Discontinuous behavior of perturbative Yang-Mills theories in the limit of dimensions $D \to 2$

A. Bassetto
Dipartimento di Fisica ''G. Galilei'' Università di Padova, Via Marzolo 8, 35131 Padova, Italy and INFN, Sezione di Padova, Italy

R. Begliuomini and G. Nardelli
Dipartimento di Fisica, Università di Trento, 38050 Povo (Trento), Italy and INFN, Gruppo Collegato di Trento, Italy

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We calculate in $D=2+\epsilon$ dimensions and in the light-cone gauge (LCG) the perturbative $\mathcal{O}(g^4)$ contribution to a rectangular Wilson loop in the (t,x) plane coming from diagrams with a self-energy correction in the vector propagator. In the limit $\epsilon \to 0$ the result is finite, in spite of the vanishing of the triple vector vertex in LCG, and provides the expected agreement with the analogous calculation in the Feynman gauge.
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I. INTRODUCTION

$SU(N)$ Yang-Mills (YM) theories exhibit peculiar and interesting features in $1+1$ dimensions $(D=2)$. The reduction from four to lower dimensions entails indeed tremendous simplifications, so that many problems can be faced, and often exactly solved [1–3]. For instance exact evaluations of vacuum to vacuum amplitudes of Wilson loop operators, that, for a suitable choice of contour and in a particular limit, provide the potential between a static $q\bar{q}$ pair [4–6], can be obtained.

YM theories without fermions in $1+1$ dimensions are considered free theories, apart from topological effects. This feature looks apparent when choosing an axial gauge. However, either when matter fields are introduced, or in Wilson loop calculations, the perturbative $(1+1)$-dimensional theory exhibits dramatic infrared (IR) singularities which need to be regularized. Unfortunately the results appear to be dependent on such regularization procedures, even when they concern gauge invariant quantities [7].

In the light cone gauge (LCG) the (IR) singular behavior is particularly apparent in the vector propagator, where the gauge pole conspires with the usual Feynman singularity to produce a double pole [8].

A Cauchy principal value (CPV) prescription for this IR singularity has often been advocated [9]. It emerges quite naturally if the theory is quantized on the light cone surface $x^+=0$ [10].

On the other hand, such a recipe is at odds with Wick's rotation. In Ref. [11] a causal prescription for the double pole has been proposed, which is nothing but the one suggested years later by Mandelstam and Leibbrandt (ML) [12], when restricted to $1+1$ dimensions. This prescription follows from equal-time quantization [13] and is mandatory in order to renormalize the theory in $1+3$ dimensions [14,10].

In view of the above-mentioned results and of the fact that ''pure'' YM theories do not immediately look free in Feynman gauge, a systematic investigation has been undertaken to clarify their properties when the two dimensional picture is reached starting from higher dimensions.

Since no exact solutions are available beyond $D=2$, the investigation has been focused on perturbative calculations, looking for consistency checks, in particular testing the gauge invariance of the theory which holds order by order in the coupling constant expansion.

Recalling that perturbative S-matrix elements cannot be consistently defined in non-Abelian gauge theories, owing to their (IR) singular mass-shell behavior, the natural gauge invariant quantities to be considered are Wilson loops.

A first test of gauge invariance in $1+3$ dimensions has been performed in Refs. [15,16] by calculating at $\mathcal{O}(g^4)$, both in Feynman and in light-cone gauge with ML prescription, a rectangular Wilson loop with lightlike sides, directed along the vectors $n_\mu=(T,-T)$, $n^*_\mu=(L,L)$ and parametrized according to the equations

$$\begin{aligned} C_1 &: x^\mu(t)=n^{*\mu}t,\\ C_2 &: x^\mu(t)=n^{*\mu}+n^\mu t,\\ C_3 &: x^\mu(t)=n^\mu+n^{*\mu}(1-t),\\ C_4 &: x^\mu(t)=n^\mu(1-t), \quad 0\leq t\leq 1. \end{aligned} \tag{1}$$

In order to perform the test, dimensional regularization $(D=2\omega)$ was used for both UV and IR singularities. Full consistency between Feynman and light-cone gauge with the ML prescription was obtained.

Since results in 2ω dimensions were available, in view of the peculiar features of Yang-Mills theories in 2 dimensions mentioned above, the interest arose in knowing the outcome of the check in the limit $\omega \to 1$. The following unexpected results were obtained in [17].

The $\mathcal{O}(g^4)$ perturbative loop expression in $d=1+(D-1)$ dimensions is *finite* in the limit $D\to 2$. The loop expression is a function only of the area $n\cdot n^*$ for any dimension D and exhibits also a dependence on C_A, the Casimir constant of the adjoint representation.

In LCG this dependence comes from two sources: diagrams with two crossed propagators [color factor $C_F(C_F - C_A/2)$, C_F being the Casimir constant of the fundamental representation]; a genuine contribution to the Wilson loop proportional to $C_F C_A$ coming from the one-loop correction to the vector propagator (self-energy diagram).

We shall concentrate our interest on the contribution due to this self-energy diagram. At a first sight, it is surprising, since, in $1+1$ dimensions, there is no triple vector vertex in axial gauges. What happens is that the vanishing strength of the vertex at $D=2$ matches the self-energy loop singularity, eventually producing a finite result. Feynman diagrams with a triple vertex but no loops tend instead smoothly to zero when inserted in the Wilson contour.

We notice that no ambiguity affects the $\mathcal{O}(g^4)$ gauge invariant result, which is finite; in addition the presence of C_A cannot be reabsorbed by a redefinition of the coupling, that, while unjustified on general grounds, would also turn out to be dependent on the area of the loop.

In order to clarify whether the appearance of C_A in the maximally non-Abelian term is indeed a pathology, one should examine the potential $V(2L)$ between a ''static'' $q\bar{q}$ pair in the fundamental representation, separated by a distance $2L$. Therefore in Ref. [18] we have considered a different Wilson loop, *viz* a rectangular loop with one side along the space direction and one side along the time direction, of length $2L$ and $2T$, respectively. Eventually the limit $T\to\infty$ at fixed L is to be taken: the potential $V(2L)$ between the quark and the antiquark is indeed related to the value of the corresponding Wilson loop amplitude $\mathcal{W}(L,T)$ through the equation [19]

$$\lim_{T\to\infty} \mathcal{W}(L,T) = \text{const} \times e^{-2iTV(2L)}. \tag{2}$$

The crucial point to notice in Eq. (2) is that dependence on the Casimir constant C_A should cancel at the leading order when $T\to\infty$ in any coefficient of a perturbative expansion of the potential with respect to coupling constant. This criterion has often been used as a check of gauge invariance [10].

In Ref. [18] the calculation has been performed in Feynman gauge, obtaining the following results.

For $D>2$ the $\mathcal{O}(g^4)$ perturbative expression of the loop depends, besides on the area, also on the ratio $\beta = L/T$. As we are eventually interested in the large-T behavior, we have always considered the region $\beta<1$; moreover we have chosen $D=2+\epsilon$ with a small $\epsilon>0$.

As long as $D>2$, agreement with Abelian-like time exponentiation (ALTE) occurs in the limit $T\to\infty$, with a pure C_F dependence in the leading coefficient. Consistency of all previous results [10] in higher dimensions is thus reestablished.

The limit $D\to 2$ for $\beta=0$ *exactly* reproduces the gauge invariant result obtained in Ref. [17] for a loop of the same area with lightlike sides; thereby we enforce the argument that in two dimensions a pure area behavior is expected, no matter the orientation and the shape of the loop. What may be surprising is that the term, which in LCG corresponds to the self-energy correction, exhibits, in the limit, a pure area dependence on its own.

However, in two dimensions at $\mathcal{O}(g^4)$, a C_A dependence is definitely there and agreement with ALTE is lost. Actually this behavior at $D=2$ persists at any order of g and affects the sum of the perturbative series [20,3].

A peculiar feature of the light-cone gauge in 2 dimensions is that individual Wilson loop diagrams do not exhibit any singularity; hence there is no need of dimensional regularization.

In Ref. [21], a $\mathcal{O}(g^4)$ perturbative calculation of the Wilson loop in LCG with ML prescription, for a rectangular loop with sides $2T\times 2L$ lying in the $x^0\times x^1$ axes, was performed at $D=2$. No agreement occurs with the result one finds in Ref. [18] when taking the limit $D\to 2$. The source of such a discrepancy is rooted in the mentioned self-energy diagram contribution, which is obviously missing at $D=2$, but provides a finite term in the limit $D\to 2$, thereby producing a discontinuity in the theory [17].

The purpose of this paper is to check explicitly this property by evaluating in LCG the relevant discontinuity for the Wilson loop of Ref. [21]. We confirm that the missing term comes from the diagram with a self-energy corrected propagator, evaluated at $D=2+\epsilon$, when eventually taking the limit $\epsilon\to 0$. We thereby reproduce for a space-time contour the phenomenon in LCG found in Ref. [17] for a contour with lightlike sides. Actually, from the computation of the self-energy diagram at $D>2$, we find, as an extra bonus, that its contribution vanishes for $\epsilon>0$ in the limit $T\to\infty$ with the same ''universal'' factor $T^{4-4\omega}$ we have obtained in Ref. [18] for the maximally non-Abelian contributions [22].

The limits $T\to\infty$ and $\epsilon\to 0$ *do not commute*.

II. THE CALCULATION

We recall some basic notions and notations. We consider, as in Ref. [18], the closed path γ parametrized by the following four segments γ_i:

$$\begin{aligned}
&\gamma_1 : \gamma_1^\mu(s) = (sT, L),\\
&\gamma_2 : \gamma_2^\mu(s) = (T, -sL),\\
&\gamma_3 : \gamma_3^\mu(s) = (-sT, -L),\\
&\gamma_4 : \gamma_4^\mu(s) = (-T, sL), \quad -1 \leqslant s \leqslant 1
\end{aligned} \tag{3}$$

describing a (counterclockwise-oriented) rectangle centered at the origin of the plane (x^1, x^0), with length sides $(2L, 2T)$, respectively.

The perturbative expansion of the Wilson loop is

$$\begin{aligned}
\mathcal{W}_\gamma(L,T) = 1 + \frac{1}{N}\sum_{n=2}^{\infty} (ig)^n & \\
\times \oint_\gamma dx_1^{\mu_1} \cdots \oint_\gamma dx_n^{\mu_n} \theta(x_1 > \cdots > x_n) & \\
\times \mathrm{Tr}[G_{\mu_1 \cdots \mu_n}(x_1, \cdots, x_n)], &
\end{aligned} \tag{4}$$

where $G_{\mu_1 \cdots \mu_n}(x_1, \ldots, x_n)$ is the Lie algebra valued n-point Green function, and the Heavyside θ functions order the points $x_1, \ldots, x_n$ along the integration path γ.

It is easy to show that the perturbative expansion of $\mathcal{W}_\gamma$ is an even power series in the coupling constant, so that we can write

$$\mathcal{W}_\gamma(L,T) = 1 + g^2 \mathcal{W}_2 + g^4 \mathcal{W}_4 + \mathcal{O}(g^6). \tag{5}$$

To have a sensitive check of gauge invariance, one has to consider at least the order g^4, (i.e., one has to evaluate $\mathcal{W}_4$), as this is the lowest order where genuinely non-Abelian $C_F C_A$ contributions may appear. In turn, in the calculation of $\mathcal{W}_4$, only the so-called maximally non-Abelian contribution $\mathcal{W}_4^{na}$ need to be evaluated, that in our case comes from the terms proportional to $C_F C_A$. The Abelian contribution, proportional to C_F^2, can be easily obtained thanks to the Abelian exponentiation theorem [22].

The diagrams contributing to $\mathcal{W}_4^{na}$ can be grouped into three families: (a) crossed diagrams ($\mathcal{C}_{(ij)(kl)}$), with a double gluon exchange in which the two (crossed) propagators join the sides (ij) and (kl) of the contour γ; (b) spider diagrams ($\mathcal{S}_{ijk}$), which are obtained by attaching a three point Green function at the tree level to the sides (ijk) of the loop; (c) bubble diagrams ($\mathcal{B}_{ij}$), that are single exchange diagrams in which the gluon propagator, corrected by a self-energy term, joins the sides (ij) of the contour.

In arbitrary dimensions, the calculation of the Wilson loop is much more awkward in LCG than in covariant gauge, due to a more complicated form of the vector propagator. However, when considering the $D \to 2$ limit, diagrams in LCG have much better analyticity properties in ω than the ones in Feynman gauge. The vector propagator in LCG with ML prescription is a tempered distribution at $D=2$, at odds with the one in Feynman gauge. Moreover it is summable along the (compact) loop contour.

Due to this property, we can conclude that all the maximally non-Abelian contributions arising from diagrams with crossed propagators sum to an expression that, in the limit $D \to 2$, reproduces the result of Ref. [21], namely,

$$\mathcal{W}^{cr} = C_A C_F \frac{(LT)^2}{3}. \tag{6}$$

Now we consider the contribution $\mathcal{W}^{bub}$ coming from bubble diagrams. In LCG and on the plane $x^0 \times x^1$, the only nonvanishing component of the two point Green function $\Delta_{\mu\nu}$ at the order $\mathcal{O}(g^2)$ is $\Delta_{++}(x) \equiv \Delta(x)$, that reads, at $x_\perp = 0$ [16],

$$\Delta(x) = -\frac{g^2}{8\pi^{2\omega}} C_A \frac{(x^-)^2}{(-x^2 + i\varepsilon)^{2\omega-2}} f(\omega), \tag{7}$$

$$f(\omega) = \frac{1}{(2-\omega)^3}\left[\frac{\Gamma^2(3-\omega)\Gamma(2\omega-3)}{\Gamma(5-2\omega)} - \frac{\Gamma(\omega-1)\Gamma(\omega)(10\omega^2 - 19\omega + 10)}{4(2\omega-3)(2\omega-1)}\right]. \tag{8}$$

Following the notations of Ref. [18], there are 10 topologically inequivalent bubble diagrams. However, due to the symmetry of the Green function and to the symmetric choice of the contour, only six of them are independent, and the $\mathcal{O}(g^4)$ contribution to the Wilson loop arising from bubble diagrams can be written as

$$\mathcal{W}^{bub} = 2(\mathcal{B}_{11} + \mathcal{B}_{22} + \mathcal{B}_{13} + \mathcal{B}_{24} + 2\mathcal{B}_{12} + 2\mathcal{B}_{14}), \tag{9}$$

where each single contribution $\mathcal{B}_{ij}$ can be calculated by replacing Eqs. (3), (7) in the formula

$$\mathcal{B}_{ij} = -\frac{1}{2} g^2 C_F \int_{-1}^{1} ds \int_{-1}^{1} dt \Delta_{\mu\nu}(\gamma_i(s) - \gamma_j(t)) \dot{\gamma}_i^\mu(s) \dot{\gamma}_j^\nu(t), \tag{10}$$

where the dot denotes derivative with respect to the variable parametrizing the segment.

The calculation being standard, we shall report only the final result

$$\begin{aligned}
\mathcal{W}^{bub} = {} & \frac{C_F C_A}{\pi^{2\omega}} f(\omega)(LT)^2 (2L)^{4-4\omega}\Bigg\{ e^{-2i\pi\omega}\beta^{4\omega-6}\Bigg[\frac{1}{(7-4\omega)(8-4\omega)}(1-(8-4\omega)\,{}_2F_1(2\omega-2,2\omega-7/2;2\omega-5/2;\beta^2) \\
& + (7-4\omega)(1-\beta^2)^{3-2\omega}) - \frac{1}{(3-2\omega)(4-2\omega)}(1-(1-\beta^2)^{4-2\omega}) + \frac{5-2\omega}{(6-4\omega)(4-2\omega)}(1-(1-\beta^2)^{3-2\omega})\Bigg] \\
& + e^{-2i\pi\omega}\beta^{4\omega-4}\Bigg[\frac{(1-\beta^2)^{3-2\omega}}{(3-2\omega)(4-2\omega)} - \frac{{}_2F_1(2\omega-2,2\omega-5/2;2\omega-3/2;\beta^2)}{(5-4\omega)} - {}_2F_1(2\omega-2,1/2;3/2;\beta^2)\Bigg] \\
& + i\beta\frac{\sqrt{\pi}(\omega-2)\Gamma(2\omega-7/2)}{\Gamma(2\omega-2)} - e^{-2i\pi\omega}\frac{\beta^{4\omega-2}}{3}\,{}_2F_1(2\omega-2,3/2;5/2;\beta^2) + \frac{\beta^2}{(7-4\omega)}\Bigg\},
\end{aligned} \tag{11}$$

where $\beta = L/T$.

Some comments are here in order. First of all there is a dependence on the dimensionless ratio β, besides the area, at variance with the analogous result in LCG for the rectangle of lightlike sides. However, in the equation above, one can easily check that the quantity $\mathcal{W}^{bub}/(LT)^2$ is not singular for $\beta \to 0$. Actually Eq. (11) exhibits, for $\omega > 1$, the expected damping factor $T^{4-4\omega}$ in the large-T limit.

In the limit $\omega \to 1$ the dependence on β disappears and the pure area law is recovered: $\mathcal{W}^{bub} = C_F C_A (LT/\pi)^2$. This is exactly the ''missing'' term to be added to the expression of Ref. [21] to obtain the final result for the maximally non-Abelian contribution to the perturbative $\mathcal{O}(g^4)$ Wilson loop in the limit $D \to 2$,

$$\mathcal{W}_4^{na} = C_F C_A \left(\frac{LT}{\pi}\right)^2 \left[1 + \frac{\pi^2}{3}\right]. \qquad (12)$$

Equation (12) is in full agreement not only with Ref. [18], where an anologous Wilson loop was calculated in Feynman gauge, but also with Ref. [17], where the loop was oriented in a different direction. Moreover, in LCG, different families of diagrams (''crossed'' and ''bubble'' diagrams) give the same contribution $[C_F C_A ((LT)^2/3)$ and $C_F C_A (LT/\pi)^2$ respectively] no matter the orientation of the loop: remarkably, invariance under area-preserving diffeomorphisms is recovered in the limit $D \to 2$, even when the Wilson loop is first evaluated in higher dimensions, and then the limit $D \to 2$ is taken.

In turn the result above implies that ''spider'' diagrams, namely diagrams with a triple vector vertex, cannot contribute in the limit $D \to 2$. This is not surprising, as the same phenomenon occurred in Ref. [17], although for a different contour (contour with lightlike sides).

In order to support this conclusion, we show that the relevant three point Green function at $\mathcal{O}(g)$, vanishes when $D \to 2$.

To this aim, let us consider the three point Green function $\mathcal{V}_{\mu\nu\rho}(x,y,z)$. Because of the LCG choice, its only nonvanishing component when considering the loop in the $x^0 \times x^1$ plane is $\mathcal{V}(x,y,z) = \mathcal{V}_{+++}(x,y,z)$; up to an irrelevant multiplicative constant, it is given by

$$\begin{aligned}\mathcal{V}(x,y,z) &= \int d^{2\omega}\zeta \frac{\partial}{\partial z^\alpha}\left[\frac{\partial}{\partial x^\alpha}\frac{\partial}{\partial y^+} - \frac{\partial}{\partial y^\alpha}\frac{\partial}{\partial x^+}\right] \\ &\quad \times F(x-\zeta)F(y-\zeta)G(z-\zeta) + \text{cycl. perm.}\{x,y,z\} \\ &\equiv (\mathcal{V}_1 - \mathcal{V}_2) + \text{cycl. perm.}\{x,y,z\}. \end{aligned} \qquad (13)$$

Here the index α runs over the transverse components and the functions G and F are the following Fourier transforms:

$$G(x) = \int d^{2\omega}p \frac{e^{ipx}}{p^2 + i\varepsilon} = -\pi^\omega \Gamma(\omega - 1)\left(-\frac{x^2}{4} + i\varepsilon\right)^{1-\omega}, \qquad (14)$$

$$\begin{aligned}F(x) &= \int d^{2\omega}p \frac{e^{ipx}}{(p^2 + i\varepsilon)(p^+ + i\varepsilon p^-)} \\ &= -i\pi^\omega \Gamma(\omega-1)\int_0^{x_+} d\rho \left(\frac{x_\perp^2 - 2x_-\rho}{4} + i\varepsilon\right)^{1-\omega}. \end{aligned} \qquad (15)$$

Let us consider, for instance, the first term in Eq. (13), that we call $\mathcal{V}_1$. Using standard Feynman integrals techniques, integrations over momenta and over the intermediate point ζ can be performed, so that $\mathcal{V}_1$ can be rewritten, after some convenient change of variables, as

$$\begin{aligned}\mathcal{V}_1 &= \frac{i\pi^\omega (4\pi)^{3\omega}}{8}\Gamma(2\omega-1)(\omega-1)\int_0^1 d\xi d\eta d\mu\, \eta[\mu(1-\mu)]^{\omega-1} \\ &\quad \times \int_0^\infty d\tau \frac{[1+\tau(\mu\xi + \eta(1-\mu))]^{2\omega-5}}{(1+\tau)^\omega} \frac{[(x-z)_+ + \tau\eta(1-\mu)(x-y)_+][(y-z)_+ + \tau\mu\xi(y-x)_+]^2}{[-\mu\xi(x-z)^2 - \eta(1-\mu)(y-z)^2 - \tau\xi\eta\mu(1-\mu)(x-y)^2 + i\varepsilon]^{2\omega-1}}. \end{aligned} \qquad (16)$$

Since $\mathcal{V}_1$ has an explicit zero at $\omega = 1$, if we show that the integral in Eq. (16) is convergent when evaluated at $\omega = 1$, we have proved that the three point Green function vanishes at $D = 2$. Integral (16) is discussed in the Appendix.

III. CONCLUSIONS

A peculiar feature of the light-cone gauge formulation of Yang-Mills theories is that they can be consistently defined in two dimensions: contrary to the covariant Feynman gauge, the light-cone gauge propagator with ML prescription for the spurious pole is a tempered distribution at $D = 2$. In particular, the large T behavior of the Wilson loop can be evaluated without the need of introducing any regulator; the finite result has been presented in Ref. [21]. This result, however, cannot be compared with the result one would obtain in Feynman gauge, as in the latter case, the free propagator is not a tempered distribution at $D = 2$. In Feynman gauge the best one can do is to evaluate the Wilson loop in D dimensions, and to take eventually the limit $D \to 2$.

In so doing one obtains again a finite result [18] that, however, is *different* from the one of Ref. [21]. In LCG the diagram with a self-energy correction in the propagator, which only exists in $D > 2$, makes the difference. It is precisely the contribution we have evaluated in this paper. It provides us with the missing term to get agreement between Refs. [21] and [18], i.e., to recover gauge invariance. Such a phenomenon was not unexpected in the light of Ref. [17]. Perturbative Yang-Mills theory in LCG looks indeed discontinuous in the limit $D\to 2$; actually, starting from a vanishing coupling at $D=2$, it exhibits a kind of ''instability'' with respect to a change of dimensions.

On one hand our result clarifies the nature of the discontinuity of Yang-Mills theories in two dimensions, on the other it raises new interesting questions for future investigations.

While in any dimension $D>2$ *perturbative* Wilson loop calculations are in agreement with Abelian-like time exponentiation, as all C_A dependent terms turn out to be depressed in the large-T limit, at $D=2$ neither the result in Ref. [21] nor the one in [18] share this property, as they both exhibit an explicit C_A dependence in the coefficient of the leading term when $T\to\infty$. At $D=2$ exponentiation in terms of C_F occurs perturbatively only in light-front formulation (Ref. [8]); in equal-time quantization, exponentiation requires full resummation of genuine nonperturbative contributions (instantons) [3].

The difference between the formulations above (and their related vacua) as well as the reason why this phenomenon seems to be crucial only at $D=2$ are under active investigation.

APPENDIX

In this appendix we show that the three point Green function tends to zero when $D\to 2$. As explained in the main text, it is sufficient to prove that the integral in Eq. (16), with the constant containing the simple zero $(\omega-1)$ factorized out, is convergent when evaluated at $\omega=1$. Such an integral, after the change of variables $\alpha=\mu\xi$, $\beta=\eta(1-\mu)$ and after explicit integration over $d\mu$, reads

$$I=\int_0^1 d\alpha d\beta\,\theta(1-\alpha-\beta)\left\{\frac{1-\alpha-\beta}{1-\alpha}+\beta\log\frac{(1-\beta)(1-\alpha)}{\alpha\beta}\right\}\times\int_0^\infty\frac{d\tau}{(1+\tau)}\frac{[(x-z)_+ +\beta\tau(x-y)_+]}{[1+\tau(\alpha+\beta)]^3}\times\frac{[(y-z)_+ +\alpha\tau(y-x)_+]^2}{[-\alpha(x-z)^2-\beta(y-z)^2-\alpha\beta\tau(x-y)^2+i\epsilon]},\tag{A1}$$

θ being the Heavyside function. The most delicate region of this integral is $\alpha\sim\beta\sim 0$, so that in order to check convergence of Eq. (A1) we can restrict ourselves to the case when the curly bracket is replaced by one. After this replacement, we set $\alpha=\rho\sigma$ and $\beta=\rho(1-\sigma)$. In the expression obtained after this change of variables, we rescale $\gamma=\rho\tau$ at fixed τ. The integral over the τ variable can be factorized providing a factor $\log(1+1/\gamma)$. Finally, renaming $\rho=1/\gamma$, Eq. (A1) with the curly bracket replaced by one can be equivalently written as

$$\mathcal{I}=-\int_0^1 d\sigma\int_0^\infty\frac{d\rho}{\rho}\frac{\log(1+\rho)}{(1+\rho)^3}\frac{[\rho(x-z)+(1-\sigma)(x-y)]_+[\rho(y-z)+\sigma(y-x)]_+^2}{[\rho\sigma(x-z)^2+\rho(1-\sigma)(y-z)^2+\sigma(1-\sigma)(x-y)^2-i\epsilon]}\ .\tag{A2}$$

Dividing the ρ integration domain as $[0,1]\cup[1,\infty)$, we split $\mathcal{I}$ as $\mathcal{I}_1+\mathcal{I}_2$. In $\mathcal{I}_1$, $\rho\in[0,1]$ and therefore we can use the following majorations: $\log(1+\rho)<\rho$ and $(1+\rho)^{-3}<1$. Thus, integration in $d\rho$ is straightforward, providing us with the estimate

$$\mathcal{I}_1\simeq-\int_0^1 d\sigma\frac{(x-z)_+(y-z)_+^2}{\sigma(x-z)^2+(1-\sigma)(y-z)^2-i\epsilon}\times\left[\frac{1}{3}+\frac{1}{2}(A-C)+(B-C)^2+B+2AB-AC+(A-C)(B-C)^2\log\left(\frac{1+C}{C}\right)\right],\tag{A3}$$

where A, B and C are defined as

$$A=(1-\sigma)(x-y)_+/(x-z)_+\,,$$

$$B=\sigma(x-y)_+/(z-y)_+\,,$$

$$C=\sigma(1-\sigma)(x-y)^2/[\sigma(x-z)^2+(1-\sigma)(y-z)^2-i\epsilon].\tag{A4}$$

In this form, it is manifest that integration over σ is convergent. The explicit result goes beyond the purpose of the paper, but it can be easily evaluated providing combinations of rational functions, logarithms and dilogarithms.

In $\mathcal{I}_2$, the ρ integration domain is $[1,\infty)$ and therefore we can use $(1+\rho)^{-3}<\rho^{-3}$. Thus, the ρ dependent part of the integrand can be approximated by

$$\frac{(\rho+A)(\rho+B)^2}{(\rho+C)}\frac{\log(1+\rho)}{\rho^4}=\frac{\log(1+\rho)}{\rho(\rho+C)}+\frac{A(\rho+B)^2+\rho(B^2+2\rho B)}{(\rho+C)\rho^3}\frac{\log(1+\rho)}{\rho}.\tag{A5}$$

To check convergence, in the second term of the right-hand side we can replace $\log(1+\rho)/\rho$ by 1. Then, integration over ρ becomes straightforward and the second term in Eq. (A5) provides integrals over $d\sigma$ of the same kind of those in $\mathcal{I}_1$, where convergence can be easily checked. The first term in the right-hand side of Eq. (A5) is more delicate. Here the majoration $\log(1+\rho)<\rho$ is too strong as it would spoil convergence in the ρ integration. An explicit integration over ρ of this term gives

$$\mathcal{I}_2^{first} \simeq \int_0^1 d\sigma \frac{(x-z)_+(y-z)_+^2}{\sigma(x-z)^2+(1-\sigma)(y-z)^2-i\epsilon} \frac{1}{C}\left[\mathrm{Li}\left(\frac{C}{C-1}\right) + \mathrm{Li}(-C) - \log 2 \log\left(\frac{1+C}{1-C}\right) - \mathrm{Li}\left(\frac{2C}{C-1}\right)\right], \quad \text{(A6)}$$

$\mathrm{Li}(z)$ being the dilogarithm function. Although cumbersome, integration over σ is finite.

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