Metric fluctuation corrections to Hawking radiation

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We study how fluctuations of the black hole geometry affect the properties of Hawking radiation. Even though we treat the fluctuations classically, we believe that the results so obtained indicate what might be the effects induced by quantum fluctuations in a self-consistent treatment. To characterize the fluctuations, we use the model introduced by York in which they are described by an advanced Vaidya metric with a fluctuating mass. Under the assumption of spherical symmetry, we solve the equation of null outgoing rays. Then, by neglecting the greybody factor, we calculate the late time corrections to the *s*-wave contributions of the energy flux and the asymptotic spectrum. We find three kinds of modifications. First, the energy flux fluctuates around its average value with amplitudes and frequencies determined by those of the metric fluctuations. Secondly, this average value receives two positive contributions, one of which can be reinterpreted as due to the ''renormalization'' of the surface gravity induced by the metric fluctuations. Finally, the asymptotic spectrum is modified by the addition of terms containing thermal factors in which the frequency of the metric fluctuations acts as a chemical potential. [S0556-2821(99)05812-9]

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I. INTRODUCTION

25 years have passed since Hawking's theoretical discovery of the quantum radiation by black holes [1]. Since then, many aspects of this phenomenon have been investigated. First, the mean value of the energy-momentum tensor has received much attention since it provides the source of the so-called semiclassical Einstein equations (see, e.g., [2,3] and references therein). Hopefully, the solutions of these equations should govern the mean evolution of the evaporating geometry. More recently, more quantum mechanical questions which raise doubts concerning the validity of this semiclassical evolution have also received attention. In particular, the controversial role of arbitrarily large (''trans-Planckian'') frequencies of vacuum fluctuations [4–10] and the gravitational back reaction due to a specific quantum [11,12] have been discussed.

In this paper, we shall consider another aspect: We study how the fluctuations of the black hole horizon geometry might affect the properties of Hawking radiation. To describe these fluctuations quantum mechanically and to determine their effects on Hawking radiation requires full quantum gravity. In addition to the ''spontaneous'' metric fluctuations there also exist so-called ''induced'' metric fluctuations, which are generated by quantum fluctuations of all other fields interacting with the gravitational one. In the regime when the ''induced'' metric fluctuations are dominating, a consistent way to describe black hole fluctuations and back reaction is to use the stochastic semiclassical theory of gravity based on the Schwinger-Keldysh effective action [13,14] and the Feynman-Vernon influence functional [15,16] methods. In stochastic gravity the semiclassical Einstein equations are generalized to Einstein-Langevin equations which contain stochastic stress-energy tensor describing metric fluctuations induced by quantized fields. (For recent review see [17–20] and references therein.)

The study of the effects connected with black hole fluctuations is a technically very complicated problem. Only some preliminary work has been done in this direction till now. Under these conditions it is natural to study simplified models. In particular, it is not unreasonable to hope that the main properties of the Hawking radiation modified by metric fluctuations can be extracted from a much simpler framework in which the fluctuations of the metric are treated classically.

The model we shall use is inspired by that proposed by York [21]. In that model, the fluctuating geometry near the horizon of the black hole is represented by a Vaidya-type metric with a fluctuating mass. The spectrum of these fluctuations is characterized by the zero point fluctuations of quantum fields. In this paper, we further simplify this model by considering only spherically symmetric fluctuations and by neglecting the scattering by the gravitational potential which occurs in the 4-dimensional Dalembertian. Then we determine how these fluctuations modify the energy flux and the asymptotic spectrum of *s*-waves.

The paper is organized as follows. York's model is described in Sec. II. Section III contains a perturbation analysis of the equation of radial null ray propagation in the fluctuating geometry. The solution of this equation is obtained in Sec. IV and used to obtain the modified energy flux in Sec. V and the spectrum in Sec. VI. The results are discussed in Sec. VII. In our work we use dimensionless units where $G=c=\hbar=1$ and the sign conventions of [22].

II. MODEL

In order to study the influence of metric fluctuations on Hawking radiation, we consider a simplified version of the

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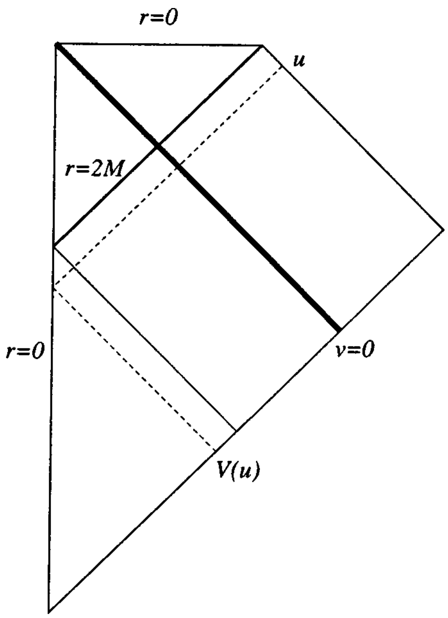

FIG. 1. Conformal diagram for a black hole created by a collapse of a massive null shell. Solid dark line $v=0$ represents the collapsing massive null shell.

model proposed by York [21]. In his model, the fluctuations of the black hole geometry are approximated by an incoming Vaidya metric with a fluctuating mass. The fluctuating part of the mass function can be decomposed into spherical harmonics. Upon quantizing the gravitational field, only components with $l \geqslant 2$ are important. However quantum fluctuations of matter fields may induce fluctuations of the geometry with all l.

In what follows we shall only consider spherical modes of fluctuations. Therefore, the metric for a neutral nonrotating black hole can be taken of the form

$$ds^2 = -A\,dv^2 + 2\,dv\,dr + r^2 dS_2^2, \tag{2.1}$$

where dS_2^2 is the metric of a unit 2-sphere and

$$A = 1 - \frac{2m}{r}, \tag{2.2}$$

$$m = m(v) = M[1+\mu(v)]\,\vartheta(v), \tag{2.3}$$

$$\mu(v) = \mu_0 \sin(\omega v). \tag{2.4}$$

This is the standard Vaidya metric in advanced time coordinates (v,r). The function $\mu(v)$ encodes the fluctuations of dimensionless amplitude μ_0. The step function $\vartheta(v)$ in relation (2.3) indicates that the black hole results from the gravitational collapse of a massive (with mass M) null shell propagating along $v=0$. Therefore inside the collapsing null shell the spacetime is flat.

The conformal diagram of the whole geometry (2.1) in the absence of fluctuations (that is for $\mu_0 = 0$) is schematically shown in Fig. 1. The dashed line on this figure shows a radial null ray which reaches $\mathcal{J}^+$ at the moment of the retarded time u and which was sent from $\mathcal{J}^-$ at $v = V(u)$ of advanced time. In the presence of the fluctuations, the conformal structure of the spacetime remains the same, but the function $V(u)$ is modified. We shall study this modification in Sec. IV.

Upon substituting the metric (2.1) into Einstein's equations, one easily gets that the right-hand side of these equations takes the form

$$T_{\mu\nu} = \frac{1}{4\pi r^2}[M\delta(v) + M\mu(v)\vartheta(v)]\,l_\mu l_\nu, \tag{2.5}$$

where $l_\mu = -v_{,\mu}$ is a future-directed null vector tangent to a radial in-going null ray. From a ''classical'' viewpoint, $\mu(v)l_\mu l_\nu$ is a fluctuating flux of positive [for $2\pi n < \omega v < 2\pi(n+1)$] and negative [for $2\pi(n+1) < \omega v < 2\pi(n+2)$] energy density. Correspondingly, the position of the apparent horizon $r_{\mathrm{AH}} = 2M(1+\mu(v))$ also fluctuates near its average value $\overline{r}_{\mathrm{AH}} = 2M$.

Following York [21] we assume that the dimensionless amplitude $\mu_0 = \alpha(m_{\mathrm{Planck}}/M)$, where α is a pure number. In particular this assumption means that $\mu_0 \ll 1$ for black holes of mass $M \gg m_{\mathrm{Planck}}$. It is also assumed that in order to get a more realistic result one should average over a spectrum of metric fluctuations.

III. RADIAL RAY PROPAGATION

A. Radial null rays in perturbed geometry

We first study the propagation of radial null rays in the fluctuating black hole geometry (2.1) since we shall use, as usually done, geometric optics to construct the solutions of the wave equation.

In-going rays are given by $v=$const and out-going rays obey the equation

$$A\,dv = 2\,dr. \tag{3.1}$$

In order to solve this equation, we use a method of perturbations and write

$$\begin{aligned} r = r(v) &= R(v) + \rho(v) + \sigma(v) + \cdots \\ &= 2M[\tilde{R}(v) + \tilde{\rho}(v) + \tilde{\sigma}(v)] + \cdots . \end{aligned} \tag{3.2}$$

$R(v)$ is the solution of Eq. (3.1) in the absence of fluctuations, and $\rho(v)$ and $\sigma(v)$ are respectively the first and second order perturbation in μ_0. Higher order corrections are denoted by dots. In what follows, we shall also often use the dimensionless versions of R, ρ, and σ which we mark by a tilde.

The equation for out-going rays in the unperturbed metric $[(\quad)^{\cdot} \equiv d/dv]$,

$$\dot{R} = \frac{1}{2}\left(1 - \frac{2M\,\vartheta(v)}{R}\right), \tag{3.3}$$

can be easily integrated. Let us choose the value of the retarded time u and denote by $r = R(v;u)$ the unperturbed trajectory of a radial ray which arrives to $\mathcal{J}^+$ at the chosen time u. This trajectory can be found by solving the equation

$$u = v - 2R_* = \text{const.} \tag{3.4}$$

Here

$$R_* = R - 2M + 2M \ln\frac{R-2M}{2M} \tag{3.5}$$

is the usual tortoise radial coordinate.

The equations for the perturbations $\rho(v)$ and $\sigma(v)$ are obtained by linearizing Eq. (3.1). Both functions obey the same equation

$$\dot{f} = \frac{M}{R^2} f + F, \tag{3.6}$$

for $v>0$ and

$$\dot{f} = 0, \tag{3.7}$$

for $v<0$. For the first order perturbation, one has

$$f = \rho, \quad F = -\frac{M}{R}\mu, \tag{3.8}$$

and for the second order perturbation

$$f = \sigma, \quad F = \frac{M}{R^2}\mu\rho - \frac{M}{R^3}\rho^2. \tag{3.9}$$

In these equations, the retarded time u is a fixed parameter which specifies the unperturbed ray under consideration and $R = R(v;u)$.

B. Perturbed horizon

Before giving the general solution of the equations for the perturbations $\rho(v)$ and $\sigma(v)$ we discuss the particular solution which describes the event horizon in the fluctuating geometry.

First notice that $R = 2M$ satisfies the unperturbed equation (3.3). This degenerate solution describes an outgoing null ray propagating along the unperturbed event horizon. Starting with this solution we easily obtain the following solutions for the dimensionless perturbations:

$$\tilde{\rho}_{EH} = \mu_0 \frac{\Omega\cos(\Omega\tilde{v}) + \sin(\Omega\tilde{v})}{1+\Omega^2}, \tag{3.10}$$

$$\tilde{\sigma}_{EH} = \mu_0^2 \frac{2\Omega^2(2-\Omega^2)\cos(2\Omega\tilde{v}) + \Omega(1-5\Omega^2)\sin(2\Omega\tilde{v})}{2(1+\Omega^2)^2(1+4\Omega^2)}, \tag{3.11}$$

where

$$\Omega = \omega/\kappa, \quad \tilde{v} = \kappa v, \tag{3.12}$$

and $\kappa = (4M)^{-1}$ is the unperturbed surface gravity of the black hole.

For both solutions it is possible to add a solution of the homogeneous equation. Such a solution corresponds to radial null rays propagating near the horizon in the unperturbed geometry. For these solutions the absolute value of ρ and σ is infinitely growing. We choose the integration constants to exclude these solutions so that the perturbed radial null ray neither goes to infinity nor to $r=0$. Therefore it describes the position of the event horizon in the perturbed geometry. It is easy to verify that Eqs. (3.10) and (3.11) coincide with the solution obtained by York [21] [see Eqs. (4.8) and (4.9) of York's paper].

It is also interesting to compute the modified value of the surface area $\mathcal{A}$ of the event horizon. When averaging over time v, we find

$$\overline{\mathcal{A}} \equiv 4\pi\overline{(r_{\text{hor}}^2(v))} = 16\pi M^2\left[1 + \frac{\mu_0^2}{2(1+\Omega^2)}\right]. \tag{3.13}$$

Similarly the average value of the surface gravity in the fluctuating geometry is

$$\bar{\kappa} \equiv \overline{\left(\frac{m(v)}{r_{\text{hor}}^2(v)}\right)} = \kappa[1 + 3\overline{(\tilde{\rho}^2)} - 2\mu_0\overline{(\tilde{\rho}\sin(\omega v))}]$$
$$= \kappa\left[1 + \frac{\mu_0^2}{2(1+\Omega^2)}\right]. \tag{3.14}$$

Upon computing the modifications of the Hawking flux, we shall see that this ''renormalized'' surface gravity will determine the modified temperature $\overline{T_H} = \bar{\kappa}/2\pi$. The change of the area, $\delta\mathcal{A} = \overline{\mathcal{A}} - \mathcal{A}$, and the change of temperature, $\delta T_H = \overline{T_H} - T_H$, of the black hole induced by the metric fluctuations obey the relation[1]

$$\frac{\delta\mathcal{A}}{\mathcal{A}} = \frac{\delta T_H}{T_H}. \tag{3.15}$$

C. Perturbed radial rays

We now consider the general case, that is we assume that $R \neq 2M$. Equation (3.7) can be easily integrated and gives $f = \text{const}$ everywhere inside the collapsing shell. To integrate Eq. (3.6), we change variable v to $R(v;u)$, where as earlier the retarded time parameter u is fixed. This allows us to rewrite Eq. (3.6) as

$$\left(1 - \frac{2M}{R}\right)\frac{df}{dR} - \frac{2M}{R^2} f = 2F. \tag{3.16}$$

The solution of this equation is

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[1] It is interesting to note that similar corrections to the black hole surface area and temperature were obtained by Hu and Shiokawa [23] in their stationary model of metric stochasticity. The relation between $\delta\mathcal{A}$ and δT_H in their case contains an additional factor 2.

$$f=\left(1-\frac{2M}{R}\right)\left[-\int_R^\infty \frac{2F}{\left(1-\frac{2M}{R}\right)^2}dR+f_0\right]. \tag{3.17}$$

The integration constant f_0 corresponds to the possibility to add a solution of the homogeneous equation; that is, a solution of Eq. (3.16) with $F=0$. We put $f_0=0$ since this choice is equivalent to the requirement that the ray propagating in the ''perturbed'' geometry arrives to $\mathcal{J}^+$ at the same time u as the ''unperturbed'' ray $r=R(v;u)$.

In what follows it is convenient to introduce the dimensionless quantities

$$x=\frac{R-2M}{2M}, \quad \tilde{u}=\kappa u=\frac{u}{4M}, \tag{3.18}$$

$$x^*=\frac{R^*}{2M}=x+\ln x, \quad \tilde{f}=\frac{f}{2M}. \tag{3.19}$$

In these notations solutions (3.17) for the dimensionless perturbations $\tilde{\rho}$ and $\tilde{\sigma}$ take the form

$$\tilde{\rho}(x)=\frac{x}{1+x}I(x), \tag{3.20}$$

$$I(x)=\int_x^\infty \frac{d\xi}{\xi^2}(1+\xi)\hat{\mu}(\xi), \tag{3.21}$$

$$\tilde{\sigma}(x)=-\frac{x}{1+x}\int_x^\infty \frac{d\xi}{\xi^2}\tilde{\rho}(\xi)\left(\hat{\mu}(\xi)-\frac{\tilde{\rho}(\xi)}{1+\xi}\right). \tag{3.22}$$

The fluctuating mass term $\hat{\mu}(\xi)\equiv\mu(v(\xi))$ which enters these equations is

$$\hat{\mu}(\xi)=\mu_0 \sin[\Omega(\xi+\ln\xi+\tilde{u})]. \tag{3.23}$$

To study quantum black hole radiation in the fluctuating geometry, we need to solve the wave equation in this geometry. As usual, we use the geometrical optic approximation.[2] Thus we only need to solve the following problem [1]. Consider a radial ray, which leaves $\mathcal{J}^-$ at some advance time v and reaches $\mathcal{J}^+$ at some retarded time u (see Fig. 1). What is the relation between u and v? To establish the relation $v=V(u)$ we use the above solution for a ray propagating in the fluctuating geometry outside the collapsing massive null shell, and glue it to the solution inside the shell. The latter means that the values of r coordinate for both rays must be the same on the shell $v=0$. Using this condition and the reflection at $r=0$, one finds for $\tilde{V}=V/4M$

$$\tilde{V}=-[1+x_0+\tilde{\rho}(x_0)+\tilde{\sigma}(x_0)], \tag{3.24}$$

where x_0 is the value of x on the massive null shell (i.e., at $v=0$). It depends on u and is a solution of the following equation:

$$\tilde{u}+x_0+\ln x_0=0. \tag{3.25}$$

We shall be interested in the null rays arriving to $\mathcal{J}^+$ at late time u. For such rays Eq. (3.25) can be solved by iterations and gives

$$x_0=e^{-\tilde{u}}(1-e^{-\tilde{u}})+O(e^{-3\tilde{u}}). \tag{3.26}$$

This relation shows that for the late-time regime x_0 is very small.

IV. CALCULATION OF $V(u)$ FUNCTION

A. First order corrections

In this section we analyze the perturbations $\tilde{\rho}(x)$ and $\tilde{\sigma}(x)$ in the late-time regime. Before proceeding to the computation of these perturbations it is appropriate to make a few remarks. First, we notice that it is necessary to compute $\tilde{\sigma}$, the quadratic fluctuation in μ_0, since our aim is to obtain all quadratic corrections to the flux and to the asymptotic spectrum. Secondly, it should be noticed that the following developments for determining $V(u)$ in the perturbed geometry are quite similar to those of Refs. [24,25] which concerned the determination of $V(u)$ when the energy-momentum tensor of Hawking quanta is taken into account.

We start with the first order perturbation $\tilde{\rho}(x)$, see Eq. (3.20). To calculate $I(x)$ we notice that Eq. (3.21) can be written in the form

$$I(x)=\mu_0 \,\mathrm{Im}[PU_1(x;\Omega)+PU_2(x;\Omega)], \tag{4.1}$$

where for $n\geqslant 1$

$$PU_n(x;\Omega)\equiv e^{i\Omega\tilde{u}}P_n(x;\Omega) \tag{4.2}$$

and

$$P_n(x;\Omega)=\int_x^\infty \frac{e^{i\Omega\xi}}{\xi^{n-i\Omega}}d\xi. \tag{4.3}$$

By integrating by parts, it is easy to show that

$$P_n(x;\Omega)=\frac{e^{i\Omega x}}{(n-1-i\Omega)x^{n-1-i\Omega}}+\frac{i\Omega}{n-1-i\Omega}P_{n-1}(x;\Omega). \tag{4.4}$$

Using this relation, we can rewrite Eq. (4.1) as

$$I(x)=\mu_0\,\mathrm{Im}\left\{\frac{1}{1-i\Omega}\left[e^{i\Omega(\tilde{u}+x+\ln x)}\frac{1}{x}+PU_1(x;\Omega)\right]\right\}$$
$$=\frac{\mu_0}{x\sqrt{1+\Omega^2}}\sin[\Omega(\tilde{u}+x+\ln x)+\arctan\Omega]+\tilde{I}(x), \tag{4.5}$$

[2] The validity of this approximation follows from the fact that the initial frequencies involved in the processes occurring at large u times are much larger than the characteristic frequency $\kappa=1/4M$.

where

$$\tilde{I}(x)=\mu_0\,\mathrm{Im}\left[\frac{PU_1(x;\Omega)}{1-i\Omega}\right]. \qquad (4.6)$$

Function $P_1(x;\Omega)$ can be expressed in terms of the incomplete gamma function $\Gamma(\alpha,\zeta)$. Using the relation

$$\int_u^\infty \frac{e^{-\mu\xi}}{\xi^{1-\alpha}}d\xi=\mu^{-\alpha}\Gamma(\alpha,\mu u), \qquad (4.7)$$

we get

$$P_1(x;\Omega)=(-i\Omega)^{-i\Omega}\Gamma(i\Omega,-i\Omega x)$$
$$=e^{-\pi\Omega/2-i\Omega\ln\Omega}\Gamma(i\Omega,-i\Omega x). \qquad (4.8)$$

Furthermore, the incomplete gamma function $\Gamma(\alpha,\zeta)$ allows the following series expansion:

$$\Gamma(\alpha,\zeta)=\Gamma(\alpha)\left[1-\zeta^\alpha e^{-\zeta}\sum_{n=0}^\infty \frac{\zeta^n}{\Gamma(\alpha+n+1)}\right]. \qquad (4.9)$$

Thus for small x $(\Omega x\ll 1)$ we have

$$\Gamma(i\Omega,-i\Omega x)\approx\Gamma(i\Omega)+\frac{i}{\Omega}e^{\pi\Omega/2}x^{i\Omega}e^{i\Omega\ln\Omega} \qquad (4.10)$$

and

$$P_1(x;\Omega)\approx e^{-\pi\Omega/2-i\Omega\ln\Omega}\Gamma(i\Omega)+\frac{ix^{i\Omega}}{\Omega}. \qquad (4.11)$$

For the calculation of $V(u)$ we need to know $I(x_0)$ where x_0 is the solution of Eq. (3.25). Using Eq. (4.5), one gets

$$I(x_0)=\frac{\mu_0\Omega}{x_0(1+\Omega^2)}+\tilde{I}(x_0), \qquad (4.12)$$

where

$$\tilde{I}(x_0)=\mu_0\,\mathrm{Im}\left[\frac{ie^{-i\Omega x_0}}{\Omega(1-i\Omega)}\right]$$
$$+\mu_0\,\mathrm{Im}\left[e^{-\pi\Omega/2-i\Omega\ln\Omega+i\Omega\tilde{u}}\frac{\Gamma(i\Omega)}{1-i\Omega}\right]. \qquad (4.13)$$

For small x_0 the first term in the right-hand side gives $\mu_0/[\Omega(1+\Omega^2)]$. To calculate the second term contribution, we notice that

$$e^{-\pi\Omega/2-i\Omega\ln\Omega}\Gamma(i\Omega)=q(\Omega)e^{-i\varphi_\Gamma(\Omega)}, \qquad (4.14)$$

where

$$q(\Omega)=\frac{\sqrt{2\pi}}{\sqrt{\Omega(e^{2\pi\Omega}-1)}}, \qquad (4.15)$$

and $\varphi_\Gamma(\Omega)$ is a real function which for large values of Ω, $\Omega\to\infty$, has the following expansion:

$$\varphi_\Gamma(\Omega)\approx\Omega+\frac{1}{4}\pi+\sum_{n=1}^\infty\frac{(-1)^{n-1}B_{2n}}{(2n-1)(2n)\Omega^{2n-1}}, \qquad (4.16)$$

where B_n are Bernoulli numbers.

Inserting Eq. (4.14) into Eq. (4.13) one gets, for small x_0,

$$\tilde{I}(x_0)\approx\frac{\mu_0}{\Omega(1+\Omega^2)}+\mu_0\frac{q(\Omega)}{\sqrt{1+\Omega^2}}\sin\Phi_1, \qquad (4.17)$$

where

$$\Phi_1=\Phi(\Omega)+\arctan\Omega,\quad \Phi(\Omega)=\Omega\tilde{u}-\varphi_\Gamma(\Omega). \qquad (4.18)$$

Collecting all the results, we finally obtain

$$I(x_0)\approx\mu_0\left[\frac{\Omega}{x_0(1+\Omega^2)}+\frac{1}{\Omega(1+\Omega^2)}+\frac{q(\Omega)}{\sqrt{1+\Omega^2}}\sin\Phi_1\right]. \qquad (4.19)$$

This result can be used to obtain the asymptotic form of the function $\tilde{V}(u)$. Using Eqs. (3.20), (3.24), (4.19), and (4.20), we find, up to the first order in $e^{-\tilde{u}}$,

$$-\tilde{V}\approx\hat{V}_0+e^{-\tilde{u}}[\hat{C}_1+\hat{C}_2\sin(\Phi_1)]+\tilde{V}_2(\tilde{u}), \qquad (4.20)$$

where

$$\hat{V}_0=1+\mu_0\frac{\Omega}{1+\Omega^2},\quad \hat{C}_1=1-\mu_0\frac{\Omega^2-1}{\Omega(1+\Omega^2)},$$

$$\hat{C}_2=\mu_0\frac{q(\Omega)}{\sqrt{1+\Omega^2}}, \qquad (4.21)$$

and

$$\tilde{V}_2(\tilde{u})=\tilde{\sigma}(x_0(\tilde{u})) \qquad (4.22)$$

is the second order correction.

B. Second order corrections

To calculate the second order correction $\tilde{\sigma}$, we must estimate the integrals of Eq. (3.22). By integrating by parts, $\tilde{\sigma}$ can be rewritten as

$$\tilde{\sigma}(x)=\frac{x}{1+x}\left(\frac{I^2(x)}{2(1+x)^2}-\int_x^\infty\frac{d\xi}{\xi^2}\hat{\mu}(\xi)I(\xi)\right), \qquad (4.23)$$

where as earlier

$$I(x)=I_1(x)+I_2(x) \qquad (4.24)$$

and

$$I_n(x)=\int_x^\infty\frac{d\xi}{\xi^n}\hat{\mu}(\xi). \qquad (4.25)$$

It is easy to see that

$$\int_x^\infty \frac{d\xi}{\xi^2}\hat{\mu}(\xi)I_2(\xi)=\frac{1}{2}[I_2(x)]^2. \tag{4.26}$$

Thus we have

$$\tilde{\sigma}(x)=\frac{x}{1+x}\left[\frac{(I_1(x)+I_2(x))^2}{2(1+x)^2}-\frac{1}{2}[I_2(x)]^2-T(x)\right], \tag{4.27}$$

where

$$T(x)=\mu_0\,\mathrm{Im}[e^{i\Omega\tilde{u}}Q_2(x;\Omega)] \tag{4.28}$$

and

$$Q_n(x;\Omega)=\int_x^\infty \frac{d\xi}{\xi^{n-i\Omega}}I_1(\xi). \tag{4.29}$$

The functions $I_n(x)$ are related to $PU_n(x;\Omega)$ defined by Eq. (4.2) as follows:

$$I_n(x)=\mu_0\,\mathrm{Im}[PU_n(x;\Omega)]. \tag{4.30}$$

Using the results of the previous section, we can obtain the expansions of $I_n(x_0)$ for small x_0. Thus, the problem of finding $\tilde{\sigma}(x_0)$ for small x_0 is reduced to study the function $T(x)$. For this purpose we first obtain a recursion relation for $Q_n(x;\Omega)$. By integrating by parts, we have

$$Q_n(x;\Omega)=\frac{e^{i\Omega x}I_1(x)}{(n-1-i\Omega)x^{n-1-i\Omega}}-\frac{1}{(n-1-i\Omega)}\int_x^\infty\frac{d\xi\, e^{i\Omega\xi}\hat{\mu}(\xi)}{\xi^{n-i\Omega}}+\frac{i\Omega}{n-1-i\Omega}Q_{n-1}(x;\Omega). \tag{4.31}$$

Using Eq. (3.23) and the definition (4.2) of $PU_n(x;\Omega)$ we can write the second term of Eq. (4.31) as

$$\int_x^\infty\frac{d\xi\, e^{i\Omega\xi}\hat{\mu}(\xi)}{\xi^{n-i\Omega}}=-\frac{i}{2}\mu_0 e^{-i\Omega\tilde{u}}\left[PU_n(x;2\Omega)-\frac{1}{(n-1)x^{n-1}}\right]. \tag{4.32}$$

Relations (4.31) and (4.32) allow us to write $Q_2(x;\Omega)$ as

$$Q_2(x;\Omega)=\frac{e^{i\Omega x}I_1(x)}{(1-i\Omega)x^{1-i\Omega}}-\frac{\mu_0 e^{-i\Omega\tilde{u}}}{2i(1-i\Omega)}\left[PU_2(x;2\Omega)-\frac{1}{x}\right]+\frac{i\Omega}{1-i\Omega}Q_1(x;\Omega). \tag{4.33}$$

We also have

$$\begin{aligned}Q_1(x;\Omega)&=\frac{\mu_0}{2i}\left[e^{i\Omega\tilde{u}}\int_x^\infty\frac{d\xi\, e^{i\Omega\xi}}{\xi^{1-i\Omega}}\int_\xi^\infty\frac{d\eta\, e^{i\Omega\eta}}{\eta^{1-i\Omega}}-e^{-i\Omega\tilde{u}}\int_x^\infty\frac{d\xi\, e^{i\Omega\xi}}{\xi^{1-i\Omega}}\int_\xi^\infty\frac{d\eta\, e^{-i\Omega\eta}}{\eta^{1+i\Omega}}\right]\\&=\frac{\mu_0}{2i}e^{-i\Omega\tilde{u}}\left[\frac{1}{2}(PU_1(x;\Omega))^2-S(x;\Omega)\right],\end{aligned} \tag{4.34}$$

where

$$S(x;\Omega)=\int_x^\infty\frac{d\xi\, e^{i\Omega\xi}}{\xi^{1-i\Omega}}\int_\xi^\infty\frac{d\eta\, e^{-i\Omega\eta}}{\eta^{1+i\Omega}}. \tag{4.35}$$

This function allows the following representation (see Appendix A):

$$\begin{aligned}S(x;\Omega)=&\frac{i}{\Omega}e^{-\pi\Omega/2+i\Omega\ln\Omega}\\&\times\Gamma(-i\Omega)x^{i\Omega}{}_1F_1(i\Omega;1+i\Omega;i\Omega x)\\&+\frac{i}{\Omega}[\ln(i\Omega x)+f(-i\Omega;i\Omega x)-\psi(i\Omega)],\end{aligned} \tag{4.36}$$

where ${}_1F_1$ is a hypergeometric function, $\psi(z)=[d\ln\Gamma(z)/dz]$, and

$$f(\alpha;y)=\sum_{n=1}^{\infty}\frac{y^n}{n(1+\alpha)_n}. \tag{4.37}$$

Here

$$(1+\alpha)_n=\frac{\Gamma(1+\alpha+n)}{\Gamma(1+\alpha)}. \tag{4.38}$$

Combining these results, we can rewrite Eq. (4.33) as

$$\begin{aligned}e^{i\Omega\tilde{u}}Q_2(x;\Omega)=&\frac{e^{i\Omega(\tilde{u}+x+\ln x)}I_1(x)}{(1-i\Omega)x}\\&-\frac{\mu_0}{2i(1-i\Omega)}\left[PU_2(x;2\Omega)-\frac{1}{x}\right]\\&+\frac{\mu_0\Omega}{2(1-i\Omega)}\left[\frac{1}{2}(PU_1(x;\Omega))^2-S(x)\right].\end{aligned} \tag{4.39}$$

The final expression for the second order perturbation $\tilde{\sigma}(x_0)$ is obtained by inserting Eqs. (4.28), (4.39), and (4.30) into Eq. (4.27).

Since we are interested in $\tilde{\sigma}(x_0)$ where x_0 is small we need only to know the first terms of its expansion in powers of x_0. To do this we use the following asymptotics at small value of x_0:

$$PU_1(x_0;\Omega)\approx\frac{i}{\Omega}+q(\Omega)e^{i\Phi(\Omega)},\tag{4.40}$$

$$PU_2(x_0;\Omega)\approx\frac{1}{1-i\Omega}\left[\frac{1}{x_0}+i\Omega PU_1(x_0;\Omega)\right],\tag{4.41}$$

$$S(x_0;\Omega)\approx\frac{i}{\Omega}\left[q(\Omega)e^{-i\Phi(\Omega)}+\ln(\Omega x_0)+\frac{i\pi}{2}-\psi(i\Omega)\right].\tag{4.42}$$

The approximative values of $I_1(x_0)$ and $I_2(x_0)$ can be easily obtained by using the relation (4.30). The calculations the asymptotic value of $\sigma(x_0)$ is hence straightforward but quite long. We use Maple to perform these calculations. The result is

$$\tilde{\sigma}(x_0)\approx\mu_0^2[\sigma_0+\sigma_1 x_0+\sigma'],\tag{4.43}$$

where

$$\sigma_0=\frac{\Omega^2(2-\Omega^2)}{(1+\Omega^2)^2(1+4\Omega^2)},\tag{4.44}$$

$$\sigma_1=\sigma_1^0+\sigma_1^1 q(\Omega)+\sigma_1^2 q^2(\Omega)+\sigma_1^3 q(2\Omega),\tag{4.45}$$

$$\sigma_1^0=\frac{2+3\Omega^2-33\Omega^4+20\Omega^6}{4\Omega^2(1+\Omega^2)^2(1+4\Omega^2)},\tag{4.46}$$

$$\sigma_1^1=\frac{(\Omega^2-1)[\Omega\cos(\Phi(\Omega))+\sin(\Phi(\Omega))]}{\Omega(1+\Omega^2)^2},\tag{4.47}$$

$$\sigma_1^2=\frac{(1-\Omega^2)+[\Omega\sin(2\Phi(\Omega))-\cos(2\Phi(\Omega))]}{4(1+\Omega^2)},\tag{4.48}$$

$$\sigma_1^3=-\frac{\Omega[(2\Omega^2-1)\sin(\Phi(2\Omega))-3\Omega\cos(\Phi(2\Omega))]}{(1+\Omega^2)(1+4\Omega^2)},\tag{4.49}$$

and

$$\sigma'=-\frac{x_0}{4(1+\Omega^2)}[\pi\Omega-2\ln(\Omega x_0)-2\,\mathrm{Im}[(\Omega-i)\psi(i\Omega)]].\tag{4.50}$$

Let us recall that in these expressions x_0 is a function of the retarded time u given by Eq. (3.26). We can now include the second order corrections (4.22) into the expression for $\tilde{V}(u)$ and write it in a form similar to Eq. (4.20)

$$-\tilde{V}\approx\tilde{V}_0+e^{-\tilde{u}}[C_1+C_2\sin(\Phi_1)+C_3\sin(\Phi_2)+C_4\sin(\Phi_3)+C\tilde{u}],\tag{4.51}$$

where

$$\tilde{V}_0=1+\mu_0\frac{\Omega}{1+\Omega^2}+\mu_0^2\frac{\Omega^2(2-\Omega^2)}{(1+\Omega^2)^2(1+4\Omega^2)},\tag{4.52}$$

$$C_1=1-\mu_0\frac{\Omega^2-1}{\Omega(1+\Omega^2)}+\mu_0^2\left[\frac{2+3\Omega^2-33\Omega^4+20\Omega^6}{4\Omega^2(1+\Omega^2)^2(1+4\Omega^2)}+\frac{1-\Omega^2}{4(1+\Omega^2)}q^2(\Omega)-\frac{(\pi\Omega-2\ln\Omega-2\,\mathrm{Im}[(\Omega-i)\psi(i\Omega)])}{4(1+\Omega^2)}\right],\tag{4.53}$$

$$C_2=\mu_0\frac{q(\Omega)}{\sqrt{1+\Omega^2}}\left[1+\mu_0\frac{\Omega^2-1}{\Omega(1+\Omega^2)}\right],\tag{4.54}$$

$$C_3=\mu_0^2\frac{q^2(\Omega)}{4\sqrt{1+\Omega^2}},\tag{4.55}$$

$$C_4=-\mu_0^2\frac{\Omega q(2\Omega)}{\sqrt{(1+\Omega^2)(1+4\Omega^2)}},\tag{4.56}$$

$$C=-\frac{\mu_0^2}{2(1+\Omega^2)},\tag{4.57}$$

and the phases Φ_i are defined as follows:

$$\Phi_1=\Omega\tilde{u}-\varphi_\Gamma(\Omega)+\arctan\Omega,\tag{4.58}$$

$$\Phi_2=2\Omega\tilde{u}-2\varphi_\Gamma(\Omega)-\arctan\left(\frac{1}{\Omega}\right),\tag{4.59}$$

$$\Phi_3=2\Omega\tilde{u}-\varphi_\Gamma(2\Omega)-\arctan\left(\frac{3\Omega}{2\Omega^2-1}\right).\tag{4.60}$$

Notice, that the coefficients $\hat{V}_0$, $\hat{C}_1$, and $\hat{C}_2$ which appeared in the first order expression (4.20) are now replaced in Eq. (4.51) by the new coefficients $\tilde{V}_0$, C_1, and C_2 with the only difference that the corresponding coefficients get second order corrections. Notice also that in Eq. (4.51), the terms with double frequency 2Ω and a term which is linear in $\tilde{u}$ are new with respect to the first oder result (4.20).

V. CALCULATION OF THE FLUX OF HAWKING RADIATION

Now we derive the s-mode contribution to Hawking radiation. In what follows, we shall neglect the scattering by the gravitational potential barrier which appears in the 4D Dalembertian. In other words, we use 2D approximation in which ingoing and outgoing modes completely decouple. This strong hypothesis requires some explanations. The decoupling of the modes greatly simplifies the calculation of the asymptotic flux when the metric is no longer static. Indeed, the height of the potential barrier now depends on v in the metric Eq. (2.1). Therefore one loses the fact that the transmission coefficients are diagonal in energy. Moreover, the new coefficients will also mix positive and negative frequency modes. This will lead to additional pair creation probabilities. Thus there will be interference effects between the usual pair creation amplitudes induced by the frequency mixing governed by Eq. (4.51) and these new coefficients.

To determine the importance of these interesting effects is complicated and goes beyond the scope of the present paper which is to describe the effects on Hawking radiation induced by the fluctuations of the geometry in the very close vicinity of the horizon. In this respect, we wish to emphasize that our classical metric has been chosen to mimic the near horizon quantum fluctuations and not the fluctuations of the height of the barrier around $r=3M$. On physical grounds, in a self-consistent treatment, one might expect that the residual fluctuations around $3M$ be much smaller that the near horizon ones. Therefore the neglection of the time dependence of the barrier might turn out to be physically legitimate.

In the two-dimensional simplified description, when the field is in its vacuum state before the formation of the black hole, the mean energy flux at $\mathcal{J}^+$ is

$$\frac{dE}{du}\equiv 4\pi r^2\langle T_{uu}\rangle^{\mathrm{ren}}=\frac{\kappa^2}{12\pi}\left(\frac{d\tilde{V}}{d\tilde{u}}\right)^{1/2}\frac{d^2}{d\tilde{u}^2}\left[\left(\frac{d\tilde{V}}{d\tilde{u}}\right)^{-1/2}\right]. \tag{5.1}$$

Here $\tilde{V}(\tilde{u})$ is the function calculated in the previous section. Notice also, that u in this relation is the proper time at $\mathcal{J}^+$ in the perturbed geometry, see the remark made after Eq. (3.17). Thus this is the time which defines positive frequency at $\mathcal{J}^+$.

Before presenting the results of the calculations of dE/du we make several remarks. First, it is evident that the expression for dE/du does not depend on the value of constant $\tilde{V}_0$ in Eq. (4.51). For this reason we can put it equal to zero. This corresponds to a simple redefinition $\tilde{V}\to\tilde{V}+\tilde{V}_0$. Moreover, dE/du is not changed if we multiply $\tilde{V}$ by an arbitrary constant. For these reasons the calculations of dE/du can be performed with the simplified form for $\tilde{V}$

$$\tilde{V}=-e^{-\tilde{u}}[1+A_1\sin(\Omega\tilde{u}+\varphi_1)+A_2\sin(2\Omega\tilde{u}+\varphi_2)+C\tilde{u}]. \tag{5.2}$$

In this expression, we have kept all terms up to second order in μ_0 and introduced the following notations:

$$A_1\equiv\mu_0 a_1+\mu_0^2 b_1=\mu_0\frac{q(\Omega)}{\sqrt{1+\Omega^2}}\left[1+2\mu_0\frac{\Omega^2-1}{\Omega(1+\Omega^2)}\right], \tag{5.3}$$

$$\varphi_1=-\varphi_\Gamma+\arctan\Omega. \tag{5.4}$$

In the same way the two terms in Eq. (4.51) with coefficients C_3 and C_4 having the same dependence $2\Omega\tilde{u}$ on the retarded time $\tilde{u}$ have been combined into the following single term:

$$A_2\sin(2\Omega\tilde{u}+\varphi_2)\equiv C_3\sin(\Phi_2)+C_4\sin(\Phi_3). \tag{5.5}$$

A_2 is of second order in μ_0 and, with notations to Eq. (5.3), can be written as $A_2\equiv\mu_0^2 b_2$. The explicit expressions for b_2 and φ_2 can be obtained easily, but since they are very long and are not important for our final result we do not reproduce them. Note also, that since C is already a quantity of second order in μ_0^2, the redefinition of $\tilde{V}$ does not affect it.

The calculation of dE/du from Eqs. (5.1) and (5.2) is straightforward but long and was performed by using Maple. It is convenient to write the result in the form

$$dE/du=(dE/du)^{\mathrm{perm}}+(dE/du)^{\mathrm{fluct}}, \tag{5.6}$$

where $(dE/du)^{\mathrm{perm}}$ is the mean value of the flux and $(dE/du)^{\mathrm{fluct}}$ is its fluctuating part. The latter will of course not contribute to the total energy received on $\mathcal{J}^+$.

The constant part is

$$(dE/du)^{\mathrm{perm}}=\frac{\kappa^2}{48\pi}\left[1+\frac{1}{2}\mu_0^2\Omega^2 q^2(\Omega)-2C\right], \tag{5.7}$$

and the fluctuating part is

$$\begin{aligned}(dE/du)^{\mathrm{fluct}}=&-\frac{\mu_0}{2}\Omega\sqrt{1+\Omega^2}q(\Omega)\cos(\Omega\tilde{u}+\varphi_1)\\&+\mu_0^2\Bigg[q(\Omega)\frac{1-\Omega^2}{\sqrt{1+\Omega^2}}\cos(\Omega\tilde{u}+\varphi_1)\\&+q^2(\Omega)\frac{\Omega^2(1-5\Omega^2)}{8(1+\Omega^2)}\cos(2\Omega\tilde{u}+2\varphi_1)\\&+q^2(\Omega)\frac{\Omega(1+4\Omega^2)}{4(1+\Omega^2)}\sin(2\Omega\tilde{u}+2\varphi_1)\\&-b_2\Omega(1+4\Omega^2)\cos(2\Omega\tilde{u}+\varphi_2)\Bigg].\end{aligned} \tag{5.8}$$

The function $q(\Omega)$ is given by Eq. (4.15) and, in the last term, $b_2\cos(2\Omega\tilde{u}+\varphi_2)$ is equal to

$$\begin{aligned}b_2\cos(2\Omega\tilde{u}+\varphi_2)=&\frac{q^2(\Omega)}{4\sqrt{1+\Omega^2}}\cos(\Phi_2)\\&-\frac{\Omega q(2\Omega)}{\sqrt{(1+\Omega^2)(1+4\Omega^2)}}\cos(\Phi_3).\end{aligned} \tag{5.9}$$

The remarkable fact is that, to second order in μ_0, the correction term which is linear in $\tilde{u}$ in Eq. (5.2) does not give any time-dependent contribution. It only gives an additional constant to $(dE/du)^{\mathrm{perm}}$ in Eq. (5.7). This has the following simple explanation: the term $C\tilde{u}$ can be removed from Eq. (5.2) by absorbing it into $e^{-\tilde{u}}$ without changing the other terms in our second order expression. This transformation corresponds to the "renormalization" of the surface gravity

$$\kappa\to\kappa_r=\kappa(1-C)=\kappa\left[1+\frac{\mu_0^2}{2(1+\Omega^2)}\right]. \tag{5.10}$$

Hence the expression for $(dE/du)^{\rm perm}$ can be identically rewritten as

$$(dE/du)^{\rm perm}=\frac{\kappa_r^2}{48\pi}\left[1+\frac{1}{2}\mu_0^2\Omega^2 q^2(\Omega)\right]=\frac{\kappa_r^2}{48\pi}\left[1+\mu_0^2\frac{\pi\Omega}{\exp(2\pi\Omega)-1}\right]. \quad (5.11)$$

It is interesting to note that the renormalized surface gravity κ_r which is introduced here coincides with the average value of the surface gravity $\overline{\kappa}$ defined by Eq. (3.14).

VI. THE MODIFIED ASYMPTOTIC SPECTRUM

Instead of focusing on integrated quantities, we shall now consider how the asymptotic spectrum is modified by the fluctuating part of the metric. As usual, the asymptotic spectrum is characterized by the angular momentum (that we take to zero) and the asymptotic energy, λ, the eigenvalue of $i\partial_u$. Indeed the fluctuations we are considering do not affect the stationary character of the asymptotic (large r) metric.

To obtain the modified spectrum, we need to compute the Bogoliubov coefficients in the modified geometry. Before proceeding to this calculation, it should be noticed that the 2D expressions we shall use are exact only for large λ, i.e. $\kappa\lambda \gg 1$. At lower frequencies, there is indeed a potential barrier in the 4D Dalembertian for s-waves which reduces the transmitted flux in a static geometry, cf. the discussion at the beginning of the former section.

We first recall how the well known properties of Hawking spectrum are extracted from the Bogoliubov coefficients. The latter are given by the overlap of the initial (infalling) modes which are specified on $\mathcal{J}^-$ and the final (outgoing) modes specified on $\mathcal{J}^+$. Both are solutions of the Dalembertian equation in the metric (2.1). For s-waves and under the neglection of the potential barrier, these modes satisfy the 2D equation $\partial_u\partial_v\phi=0$. Thus the in-modes can be decomposed in terms of plane waves

$$\phi_\nu(v)=\frac{e^{-i\nu v}}{\sqrt{4\pi\nu}}, \quad (6.1)$$

where ν is the energy measured on $\mathcal{J}^-$. Similarly, the out-modes are

$$\phi_\lambda(u)=\frac{e^{-i\lambda u}}{\sqrt{4\pi\lambda}}, \quad (6.2)$$

where λ is the energy measured on $\mathcal{J}^+$.

The scattering of in-modes in the time dependent geometry simply follows from the ‘‘reflection’’ condition on $r=0$ wherein the Wronskian must vanish. This implies that the scattered in-modes are given by $\phi_\nu(V(u))$. Then the Bogoliubov coefficients are

$$\alpha_{\nu,\lambda}=\int du\,\phi_\nu^*(V(u))\,i\overleftrightarrow{\partial}_u\phi_\lambda(u)=\int du\,\frac{e^{i\nu V(u)}}{\sqrt{4\pi\nu}}\,\frac{e^{-i\lambda u}}{\sqrt{\pi\lambda^{-1}}},$$

$$\beta_{\nu,\lambda}=\int du\,\phi_\nu(V(u))\,i\overleftrightarrow{\partial}_u\phi_\lambda(u)=\int du\,\frac{e^{-i\nu V(u)}}{\sqrt{4\pi\nu}}\,\frac{e^{-i\lambda u}}{\sqrt{\pi\lambda^{-1}}}. \quad (6.3)$$

The second expressions follow from integration by part and the neglect of the end-point contributions.

In the unperturbed geometry, for large u, $\kappa V(u)$ is given by $-1-e^{-\kappa u}$, see Eq. (3.24). The validity of this asymptotic expression requires that the initial frequencies ν be large enough. Indeed, upon applying the stationary phase condition to the integrand of the coefficient $\alpha_{\nu,\lambda}$ one obtains the following relation for the position of the saddle point u_{sp}:

$$\lambda=\nu e^{-\kappa u_{sp}}. \quad (6.4)$$

This equation simply specifies the value of u at which the Doppler shifted initial frequency ν resonates with λ. Notice in passing that it is this exponentially small Doppler factor which leads to the necessity of considering arbitrary large initial (‘‘trans-Planckian’’) frequencies. From Eq. (6.4), one deduces that large ν means that the corresponding value of $e^{-\kappa u_{sp}}$ satisfies $e^{-\kappa u_{sp}} \gg e^{-2\kappa u_{sp}}$. Notice also that the stationary phase condition applied to the β coefficient, leads to the same condition up to an overall negative sign. This implies that the location of the saddle point in the complex u-plane receives an imaginary contribution equal to $-i\pi/\kappa$. The determination of the sign of this imaginary part follows from the fact that $V(u)$ appearing in $e^{-i\nu V}$ must belong to the lower half complex V-plane. Physically this amounts to specifying that the in-vacuum contains no excitation characterized by positive ν. Mathematically it gives $|\beta_{\nu,\lambda}/\alpha_{\nu,\lambda}|^2 = e^{-2\pi\lambda/\kappa}$.

These considerations based on a saddle point analysis are confirmed by the exact integration of Eqs. (6.3). It will be found useful to express the exact expressions in terms of the following function:

$$B(\nu,\lambda)=\Gamma\left(i\frac{\lambda}{\kappa}\right)\sqrt{\frac{\lambda}{(2\pi\kappa)}}\left(\frac{\nu}{\kappa}\right)^{-i\lambda/\kappa} e^{-\pi\lambda/2\kappa}, \quad (6.5)$$

where $\Gamma(x)$ is the Euler complete gamma function. The norm of this function gives $|B|^2=(e^{2\pi\lambda/\kappa}-1)^{-1}$. Upon extending the domain of validity of the asymptotic behavior of $V(u)$ for all u, one finds

$$\alpha_{\nu,\lambda}=B(\nu,\lambda)\sqrt{\frac{1}{2\pi\kappa\nu}}e^{\pi\lambda/\kappa}e^{-i\nu/\kappa},$$

$$\beta_{\nu,\lambda}=B(\nu,\lambda)\sqrt{\frac{1}{2\pi\kappa\nu}}e^{i\nu/\kappa}. \quad (6.6)$$

Two crucial properties follow from Eq. (6.6): first, the Planck distribution characterizing the mean number of out-quanta in the in-vacuum (up to a normalization factor, it is obtained from $|\beta_{\nu,\lambda}|^2$), and secondly, the existence of a constant flux of out-quanta. The stationarity follows from the fact that the phases of β and α are both proportional to $\nu^{-i\lambda/\kappa}$. This implies indeed that the value of the energy flux is constant. To prove it, we recall that the renormalized value of the energy flux in the in-vacuum, when expressed in terms of the Bogoliubov coefficients, is

$$\frac{dE}{du}=\int_0^\infty d\lambda\int_0^\infty d\lambda'\frac{\sqrt{\lambda\lambda'}}{2\pi}\left[e^{-i(\lambda-\lambda')u}\left(\int_0^\infty d\nu\,\beta^*_{\nu,\lambda}\beta_{\nu,\lambda'}\right)-\mathrm{Re}\left(e^{i(\lambda+\lambda')u}\int_0^\infty d\nu\,\alpha_{\nu,\lambda}\beta_{\nu,\lambda'}\right)\right]. \tag{6.7}$$

By using Eq. (6.6) and performing the integral over ν, one immediately obtains that the second term, the interfering one, vanishes and that the first term is constant.

We shall now determine how these properties are affected by the fluctuations of the geometry described by Eq. (2.1). Under our restriction to s-waves and neglection of the potential barrier, it suffices to repeat the same procedure with the modified function $V(u)$, as if we were considering 2D propagation. The asymptotic expression we shall use is given by

$$-\kappa_r V(u)=\tilde{V}_0+e^{-\kappa_r u}[1+A_1\sin(\omega u+\varphi_1)+A_2\sin(2\omega u+\varphi_2)], \tag{6.8}$$

where the values of $\tilde{V}_0$, A_1, and A_2 can be found in Eqs. (4.52), (5.3), and (5.5). We have kept the constant shift in V since it will modify the absolute phase of the Bogoliubov coefficients. The symbol κ_r designates the "renormalized" surface gravity introduced in Eq. (5.10). We have indeed absorbed the linear term in u of Eq. (5.2) in the redefinition of κ. As before, since $\kappa_r-\kappa=O(\mu_0^2)$ and since we shall work up to quadratic corrections in μ_0, the modification of the terms proportional to A_1 and A_2 are irrelevant.

To reveal the nature of the modifications induced by the time dependent fluctuations of the metric, it is appropriate to analyze the β coefficient. Up to quadratic order in the metric fluctuations of amplitude μ_0, the modified β coefficient reads

$$\begin{aligned}\beta^{mod}_{\nu,\lambda}&=\int_{-\infty}^{\infty}du\,\frac{e^{-i\lambda u}}{\sqrt{\pi\lambda^{-1}}}\,\frac{e^{-i\nu V(u)}}{\sqrt{4\pi\nu}}\\&=\int du\,\frac{e^{-i\lambda u}}{\sqrt{\pi\lambda^{-1}}}\,\frac{e^{i\nu(\tilde{V}_0+e^{-\kappa_r u})/\kappa_r}}{\sqrt{4\pi\nu}}\left[1-\frac{1}{2}\left(\frac{\nu}{\kappa_r}\right)^2e^{-2\kappa_r u}A_1^2\sin^2(\omega u+\varphi_1)\right.\\&\quad\left.+i\frac{\nu}{\kappa_r}e^{-\kappa_r u}\{A_1\sin(\omega u+\varphi_1)+A_2\sin(2\omega u+\varphi_2)\}\right].\end{aligned} \tag{6.9}$$

By decomposing the sines into imaginary exponentials and by using several times the relation $\Gamma(x+1)=x\Gamma(x)$ we get

$$\begin{aligned}\beta^{mod}_{\nu,\lambda}=\sqrt{\frac{1}{2\pi\kappa_r\nu}}e^{i\nu\tilde{V}_0/\kappa_r}\Bigg\{&B_r(\nu,\lambda)\left[1-\frac{A_1^2}{4\kappa_r^2}(\lambda-i\kappa_r)\lambda\right]-\sqrt{\lambda}\,\frac{A_1}{2\kappa_r}[B_r(\nu,\lambda-\omega)\sqrt{\lambda-\omega}e^{i\varphi_1}-B_r(\nu,\lambda+\omega)\sqrt{\lambda+\omega}e^{-i\varphi_1}]\\&-\sqrt{\lambda}\,\frac{A_2}{2\kappa_r}[B_r(\nu,\lambda-2\omega)\sqrt{\lambda-2\omega}e^{i\varphi_2}-B_r(\nu,\lambda+2\omega)\sqrt{\lambda+2\omega}e^{-i\varphi_2}]\\&+\sqrt{\lambda}\,\frac{A_1^2}{8\kappa_r^2}[B_r(\nu,\lambda-2\omega)(\lambda-2\omega-i\kappa_r)\sqrt{\lambda-2\omega}e^{2i\varphi_1}+B_r(\nu,\lambda+2\omega)(\lambda+2\omega-i\kappa_r)\sqrt{\lambda+2\omega}e^{-2i\varphi_1}]\Bigg\}.\end{aligned} \tag{6.10}$$

Here $B_r(\nu,\lambda)$ designates the function $B(\nu,\lambda)$ defined in Eq. (6.5) with the renormalized surface gravity.

The replacement of $B(\nu,\lambda)$ by $B(\nu,\lambda\pm\omega)$ in the last three terms of Eq. (6.10) indicates that ω, the frequency of the fluctuating metric, enters into the expressions in such a way that the "effective" frequency, i.e., the one which weighs the new amplitudes, is $\lambda\pm\omega$.

Physically, this leads to a modification of the mean (quantum averaged and time averaged) number of quanta which reach $\mathcal{J}^+$ per unit u time. This averaged flux is

$$\langle\overline{n_\lambda}\rangle=\frac{\int^N d\nu|\beta^{mod}_{\nu,\lambda}|^2}{\int^N d\nu/\kappa_r\nu}. \tag{6.11}$$

We have implemented time average by integrating over ν up to the cut-off frequency N and dividing the resulting expression by $\int^N d\nu/\kappa_r\nu=\Delta u$. This last equality follows from the resonance condition, Eq. (6.4), and its validity requires that $\Delta u\gg 1/\kappa_r$, as in usual golden rule estimates.

In this ratio, to quadratic order in μ_0, only four terms contribute. The first two terms of the first line of Eq. (6.10) and those resulting from the square of the second line. This is because the integral of all other terms are crossed terms of the form $B(\nu,\lambda)B^*(\nu,\lambda\pm\omega)$ which lead to oscillatory integrands containing $e^{\pm i\omega u_{sp}(\nu)}$. These terms do not contribute to the production rate, Eq. (6.11), in the limit of large $\kappa\Delta u$.

By combining the four nonvanishing contributions we get

$$\langle\overline{n_\lambda}\rangle=\frac{1}{2\pi}\left\{\frac{1}{e^{2\pi\lambda/\kappa_r}-1}\left(1-\left(\frac{A_1}{\kappa_r}\right)^2\frac{\lambda^2}{2}\right)+\left(\frac{A_1}{2\kappa_r}\right)^2\lambda\left[\frac{\lambda-\omega}{e^{2\pi(\lambda-\omega)/\kappa_r}-1}+\frac{\lambda+\omega}{e^{2\pi(\lambda+\omega)/\kappa_r}-1}\right]\right\}. \tag{6.12}$$

Having obtained the spectrum of the particle number flux, the spectrum of the energy flux is

$$\frac{dE}{du\,d\lambda}=\lambda\langle\overline{n_\lambda}\rangle=\frac{\kappa_r}{2\pi}\left[f(\Lambda)+\frac{\mu_0^2}{2}F(\Lambda;\Omega)\right], \tag{6.13}$$

where $\Lambda=\lambda/\kappa_r$, $\Omega=\omega/\kappa_r$,

$$f(\Lambda)=\frac{\Lambda}{\exp(2\pi\Lambda)-1} \tag{6.14}$$

and

$$F(\Lambda;\Omega)=\frac{\pi\Lambda^2}{\Omega^2(1+\Omega^2)}f(\Omega)[f(\Lambda-\Omega)+f(\Lambda+\Omega)-2f(\Lambda)]. \tag{6.15}$$

Plots of the function $F(\Lambda;\Omega)$ for different values of Ω are given in Fig. 2.

In order to obtain the constant part of the density of energy flux one must integrate Eq. (6.13) over the frequency λ

$$\frac{dE}{du}=\frac{\kappa_r^2}{2\pi}\int_0^\infty d\Lambda\left[f(\Lambda)+\frac{\mu_0^2}{2}F(\Lambda;\Omega)\right]. \tag{6.16}$$

Using formulas (B1) and (B10) from Appendix B, it is easy to verify that Eq. (6.16) coincides with the expression (5.11) for $(dE/du)^{\rm perm}$ we obtained earlier.

The other physical consequence of the correction terms to α and β is the following. The mean instantaneous energy flux now oscillates around its time average with the harmonics of ω. This can be seen from the oscillatory terms $B(\nu,\lambda)B^*(\nu,\lambda\pm(2)\omega)$ which behave in $e^{\pm i(2)\omega u_{sp}(\nu)}$ when parametrized by $\kappa u_{sp}=\ln\nu/\lambda$, the location of the dominant contribution to $\alpha_{\nu,\lambda}$. The amplitudes of the fluctuations are linear in A_1 and A_2. Moreover, the phase shifts of the modified coefficients with respect the unperturbed ones differ for α and β. This implies that the second term in Eq. (6.7) no longer vanishes even though it still does not contribute to the mean (time average) energy flux.

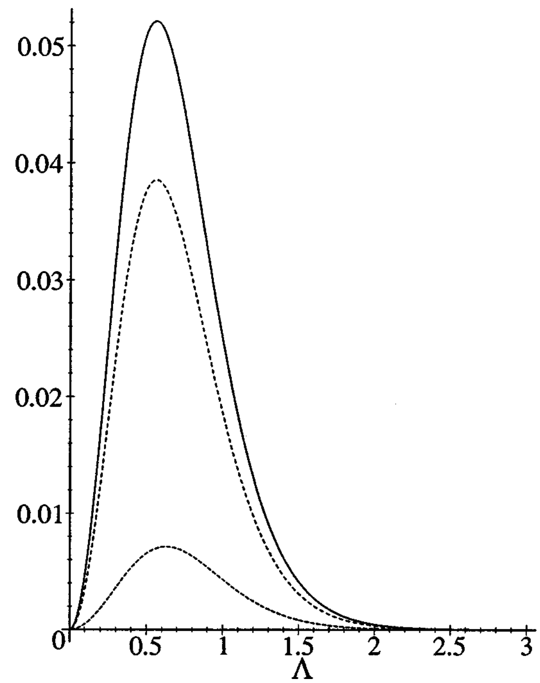

FIG. 2. Function $F(\Lambda,\Omega)$ for different values of dimensionless frequency Ω: $\Omega=0.01$, solid line; $\Omega=0.1$, dashed line; and $\Omega=0.5$, dotted line.

VII. CONCLUSION

In this paper we studied how the fluctuations of the geometry near the black-hole horizon affect Hawking radiation. To characterize the fluctuations of geometry, we used York's model [21] in which they are described by an in-coming Vaydia metric with a time dependent mass $m(v)$ which fluctuates around its mean with amplitude μ_0 and period $2\pi/\omega$. For further simplicity, we only considered the s-modes of a quantum scalar massless field and we also neglected the scattering by the gravitational potential barrier. Using these simplifying assumptions, we reached the following conclusions.

First, the expectation value of the outgoing flux of energy is no longer constant. It now fluctuates around its time averaged value with frequencies given by harmonics of ω and with amplitudes starting with a term linear in μ_0, see Eq. (5.8). The fact that the phase of the fluctuations of the expectation value of dE/du is well defined results from the fact that the fluctuations we considered were treated classically. In a more quantum mechanical treatment, these well defined phases will probably be replaced by a more diffuse ensemble of phases.

Secondly, the time averaged value of the outgoing flux of energy is modified. One part of this modification is connected with the renormalization of the surface gravity of the fluctuating black hole given by expression (5.10). The other part is an additional factor given by $\mu_0^2\pi\Omega/(\exp(2\pi\Omega)-1)$, see Eq. (5.11). Both changes are second order in μ_0. More surprisingly, they decrease for large Ω. Indeed, one might have feared that fast metric fluctuations would lead to copious pair creation.[3] We conjecture that we do not find abundant production since we worked in a two-dimensional model in which the time dependence of the metric does not affect directly massless modes, thanks to conformal invariance.

Thirdly, the asymptotic spectrum of Hawking radiation is also modified. Besides the renormalization of the surface gravity which shifts the temperature, the modified spectrum (6.12) contains three additional correction terms. The two last terms in that equation contain Bose thermal factors of the form $1/(\exp[2\pi(\lambda\pm\omega)/\kappa]-1)$. In these relations, the frequency of geometry fluctuations, $\pm\omega$, plays the role of a chemical potential. The presence of such chemical potential is reminiscent to superradiance. This fact supports the general ideas proposed by York since the appearance of these factors might be expected from the existence of a *quantum ergoshpere*. Indeed, due to quantum fluctuations, the average position of the event horizon is moved by a term proportional to the second power μ_0^2 of the amplitude of fluctuations, while the temporal position of the apparent horizon is fluctuating with amplitude μ_0. An alternative way to describe these fluctuations is to say that there exists a blurring of the physical null cone at the unperturbed horizon. Because of the existence of negative energy states inside the unperturbed black hole matter can escape from the narrow region close to the horizon. This leakage of energy is seen as Hawking radiation [21]. Under the same conditions one can expect an additional amplification of Hawking quanta while they are propagating close to the fluctuating horizon. The amplification factor we got in the expression for the modified spectrum of Hawking radiation may be considered as an indication to this effect.

The modifications in the black hole temperature and surface area in the presence of metric fluctuations raise the question about the modifications of black hole thermodynamics. If we identify the energy of the system E with the averaged mass of the black hole M, and the temperature of the black hole, T, with $\kappa_r/2\pi$, then the first law $dE=T\,dS$ and Eqs. (3.13) and (5.10) define the averaged entropy to be

$$\overline{S}=\frac{\overline{\mathcal{A}}}{4}\left[1+\frac{\mu_0^2}{2(1+\Omega^2)}\right]^{-2}\sim\frac{\overline{\mathcal{A}}}{4}\left(1-\frac{\mu_0^2}{1+\Omega^2}\right). \quad (7.1)$$

Therefore, one loses the relationship between the entropy and a fourth of the area.

If one writes the amplitude of the fluctuations as $\mu_0=\alpha m_{\mathrm{Planck}}/M$ where α is dimensionless, one has

$$\overline{S}=4\pi\frac{M^2}{m_{\mathrm{Planck}}^2}-s, \quad (7.2)$$

where

$$s=\frac{4\pi\alpha^2}{1+\Omega^2} \quad (7.3)$$

does not depend on the black hole mass. It is worth noticing that this modification of the black hole entropy is exactly of the same form as in theories with corrections quadratic in the curvature [27]. We recall that these corrections arise in the effective action by taking, in the one-loop approximation, the average of the gravitational equations over quantum fluctuations of the metric. This observation might give a possible explanation to the origin of the similarity between Eq. (7.2) and the results of Ref. [27]. Similarily, it would be interesting to find the relation between the results obtained in this paper and the quantum treatments of [28–31].

Even though these results were obtained in an extremely simplified model in which the metric fluctuations were treated classically, we believe that they indicate what might be the impact of the quantum fluctuations of the near horizon geometry on black hole radiance.

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APPENDIX A: CALCULATION OF FUNCTION $S(x)$

To calculate function

$$S(x)=\int_x^\infty \frac{d\xi\, e^{i\Omega\xi}}{\xi^{1-i\Omega}}\int_\xi^\infty \frac{d\eta\, e^{-i\Omega\eta}}{\eta^{1+i\Omega}}$$

$$=(i\Omega)^{i\Omega}\int_x^\infty \frac{d\xi\, e^{i\Omega\xi}}{\xi^{1-i\Omega}}\Gamma(-i\Omega,i\Omega\xi), \quad (A1)$$

we use the following general result, which can be found in [26] (volume II relation 2.10.3.6):

$$\int_a^\infty x^{\alpha-1}(x-a)^{\beta-1}e^{cx}\Gamma(\nu,cx)dx$$

$$=a^{\alpha+\beta-1}\Gamma(\nu)B(\beta,1-\alpha-\beta)\,{}_1F_1(\alpha;\alpha+\beta;ac)$$

$$-\frac{a^{\alpha+\beta+\nu-1}c^\nu}{\nu}B(\beta,1-\alpha-\beta-\nu)$$

$$\times{}_2F_2(\alpha+\nu,1;\nu+1,\alpha+\beta+\nu;ac)$$

$$-\frac{\pi c^{1-\alpha-\beta}}{\sin[(\alpha+\beta+\nu)\pi]}\frac{\Gamma(\alpha+\beta-1)}{\Gamma(1-\nu)}$$

$$\times{}_1F_1(1-\beta;2-\alpha-\beta;ac). \quad (A2)$$

Here

$$B(x,y)=\frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)}. \quad (A3)$$

This relation is valid for $a>0$, $\mathrm{Re}\,\beta>0$, $\mathrm{Re}(\alpha+\beta+\nu)<2$; $|\arg c|<\pi$. For a particular case $\beta=1$ one has

[3]We are grateful to T. Jacobson for pointing out this fact to us.

$$\int_a^\infty x^{\alpha-1}e^{cx}\Gamma(\nu,cx)dx$$
$$=-\frac{a^\alpha\Gamma(\nu)}{\alpha}\ {}_1F_1(\alpha;1+\alpha;ac)+\frac{a^{\alpha+\nu}c^\nu}{\nu(\alpha+\nu)}\,{}_2F_2(\alpha+\nu,1;1+\nu,1+\alpha+\nu;ac)+\frac{\pi c^{-\alpha}}{\sin[(\alpha+\nu)\pi]}\frac{\Gamma(\alpha)}{\Gamma(1-\nu)}. \tag{A4}$$

The integral in Eq. (A1) is of the form (A4) with

$$\alpha=i\Omega,\quad \nu=-i\Omega,\quad a=x,\quad c=i\Omega. \tag{A5}$$

For this case $\alpha+\nu=0$, and the last two terms in Eq. (A4) are divergent. To consider this limit, we define $\gamma=\alpha+\nu$ and rewrite the sum of these two divergent terms in the following form:

$$\frac{a^{\alpha+\nu}c^\nu}{\alpha(\alpha+\nu)}\,{}_2F_2(\alpha+\nu,1;1+\nu,1+\alpha+\nu;ac)+\frac{\pi c^{-\alpha}}{\sin[(\alpha+\nu)\pi]}\frac{\Gamma(\alpha)}{\Gamma(1-\nu)}=\frac{c^{\nu-\gamma}}{\nu}Z, \tag{A6}$$

where

$$Z=\frac{1}{\gamma}\left[y^\gamma{}_2F_2(\gamma,1;1+\nu,1+\gamma;y)+\frac{\pi\gamma\nu}{\sin(\gamma\pi)}\frac{\Gamma(\gamma-\nu)}{\Gamma(1-\nu)}\right], \tag{A7}$$

and $y=ac$.

Using the expression for the hypergeometric function

$${}_2F_2(\gamma,1;1+\nu,1+\gamma;y)=1+\gamma\sum_{n=1}^\infty\frac{y^n}{(\gamma+n)(1+\nu)_n}, \tag{A8}$$

where $(1+\nu)_n=\Gamma(1+n+\nu)/\Gamma(1+\nu)$, we can write for small γ

$${}_2F_2(\gamma,1;1+\nu,1+\gamma;y)\approx 1+\gamma f(\nu;y), \tag{A9}$$

where

$$f(\nu;y)=\sum_{n=1}^\infty\frac{y^n}{n(1+\nu)_n}. \tag{A10}$$

We also have

$$\frac{\Gamma(\gamma-\nu)}{\Gamma(1-\nu)}\approx-\frac{1}{\nu}+\frac{\Gamma'(-\nu)}{\Gamma(1-\nu)}\gamma=-\frac{1}{\nu}-\frac{1}{\nu}\psi(-\nu)\gamma, \tag{A11}$$

where $\psi(z)=d\ln\Gamma(z)/dz$.

Substituting expressions (A9) and (A11) into (A7) and expanding $\sin(\gamma\pi)=\gamma\pi(1-\frac{1}{6}\gamma^2\pi^2+\cdots)$, we get

$$Z=\frac{1}{\gamma}[y^\gamma(1+\gamma f(\nu,y))-1-\gamma\psi(-\nu)]$$
$$=\frac{y^\gamma-1}{\gamma}+f(\nu,y)-\psi(-\nu)\approx\ln y+f(\nu,y)-\psi(-\nu). \tag{A12}$$

Thus for $\nu=-\alpha$ we have

$$\int_a^\infty x^{\alpha-1}e^{cx}\Gamma(-\alpha,cx)dx$$
$$=-\frac{a^\alpha\Gamma(-\alpha)}{\alpha}\,{}_1F_1(\alpha;1+\alpha;ac)-\frac{c^{-\alpha}}{\alpha}[\ln(ac)+f(-\alpha;ac)-\psi(\alpha)]. \tag{A13}$$

Using this result, we obtain for $S(x)$ the following expression:

$$S(x)=\frac{i}{\Omega}e^{i\Omega\ln\Omega+i\Omega\ln x}e^{-\pi\Omega/2}\times\Gamma(-i\Omega)\,{}_1F_1(i\Omega;1+i\Omega;i\Omega x)+\frac{i}{\Omega}[\ln(i\Omega x)+f(-i\Omega;i\Omega x)-\psi(i\Omega)]. \tag{A14}$$

APPENDIX B: CALCULATION OF INTEGRALS

In this appendix we demonstrate that

$$J\equiv\int_0^\infty\frac{d\lambda\,\lambda^2(\lambda+\Omega)}{\exp[2\pi(\lambda+\Omega)]-1}+\int_0^\infty\frac{d\lambda\,\lambda^2(\lambda-\Omega)}{\exp[2\pi(\lambda-\Omega)]-1}=\frac{1}{120}(10\Omega^4+10\Omega^2+1). \tag{B1}$$

First we notice that

$$J=\lim_{p\to0}[J_+(p)+J_-(p)], \tag{B2}$$

where

$$J_\pm(p)=\int_0^\infty\frac{d\lambda\,\lambda^2(\lambda\pm\Omega)\exp(-p|\lambda|)}{\exp[2\pi(\lambda\pm\Omega)]-1}. \tag{B3}$$

Making change of variable of integration $\lambda\to-\lambda$ in the expression for $J_-(p)$ we get

$$J_-(p)=\int_{-\infty}^0 d\lambda\,\lambda^2(\lambda+\Omega)\exp(p\lambda)+\int_{-\infty}^0\frac{d\lambda\,\lambda^2(\lambda+\Omega)\exp(-p|\lambda|)}{\exp[2\pi(\lambda+\Omega)]-1}. \tag{B4}$$

Thus we have

$$J_{+}(p)+J_{-}(p)=\int_{-\infty}^{0}d\lambda\,\lambda^{2}(\lambda+\Omega)\exp(p\lambda)+\int_{-\infty}^{\infty}\frac{dx\,(x-\Omega)^{2}\,x\exp(-p|x-\Omega|)}{\exp(2\pi x)-1}. \tag{B5}$$

In the last integral in the right-hand side we made change of variables $\lambda=x-\Omega$.

Consider now the integral

$$\int_{-\infty}^{0}\frac{dx(x-\Omega)^{2}x\exp(-p|x-\Omega|)}{\exp(2\pi x)-1}. \tag{B6}$$

By changing variables $x\rightarrow -x$ it can be identically rewritten as

$$\exp(-p\Omega)\int_{0}^{\infty}dx\,x(x+\Omega)^{2}\exp(-px)+\exp(-p\Omega)\int_{0}^{\infty}\frac{dx\,x(x+\Omega)^{2}\exp(-px)}{\exp(2\pi x)-1}. \tag{B7}$$

Using Eqs. (B5) and (B7) and taking the limit $p\rightarrow 0$ we get

$$J=2\int_{0}^{\infty}\frac{dx\,x(x^{2}+\Omega^{2})}{\exp(2\pi x)-1}+\Delta J, \tag{B8}$$

where

$$\begin{aligned}\Delta J&=\lim_{p\rightarrow 0}\left[\int_{-\infty}^{0}dx\,x^{2}(x+\Omega)\exp(px)+\exp(-p\Omega)\int_{0}^{\infty}dx\,x(x+\Omega)^{2}\exp(-px)\right]\\&=\lim_{p\rightarrow 0}\left[\left(-\frac{6}{p^{4}}+\frac{2\Omega}{p^{3}}\right)+e^{-p\Omega}\left(\frac{6}{p^{4}}+\frac{4\Omega}{p^{3}}+\frac{\Omega^{2}}{p^{2}}\right)\right]\\&=\frac{1}{12}\Omega^{4}.\end{aligned} \tag{B9}$$

The integrals which enter Eq. (B8) can be easily calculated:

$$\int_{0}^{\infty}\frac{dx\,x}{\exp(2\pi x)-1}=\frac{1}{24},\quad\int_{0}^{\infty}\frac{dx\,x^{3}}{\exp(2\pi x)-1}=\frac{1}{240}. \tag{B10}$$

Combining these results we get Eq. (B1).

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