

# Matter creation via vacuum fluctuations in the early Universe and observed ultrahigh energy cosmic ray events

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Cosmic rays of the highest energy, above the Greisen-Zatsepin-Kuzmin cutoff of the spectrum, may originate in decays of superheavy long-living  $X$  particles. These particles may be produced in the early Universe from vacuum fluctuations during or after inflation and may constitute a considerable fraction of cold dark matter. We calculate numerically their abundance for a wide range of models.  $X$  particles are considered to be either bosons or fermions. Particles that are several times heavier than the inflaton,  $m_{\text{inflaton}} \approx 10^{13}$  GeV, and were produced by this mechanism, can account for the critical mass in the Universe naturally. In some cases induced isocurvature density fluctuations can leave an imprint in the anisotropy of cosmic microwave background radiation. [S0556-2821(99)07010-1]

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## I. INTRODUCTION

According to the Greisen-Zatsepin-Kuzmin [1] (GZK) observation, the energy spectrum of ultrahigh energy (UHE) cosmic rays produced at extragalactic distances should exhibit an exponential cutoff at energy  $E \sim 5 \times 10^{10}$  GeV. However, a number of cosmic ray events with energies well beyond the predicted GZK cutoff were recently observed by the various experimental groups [2]. This is in obvious contradiction with the standard cosmological and particle physics models and clearly requires some new physics beyond the standard model.

Conceptually, the simplest explanation [3,4] could be that the highest energy cosmic rays are produced in the decays of heavy long-living particles in a cosmologically local part of the Universe. We will call these progenitors of the cosmic rays  $X$  particles. The mass of the  $X$  particles has to be very large,  $m_X \gtrsim 10^{13}$  GeV, for them to be responsible for cosmic rays events in the energy range  $E \gtrsim 10^{11}$  GeV. It was noticed [5,6] that such heavy particles are produced in the early Universe from vacuum fluctuations and their abundance can be correct naturally, if the standard Friedmann epoch in the evolution of the Universe was preceded by an inflationary stage. This is a fundamental process of particle creation unavoidable in the time varying background (space-time metric in the present situation) and it requires no interactions. The temporal change of the metric is the single cause of particle production. Basically, it is the same process which during inflation had generated primordial large scale density perturbations and seeded the formation of galaxies and galaxy clusters. However, there is a difference. An inflationary stage is not required to produce superheavy particles from the

vacuum, unlike the case of the generation of long-wavelength perturbations. Rather, inflation provides a cutoff in excessive production of heavy particles which would happen in the Friedmann universe if it would start from an initial singularity [5]. Note that we consider sterile or almost sterile  $X$  particles, so that their production in usual plasma interactions and decays can be neglected. Thermal production of heavy  $X$  particles was discussed in Refs. [3,4] and production during reheating was considered recently in [7].

Particle production from vacuum fluctuations in the Friedmann universe during matter- or radiation-dominated stages was considered long ago [8,9,10] and this gave the basis for simple estimates of  $X$ -particle abundance [5], which have the mass of order or smaller than the Hubble constant at the end of inflation,  $m_X \lesssim H_i$ . However, the more interesting case of heavier  $X$  particles,  $m_X > H_i$ , requires detailed calculations. Relevant calculations were done already by Chung, Kolb, and Riotto [6]. However, to describe the end of inflation and the transition to the Friedmann universe, they match the fixed de Sitter background to the subsequent radiation or matter-dominated expansion either as an instantaneous junction or with the help of a smoothing function. The result is especially sensitive to the details of the junction procedure in the case of large  $m_X$  which is of interest for us here. Further, the range of models considered in Ref. [6] was restricted to the case of scalar particles with conformal coupling to gravity.

The purpose of the present paper is to calculate the production of superheavy particles from the vacuum in the inflationary Universe for a wide range of models, and to give a detailed and extended discussion of our other results already reported in Ref. [5]. When considering particle creation, we do not make any approximations. To this end we find numerically the exact evolution of the scale factor in the model of the ‘‘chaotic’’ inflation [11]. Our basic formalism relies on the method of Bogolyubov transformations, which for the case of particle creation by nonstationary gravitational field

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was developed in Refs. [8,9,10]. We consider the possibility for  $X$  particles to be either bosons or fermions.

In Sec. II we remind the reader about the explicit relations between mode functions of the field, coefficients of the Bogolyubov transformation and the particle number. In Sec. III we present the results of our numerical calculations of particle production from the vacuum fluctuations both for Friedmann and inflationary stages in the universe evolution. In Sec. IV we relate the results of the previous section to the present day density of  $X$  particles. In Sec. V we discuss relevance of our results to the UHE cosmic rays events, and Sec. VI contains our conclusions.

## II. PARTICLE PRODUCTION BY A NONSTATIONARY GRAVITATIONAL FIELD

Here we summarize the basic formalism of gravitational particle creation in an expanding universe which we employed in our analysis. For more details see Refs. [8,9,10]. We chose the metric to be conformally flat at an early cosmological epoch,  $ds^2 = a(\eta)^2(d\eta^2 - d\mathbf{x}^2)$ . The number density of  $X$  particles created from the vacuum in a time-varying cosmological background can be written as

$$n_X = \frac{1}{2\pi^2 a^3} \sum_s \int |\beta_k|^2 k^2 dk, \quad (1)$$

where  $\beta_k$  are the Bogolyubov's coefficients which relate "in" and "out" mode functions,  $k$  is the comoving momentum, and  $\sum_s$  is the sum over spin states. The expression (1) gives the number density of particles only, with the equal amount of antiparticles being created in the case of charged fields. The creation of Bose and Fermi particles is to be considered separately.

(i) *Bosons*. The mode functions,  $\chi_k \equiv \chi_k(\eta)$  of a scalar Bose field are solutions of the oscillator equation

$$\chi_k'' + \omega_k^2 \chi_k = 0, \quad (2)$$

with the time-dependent frequency

$$\omega_k^2 = k^2 - \frac{a''}{a}(1 - 6\xi) + m_X^2 a^2. \quad (3)$$

Here  $' \equiv d/d\eta$ . The constant  $\xi$  describes the direct coupling to the curvature, with  $\xi=0$  corresponding to the minimal coupling and  $\xi=1/6$  being the case of conformal coupling. Equations for massless conformally coupled quanta are reduced to the equations in flat spacetime and such particles are not created. For massive particles conformal invariance is broken and particles are created regardless of the value of  $\xi$ . Given the initial (vacuum) conditions

$$\chi_k(0) = \omega^{-1/2}, \quad \chi_k'(0) = -i\omega\chi_k, \quad (4)$$

the Bogolyubov's coefficients at any time moment  $\eta$  are found to be

$$|\beta_k|^2 = \frac{|\chi_k'|^2 + \omega^2 |\chi_k|^2 - 2\omega}{4\omega}. \quad (5)$$

(ii) *Fermions*. The relevant mode functions of the Fermi field satisfy the oscillator equation with the complex frequency

$$\chi_k'' + (\omega_k^2 - im_X a') \chi_k = 0, \quad (6)$$

where the real part of the frequency is given by  $\omega_k^2 = k^2 + m_X^2 a^2$ . We choose

$$\chi_k(0) = \sqrt{1 - \frac{m_X a}{\omega}}, \quad \chi_k'(0) = -i\omega\chi_k, \quad (7)$$

as the initial conditions. In this case we find per spin state

$$|\beta_k|^2 = \frac{\omega - m_X a - \text{Im}(\chi_k \chi_k'^*)}{2\omega}. \quad (8)$$

The summation over spins gives a factor of 2 in Eq. (1).

## III. NUMERICAL RESULTS

### A. Power-law cosmology

In this section we consider particle creation during the regular, i.e., noninflationary, stage of the Universe expansion. We shall refer to this stage as the Friedmann stage of the Universe evolution. During the Friedmann stage the scale factor is given by the expression  $a(t) \propto t^\alpha$  with  $\alpha < 1$ . This case is of interest since  $\alpha = 1/2$  and  $\alpha = 2/3$  corresponds to the radiation- and matter-dominated stages of the Universe expansion, which are inevitable. Moreover, in some cases (and we shall encounter such situations below) particle creation during this stage gives a dominant contribution, while creation during inflation (or at the transition between stages) is negligible.

Let us first consider massive particles conformally coupled to gravity. It is the particle mass which couples the system to the background expansion and serves as the source of particle creation. Therefore, we expect  $n_X \propto m_X^3 a^{-3}$  at late times when particle creation diminishes and the number of particles conserves in a comoving volume. Because the scale factor has an arbitrary normalization, for definiteness it is more convenient to rewrite this expression using relations  $a \propto (mt)^\alpha$ , or  $a \propto (m/H)^\alpha$ . We prefer the latter choice since the exact value of time depends on the previous history in the inflationary cosmology and consequently the time coordinate  $t$  is ambiguously defined. Moreover, it is a nonzero value of the Hubble constant which is responsible for particle creation in an expanding universe. The particle production is expected to stop when  $H \ll m_X$ . Therefore, at the late epoch we can parametrize the anticipated formulas for  $n_X$  as

$$n_X = C_\alpha m_X^3 \left( \frac{H}{m_X} \right)^{3\alpha}, \quad (9)$$

where the constant  $C_\alpha$  depends only upon the background cosmology, i.e.,  $\alpha$ , and it can be found numerically. Note that for the radiation-dominated case it was found in Ref. [8],  $n_X \approx 5.3 \times 10^{-4} m_X^3 (m_X t)^{-3/2}$ . We confirm this result and extend it here to a broad range of cosmologically interesting

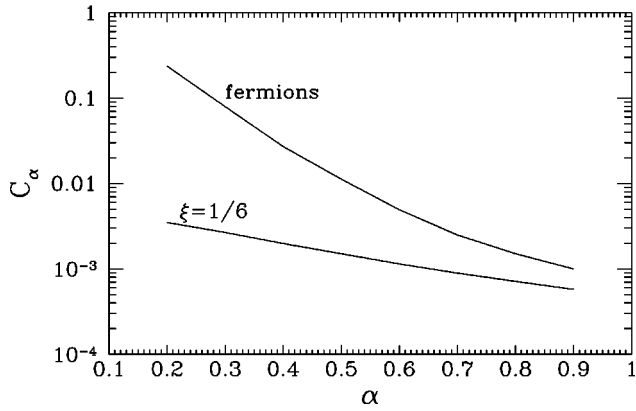


FIG. 1. Coefficient  $C_\alpha$ , defined in Eq. (9), is shown as a function of  $\alpha$  for background cosmology with a power-law scale factor  $a \propto t^\alpha$ .

values of  $\alpha$ , which includes the matter-dominated case as well. Our results are summarized in Fig. 1 where we plot  $C_\alpha$  as a function of  $\alpha$ . The particle number reaches this asymptotic value, Eq. (9), if the initial vacuum state was defined at the sufficiently early epoch,  $H(0) \gg m_X$ . We found that  $H(0) \approx 10^4 m_X$  is sufficient for the results to become practically independent of the choice of the initial time if  $\alpha = 1/2$ , with a smaller (larger) ratio of  $H(0)/m_X$  required at larger (smaller)  $\alpha$ .

For the radiation-dominated Universe one finds  $\Omega_X \equiv \rho_X/\rho_c = m_X n_X 32\pi G t^2/3$  with the present value of  $\Omega_X$  being equal to  $\Omega_X \sim 2 \times 10^{-2} (m_X^2/M_{\text{pl}}^2) \sqrt{m_X t_e}$ , where  $t_e$  is the time of equal densities of radiation and matter in the  $\Omega = 1$  universe. This gives  $\Omega_X \sim (m_X/10^9 \text{ GeV})^{5/2}$ . We see that stable particles with  $m_X \geq 10^9 \text{ GeV}$  will overclose the Universe even if initially they were in a vacuum state and were created from the vacuum during the regular Friedmann radiation-dominated stage of the evolution. (It is possible to separate the vacuum creation from the creation in collisions in plasma since  $X$  particles may be effectively sterile.)

However, this restriction will be not valid<sup>1</sup> if the evolution of the Universe, as it is believed, was more complicated than the simple radiation-dominated expansion from a singularity. The Hubble constant may have never exceeded  $m_X$ , which is the case of inflation,  $H(0) \approx m_\phi \approx 10^{13} \text{ GeV}$ . Moreover, compared to the case considered above, the density of  $X$  particles created during inflation is additionally diluted by the entropy release in reheating after inflation.

### B. Inflationary cosmology

Any viable modern cosmological model invokes the hypothesis of inflation (for a review and list of references, see Refs. [12,13]). During inflation the Universe expands exponentially which solves the horizon and flatness problems of standard big-bang cosmology. Inflation is generally assumed

to be driven by a special scalar field  $\phi$  known as the *inflaton*. Fluctuations generated at the inflationary stage may have the strength and the power spectrum suitable for the generation of the large-scale structure. This fixes the range of parameters of the inflaton potential. For example, the mass of the inflaton field has to be  $m_\phi \sim 10^{13} \text{ GeV}$ . During inflation, the inflaton field slowly rolls down towards the minimum of its potential. Inflation ends when the potential energy associated with the inflaton field becomes smaller than the kinetic energy. Coherent oscillations of the inflaton field contained all the energy of the Universe at that time. It is possible [14] that a significant fraction of this energy was released to other boson species after only a dozen oscillations of the inflaton field, in the regime of a broad parametric resonance. This process was studied in detail [15,16]. It was shown that even rather heavy particles with masses by an order of magnitude larger than the inflaton mass can be produced quite copiously (see also [17]). Applying these results to the case of our interest, we find that the stable very heavy particles,  $m_\phi \leq m_X \leq 10 m_\phi$ , will be generally produced in excess in this process.

However, if the parametric resonance is ineffective for some reason, and one estimates the particle number density after inflation at the level of initial conditions used in Ref. [15], one finds that  $\Omega_X$  might prove to be of acceptable magnitude. This level is saturated by the fundamental process of particle creation during inflation from vacuum fluctuations and it is the same process which generated primordial large-scale density perturbations. Parametric resonance for  $X$  particles is turned off if the  $X$  field is either a fermion field or its coupling to the inflaton is small,  $g^2 \ll 10^4 (m_X/m_\phi)^4 (m_\phi/M_{\text{pl}})^2$  [15]. (Parametric creation of fermions was considered in Refs. [18]. While it is not a ‘‘resonance’’ process, it might be useful for creation of very heavy fermions.)

Particle creation from vacuum fluctuations during inflation (or in the de Sitter space) was extensively studied [19,20], usually in the case of small  $m_X$  and in application to generation of density fluctuations necessary for the large-scale structure formation. The characteristic quantity which is usually cited, the variance of the field,  $\langle X^2 \rangle$ , is defined by an expression similar to Eq. (1). In the typical case the difference is given by the factor  $2 \sin^2(\omega_k \eta)/\omega_k$  in the integrand. For example, for the scalar Bose field with minimal coupling to the curvature,  $\langle X^2 \rangle = 3H_i^4/8\pi^2 m_X^2$  if  $m_X \ll H_i$  [19,20]. For a massless self-interacting field  $\langle X^2 \rangle \approx 0.132 H_i^2/\sqrt{\lambda}$  [21]. Particle creation in the settings close to our problem, but for the specific case of the Hubble-dependent effective mass,  $m_X(t) \propto H(t)$ , was considered in Ref. [22].

In general, if  $m_X \sim H_i \approx m_\phi$ , we can again expect [5] that the particle density will be given by Eq. (9). In general, the coefficient  $C$  in this equation will be a function of the ratio  $H_i/m_X$ , of the coupling constant  $\xi$ , and will depend on details of the transition between inflationary and matter- (or radiation-) dominated phases, etc. For  $m_X > H_i$  an exponential suppression is expected. Detailed numerical study of  $X$  particle abundance created from the vacuum in inflationary cosmology is the subject of this section of the paper.

<sup>1</sup>Another possibility is the late-time entropy release.

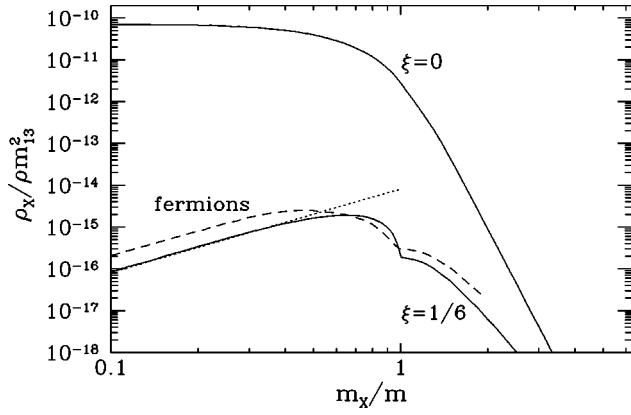


FIG. 2. Ratio of the energy density in  $X$  particles to the total energy density at late times in a model with the massive inflaton,  $V(\phi) = m^2 \phi^2/2$ , as a function of  $X$  particle mass,  $m_X$ . For the inflaton mass we defined  $m_{13} \equiv m/10^{13}$  GeV. The dotted line is the low mass asymptotic, Eq. (9).

The case of large  $m_X$  with application to the dark-matter problem was already considered in Ref. [6]. However, the analysis was restricted to the case of conformally coupled scalar particles only, and approximations were made to the extent that the fixed de Sitter background was matched to the subsequent radiation- or matter-dominated expansion either as an instantaneous transition or with the help of some smoothing function. Particle creation in the case of large  $m_X$  can be especially sensitive to (some extent arbitrary) the details of such a junction procedure.

In the present paper, considering particle creation as described by Eqs. (1)–(8), we do not make any approximations. We assume a specific model of inflation, namely “chaotic” inflation [11], and find numerically the exact evolution of the scale factor in this model using a classical description for the inflaton field and neglecting backreaction of created particles. We consider both the “massive” inflaton with the scalar potential  $V(\phi) = m^2 \phi^2/2$ , and the “massless” inflaton with the potential  $V(\phi) = \lambda \phi^4/4$ . The normalization to the observable large-scale structure requires  $m^2/M_{\text{Pl}}^2 \approx 10^{-12}$  in the former model and  $\lambda \approx 10^{-13}$  in the latter model. We consider creation of conformally and minimally coupled scalar  $X$  particles, as well as creation of  $X$  fermions.

Our results are summarized in Fig. 2 (massive inflaton) and Fig. 3 (massless inflaton). We define the energy density in  $X$  particles at late times as  $\rho_X = m_X n_X$ . In the case of the massive inflaton we normalized  $\rho_X$  by the total-energy density. The ratio of those two quantities reaches some asymptotic value and then remains constant in a matter-dominated universe, and can be measured at any sufficiently late time in the numerical simulation which does not include the inflaton decay. For each value of  $m_X$  we integrated Eqs. (1)–(8) until the asymptotic constant value of this ratio was reached. Similarly, the convenient quantity to be measured in the massless inflaton model is proportional to  $\rho_X / \rho^{3/4}$ , which also becomes constant at late times. The curves labeled by  $\xi=0$  and  $\xi=1/6$  correspond to the minimal and the conformal coupling to gravity, respectively. The dashed line describes the creation of fermions.

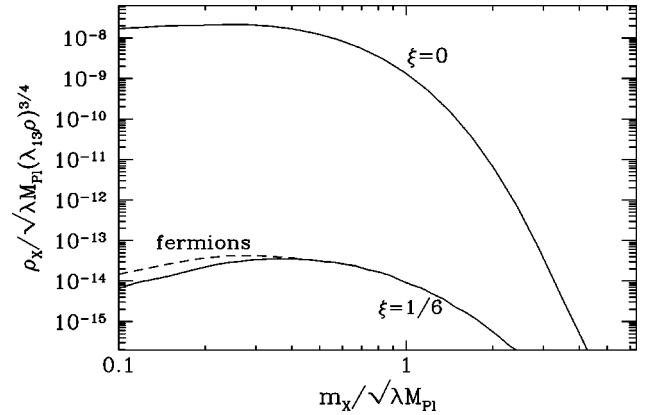


FIG. 3. Energy density of created  $X$  particles in the massless inflaton model,  $V(\phi) = \lambda \phi^4/4$ . We defined  $\lambda_{13} \equiv \lambda/10^{-13}$ .

We see that particle production is most effective in the case of scalar particles with minimal coupling, exceeding  $\rho_X$  generated in other models by many orders of magnitude if  $m_X \lesssim m$ . Note also that the ratio of the energy density in  $X$  particles to the total energy density is independent upon  $m_X$  at  $m_X < m$  in this model. This might seem to be easily understandable. Indeed, the magnitude of fluctuations generated during inflation in de Sitter space is  $\langle X^2 \rangle \propto m_X^{-2}$  if  $m_X \ll H_i$ . Multiplying this by  $m_X^2$  one would find  $\rho_X \approx m_X^2 \langle X^2 \rangle = \text{const}$ , seemingly in agreement with the observed behavior, Fig. 2. However, this agreement is rather coincidental since it is possible to relate  $\rho$  and  $\langle X^2 \rangle$  in this simple manner at late times only.

In our numerical calculations we had found that the variance  $\langle X^2 \rangle$  of the field  $X$  measured at the end of inflation is independent upon  $m_X$  if the mass of  $X$  is small. At some later epoch when  $H \approx m_X$ , which will be long after the end of inflation if  $X$  is a light field, the field  $X$  starts to oscillate on all scales, including  $k=0$ . Only at this time, which we denote by  $t_X$ , all field fluctuations are transformed into nonzero particle density and we can use  $\rho_X = m_X n_X \approx m_X^2 \langle X^2 \rangle$ . The variance of  $X$  fluctuations was unchanged on large scales, starting from the end of inflation down to the time  $t_X$ . So, when the field starts to oscillate,  $\rho_X \propto m_X^2$ . However, the energy density of the inflaton field,  $\rho = 3H^2/8\pi G$  decreased during this time interval in proportion to  $H^2(t_X)/H^2(0) \approx m_X^2/H^2(0)$ . That is why the ratio of the energy density in  $X$  particles to the total-energy density does not depend on  $m_X$  when measured at  $t > t_X$ .

Variance of the field  $X$  is different from that usually calculated for the fixed de Sitter inflationary background because we consider the actual evolution of the scale factor, and the value of the Hubble parameter is not constant during inflation, being larger at earlier times. Correspondingly, the number of created particles per decade of  $k$  grows logarithmically towards small  $k$  if  $m_X$  is small. (The power spectrum behaves similarly.) For definiteness, we restricted our calculations to  $\sim 10^{15}$  for the expansion factor during inflation. The examples of the particle number,  $k^3 n_X(k)$  for several values of  $m_X$  are shown in Fig. 4 at the moment corresponding to ten completed inflaton oscillations. The particle mo-

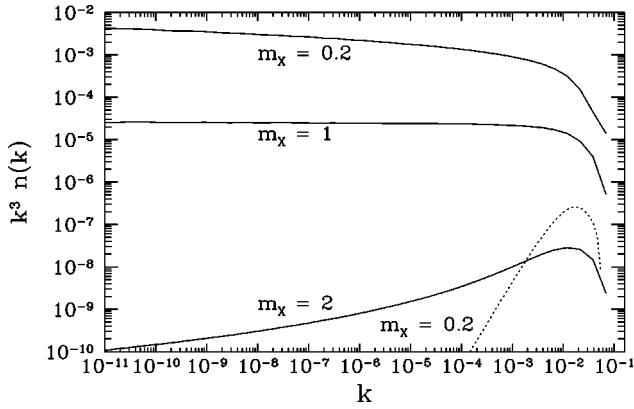


FIG. 4. The spectrum of created particles,  $k^3 n(k)$ , in a model with massive inflaton is shown for several choices of the mass of scalar  $X$  particles with the minimal coupling (solid lines) and the conformal coupling (dotted line). Masses and momenta  $k$  are given in units of the inflaton mass.

mentum is measured in units of the inflaton mass. In contrast to this, in the fixed de Sitter background  $4\pi^2 \langle X^2 \rangle \approx H^2 \int d \ln k (k/H)^{3-2\nu}$  with  $3-2\nu \approx 2m_X^2/3H^2$  at small  $m_X$  and consequently  $\langle X^2 \rangle \propto 1/m_X^2$ . When our code was tested on the fixed de Sitter background it reproduced the proper power spectrum exactly. Note that the power spectrum in the fixed de Sitter background grows towards large values of  $k$ , which is opposite to the behavior of Fig. 4.

Therefore, calculations which would be based on the customary procedure of matching a fixed de Sitter background to a subsequent Friedmann stage would give wrong results in this case. Note, however, that the result depends upon the duration of the inflationary stage, so our results in the case of scalars with minimal coupling and  $m_X < m$  should be considered only as a lower bound on  $X$ -particle abundance.

Matching is also dangerous in the case of large  $m_X$ , and its influence was studied in detail in Ref. [6]. When the change is too abrupt, it generates artificial particles. This can easily happen for  $m_X > m$ , see, e.g., [23] where excessive production was found. At  $m_X \geq m$  the number of created particles decreases exponentially with  $m_X$ . For the case of massive inflaton, our result shown in Fig. 2 can be fitted as

$$\frac{\rho_X}{\rho} \approx 10^{-10} \frac{m_X}{m} m_{13}^2 \exp(-2\pi m_X/m), \quad (10)$$

where  $m_{13} \equiv m/10^{13}$  GeV. (Note that the fitting formulas are approximate and arbitrary to some extent.)

As Fig. 4 shows, the power spectrum of fluctuations in  $X$  particles is almost scale independent at small  $k$  if  $m_X/m \approx 1$ . Therefore, if such particles constitute a considerable fraction of dark matter, these fluctuations will be transformed into isocurvature density perturbations at late times and can affect the large-scale structure formation. Isocurvature fluctuations produce six times larger angular temperature fluctuations in cosmic microwave background radiation (CMBR) for the same amplitude of long-wavelength density perturbations compared to the adiabatic case [24]. Therefore, to fit observations by a single spectrum, the mass fluctuation spec-

trum in isocurvature cold-dark-matter cosmology must be tilted (with respect to scale invariant spectrum) to favor smaller scales.

Let the power spectrum of the field fluctuations be  $k^3 P_X(k) \propto k^\beta$ . To fit to the second moments of the large-scale mass and (not contradict) cosmic microwave background distributions requires  $\beta \geq 0.25$ . Models with  $\beta$  ranging from 0.3 to 0.6 were constructed and considered in Refs. [25,26]. It is interesting that in our case the spectrum is indeed correctly tilted, see Fig. 4, for  $m_X$  which is somewhat larger than the inflaton mass. On the largest scale of our integration,  $k \approx 10^{-15}$  we find  $\beta \approx 0.1$  for  $m_X/m = 2$  and  $\beta \approx 0.3$  for  $m_X/m = 3$ . Since  $\beta$  decreases with the length scale increasing, on scales corresponding to the current horizon  $\beta$  will be smaller. Because of that the amplitude of perturbations will be also too small on cosmological scales when  $\beta$  will be reaching the desired magnitude,  $\beta \geq 0.25$ . Indeed, density fluctuations,  $\delta\rho_X/\rho_X$ , which are roughly proportional to spectra shown in Fig. 4, are reaching unity near the break in the power spectrum ( $k \sim 1$  at the end of inflation) and on the scales of interest amplitude of fluctuations became too small already at  $m_X/m = 3$ . However, their magnitude may be just right,  $\delta\rho_X/\rho_X \approx 10^{-5}$ , on scales  $k \geq 10^{-25}$  for the case  $m_X/m \approx 2$  (or slightly larger) if  $\Omega_X \approx 1$ , see Fig. 4. Magnitude cannot correspond to the observable on the galaxy scales since then there will be too many fluctuations in CMBR. This puts a constraint on the models.  $X$  particles with minimal coupling to gravity,  $m_X < 2m$  and  $\Omega_X \approx 1$  are excluded. (Such particles with a small contribution to  $\Omega$  may exist though.) However, the magnitude of the density fluctuations induced in the process of  $X$  particle creation can correspond to the observable on the horizon scale, and be responsible for fluctuations in CMBR.  $X$  particles with  $m_X/m \approx 2$  and  $\Omega_X \approx 1$  can give such a contribution naturally. (The source for the galaxies' seeds has to be different in this case.) Note that the density autocorrelation function is proportional to the square of the field autocorrelation function in the present case, and the density fluctuations are non-Gaussian.

In the case of fermions or scalar particles with conformal coupling, the number density of the created particles has a kink at  $m_X = m$  in the model with massive inflaton, see Fig. 2. This signifies that different mechanisms are responsible for particle creation at  $m_X > m$  and  $m_X < m$ . Particles heavier than the inflaton are created during inflation, while particles lighter than the inflaton are created after inflation during the regular Friedmann stage of expansion. We see that particle creation during this latter stage is actually more effective ( $n_X$  rapidly increases when  $m_X$  drops slightly below  $m$ ). Hence, we can assume that the number density of created particles at  $m_X \ll m$  is given by Eq. (9) with  $C_\alpha = C_{2/3}$ . ( $C_{2/3} \approx 9.7 \times 10^{-4}$  for the conformal scalars, see Fig. 1.) Since the total-energy density is given by  $\rho = 3H^2/8\pi G$ , we find for  $\alpha = 2/3$ ,

$$\frac{\rho_X}{\rho} = \frac{8\pi m_X^2}{3 M_{\text{Pl}}^2} C_{3/2}. \quad (11)$$

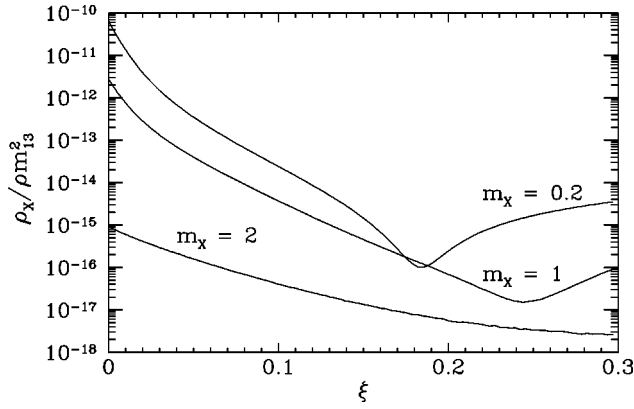


FIG. 5. Ratio of the energy density in  $X$  particles to the total energy density at late times in a model with the massive inflaton,  $V(\phi) = m^2 \phi^2/2$ , as a function of  $\xi$ .

This function is plotted in Fig. 2 by the dotted line which shows that already at  $m_X < 0.5m$  the number density of created particles is given by Eq. (9), indeed. Note that in Ref. [6] all particle creation was attributed to the transition from the inflationary to matter-dominated phase, while we find that for  $m_X < m$  the dominant particle creation occurs at a later stage, when  $H \approx k \approx m_X$  (for  $m_X < m$ ). The correct interpretation allows us to identify the low mass asymptotic, Eq. (11), see also [5].

Scalar particles with minimal coupling to gravity are produced predominantly during the inflationary stage for all  $m_X$ , and the resulting number density is much higher, see Fig. 2.

Conformal coupling,  $\xi = 1/6$ , is a stable point of renormalization-group equations. Otherwise  $\xi$  is a running coupling constant and there is no reason to expect the coupling to be, say, minimal at mass scales of inflation. The dependence of  $X$ -particle abundance on the coupling to the curvature scalar is shown in Fig. 5.

The situation in the case of the massless inflaton,  $V(\phi) = \lambda \phi^4/4$ , is quite similar. However, now there is no distinctive mass scale in the equations of motion for the scale factor, and the kink in the function  $n_X(m_X)$  does not appear. The quantity  $\sqrt{\lambda} \phi$  might be considered as an effective inflaton mass, but it changes during the evolution. For definiteness we plot  $n_X$  as a function of  $m_X/\sqrt{\lambda} M_{\text{Pl}}$ . Note that the transition from the inflationary to the radiation-dominated stage occurs at  $\phi \approx 0.35 M_{\text{Pl}}$  [15] and this may define the more natural mass scale for the ratio  $m_X/m_{\text{eff}}(\phi)$ . Indeed, we observe that  $n_X$  reaches its maximum when  $m_X \approx 0.35 \sqrt{\lambda} M_{\text{Pl}}$ , see Fig. 3.

We find that the number density of produced fermions in both cases is approximately equal to the number density of scalar particles with conformal coupling. The main difference amounts to a spin factor of 2.

#### IV. PRESENT DENSITY OF $X$ PARTICLES

Let us estimate the present day number density of  $X$  particles. Consideration is somewhat different for massive and massless inflaton models.

*Massive inflaton*,  $V(\phi) = m_\phi^2 \phi^2/2$ .

In this case the inflation is followed by the stage of matter domination which lasts until inflaton decays and thermal equilibrium is established. Let us denote the temperature at the beginning of radiation-dominated stage as  $T_*$ . Actually, a transition to the radiation-dominated universe does not necessarily occur with the final state being in thermal equilibrium. Complications arise if there are light bosons in a theory,  $m_B \ll m_\phi$ , even relatively weakly coupled to the inflaton,  $g^2 \gtrsim 10^4 m_\phi^2/M_{\text{Pl}}^2 \sim 10^{-8}$ . In this case the inflaton will decay via parametric resonance after just a few oscillations and relativistic degrees of freedom start to dominate the equation of state. This happens typically when the energy density in the inflaton oscillations is redshifted by a factor  $r \approx 10^{-6}$  compared to the value  $m_\phi^2 M_{\text{Pl}}^2$  [14,15]. Matter is still far from being in thermal equilibrium, but since Universe expansion already at this point changes to radiation-dominated it is convenient to characterize this radiation-dominated stage by an equivalent ‘‘temperature,’’  $T_* \sim r^{1/4} \sqrt{m_\phi M_{\text{Pl}}}$ . In any case,  $T_*$  is the model-dependent parameter and we leave it as such.

Until the moment when the equation of state changes from matter dominated to radiation dominated, the ratio of the energy density in  $X$  particles to the total-energy density retains its value reached at the end of inflation,  $\rho_X/\rho = \text{const}$ . This ratio is plotted in Fig. 2 for different models as a function of model parameters and it defines the present day ratio of  $\rho_X$  to the critical density in the Universe,  $\rho_c$ , as follows:

$$\Omega_X h^2 = \frac{\rho_X}{\rho} \frac{T_*}{\gamma T_0} \Omega_R h^2, \quad (12)$$

where  $T_0$  is present temperature of cosmic microwave background, and  $\Omega_R h^2 = \rho_{\text{rad}} h^2/\rho_c \approx 4.31 \times 10^{-5}$  is the fraction of critical density that is in radiation today [13]. The following definitions are used here  $\rho_c = 3H_0^2 M_{\text{Pl}}^2/8\pi$  and  $H_0 = 100h \text{ km s}^{-1} \text{ Mpc}^{-1}$ . Parameter  $\gamma$  describes the change in entropy per comoving volume for the whole cosmological history starting from  $T = T_*$  down to the present day, i.e.,  $s_0 \equiv \gamma s_*$ . In simple cosmological models,  $\gamma \sim 1$ . However, in models with subsequent latent heat release (due to phase transitions or due to the decay of long-living particles, e.g., moduli fields)  $\gamma$  can be very large.

Let us assume for illustration purposes that  $T_* = 10^9 \text{ GeV}$ , i.e.,  $(T_*/T_0)(\Omega_R h^2) \approx 1.8 \times 10^{17}$ . (This is the highest allowed reheating temperature when gravitinos in supergravity theories are not overproduced [27].) With  $\gamma \sim 1$  we find that  $\Omega_X h^2 \sim 1$  if  $m_X/m \approx 2$  in models with scalar  $X$  particles with conformal coupling, or if  $X$  particles are fermions, see Fig. 2. For minimally coupled scalars this value of  $\Omega_X$  is reached for  $m_X/m \approx 3$ .

To summarize this subsection:  $\Omega_X h^2$  is found by multiplying the data of Fig. 2 by a factor

$$1.8 \times 10^{17} (T_*/\gamma 10^9 \text{ GeV}) m_3^2.$$

*Massless inflaton*,  $V(\phi) = \lambda \phi^4/4$ .

Before and after the decay of the inflaton the Universe expands as radiation dominated. In Sec. III for the state which precedes the decay of inflaton oscillations we had calculated the quantity  $\rho_X/\sqrt{\lambda}M_{\text{Pl}}\rho^{3/4}$ , where  $\rho$  is the inflaton energy density. After the decay of the inflaton, in a state of thermal equilibrium we can use

$$s_* = \frac{4}{3} \left( \frac{\pi^2 g_*}{30} \right)^{1/4} \rho_*^{3/4}, \quad (13)$$

where  $g_*$  is the number of relativistic degrees of freedom at temperature  $T_*$ . Assuming instantaneous transition between those states,  $\rho_* = \rho$ , and again using

$$\Omega_X = (\rho_X/T_0 s_0)(T_0 s_0/\rho_c)$$

together with  $s_0 \equiv \gamma s_*$ , we find

$$\Omega_X h^2 = \frac{\sqrt{\lambda} M_{\text{Pl}}}{\gamma T_0} \left( \frac{\pi^2 g_*}{30} \right)^{-1/4} \frac{\rho_X}{\sqrt{\lambda} M_{\text{Pl}} \rho^{3/4}} \Omega_R h^2. \quad (14)$$

We see that  $\Omega_X h^2$  can be found by multiplying the data plotted in Fig. 3 by a factor  $2.5 \times 10^{20} \lambda_{13}^{5/4} / \gamma g_{200}^{1/4}$ , where we defined  $g_{200} \equiv g_*/200$ , and used  $\sqrt{\lambda} M_{\text{Pl}}/T_0 \approx 1.6 \times 10^{25} \sqrt{\lambda_{13}}$ . Note that  $\sqrt{\lambda} M_{\text{Pl}} \approx 3.9 \times 10^{12} \sqrt{\lambda_{13}}$  GeV, so that already with  $\gamma=1$  we shall have  $\Omega_X < 1$  if  $m_X \geq 4 \times 10^{13}$  GeV.

## V. X PARTICLES AND UHE COSMIC RAYS

Why were the ultrahigh-energy cosmic rays observed above the Greisen-Zatsepin-Kuzmin energy cutoff? One solution to the problem might be provided, for example, by the existence of some exotic particles which are able to propagate (evading the GZK bound) from cosmological distances and yet interact in the Earth's atmosphere like a hadron. A particle with such conflicting properties was found in a class of supersymmetric theories [28]. Alternatively, high-energy cosmic rays may have been produced locally within the GZK distance. One possibility is connected with the destruction of topological defects [29], while another one is connected with decays of primordial heavy long-living particles [3,4]. The candidate  $X$  particle must obviously obey constraints on mass, density, and lifetime.

In order to produce cosmic rays in the energy range  $E \geq 10^{11}$  GeV, the mass of  $X$  particles has to be very large,  $m_X \geq 10^{13}$  GeV [3,4]. The lifetime  $\tau_X$  cannot be much smaller than the age of the Universe,  $\tau \approx 10^{10}$  yr. With such a short lifetime, the observed flux of UHE cosmic rays will be generated with the rather low density of  $X$  particles,  $\Omega_X \sim 10^{-12}$ , where  $\Omega_X \equiv m_X n_X / \rho_{\text{crit}}$ ,  $n_X$  is the number density of  $X$  particles, and  $\rho_{\text{crit}}$  is the critical density. On the other hand,  $X$  particles must not overclose the Universe,  $\Omega_X \leq 1$ . With  $\Omega_X \sim 1$ , the  $X$  particles may play the role of cold dark matter and the observed flux of UHE cosmic rays can be matched if  $\tau_X \sim 10^{22}$  yr. In Refs. [30] the observed spectra of UHE cosmic rays were fitted to the model assuming specific abundances, the lifetime and  $m_X$  in the range  $10^{12} \lesssim m_X/\text{GeV} \lesssim 10^{14}$ . However, the statistics is poor at present to pinpoint parameters (or rule out the model). The allowed

windows are quite wide [3], but on the exotic side, which may give rise to some problems.

The possibility that  $X$  particles were generated from the vacuum by gravitational interactions [5,6] may answer the question why their abundance should lie in the interesting range  $\Omega_X \lesssim 1$ , and was considered in detail in this paper.

The problem of the particle physics mechanism responsible for a long but finite lifetime of very heavy particles can be solved in several ways. For example, the otherwise conserved quantum number carried by  $X$  particles may be broken very weakly due to instanton transitions [3], or quantum gravity (wormhole) effects [4]. If instantons are responsible for  $X$ -particle decays, the lifetime is estimated as  $\tau_X \sim m_X^{-1} \cdot \exp(4\pi/\alpha_X)$ , where  $\alpha_X$  is the coupling constant of the relevant gauge interaction. The lifetime will fit the allowed window if the coupling constant (at the scale  $m_X$ ) is  $\alpha_X \approx 0.1$  [3].

A class of natural candidates for superheavy long-living particles which arise in string and  $M$  theory was reevaluated recently in Ref. [31] and particles with desired mass and long lifetime were identified. Other interesting candidates were found among adjoint messengers in gauge-mediated supergravity models [6,32] and in models of superheavy dark matter with discrete gauge symmetries [33].

## VI. CONCLUSIONS

We have shown that the very weakly interacting superheavy  $X$  particles with  $m_X = (\text{a few}) \cdot 10^{13}$  GeV may naturally constitute a considerable fraction of cold dark matter. These particles are produced in the early Universe from vacuum fluctuations during or after inflation. Related density fluctuations may have left an imprint in fluctuations of cosmic microwave background radiation if scalar  $X$  particles with minimal coupling to gravity are approximately twice heavier than the inflaton and  $\Omega_X \sim 1$ . Decays of  $X$  particles may explain UHE cosmic ray phenomenon.

Our hypothesis has unique observational consequences. If UHE cosmic rays are indeed due to the decay of these superheavy particles, there has to be a new sharp cutoff in the cosmic ray spectrum at energies somewhat smaller than  $m_X$ . Since the number density  $n_X$  depends exponentially upon  $m_X/m_\phi$ , the position of this cutoff is fixed and can be predicted to be near  $m_\phi \approx 10^{13}$  GeV, the very shape of the cosmic ray spectrum beyond the GZK cutoff being of quite generic form following from the QCD quark/gluon fragmentation. Next generation experiments, like the Pierre Auger Project [34], High Resolution Fly's Eye [35], the Japanese Telescope Array Project [36], may prove able to discover this fundamental phenomenon.

We conclude that the observations of ultrahigh-energy cosmic rays can probe the spectrum of elementary particles in the superheavy range and can give a unique opportunity for investigation of the earliest epoch of evolution of the Universe, starting with the amplification of vacuum fluctuations during inflation through fine details of gravitational interaction and down to the physics of reheating.

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- [1] K. Greisen, Phys. Rev. Lett. **16**, 748 (1966); G. T. Zatsepin and V. A. Kuzmin, Pis'ma Zh. Éksp. Teor. Fiz. **4**, 114 (1966) [JETP Lett. **4**, 78 (1966)].
- [2] N. Hayashida *et al.*, Phys. Rev. Lett. **73**, 3491 (1994); D. J. Bird *et al.*, Astrophys. J. **424**, 491 (1994); **441**, 144 (1995); T. A. Egorov *et al.*, in *Proceedings of the Tokyo Workshop on Techniques for the Study of Extremely High Energy Cosmic Rays*, edited by M. Nagano (ICRR, University of Tokyo, 1993).
- [3] V. A. Kuzmin and V. A. Rubakov, Yad. Fiz. **61**, 1122 (1998) [Phys. At. Nucl. **61**, 1028 (1998)].
- [4] V. Berezhinsky, M. Kachelriess, and A. Vilenkin, Phys. Rev. Lett. **79**, 4302 (1997).
- [5] V. A. Kuzmin and I. I. Tkachev, Pis'ma Zh. Éksp. Teor. Fiz. **68**, 255 (1998) [JETP Lett. **68**, 271 (1998)].
- [6] D. J. H. Chung, E. W. Kolb, and A. Riotto, Phys. Rev. D **59**, 023501 (1999); Phys. Rev. Lett. **81**, 4048 (1998).
- [7] D. J. H. Chung, E. W. Kolb, and A. Riotto, hep-ph/9809453.
- [8] S. G. Mamaev, V. M. Mostepanenko, and A. A. Starobinskii, Zh. Éksp. Teor. Fiz. **70**, 1577 (1976) [Sov. Phys. JETP **43**, 823 (1976)].
- [9] S. G. Mamaev, V. M. Mostepanenko, and V. M. Frolov, Yad. Fiz. **23**, 1118 (1976) [Sov. J. Nucl. Phys. **23**, 592 (1976)]; **26**, 215 (1977) [**26**, 111 (1977)].
- [10] L. Parker, Phys. Rev. **183**, 1057 (1969); A. A. Grib and S. G. Mamaev, Yad. Fiz. **10**, 1276 (1969) [Sov. J. Nucl. Phys. **10**, 722 (1970)]; Ya. B. Zel'dovich and A. A. Starobinskii, Zh. Exp. Teor. Fiz. **61**, 2161 (1971) [Sov. Phys. JETP **34**, 1159 (1972)].
- [11] A. D. Linde, Phys. Lett. **108B**, 389 (1982); A. Albrecht and P. J. Steinhardt, Phys. Rev. Lett. **48**, 1220 (1982).
- [12] A. D. Linde, *Particle Physics and Inflationary Cosmology* (Harwood Academic, New York, 1990).
- [13] E. W. Kolb and M. S. Turner, *The Early Universe* (Addison-Wesley, Reading, MA, 1990).
- [14] L. A. Kofman, A. D. Linde, and A. A. Starobinsky, Phys. Rev. Lett. **73**, 3195 (1994); Y. Shtanov, J. Traschen, and R. Brandenberger, Phys. Rev. D **51**, 5438 (1995).
- [15] S. Yu. Khlebnikov and I. I. Tkachev, Phys. Rev. Lett. **77**, 219 (1996); Phys. Lett. B **390**, 80 (1997); Phys. Rev. Lett. **79**, 1607 (1997); Phys. Rev. D **56**, 653 (1997).
- [16] L. A. Kofman, A. D. Linde, and A. A. Starobinsky, Phys. Rev. D **56**, 3258 (1997).
- [17] E. W. Kolb, A. Riotto, and I. I. Tkachev, Phys. Lett. B **423**, 348 (1998).
- [18] J. Baacke, K. Heitmann, and C. Patzold, Phys. Rev. D **58**, 125013 (1998); P. B. Greene and L. Kofman, hep-ph/9807339.
- [19] N. A. Chernikov and E. A. Tagirov, Ann. Inst. Henri Poincaré, Sect. A **9A**, 109 (1968); E. A. Tagirov, Ann. Phys. (N.Y.) **76**, 561 (1973); T. S. Bunch and P. C. W. Davies, Proc. R. Soc. London **360**, 117 (1978).
- [20] A. D. Linde, Phys. Lett. **116B**, 335 (1982); A. A. Starobinsky, *ibid.* **117B**, 175 (1982); A. Vilenkin and L. H. Ford, Phys. Rev. D **26**, 1231 (1982); B. Allen, *ibid.* **32**, 3136 (1985).
- [21] A. A. Starobinsky and J. Yokoyama, Phys. Rev. D **50**, 6357 (1994).
- [22] D. Lyth and D. Roberts, Phys. Rev. D **57**, 7120 (1998).
- [23] K. Enqvist, K. W. Ng, and K. A. Olive, Nucl. Phys. **B303**, 713 (1988).
- [24] A. A. Starobinsky and V. Sahni, in *Proceedings of the 6th Soviet Gravitation Conference*, edited by V. N. Ponomarev (MGPI Press, Moscow, 1984); G. Efstathiou and J. R. Bond, Mon. Not. R. Astron. Soc. **218**, 103 (1986); H. Kodama and M. Sasaki, Int. J. Mod. Phys. A **2**, 491 (1987); A. A. Starobinsky, Pis'ma Astron. Zh. **14**, 394 (1988) [Sov. Astron. Lett. **14**, 166 (1988)].
- [25] A. Linde and V. Mukhanov, Phys. Rev. D **56**, 535 (1997).
- [26] P. J. E. Peebles, astro-ph/9805194; astro-ph/9805212.
- [27] J. Ellis, J. E. Kim, and D. V. Nanopoulos, Phys. Lett. **145B**, 181 (1984).
- [28] G. R. Farrar, Phys. Rev. Lett. **76**, 4111 (1996); D. J. Chung, G. R. Farrar, and E. W. Kolb, Phys. Rev. D **57**, 4606 (1998).
- [29] C. T. Hill, Nucl. Phys. **B224**, 469 (1983); C. T. Hill, D. N. Schramm, and T. P. Walker, Phys. Rev. D **36**, 1007 (1987); G. Sigl, D. N. Schramm, and P. Bhattacharjee, Astropart. Phys. **2**, 401 (1994); V. Berezhinsky, X. Martin, and A. Vilenkin, Phys. Rev. D **56**, 2024 (1997); V. Berezhinsky and A. Vilenkin, Phys. Rev. Lett. **79**, 5202 (1997).
- [30] V. Berezhinsky, astro-ph/9801046; M. Birkel and S. Sarkar, Astropart. Phys. **9**, 297 (1998).
- [31] K. Benakli, J. Ellis, and D. V. Nanopoulos, Phys. Rev. D **59**, 047301 (1999).
- [32] T. Han, T. Yanagida, and R.-J. Zhang, Phys. Rev. D **58**, 095011 (1998).
- [33] K. Hamaguchi, Y. Nomura, and T. Yanagida, Phys. Rev. D **58**, 103503 (1998); **59**, 063507 (1999).
- [34] J. W. Cronin, Nucl. Phys. B (Proc. Suppl.) **288**, 213 (1992); The Pierre Auger Observatory Design Report, 2nd ed., March 1997; M. Boratav, for the Pierre Auger Collaboration, *The Pierre Auger Observatory Project: An Overview*, Proceedings of 25th International Cosmic Ray Conference, Durban, Vol. 5, p. 205 (1997).
- [35] S. C. Corbato *et al.*, Nucl. Phys. B (Proc. Suppl.) **288**, 36 (1992).
- [36] M. Teshima *et al.*, Nucl. Phys. B (Proc. Suppl.) **288**, 169 (1992).