimensionless. The potential $V(\phi)$ is assumed to have its minimum value at $\phi=\sigma$; then the above action is reduced to the well-known Einstein-Hilbert action with gravitational constant $G_N=1/8\pi\xi\sigma^2$.

In the analogy of the $SU(2)\times U(1)$ symmetry of electroweak interactions, we can consider a symmetry which may be broken through a spontaneous symmetry breaking in the gravitational interactions. The most attractive candidate symmetry is Weyl's conformal symmetry which rejects the Einstein-Hilbert action, but admits the induced gravity action Eq. (1), with a specific conformal coupling, $\xi=\frac{1}{6}$.

In Riemann space, the conformal coupling is unique with $\xi=\frac{1}{6}$. However, introducing vector torsion, an extended conformal coupling is possible in induced gravity [5,6] because the vector torsion plays the role of a conformal gauge field in Riemann-Cartan space [3,7,8].

It is found that induced gravity at conformal coupling should have conformal invariance for consistency [6,9]. We investigate the conformal coupling in induced gravity with a gradient torsion.

II. CONFORMAL COUPLINGS IN INDUCED GRAVITY

The induced gravity action, Eq. (1), is invariant under the conformal transformation

$$g'_{\mu\nu}(x)=\exp(2\rho)g_{\mu\nu}(x),\quad \phi'(x)=\exp(-\rho)\phi(x),\qquad(2)$$

at the conformal coupling $\xi=\frac{1}{6}$ for a conformally invariant scalar potential.

In Riemann-Cartan space, an extension of conformal coupling with the torsion in induced gravity is possible. It is found that a minimal extension to Riemann-Cartan space is sufficient for our purpose.

The conformal transformation of the affine connections $\Gamma^\gamma{}_{\beta\alpha}$ is determined from the invariance of the tetrad postulation,

$$D_\alpha e^i_\beta\equiv\partial_\alpha e^i_\beta+\omega^i_{j\alpha}e^j_\beta-\Gamma^\gamma{}_{\beta\alpha}e^i_\gamma=0,\qquad(3)$$

under the following tetrads e^i_α transformations:

$$(e^i_\alpha)'=\exp(\rho)e^i_\alpha.\qquad(4)$$

The spin connections $\omega^i_{j\alpha}$ are conformally invariant like other gauge fields.

The affine connections and the torsions which are the antisymmetric components of the affine connections transform as follows:

$$(\Gamma^\gamma{}_{\beta\alpha})'=\Gamma^\gamma{}_{\beta\alpha}+\delta^\gamma_\beta\partial_\alpha\rho,$$
$$(T^\gamma{}_{\beta\alpha})'=T^\gamma{}_{\beta\alpha}+\delta^\gamma_\beta\partial_\alpha\rho-\delta^\gamma_\alpha\partial_\beta\rho.\qquad(5)$$

Therefore, the trace of the torsion $T^\gamma{}_{\gamma\alpha}$ effectively plays the role of a conformal gauge field. In general, the torsion can be decomposed into three components:

$$T^\alpha{}_{\beta\gamma}=\Sigma^\alpha{}_{\beta\gamma}+A^\alpha{}_{\beta\gamma}-\delta^\alpha_\gamma S_\beta+\delta^\alpha_\beta S_\gamma,\qquad(6)$$

where $\Sigma_{[\alpha\beta\gamma]}\equiv0$, and $\Sigma^\alpha{}_{\alpha\gamma}\equiv0$, $A_{\alpha\beta\gamma}\equiv T_{[\alpha\beta\gamma]}$. The traceless part of torsion $C^\alpha{}_{\beta\gamma}\equiv\Sigma^\alpha{}_{\beta\gamma}+A^\alpha{}_{\beta\gamma}$ is conformally invariant:

$$(S_\alpha)'=S_\alpha+\partial_\alpha\rho,\quad (C^\alpha{}_{\beta\gamma})'=C^\alpha{}_{\beta\gamma}.\qquad(7)$$

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Because a minimal extension to Riemann-Cartan space is sufficient for our purpose, we impose the conformally invariant torsionless condition:

$$C^{\alpha}{}_{\beta\gamma}\equiv 0. \tag{8}$$

This condition is the conformally invariant extension of the torsionless condition in Riemann space, $T^{\alpha}{}_{\beta\gamma}\equiv 0$. For this minimally extended Riemann-Cartan space, the affine connection can be written in terms of $g_{\mu\nu}$ and S_α:

$$\Gamma^{\alpha}{}_{\beta\gamma}=\left\{{}^{\alpha}_{\beta\gamma}\right\}+S^{\alpha}g_{\beta\gamma}-S_{\beta}\delta^{\alpha}_{\gamma}. \tag{9}$$

Introducing the conformally covariant derivative D_α for the scalar field ϕ,

$$D_\alpha\phi\equiv\partial_\alpha\phi+S_\alpha\phi, \tag{10}$$

we have an extended conformal coupling of induced gravity up to total derivatives as follow:

$$I=\int d^4x\sqrt{-g}\left\{\frac{\xi}{2}R(\Gamma)\phi^2+\frac{1}{2}D_\alpha\phi D^\alpha\phi-\frac{1}{4}H_{\alpha\beta}H^{\alpha\beta}-V(\phi)\right\}, \tag{11}$$

where we have excluded the curvature square terms. Now, the coupling ξ is a dimensionless arbitrary constant. Using Eq. (9) we can rewrite this action in terms of the Riemann curvature scalar $R(\{\})$:

$$I=\int d^4x\sqrt{-g}\left\{\frac{\xi}{2}R(\{\})\phi^2+\frac{1}{2}\partial_\alpha\phi\partial^\alpha\phi+(1-6\xi)S^\alpha(\partial_\alpha\phi)\phi+\frac{1}{2}(1-6\xi)S_\alpha S^\alpha\phi^2-\frac{1}{4}H_{\alpha\beta}H^{\alpha\beta}-V(\phi)\right\}. \tag{12}$$

The more general form of induced gravity action can be considered [10,11], but we restrict the couplings to be conformal. In the limit of $\xi\to\frac{1}{6}$, this extended conformal coupling is reduced to the ordinary conformal coupling in Riemann space decoupled from the vector torsion.

III. DILATON GRAVITY FROM INDUCED GRAVITY WITH GRADIENT TORSION

Analyzing the equations of motion for the action, Eq. (12), with an effective potential $V_{eff}(\phi;S_\alpha,g_{\beta\gamma})$ which depends on the metric and torsion in general, we obtain the following two equations of motion and a constraint for the scalar potential:

$$\nabla_\mu H^{\mu\nu}=-(1-6\xi)\{(\partial^\nu\phi)\phi+S^\nu\phi^2\}+\frac{\partial V_{eff}(\phi;S_\alpha,g_{\beta\gamma})}{\partial S_\nu}, \tag{13}$$

$$\begin{aligned}\xi\phi^2G_{\mu\nu}=&\left(H_{\mu\alpha}H_\nu{}^\alpha-\frac{1}{4}g_{\mu\nu}H_{\alpha\beta}H^{\alpha\beta}\right)-\left(\partial_\mu\phi\partial_\nu\phi-\frac{1}{2}g_{\mu\nu}\partial_\alpha\phi\partial^\alpha\phi\right)\\&-(1-6\xi)\phi^2\left(S_\mu S_\nu-\frac{1}{2}g_{\mu\nu}S_\alpha S^\alpha\right)-(1-6\xi)\\&\times(S_\mu\phi\partial_\nu\phi+S_\nu\phi\partial_\mu\phi-g_{\mu\nu}S^\alpha\phi\partial_\alpha\phi)\\&+\xi\{\nabla_\mu(\phi\partial_\nu\phi)+\nabla_\nu(\phi\partial_\mu\phi)-g_{\mu\nu}\Box\phi^2\}\\&-g_{\mu\nu}V_{eff}(\phi;S_\alpha,g_{\beta\gamma})+2\frac{\partial V_{eff}(\phi;S_\alpha,g_{\beta\gamma})}{\partial g^{\mu\nu}},\end{aligned} \tag{14}$$

$$\begin{aligned}4V_{eff}(\phi;S_\alpha,g_{\beta\gamma})-\phi\frac{\partial V_{eff}(\phi;S_\alpha,g_{\beta\gamma})}{\partial\phi}\\=2\frac{\partial V_{eff}(\phi;S_\alpha,g_{\beta\gamma})}{\partial g^{\mu\nu}}g^{\mu\nu}+\nabla_\nu\frac{\partial V_{eff}(\phi;S_\alpha,g_{\beta\gamma})}{\partial S_\nu},\end{aligned} \tag{15}$$

where all covariant derivatives are in Riemann space with the Christoffel connections [6,9].

The constraint, Eq. (15) requires that the metric independent bare potential should be quartic in the scalar field, $V_0(\phi)=(\lambda/4!)\phi^4$, and the deviation of the radiatively corrected effective potential from the quartic form is only allowed with compensation by the metric and vector torsion dependences of the effective potential. Because this constraint comes from the assumption that the bare action is conformally invariant except the potential term, if we consider nonconformal coupling in kinetic and interacting terms, such a constraint would not appear.

Let us consider a reduction of the system. If the effective potential does not have the vector torsion dependence, i.e., $V_{eff}(\phi;g_{\beta\gamma})$, Eq. (13) allows the following conformally invariant reduction:

$$D_\alpha\phi=0. \tag{16}$$

This implies that the vector torsion is a gradient form:

$$S_\alpha=-\partial_\alpha\ln(\phi/\phi_0)=-\partial_\alpha\sigma,\quad\phi\equiv\phi_0e^\sigma, \tag{17}$$

where ϕ_0 is a dimensional constant, and the field strength of the vector torsion vanishes, $H_{\alpha\beta}=0$, which is consistent with Eq. (13). In this reduction, the bare action of Eq. (11) becomes

$$I=\int d^4x\sqrt{-g}\,\phi_0^2e^{2\sigma}\left\{\frac{\xi}{2}R(\{\ \})+3\xi\partial_\alpha\sigma\partial^\alpha\sigma-\frac{\lambda}{4!}\phi_o^2e^{2\sigma}\right\}. \tag{18}$$

This is the form of the conformal factor theory of dilaton gravity [12,13]. If the antisymmetric torsion term

$\frac{1}{12}C_{\alpha\beta\gamma}C^{\alpha\beta\gamma}$ is included, this action is the form of string gravity with some redefinition of fields except the quartic potential term [14].

For the reduction of the effective action, the potential term is replaced by $e^{-2\sigma}V_{eff}(\phi_0 e^{\sigma}, g_{\alpha\beta})$. In this reduction, a dimensional constant ϕ_0 is introduced, but the action is invariant under the global scaling $dx^{\mu} \to a dx^{\mu}$, $\phi_0 \to \phi_0/a$ if no conformal anomaly is introduced in the effective action. However, the appearance of conformal anomaly in the conformally induced gravity is not allowed due to the constraint, Eq. (15), which is the requirement of conformal invariance [9] from the consistency of equations of motion for the bare and the effective action in conformally induced gravity [6]. In this reduction, the constraint which the effective potential should satisfy is

$$4V_{eff}(\phi;g_{\beta\gamma}) - \phi\frac{\partial V_{eff}(\phi;g_{\beta\gamma})}{\partial\phi} = 2\frac{\partial V_{eff}(\phi;g_{\beta\gamma})}{\partial g^{\mu\nu}}g^{\mu\nu}. \tag{19}$$

Therefore, only the metric independent quartic potential with effective coupling λ_{eff} seems to be allowed. But, in general, the quantum effect by quantum fluctuation of the scalar field on the classical metric and torsion background [12,13] gives the following corrections to the scalar mass m^2, the coupling λ, and the cosmological constant Λ, respectively [15,16]:

$$\delta m^2 \propto \lambda m^2, \quad \delta\lambda \propto \lambda^2, \quad \delta\Lambda \propto a m^2 + b\lambda. \tag{20}$$

Therefore, $\lambda = 0$ would be the only solution of Eq. (19) as a trivial fixed point of the renormalization group. However, the definite claim about $\lambda = 0$ is possible only after consideration of the full quantum effects of the theory, which is of course far beyond our scope yet.

Redefining the metric

$$g_{\alpha\beta}e^{2\sigma} \to g_{\alpha\beta}, \tag{21}$$

the above dilaton gravity action can be written in the following standard Einstein action:

$$I = \int d^4x\sqrt{-g}\left\{\frac{\xi}{2}\phi_0^2 R(\{\ \}) - \frac{\lambda}{4!}\phi_0^4\right\}, \tag{22}$$

where the gravitational constant $G_N = 1/8\pi\xi\phi_0^2$, and the cosmological constant $\Lambda = (\lambda/4!)(1/8\pi\xi G_N)^2$. If the effective potential deviates from the quartic form, then there would remain an explicit σ dependence on the action after this redefinition of the metric even though it is not plausible due to the constraint, Eq. (19).

In this Einstein frame the conformal symmetry is hidden and the cosmological constant term has the origin of the quartic potential term in the original induced gravity action. Recalling the constraint, Eq. (19), which the effective potential should satisfy, the nonzero coupling λ can be hardly expected after the radiative correction by the scalar field in the original induced gravity action [15,16]. Therefore, we can say that if Einstein gravity has its roots in conformally induced gravity, the nonzero cosmological constant is not plausible due to the constraint from the conformal coupling.

In this discussion, we have not considered torsions generated by matter fields because vector torsion couplings are not expected in the standard minimal action for Dirac and gauge fields [11].

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