Trace identity in a model with broken symmetry

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(Received 2 February 1999; published 18 May 1999)

Considering the simple chiral fermion meson model when the chiral symmetry is explicitly broken, we show the validity of a trace identity—to all orders of perturbation theory—playing the role of a Callan-Symanzik equation and which allows us to identify directly the breaking of dilatations with the trace of the energy-momentum tensor. More precisely, by coupling the quantum field theory considered to a classical curved space background, represented by the nonpropagating external vielbein field, we can express the conservation of the energy-momentum tensor through the Ward identity which characterizes the invariance of the theory under the diffeomorphisms. Our ''Callan-Symanzik equation'' then is the anomalous Ward identity for the trace of the energy-momentum tensor, the so-called ''trace identity.'' [S0556-2821(99)04512-9]

PACS number(s): 11.10.Gh, 11.25.Db

I. INTRODUCTION

A classical field theory is scale invariant precisely when it has the following properties: it contains no mass terms and all coupling constants are dimensionless. For a theory of this type it is possible to define an energy-momentum tensor with a vanishing trace (see [1] for a review). On the other hand, it is natural to expect that the scale symmetry is broken to provide us a scale to live on. There are many different ways to break scale invariance. The most ordinary way is that scale invariance is broken due to quantum effects.

As is well known, in the context of a renormalized perturbation theory, integrals associated with the Feynman graphs are generally UV divergents. To give a proper meaning to such expressions we have to adopt a suitable subtraction scheme. The effect of the latter is to render the integrals convergent. The renormalization is performed through the addition of a certain number of counterterms to the initial action considered, whose coefficients are left arbitrary. These coefficients have however to be fixed by a set of normalization conditions, which are applied at a certain momentum scale. In this point, the important feature is the appearance of a normalization parameter κ; it has the dimension of a mass and will thus cause the breaking of scale invariance. The natural choice $\kappa = m$, where m is the physical mass, cannot be used because it leads to singularities at the massless limit. In this way, there is no way to preserve quantum scale invariance at the massless limit: scale invariance is anomalous[1] [2–6]. This is called the trace anomaly induced by radiative corrections.

In order to get insight into this problem, a systematic study of renormalization properties can be achieved via the Callan-Symanzik (CS) equation [3]. It describes the behavior of the quantum theory under scale transformations, being successfully applied in symmetric models. If the theory is broken the construction of the CS equation is more involved [13–15]. Because of the shifts by constant amounts in certain fields, the dilatation Ward operator does not will commute with the Ward operator for broken symmetry. Therefore the breaking of dilatations is not symmetric but has a certain covariance under the symmetry transformations already at the classical level. As alluded in Ref. [14], to identify the breaking of dilatations with the trace of the energy-momentum tensor is thus complicated at the quantum level. In addition, in a purely physical parametrization, the effect of the breaking induces the appearance of physical mass β-functions [13–15], a situation which asks for a deeper understanding. Of a particular kind is the case for a realistic supersymmetric gauge field theory [15].

The purpose of the present paper is to supplement the works of [13–15] and, exploiting the techniques developed in [11], to provide an alternative way of deriving the CS equation, which allows us to identify the breaking of dilatations with the trace of the energy-momentum tensor in a model with broken symmetry. More precisely, by coupling the quantum field theory considered to a classical curved space background, represented by the nonpropagating external vielbein field, we can express the conservation of the energy-momentum tensor through the Ward identity which characterizes the invariance of the theory under the diffeomorphisms. Our ''Callan-Symanzik equation'' then is the anomalous Ward identity for the trace of the energy-momentum tensor, the so-called ''trace identity.''[2] As a by-product, the approach will allow us to bring to an end that

[1]Breaking of quantum scale invariance by an anomaly is quite general. Only some models remain scale invariant at the quantum level (see for instance [7–11]). Within perturbative theory, a necessary and sufficient condition to obtain the ''scale invariance,'' it is the vanishing to all orders of the β-functions—anomalous dimensions are allowed to be different of zero. The latter corresponding to field redefinitions, are physically trivial and hence vanish on the mass shell [12].

[2]There exist studies in the literature concerning the local dilatation properties—general coordinate transformations in the Weyl's sheme—as discussed by Bandelloni *et al.* [16], where a more difficult task was taken up: the radiative mass generation due to the trace anomaly. Even though their approach is related to ours, here instead of introducing an external dilatation field, beyond the external metric or vielbein field, we only consider the latter.

the β- and γ-functions in curved space-time are the same as in flat one.

Since we are working with an external vielbein which is not necessary flat, our results hold for a curved manifold, as long as its topology remains that of flat $\mathcal{R}^4$, with asymptotically vanishing curvature. It is the latter two restrictions which allow us to use the general results of renormalization theory, established in flat space. Indeed, we may then expand in the powers of $\overline{e_\mu^m}=e_\mu^m-\delta_\mu^m$, considering $\overline{e_\mu^m}$ as a classical background field in flat $\mathcal{R}^4$, and thus make use of the general theorems of renormalization theory actually proved for flat space-time [17,18].

We shall consider the model of chiral fermion meson with explicit breaking of the chiral symmetry. In contrast with Ref. [19], which have considered the explicit breaking term in order to be able to treat the massless Goldstone particle in its massless limit, we are interested in the case where the pion fields are massive. The importance of this lies in fact that the chiral fermion meson model allows for the possibility of a simultaneous description of the baryonic and mesonic low energy sector in hadron physics.[3] Consistency conditions for chiral symmetry and scale invariance, in absence of fermions and charged pions, were studied in [21].[4]

The outline of the paper is as follows. The model in a curved Riemannian manifold described in terms of external vielbein and spin connection fields, is introduced in Sec. II, together with its symmetries. The trace identity for the classical theory is derived in Sec. III. The extension of this identity to the quantum level is discussed in Sec. IV, followed by a summary. For the sake of completeness, we add two appendices: The renormalizability of the model is sketched in Appendix A. Assuming the limit of flat space-time, the CS equation is derived in Appendix B.

II. GENERALITIES OF THE MODEL

In this section, we give a brief description of the model in a curved space-time. The chiral fermion meson model involves a fermionic isodoublet, ψ, of zero mass, a scalar sigma field, σ, and a triplet of charged pseudscalar pion fields, π^a. Because fermions are represented by spinor fields which are subjected to the Lorentz group—and not to the diffeomorphisms group—we must refer these fields to the tangent frame and treat the fermions as scalars with respect to the diffeomorphisms. We achieve this with the help of the vierbein. Space-time is a 4-dimensional Riemannian manifold $\mathcal{M}$, with coordinates x^μ, $\mu=0,1,2,3$. It is described by a vierbein field $e_\mu^m(x)$ and its inverse $e_m^\mu(x)$, μ being a world index and m a tangent space index. The spin connection $\omega_\mu^{mn}(x)$ is not an independent field, but depends on the vierbein due to the vanishing torsion condition: $\omega_{lmn}=\frac{1}{2}(\Omega_{lmn}+\Omega_{mnl}-\Omega_{nlm})$, with $\Omega_{lmn}=e_l^\mu e_m^\nu(\partial_\mu e_{\nu n}-\partial_\nu e_{\mu n})$.

The metric tensor reads $g_{\mu\nu}(x)=\eta_{mn}e_\mu^m(x)e_\nu^n(x)$; η_{mn} being the tangent space flat metric. We denote by e the determinant of e_μ^m. As explained in the Introduction, we assume the manifold $\mathcal{M}$ to be topologically equivalent to $\mathcal{R}^4$ and asymptotically flat.

We shall take into account the physically interesting case when the chiral symmetry is broken adding a linear term to the action. The linear breaking term implies that the quantum σ field has a nonvanishing vacuum expectation value $\langle\sigma\rangle=v$. If one wishes to interpret the theory in terms of particles, it is necessary to perform a field translation $\sigma\to\sigma+v$ such that $\delta\Sigma/\delta\sigma|_{\sigma=v}=0$. As an effect the mass degeneracy between fields σ and π^a disappears and the fermionic isodoublet ψ acquires a mass.

The corresponding action in a curved manifold is given by

$$\Sigma=\int d^4xe\Big\{\overline{\psi}i\gamma^\mu\mathcal{D}_\mu\psi+\frac{1}{2}(\partial_\mu\sigma\partial^\mu\sigma+\partial_\mu\pi^a\partial^\mu\pi^a) -\frac{1}{2}\mu^2((\sigma+v)^2+\pi^a\pi^a)-\frac{1}{4}\lambda((\sigma+v)^2+\pi^a\pi^a)^2 -g\overline{\psi}((\sigma+v)+i\gamma^5\tau^a\pi^a)\psi+c\sigma\Big\}, \tag{1}$$

with $\gamma^\mu=e_m^\mu\gamma^m$, where γ^m are the Dirac matrices in the tangent space. g and λ are the pion-fermion and pion-pion coupling constants, respectively. The covariant derivative is defined by

$$\mathcal{D}_\mu\psi(x)\equiv\left(\partial_\mu+\frac{1}{2}\omega_\mu^{mn}(x)\Omega_{mn}\right)\psi(x), \tag{2}$$

with $\Omega^{[mn]}$ acting on $\psi(x)$ as an infinitesimal Lorentz matrix in the appropriate representation.

The masses arising from the action (1) for ψ, π^a and σ are given by

$$m_\psi=gv,\quad m_\pi^2=\mu^2+\lambda v^2,\quad m_\sigma^2=\mu^2+3\lambda v^2. \tag{3}$$

Moreover, from the Born approximation to the vacuum expectation value [22], we obtain

$$c=v(\mu^2+\lambda v^2)=vm_\pi^2. \tag{4}$$

The isospin, chiral, diffeomorphisms and local Lorentz transformations read as follows.

(i) Isospin:

$$\delta_{\rm iso}\sigma=0,\quad \delta_{\rm iso}\psi=-i\frac{\alpha^a\tau^a}{2}\psi,\quad \delta_{\rm iso}\pi^a=\epsilon^{abc}\alpha^b\pi^c,\quad \delta_{\rm iso}\overline{\psi}=i\overline{\psi}\frac{\alpha^a\tau^a}{2}. \tag{5}$$

[3]See e.g. [20] for current study of the model in a somewhat phenomenological basis when temperature effects are considered, in absence of fermions.

[4]I thank Professor R. Jackiw for drawing these works to my attention.

(ii) Chiral:

$$\delta_{\rm chiral}\sigma=-\alpha^a\pi^a,\quad \delta_{\rm chiral}\psi=-i\frac{\alpha^a\tau^a}{2}\gamma^5\psi,$$

$$\delta_{\rm chiral}\pi^a=\alpha^a(\sigma+v),\quad \delta_{\rm chiral}\bar\psi=-i\bar\psi\gamma^5\frac{\alpha^a\tau^a}{2}. \tag{6}$$

(iii) Diffeomorphisms:

$$\delta^{(\varepsilon)}_{\rm diff}e^m_\mu=\mathcal{L}_\varepsilon e^m_\mu=\varepsilon^\lambda\partial_\lambda e^m_\mu+(\partial_\mu\varepsilon^\lambda)e^m_\lambda,$$

$$\delta^{(\varepsilon)}_{\rm diff}\Phi=\mathcal{L}_\varepsilon\Phi=\varepsilon^\lambda\partial_\lambda\Phi,\quad \Phi=\sigma,\pi^a,\psi, \tag{7}$$

where $\mathcal{L}_\varepsilon$ is the Lie derivative along the vector field $\varepsilon^\mu(x)$ — the infinitesimal parameter of the transformation.

(iv) Local Lorentz transformations

$$\delta^{(\lambda)}_{\rm Lorentz}\Phi=\frac{1}{2}\lambda_{mn}\Omega^{mn}\Phi,\quad \Phi=\text{any field}, \tag{8}$$

with infinitesimal parameters $\lambda_{[mn]}$.

The Ward identities corresponding to the isospin, chiral, diffeomorphisms and local Lorentz symmetries for the action (1) can be expressed introducing the functional differential Ward operators, as given below:

$$W^a_{\rm iso}\Sigma=\int d^4x\left(\epsilon^{abc}\pi^b\frac{\delta}{\delta\pi^c}+i\bar\psi\frac{\tau^a}{2}\frac{\delta}{\delta\bar\psi}-i\frac{\overleftarrow{\delta}}{\delta\psi}\frac{\tau^a}{2}\psi\right)\Sigma=0, \tag{9}$$

$$W^a_{\rm chiral}\Sigma=\int d^4x\left((\sigma+v)\frac{\delta}{\delta\pi^a}-\pi^a\frac{\delta}{\delta\sigma}-i\bar\psi\gamma^5\frac{\tau^a}{2}\frac{\delta}{\delta\bar\psi}-i\frac{\overleftarrow{\delta}}{\delta\psi}\frac{\tau^a}{2}\gamma^5\psi\right)\Sigma=\Delta^a_{\rm class}, \tag{10}$$

where

$$\Delta^a_{\rm class}=-\int d^4x\, e c\,\pi^a \tag{11}$$

is the breaking term which, being linear in the quantum field π^a, will not be renormalized, i.e., it will remain a classical breaking [23]:

$$W_{\rm diff}\Sigma=\int d^3x\sum_\Phi \delta^{(\varepsilon)}_{\rm diff}\Phi\frac{\delta\Sigma}{\delta\Phi}=0, \tag{12}$$

and

$$W_{\rm Lorentz}\Sigma=\int d^3x\sum_\Phi \delta^{(\lambda)}_{\rm Lorentz}\Phi\frac{\delta\Sigma}{\delta\Phi}=0, \tag{13}$$

where the summations run over all quantum and external fields.

Finally, the classical action (1) is constrained, besides the Ward identities (9), (10), (12) and (13) by a set of discrete symmetries, i.e., parity P and charge conjugation C, whose action on the fields is given as below.

(i) Parity P:

$$x\xrightarrow{P}(x^0,-\vec{x}),$$
$$\psi\xrightarrow{P}\gamma^0\psi,$$
$$\bar\psi\xrightarrow{P}\bar\psi\gamma^0, \tag{14}$$
$$\pi^a\xrightarrow{P}-\pi^a,$$
$$\sigma\xrightarrow{P}\sigma.$$

(ii) Charge conjugation C:

$$\psi\xrightarrow{C}\psi^c=C\bar\psi^t,$$
$$\bar\psi\xrightarrow{C}\bar\psi^c=-\psi^tC^{-1},$$
$$\pi^{1,3}\xrightarrow{C}\pi^{1,3}, \tag{15}$$
$$\pi^2\xrightarrow{C}-\pi^2,$$
$$\sigma\xrightarrow{C}\sigma,$$

where $C=i\gamma^0\gamma^2$ is the charge conjugation matrix.

Ultraviolet dimensions of all fields are collected in Table I.[5]

TABLE I. Ultraviolet dimension d_Φ.

	ψ	σ	π^a	e^m_μ
d_Φ	3/2	1	1	0

III. TRACE IDENTITY FOR THE CLASSICAL THEORY

From now on we change from unphysical parametrization (μ^2,λ,g,c) to physical one $(m^2_\pi,m^2_\sigma,m_\psi,v)$, with the change of variables given by Eqs. (3) and (4). In this way, the classical action (1) takes the form

[5]The ultraviolet dimensions determine the ultraviolet power-counting. If there are massless fields in the theory, one should take special care of the infrared convergence [24].

$$\Sigma=\int d^4xe\Bigg\{\bar{\psi}i\gamma^\mu\mathcal{D}_\mu\psi+\frac{1}{2}(\partial_\mu\sigma\partial^\mu\sigma+\partial_\mu\pi^a\partial^\mu\pi^a)-\frac{(3m_\pi^2-m_\sigma^2)}{4}((\sigma+v)^2+\pi^a\pi^a)-\frac{(m_\sigma^2-m_\pi^2)}{8v^2}((\sigma+v)^2+\pi^a\pi^a)^2-\frac{m_\psi}{v}\bar{\psi}((\sigma+v)+i\gamma^5\tau^a\pi^a)\psi+m_\pi^2v\sigma\Bigg\}. \quad (16)$$

For a given field theory the energy-momentum tensor is defined as the functional derivative:

$$\Theta_\lambda^\mu=e^{-1}e_\lambda^m\frac{\delta\Sigma}{\delta e_\mu^m}. \quad (17)$$

The conservation of the energy-momentum tensor Θ_λ^μ is a consequence of the diffeomorphism Ward identity (12) and of the definition (17), yielding the following equation:

$$\int d^4x\,\varepsilon(x)(e\nabla_\mu\Theta_\lambda^\mu(x)-w_\lambda(x)\Sigma)=0, \quad (18)$$

where ∇_μ is the covariant derivative with respect to the diffeomorphisms, with the differential operator $w_\lambda(x)$ acting on Σ representing contact terms:

$$w_\lambda(x)=\sum_{\sigma,\pi,\psi}(\nabla_\lambda\Phi)\frac{\delta}{\delta\Phi} \quad (19)$$

(becoming the translation Ward operator in the limit of flat space).

The integral of the trace of the tensor Θ_λ^μ,

$$\int d^4xe\Theta_\mu^\mu=\int d^4xe_\mu^m\frac{\delta\Sigma}{\delta e_\mu^m}\equiv\mathcal{N}_e\Sigma, \quad (20)$$

turns out to be an equation of motion, up to soft breakings — which means that Θ_λ^μ is the improved energy-momentum tensor. This follows from the identity, which is easily checked by inspection of the classical action

$$\mathcal{N}_e\Sigma=\left(\frac{3}{2}\mathcal{N}_\psi+\mathcal{N}_\Phi^{\text{hom}}+m_\sigma\frac{\partial}{\partial m_\sigma}+m_\pi\frac{\partial}{\partial m_\pi}+m_\psi\frac{\partial}{\partial m_\psi}+v\frac{\partial}{\partial v}\right)\Sigma, \quad (21)$$

where $\mathcal{N}_e$ is the counting operator of the vierbein e_μ^m. $\mathcal{N}_\psi$ and $\mathcal{N}_\Phi$ are counting operators defined by

$$\mathcal{N}_\psi=\int d^4x\left(\bar{\psi}\frac{\delta}{\delta\bar{\psi}}+\frac{\overleftarrow{\delta}}{\delta\psi}\psi\right), \quad (22)$$

$$\mathcal{N}_\Phi^{\text{hom}}=\int d^4x\left(\sigma\frac{\delta}{\delta\sigma}+\pi^a\frac{\delta}{\delta\pi^a}\right). \quad (23)$$

$\mathcal{N}_\Phi^{\text{hom}}$ is the unshifted counting operator for scalar fields.

It is interesting to note that Eq. (21) is nothing but the Ward identity for rigid Weyl symmetry [25]—broken by the mass terms and dimensionful couplings.

Our classical ''trace identity'' is defined by

$$W^{\text{trace}}\Sigma=\Lambda, \quad (24)$$

where

$$W^{\text{trace}}=\mathcal{N}_e-\frac{3}{2}\mathcal{N}_\psi-\mathcal{N}_\Phi^{\text{hom}}, \quad (25)$$

and

$$\Lambda=\int d^4xe\left(-m_\sigma^2\sigma^2-m_\pi^2\pi^a\pi^a-m_\psi\bar{\psi}\psi-\frac{(m_\sigma^2-m_\pi^2)}{2v}\sigma(\sigma^2+\pi^a\pi^a)\right). \quad (26)$$

The latter is the effect of the breaking of scale invariance due to the dimensionful parameters. The dimension of Λ—the dimensions of m_π^2, m_σ^2, m_ψ and v not being taken into account—is lower than four: it is a soft breaking.

In order to make the connection between the trace identity and the dilatational Ward identity — which is just the scaling Ward identity—let us consider a while the limit of flat space, where rigid dilatation symmetry makes sense. In this limit (18) holds with $e=1$ and $\nabla_\mu=\partial_\mu$. The classical dilatation current $\mathcal{S}^\mu$ can now be defined as

$$\mathcal{S}^\mu(x)=x^\lambda\Theta_\lambda^\mu(x). \quad (27)$$

It obeys the conservation identity

$$\partial_\mu\mathcal{S}^\mu=\Theta_\mu{}^\mu+x^\lambda\partial_\mu\Theta_\lambda^\mu=\left(\frac{3}{2}+x\cdot\partial\right)n_\psi\Sigma+(1+x\cdot\partial)n_\Phi^{\text{hom}}\Sigma+\Lambda(x), \quad (28)$$

where $\Lambda(x)$ is the integrand of Eq. (26). n_ψ and n_Φ^{hom} are local operators given by

$$n_\psi(x)=\left(\bar{\psi}\frac{\delta}{\delta\bar{\psi}}+\frac{\overleftarrow{\delta}}{\delta\psi}\psi\right),$$

$$n_\Phi^{\text{hom}}(x)=\left(\sigma\frac{\delta}{\delta\sigma}+\pi^a\frac{\delta}{\delta\pi^a}\right).$$

Integrating Eq. (28) we get the broken dilatation Ward identity for the classical theory:

$$W_{\text{D}}\Sigma=\int d^4x\sum_{\Phi=\sigma,\pi^a,\psi}[(d_\Phi+x\cdot\partial)\Phi]\frac{\delta\Sigma}{\delta\Phi}=\Lambda, \quad (29)$$

where d_Φ is the dimension of the field Φ (see Table I).

The trace Ward operator defined in Eq. (24), together with the operators (9), (10), (12) and (13) satisfy the algebra

$$[W^{\rm trace},W^{a}_{\rm chiral}]\mathcal{F}=\int d^4x v\,\frac{\delta\mathcal{F}}{\delta\pi^a}, \tag{30}$$

$$[W^{\rm trace},W_{\rm X}]\mathcal{F}=0,\quad {\rm X}={\rm iso,\ Lorentz,\ diff}, \tag{31}$$

where $\mathcal{F}$ is an arbitrary functional.

IV. TRACE IDENTITY FOR THE QUANTUM THEORY

We have now to extend the construction of the preceding section to all orders of perturbation theory. As a starting point, we must be able to write the breaking of scale invariance (26) in form of a differential operator. This will be possible with the help of additional external fields η and ρ, which transform invariantly under isospin and chiral symmetries—*P*- and *C*-even—introducing the new classical action

$$\Sigma^{\natural}=\Sigma+\int d^4x e(a\rho\mathcal{Q}^{\rm inv}+3m_\pi^2 v\,\eta\sigma), \tag{32}$$

with

$$\mathcal{Q}^{\rm inv}=(\sigma+v)^2+\pi^a\pi^a,$$

an invariant polynomial of dimension two. The ultraviolet dimension of η and ρ is 2.

Following along the lines of [14], note that

$$\left[\int d^4x\left(v\,\frac{\delta}{\delta\sigma}+\frac{\delta}{\delta\eta}+a\,\frac{\delta}{\delta\rho}\right),W^a_{\rm chiral}\right]\mathcal{F}=\int d^4x v\,\frac{\delta\mathcal{F}}{\delta\pi^a}. \tag{33}$$

Taking into account (30) and (33), for $\mathcal{F}=\Sigma^{\natural}$ (setting at the end $\eta=\rho=0$), one finds

$$\begin{aligned}W^a_{\rm chiral}W^{\rm trace}\Sigma^{\natural}|_{\eta=\rho=0}\\ &=W^a_{\rm chiral}\left(\int d^4x\left(v\,\frac{\delta}{\delta\sigma}+\frac{\delta}{\delta\eta}+a\,\frac{\delta}{\delta\rho}\right)\Sigma^{\natural}|_{\eta=\rho=0}\right)\\ &=W^a_{\rm chiral}\Lambda.\end{aligned} \tag{34}$$

Therefore, in the tree approximation one gets the expression

$$\Lambda=\int d^4x\left(v\,\frac{\delta}{\delta\sigma}+\frac{\delta}{\delta\eta}+a\,\frac{\delta}{\delta\rho}\right)\Sigma^{\natural}|_{\eta=\rho=0}, \tag{35}$$

with

$$a=\frac{(m_\sigma^2-3m_\pi^2)}{2}, \tag{36}$$

determined by the normalization conditions.

We now define the ‘‘symmetrized’’ form of the classical trace identity

$$\begin{aligned}\hat{W}^{\rm trace}\Sigma^{\natural}|_{\eta=\rho=0}\\ &=\left(W^{\rm trace}-\int d^4x\left(v\,\frac{\delta}{\delta\sigma}+\frac{\delta}{\delta\eta}+a\,\frac{\delta}{\delta\rho}\right)\right)\Sigma^{\natural}|_{\eta=\rho=0}=0.\end{aligned} \tag{37}$$

With the new operator $\hat{W}^{\rm trace}$, we have

$$\begin{aligned}[\hat{W}^{\rm trace},W^a_{\rm chiral}]\mathcal{F}=0,\\ [\hat{W}^{\rm trace},W_{\rm X}]\mathcal{F}=0,\end{aligned} \tag{38}$$

where X = iso, Lorentz, diff.

Let us note that

$$W^a_{\rm chiral}\Sigma^{\natural}=\Delta^{\natural a}_{\rm class}, \tag{39}$$

where

$$\Delta^{\natural a}_{\rm class}=-\int d^4x e(1+3\,\eta)m_\pi^2 v\,\pi^a, \tag{40}$$

is a breaking term which stays linear in the quantum field π^a, and will remain a classical breaking [23]. Thus, the proof of the renormalizability for Eq. (32) remains the same as the one sketched in Appendix A.

The corresponding quantum theory is described introducing the new vertex functional

$$\Gamma^{\natural}=\Sigma^{\natural}+O(\hbar). \tag{41}$$

In this way, the trace identity (37) takes the form

$$\hat{W}^{\rm trace}\Gamma^{\natural}|_{\eta=\rho=0}=\Delta\cdot\Gamma^{\natural}|_{\eta=\rho=0}=\Delta+O(\hbar). \tag{42}$$

The insertion in the right-hand side represents the breaking due to the effect of the radiative corrections we want to study, and Δ is their lowest order contribuition. From quantum action principle (QAP) [17], Δ is an integrated field polynomial compatible with ultraviolet dimension 4 and even under the parity *P* and charge conjugation *C*.

According also to the QAP, applying the algebraic structure (38) to the vertex functional, one gets

$$\begin{aligned}W^a_{\rm chiral}\hat{W}^{\rm trace}\Gamma^{\natural}|_{\eta=\rho=0}=W^a_{\rm chiral}\Delta=0,\\ W_{\rm X}\hat{W}^{\rm trace}\Gamma^{\natural}|_{\eta=\rho=0}=W_{\rm X}\Delta=0\end{aligned} \tag{43}$$

with X = iso, Lorentz diff.

For that reason, Δ is an invariant insertion which can be expanded in a suitable basis. It is remarkable that, in perturbation theory, any such basis of renormalized insertions is completely characterized by the corresponding classical basis [23]. Such a basis is given in the classical approximation by Eq. (A4)—see Appendix A. An appropriate quantum extension of this basis is obtained through the introduction of a set of symmetric differential operators acting on $\Gamma^{\natural}$—setting at the end $\eta=\rho=0$—and in one-to-one correspondence to the basis of integrated polynomials in Eq. (A4). We define a symmetric operator as an operator ∇ which satisfies the condition

$$[\nabla, W_{\rm X}]=0, \quad {\rm X}=\text{iso, chiral, Lorentz, diff.} \tag{44}$$

The set

$$\left\{ \int d^4x a \frac{\delta}{\delta\rho}, v\frac{\partial}{\partial v}, m_\psi \frac{\partial}{\partial m_\psi}, \mathcal{N}_\Phi, \mathcal{N}_\psi \right\}, \tag{45}$$

with $\mathcal{N}_\psi$ given by Eq. (22) and

$$\mathcal{N}_\Phi = \int d^4x \left((\sigma+v)\frac{\delta}{\delta\sigma} + \pi^a \frac{\delta}{\delta\pi^a}\right), \tag{46}$$

forms a basis for the symmetric operators of the model, taking into account the physical parametrization.

Thus, the expansion of Δ in the basis (45), we have just constructed, yields the quantum trace identity in the curved space-time:

$$\begin{aligned}\int d^4x e \Theta^\mu_\mu \cdot \Gamma^\natural|_{\eta=\rho=0} &= \Bigg(\beta_{m_\psi} m_\psi \frac{\partial}{\partial m_\psi} + \beta_v v \frac{\partial}{\partial v} + \left(\frac{3}{2} - \gamma_\psi\right)\mathcal{N}_\psi \\ &\quad + (1-\gamma_\Phi)\mathcal{N}_\Phi^{\rm hom} + (1-\beta_v - \gamma_\Phi)\int d^4x v \frac{\delta}{\delta\sigma} \\ &\quad + (1+\delta)\int d^4x a\frac{\delta}{\delta\rho} + (1+\beta_v)\int d^4x \frac{\delta}{\delta\eta}\Bigg)\Gamma^\natural|_{\eta=\rho=0} \\ &\quad + \text{terms vanishing in the flat limit,}\end{aligned} \tag{47}$$

where $\mathcal{N}_\Phi^{\rm hom}$ is an unshifted counting operator.

In the flat space, Eq. (47) is equivalent to the Callan-Symanzik equation, which is the Ward identity for anomalous dilatation invariance—see Appendix B. It is worthwhile to note that this result allows us to conclude which the β-functions and anomalous dimensions in curved space are the same as in flat space. The presence of the β_{m_ψ}-function corresponds to renormalization of the physical mass of fermionic fields, with the consequence that the hard breaking of dilatations depends on the normalization point also in the asymptotic region.

V. SUMMARY

In this paper, we show—by using the techniques developed in [11]—as directly to identify the breaking of dilatations with the trace of the energy-momentum tensor in a model with explicitly broken symmetry. This is not a trivial task due the shifts by constant amounts in certain fields: the dilatational operator does not commute with the Ward operator for broken symmetry, but has a certain covariance under the symmetry transformations already at the classical level. Most remarkable is the presence of the β-function associated with the physical mass of fermions. According Becchi [13], a ''true'' CS equation does not exist in such situation. By ''true'' it should be understood an equation which does not contain β-functions belonging to the physical mass differential operators. In any case this requires an analysis. This effort is essential if one aims at having contact with phenomenology. In particular, this is the case for a realistic supersymmetric gauge field theory [15]. The reader may convince himself that our algorithm also works for the case of spontaneous symmetry breaking. In this case, due to the eventual appearance of Goldstone modes, infrared anomalies may be picked up, and in higher order have to be proven to be absent [14]. As a by-product, the approach has allowed us to conclude that the β-functions and the anomalous dimensions in curved space are the same as in flat space — evidently this is valid for a class of curved manifolds with topology remains that of flat $\mathcal{R}^4$ and with asymptotically vanishing curvature. It is only in this case we can use the general results of renormalization theory, established in flat space.

ACKNOWLEDGMENTS

I am deeply indebted to Professor O. Piguet for helpful comments on a preliminary draft of this paper. I wish to thank Professor L. Bonora for his kind invitations at the Scuola Internazionale Superiore di Studi Avanzati (SISSA) in Trieste, Italy. Thanks are also due to J.A. Helayël-Neto, E.A. Pereira, and O.M. Del Cima for encouragement.

This work was supported by the Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq) under the contract number 201142/97-0.

APPENDIX A: ALGEBRAIC PROOF OF RENORMALIZABILITY

In this appendix, we sketch a proof of renormalizability of the chiral fermion meson model on the light of the regularization-independent algebraic method.[6]

In the first step, we study the stability of the classical action. For the quantum theory the stability corresponds to the fact that the radiative corrections can be reabsorbed by a redefinition of the initial parameters of the theory. Next, one computes the possible anomalies through an analysis of the Wess-Zumino condition, then one checks if the possible breakings induced by radiative corrections can be fine-tuned by a suitable choice of non-invariant local counterterms.

1. Stability

In order to study the stability of the model under radiative corrections, we introduce an infinitesimal perturbation in the classical action Σ by means of a integrated local functional $\tilde{\Sigma}$ that satisfies the constraint of a quantum correction

[6]In fact, this has already been considered in [13] via Bogoliubov-Parasiuk-Hepp-Zimmermann (BPHZ) renormalization scheme and recently in [26] via ''algebraic'' renormalization for the theory in flat space only. The generalization to curved space is straightforward.

$$\Sigma \rightarrow \Sigma + \epsilon \tilde{\Sigma}, \tag{A1}$$

where ϵ is an infinitesimal parameter.

The perturbed action must satisfy, to the order ϵ, the same equations as Σ:

$$W_{\mathrm{X}}(\Sigma + \epsilon \tilde{\Sigma}) = W_{\mathrm{X}}(\Sigma) + \epsilon W_{\mathrm{X}} \tilde{\Sigma} + O(\epsilon^2) = 0,$$

$$W^a_{\mathrm{iso}}(\Sigma + \epsilon \tilde{\Sigma}) = W^a_{\mathrm{iso}}(\Sigma) + \epsilon W^a_{\mathrm{iso}} \tilde{\Sigma} + O(\epsilon^2) = 0,$$

$$W^a_{\mathrm{chiral}}(\Sigma + \epsilon \tilde{\Sigma}) = W^a_{\mathrm{chiral}}(\Sigma) + \epsilon W^a_{\mathrm{chiral}} \tilde{\Sigma} + O(\epsilon^2) = \Delta^a_{\mathrm{class}}, \tag{A2}$$

with X = Lorentz, diff.

To first order in ϵ, one obtains

$$W_{\mathrm{X}} \tilde{\Sigma} = 0, \quad W^a_{\mathrm{iso}} \tilde{\Sigma} = 0, \quad W^a_{\mathrm{chiral}} \tilde{\Sigma} = 0, \tag{A3}$$

consequently all counterterms required by renormalization have to be symmetric.

Let us look for the most general invariant counterterm $\tilde{\Sigma}$, i.e., the most general field polynomial of UV dimension $\leqslant 4$, respecting parity and charge conjugation symmetries and the conditions (A3). An explicit computation, shows that $\tilde{\Sigma}$ can be written in the following way:

$$\tilde{\Sigma} = \int d^4 x e \sum_{i=1}^{5} a_i \mathcal{P}_i(x), \tag{A4}$$

where

$$\mathcal{P}_1 = \bar{\psi} i \gamma^\mu \mathcal{D}_\mu \psi, \quad \mathcal{P}_2 = (\partial_\mu \sigma \partial^\mu \sigma + \partial_\mu \pi^a \partial^\mu \pi^a),$$

$$\mathcal{P}_3 = ((\sigma + v)^2 + \pi^a \pi^a), \quad \mathcal{P}_4 = ((\sigma + v)^2 + \pi^a \pi^a)^2,$$

$$\mathcal{P}_5 = \bar{\psi}((\sigma + v) + i \gamma^5 \tau^a \pi^a) \psi,$$

with $a_1, \ldots, a_5$ arbitrary coefficients. We have neglected terms such as $\int d^4 e R(\sigma^2 + \pi^a \pi^a)$, which do not contribute in the limit of flat space.

The arbitrary coefficients are fixed in such a way that they hold order by order in perturbation theory by normalization conditions. Considering the physical parametrization adopted in the main text and since we will have a particle interpretation only if the vaccum expectation value of the fields vanish, we impose the following nonsingular system of normalization conditions:

$$\Gamma_\sigma = 0, \quad \frac{\partial}{\partial p^2} \Gamma_{\sigma\sigma} \bigg|_{p^2 = \kappa} = 1, \quad \frac{\partial}{\partial \not{p}} \Gamma_{\bar{\psi}\psi} \bigg|_{\not{p} = \kappa} = 1,$$

$$\Gamma_{\sigma\sigma}|_{p^2 = m^2_\sigma} = 0, \quad \Gamma_{\bar{\psi}\psi}|_{\not{p} = m_\psi} = 0, \tag{A5}$$

where κ is a energy scale and $p(\kappa)$ some reference set of 4-momenta at this scale. m^2_π is determined through the chiral Ward identity (10). With the normalization conditions (A5), the most general action becomes identical to the action (16).

2. Anomalies

Because the classical stability does not imply in general the possibility of extending the theory to the quantum level, our second task is to infer for possible anomalies. Then, a generating functional for vertex functions, Γ, is constructed

$$\Gamma = \Sigma + O(\hbar), \tag{A6}$$

such that

$$W_{\mathrm{X}} \Gamma = 0, \quad W^a_{\mathrm{iso}} \Gamma = 0, \quad W^a_{\mathrm{chiral}} \Gamma = \Delta^a_{\mathrm{class}}. \tag{A7}$$

The validity to all orders of the Ward identities of diffeomorphisms and local Lorentz will be assumed in the following. In fact, in the absence of gauge fields, these anomalies can exist only in $D = 4k + 2$ dimensions, with ($k = 0,1,2, \ldots$), represented by a local polynomial in the curvature only (see [27] and references cited therein).[7]

It remains now to show the possibility of implementing the isospin and chiral Ward identities for the vertex functional Γ. The proof is recursive. We shall admit the assumption that there exists a vertex functional $\Gamma^{(n-1)}$ obeying the Ward identities (A7) until the order $n-1$ in $\hbar$,

$$W^a_{\mathrm{iso}} \Gamma^{(n-1)} = O(\hbar^n), \tag{A8}$$

$$W^a_{\mathrm{chiral}} \Gamma^{(n-1)} = \Delta^a_{\mathrm{class}} + O(\hbar^n). \tag{A9}$$

As a result of QAP [17], the forms (A8) and (A9) will be broken at the n-order as follows:

$$W^a_{\mathrm{iso}} \Gamma^{(n-1)} = \hbar^n \Delta \cdot \Gamma = \hbar^n \Delta^a_{\mathrm{iso}} + O(\hbar^{n+1}), \tag{A10}$$

$$W^a_{\mathrm{chiral}} \Gamma^{(n-1)} = \Delta^a_{\mathrm{class}} + \hbar^n \Delta \cdot \Gamma = \Delta^a_{\mathrm{class}} + \hbar^n \Delta^a_{\mathrm{chiral}} + O(\hbar^{n+1}), \tag{A11}$$

where Δ^a_{iso} and $\Delta^a_{\mathrm{chiral}}$, are integrated local functionals with UV dimension $\leqslant 4$.

Because of the invariance under parity P and charge conjugation C, the Ward operators and the quantum breakings satisfy the following properties.

(i) Parity P:

$$W^a_{\mathrm{iso}} \xrightarrow{P} W^a_{\mathrm{iso}}, \qquad \Delta^a_{\mathrm{iso}} \xrightarrow{P} \Delta^a_{\mathrm{iso}},$$

$$W^a_{\mathrm{chiral}} \xrightarrow{P} -W^a_{\mathrm{chiral}}, \qquad \Delta^a_{\mathrm{chiral}} \xrightarrow{P} -\Delta^a_{\mathrm{chiral}}. \tag{A12}$$

(ii) Charge conjugation C:

[7] See e.g. [28] where the authors study the cohomology problem of the overall local symmetry group of theories with external gravity, including diffeomorphisms, local Lorentz and gauge transformations, in order to determine all possible anomalies.

$$W^{1,3}_{\rm iso}\stackrel{C}{\rightarrow}-W^{1,3}_{\rm iso},\quad \Delta^{1,3}_{\rm iso}\stackrel{C}{\rightarrow}-\Delta^{1,3}_{\rm iso},$$

$$W^{2}_{\rm iso}\stackrel{C}{\rightarrow}W^{2}_{\rm iso},\qquad \Delta^{2}_{\rm iso}\stackrel{C}{\rightarrow}\Delta^{2}_{\rm iso},$$

$$W^{1,3}_{\rm chiral}\stackrel{C}{\rightarrow}W^{1,3}_{\rm chiral},\qquad \Delta^{1,3}_{\rm chiral}\stackrel{C}{\rightarrow}\Delta^{1,3}_{\rm chiral},$$

$$W^{2}_{\rm chiral}\stackrel{C}{\rightarrow}-W^{2}_{\rm chiral},\qquad \Delta^{2}_{\rm chiral}\stackrel{C}{\rightarrow}-\Delta^{2}_{\rm chiral}. \tag{A13}$$

Using the commutation relation $[\tau^a,\tau^b]=2i\epsilon^{abc}\tau^c$, it is easy to check that the Ward operators obey the following commutation rules of the Lie algebra:

$$[W^a_{\rm iso},W^b_{\rm iso}]\mathcal{F}=-\epsilon^{abc}W^c_{\rm iso}\mathcal{F},$$
$$[W^a_{\rm iso},W^b_{\rm chiral}]\mathcal{F}=-\epsilon^{abc}W^c_{\rm chiral}\mathcal{F}, \tag{A14}$$
$$[W^a_{\rm chiral},W^b_{\rm chiral}]\mathcal{F}=-\epsilon^{abc}W^c_{\rm iso}\mathcal{F},$$

with $\mathcal{F}$ an arbitrary functional.

Applying the algebraic structure above displayed to the vertex functional, we obtain the Wess-Zumino consistency conditions [29]

$$W^a_{\rm iso}\Delta^b_{\rm iso}-W^b_{\rm iso}\Delta^a_{\rm iso}=-\epsilon^{abc}\Delta^c_{\rm iso},$$
$$W^a_{\rm iso}\Delta^b_{\rm chiral}-W^b_{\rm chiral}\Delta^a_{\rm iso}=-\epsilon^{abc}\Delta^c_{\rm chiral}, \tag{A15}$$
$$W^a_{\rm chiral}\Delta^b_{\rm chiral}-W^b_{\rm chiral}\Delta^a_{\rm chiral}=-\epsilon^{abc}\Delta^c_{\rm iso}.$$

Solving constraints such as Eq. (A15) is technically known as a problem of Lie algebra cohomology. Its solution can always be written as a sum of a trivial cocycle $W^a_{\rm iso(chiral)}\Delta$, and of nontrivial elements belonging to the cohomology of $W^a_{\rm iso(chiral)}$:

$$\Delta^a_{\rm iso(chiral)}=\mathcal{A}^a_{\rm iso(chiral)}+W^a_{\rm iso(chiral)}\Delta. \tag{A16}$$

As it is well known the theory will be anomaly free if conditions (A15) admit only the trivial solution

$$\Delta^a_{\rm iso(chiral)}=W^a_{\rm iso(chiral)}\Delta, \tag{A17}$$

with Δ an integrated local functional even under parity and charge conjugation. On the other hand, nontrivial cocycles, i.e.,

$$\mathcal{A}^a_{\rm iso(chiral)}\neq W^a_{\rm iso(chiral)}\Delta, \tag{A18}$$

cannot be reabsorbed as local counterterms and represent an obstruction in order to have an invariant quantum vertex functional.

A direct inspection shows that there is no such a functional satisfying Eqs. (A12) and (A13) for $\Delta^a_{\rm iso}$. Hence, its cohomology is empty. On the other hand, the chiral breaking, $\Delta^a_{\rm chiral}$, can be expanded in the basis

$$\int d^4x e(\pi^a,\pi^a\sigma^2,\pi^a\sigma^3,\partial_\mu\pi^a\partial^\mu\sigma,\bar{\psi}\gamma^5\tau^a\psi,\pi^a\bar{\psi}\psi), \tag{A19}$$

parity and charge conjugation being taken into account.

The consistency conditions (A15) reduce this basis to

$$\int d^4x e(\pi^a,\bar{\psi}\gamma^5\tau^a\psi). \tag{A20}$$

Such a basis can be obtained by applying $W^a_{\rm chiral}$ to

$$\int d^4x e(\sigma,\bar{\psi}\psi), \tag{A21}$$

i.e., it can be reabsorbed as local counterterms.

Denoting the latter field monomials by Δ_i, we can write (A11) as

$$W^a_{\rm chiral}\Gamma^{(n-1)}=\Delta^a_{\rm class}+\hbar^n\Delta\cdot\Gamma$$
$$=\Delta^a_{\rm class}+\hbar^n W^a_{\rm chiral}\Delta+O(\hbar^{n+1}), \tag{A22}$$

where $\Delta=\Sigma_{i=1}^2 r^i\Delta_i$.

Defining $\Sigma^{(n-1)}$ the action, with all its couterterms until the order $n-1$, which leads to the functional $\Gamma^{(n-1)}$, then replacing the action $\Sigma^{(n-1)}$ by the new action

$$\Sigma^{(n)}=\Sigma^{(n-1)}-\hbar^n\Delta, \tag{A23}$$

lead to the new vertex functional

$$\Gamma^{(n)}=\Gamma^{(n-1)}-\hbar^n\Delta+O(\hbar^{n+1}). \tag{A24}$$

Thus, with the results obtained above, we get

$$W^a_{\rm iso}\Gamma^{(n)}=O(\hbar^{n+1}), \tag{A25}$$

$$W^a_{\rm chiral}\Gamma^{(n)}=\Delta^a_{\rm class}+O(\hbar^{n+1}), \tag{A26}$$

which is the next order Ward identities we wanted to prove.

APPENDIX B: CALLAN-SYMANZIK EQUATION

In this appendix we wish to derive the CS equation. This allows us to identify the coefficients β and γ of the expression (47) with those of the CS equation, when we take the limit of flat space-time. Our starting point is Eq. (29), the broken dilatation Ward identity, with Σ replaced by $\Sigma^\natural$

$$W_{\rm D}\Sigma^\natural|_{\eta=\rho=0}=\int d^4x\sum_{\Phi=\sigma,\pi^a,\psi}[(d_\Phi+x\cdot\partial)\Phi]\frac{\delta\Sigma^\natural}{\delta\Phi}\bigg|_{\eta=\rho=0}$$
$$=\Lambda. \tag{B1}$$

With the help of Eq. (35), we can define the ‘‘symmetrized’’ form of Eq. (B1)

$$\left(W_{\rm D}-\int d^4x\left(v\frac{\delta}{\delta\sigma}+\frac{\delta}{\delta\eta}+a\frac{\delta}{\delta\rho}\right)\right)\Sigma^\natural|_{\eta=\rho=0}=0. \tag{B2}$$

Applying QAP one derives from Eq. (B2) that the dilatations in higher order are broken by

$$\left(W_D - \int d^4x \left(v\,\frac{\delta}{\delta\sigma} + \frac{\delta}{\delta\eta} + a\,\frac{\delta}{\delta\rho} \right)\right)\Gamma^{\natural}|_{\eta=\rho=0} = \Delta \cdot \Gamma^{\natural}|_{\eta=\rho=0} = \Delta + O(\hbar), \tag{B3}$$

where $\Gamma^{\natural}$ is the vertex functional defined in Eq. (41). Δ represents the breaking in the lowest order.

According to the fact that the left-hand side of Eq. (B3) is symmetric with respect to isospin and chiral symmetries, one gets

$$W^a_{\rm iso}\Delta = W^a_{\rm chiral}\Delta = 0. \tag{B4}$$

The invariant insertion Δ can be expanded in a suitable basis of symmetric operators of the theory—parity and charge conjugation being taken into account. Assuming the physical parametrization, this basis is given by set of operators (45), yielding

$$\left(W_D - \int d^4x \left(v\,\frac{\delta}{\delta\sigma} + \frac{\delta}{\delta\eta} + a\,\frac{\delta}{\delta\rho} \right)\right)\Gamma^{\natural}|_{\eta=\rho=0} = \Bigg(\beta_{m_\psi} m_\psi \frac{\partial}{\partial m_\psi} + \beta_v v \frac{\partial}{\partial v} - \gamma_\psi \mathcal{N}_\psi - \gamma_\Phi \mathcal{N}_\Phi - \beta_v \int d^4x v\,\frac{\delta}{\delta\sigma} + \delta \int d^4x a\,\frac{\delta}{\delta\rho} + \beta_v \int d^4x \frac{\delta}{\delta\eta}\Bigg)\Gamma^{\natural}|_{\eta=\rho=0}, \tag{B5}$$

where $\mathcal{N}_\psi$ and $\mathcal{N}_\Phi$ are counting operators given by Eqs. (22) and (46), respectively.

The latter can be rewritten in a more explicit form with the help of the dimensional analysis identity

$$(\mathcal{D} + W_D)\Gamma^{\natural}|_{\eta=\rho=0} = 0, \tag{B6}$$

where

$$\mathcal{D} = \sum_{\mu = \kappa, v, m_\sigma, m_\pi, m_\psi} \mu \frac{\partial}{\partial \mu}, \tag{B7}$$

with κ the mass scale at which the normalization conditions defining the parameters of the quantum theory are taken.

This yields the Callan-Symanzik equation

$$\left(\mathcal{D} + \beta_{m_\psi} m_\psi \frac{\partial}{\partial m_\psi} + \beta_v v \frac{\partial}{\partial v} - \gamma_\psi \mathcal{N}_\psi - \gamma_\Phi \mathcal{N}^{\rm hom}_\Phi \right)\Gamma^{\natural}|_{\eta=\rho=0} = \Bigg(-(1-\beta_v-\gamma_\Phi)\int d^4x v \frac{\delta}{\delta\sigma} - (1+\delta)\int d^4x a\,\frac{\delta}{\delta\rho} - (1+\beta_v)\int d^4x \frac{\delta}{\delta\eta}\Bigg)\Gamma^{\natural}|_{\eta=\rho=0}, \tag{B8}$$

where $\mathcal{N}^{\rm hom}_\Phi$ is given by Eq. (23).

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