New experimental tests of sum rules for charmed baryon masses

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(Received 13 January 1999; published 13 April 1999)

New experimental measurements are used to test model independent sum rules for charmed baryon masses. Sum rules for medium-strong mass differences are found to be reasonably well satisfied with increasing accuracy, and the new measurements permit an improved prediction of 2778±9 MeV for the mass of the Ω_c^{*0}. But an isospin breaking sum rule for the Σ_c mass splitting is still in significant disagreement posing a serious problem for the quark model of charmed baryons. Individual Σ_c mass splittings are investigated, using the new CLEO measurement of the Ξ_c' mass splitting, but the accuracy is not yet sufficient for a good test. [S0556-2821(99)01411-3]

PACS number(s): 12.40.Yx, 14.20.−c, 14.40.−n

Model independent sum rules [1–3] were derived some time ago for heavy-quark baryon masses using minimal assumptions within the quark model. The sum rules depend on standard quark model assumptions, and an additional assumption that the interaction energy of a pair of quarks in a particular spin state does not depend on which baryon the pair of quarks is in (''baryon independence''). This is a somewhat weaker assumption than full SU(3) symmetry of the wave function, which would require the same spatial wave function for each octet baryon, and each individual wave function to the SU(3) symmetrized. Instead we use wave functions with no SU(3) symmetry, as described in Ref. [4]. The wave functions can also be different for different quarks. For instance, a *u-s* pair in the Σ^+ hyperon can have a different spatial wave function than a *u-d* pair in the proton, but is assumed to have the same interaction energy as a *u-s* pair in the Ξ^0 hyperon.

In deriving the sum rules, no assumptions are made about the type of potential, and no internal symmetry beyond baryon independence is assumed. The sum rules allow any amount of symmetry breaking in the interactions and individual wave functions, but do rest on baryon independence for each quark-quark interaction energy. Several of the sum rules [Eqs. (4), (5), and (6) below] also rely on the assumption that there is no orbital angular momentum so that the three spin-$\frac{1}{2}$ quark spins add directly to spin-$\frac{1}{2}$ or spin-$\frac{3}{2}$. More detailed discussion of the derivation of the sum rules is given in Refs. [1] and [4].

We have previously tested these sum rules in Refs. [2] and [3] using early measurements of heavy-quark baryon masses. Those tests showed reasonable agreement within fairly large experimental errors for two sum rules for medium-strong charmed baryon mass differences and for one sum rule for bottom baryon mass differences. But there was a relatively large, and worrisome, discrepancy for the isospin breaking mass differences between the Σ_c charge states. Since those tests, there have been a number of new experiments [6–11] resulting in more accurate and more reliable values for some of the charmed baryon masses used in the sum rules. In this paper we look at the effect on the sum rules of these new experiments, especially the recent CLEO II measurement [10] of the $\Xi_c'^{+}$ and $\Xi_c'^{0}$ masses.

The measured charmed baryon masses that will be used in the sum rules are listed in Table I for the expected baryon assignments. The Ξ_c^+ baryon and the $\Xi_c'^{+}$ baryon are distinguished, in the quark model, by having different spin states for the *u-s* quark pair. The Ξ_c^+ is the spin-$\frac{1}{2}$ *usc* baryon having the *u-s* quarks in a spin zero state, and the $\Xi_c'^{+}$ has the *u-s* quarks in a spin one state. A similar distinction is made for the *d-s* quark pair in the Ξ_c^0 and $\Xi_c'^{0}$ charmed baryons. The numerical values in Table I are given in terms of appropriate mass differences when that corresponds to how the measurement was made. Where new experiments have given more accurate numbers since our previous test of the sum rules, an asterisk has been put after the reference. Masses for light quark (*u*,*d*,*s*) baryons are all taken from the Review of Particle Physics [5].

The isospin breaking sum rule for the Σ_c masses is [2]

$$\begin{array}{ccccc} \Sigma^+ + \Sigma^- - 2\Sigma^0 & = & \Sigma^{*+} + \Sigma^{*-} - 2\Sigma^{*0} & = & \Sigma_c^{++} + \Sigma_c^0 - 2\Sigma_c^+ , \\ (1.7\pm .2) & & (2.6\pm 2.1) & & (-2.2\pm 1.2) \end{array} \tag{1}$$

TABLE I. Charmed baryon masses used in the sum rules. The asterisk indicates where new experiments have given more accurate numbers since our previous test of the sum rules.

Baryon	Mass (MeV)	Reference
Λ_c^+	2284.9 ± 0.6	[5]
Σ_c^{++}	$\Lambda_c^+ + 167.9\pm 0.2$	[5,6,7]*
Σ_c^0	$\Sigma_c^{++} - 0.6\pm 0.2$	[5,6]*
Σ_c^+	$\Sigma_c^0 + 1.4\pm 0.6$	[12]
Σ_c^{*++}	$\Lambda_c^+ + 234.5\pm 1.4$	[8]*
Σ_c^{*0}	$\Lambda_c^+ + 232.6\pm 1.3$	[8]*
Ξ_c^+	2465.6 ± 1.4	[5,9]*
Ξ_c^0	2470.3 ± 1.8	[5]
$\Xi_c'^{+}$	$\Xi_c^+ + 107.8\pm 3.0$	[10]*
$\Xi_c'^{0}$	$\Xi_c^0 + 107.0\pm 2.9$	[10]*
Ξ_c^{*+}	$\Xi_c^0 + 174.3\pm 1.1$	[11]*
Ξ_c^{*0}	$\Xi_c^+ + 178.2\pm 1.1$	[13]
Ω_c^0	2704 ± 4	[5]

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where we have written the experimental values in MeV below each equation. There is reasonable agreement for the $\Sigma - \Sigma^*$ sum rule, as well as for several other isospin breaking sum rules for light quark baryons [1,4]. But the Σ_c isospin splitting combination is significantly different from the other two combinations in Eq. (1). As noted in Ref. [2], this disagreement poses a serious problem because it is difficult to see how any reasonable quark model of charmed baryons could lead to the relatively large negative value for the Σ_c combination in Eq. (1). A large number of specific quark model calculations [14] of charmed baryon masses generally satisfy the Σ_c sum rule, and all predict large positive values for the Σ_c mass combination in Eq. (1).

The experimental input that has been used for this combination of Σ_c masses are the two separate mass difference measurements:

$$\Sigma_c^{++} - \Sigma_c^0 = 0.6 \pm 0.2 \quad \text{Ref. [5]}, \tag{2}$$

$$\Sigma_c^{+} - \Sigma_c^0 = 1.4 \pm 0.6 \quad \text{Ref. [12]}. \tag{3}$$

The $\Sigma_c^{++} - \Sigma_c^0$ mass difference results from four separate experiments that are reasonably consistent with one another, while there is only one experiment [12] that has measured the $\Sigma_c^{+} - \Sigma_c^0$ difference. There is no reason to question this experimental measurement of $\Sigma_c^{+} - \Sigma_c^0$, and the result of Ref. [12] for $\Sigma_c^{++} - \Sigma_c^0$ agrees well with the other experiments [15]. However, the extreme importance of the large discrepancy in the Σ_c sum rule of Eq. (1) should make a new experimental measure of the mass difference $\Sigma_c^{+} - \Sigma_c^0$ a high priority.

The new experimental measurement of the Ξ_c' masses [10] makes it possible, in principle, to test sum rules for separate mass differences of the Σ_c. These are

$$\begin{array}{l}\Sigma_c^{++} - \Sigma_c^0 = \Sigma^{*+} - \Sigma^{*-} + 2[(\Xi^{*-} - \Xi^{*0}) + (\Xi_c'^{+} - \Xi_c'^{0})] \\ (0.6 \pm 0.2) \quad (-6.2 \pm 9.7)\end{array} \tag{4}$$

$$\begin{array}{l}\Sigma_c^{+} - \Sigma_c^0 = \Sigma^{*0} - \Sigma^{*-} + (\Xi^{*-} - \Xi^{*0}) + (\Xi_c'^{+} - \Xi_c'^{0}). \\ (1.4 \pm 0.6) \quad (-4.2 \pm 4.9)\end{array} \tag{5}$$

Unfortunately, the experimental errors on the Ξ_c' mass differences are still too large at this point to make an accurate comparison with the Σ_c mass differences.

Although the discrepancy noted above for the Σ_c mass differences puts any other quark model study of charmed baryons into question, we now look at sum rules for medium-strong mass differences, anticipating some eventual resolution (theoretical or experimental) of the difficulties posed by the Σ_c mass splitting. A new measurement [8] of the masses of the Σ^{*++} and Σ^{*0} baryons makes possible a more accurate test of the sum rule [3]

$$\begin{array}{l}(\Sigma_c^{*+} - \Lambda_c^{+}) + \frac{1}{2}(\Sigma_c^{+} - \Lambda_c^{+}) = (\Sigma^{*0} - \Lambda^0) + \frac{1}{2}(\Sigma^0 - \Lambda^0). \\ (319 \pm 2) \qquad\qquad (307)\end{array} \tag{6}$$

We use the measured Σ_c^{*++} mass for the Σ_c^{*+}mass, but that difference is probably small. A corresponding sum rule [3] for the *b*-quark baryons $\Sigma_b^{*0}, \Sigma_b^0, \Lambda_b^0$ has not changed, and is in good agreement.

In Ref. [2] we used a sum rule to predict 2583 ± 3 MeV for the Ξ_c^{*+} mass. This mass has now been measured [10], and is listed in Table I. This permits a test of the sum rule, which we write here as

$$\begin{array}{l}\Sigma_c^{++} + \Omega_c^0 - 2\Xi_c'^{+} = \Sigma^{+} + \Omega^{-} - \Xi^0 - \Xi^{*0}. \\ (10 \pm 8) \qquad\qquad (15)\end{array} \tag{7}$$

The two sum rules in Eqs. (6) and (7) are satisfied to about the same extent as light-quark baryon sum rules relating spin-$\frac{1}{2}$ baryon masses to spin-$\frac{3}{2}$ baryon masses [1,4].

The new experimental measurements can be used to improve the accuracy of our previous prediction [3] of the as yet unmeasured Ω_c^{*0} mass

$$\Omega_c^{*0} = \Omega_c^0 + 2(\Xi_c^{*+} - \Xi_c'^{+}) - (\Sigma_c^{*++} - \Sigma_c^{++}) = 2779 \pm 9. \tag{8}$$

In conclusion, we can say that increasingly accurate experimental mass determinations are making the model independent sum rule discussed here increasingly useful tests of the quark model for charmed baryons. We see that sum rules for medium-strong energy differences are satisfied at least as well for heavy-quark baryon as for light-quark baryons. However there remains a serious disagreement for the Σ_c isospin breaking sum rule, which is violated by three standard deviations. Since sum rules in disagreement are of more concern than those which are satisfied, resolving the Σ_c mass differences is of prime importance. Thus far no theoretical suggestion has been forthcoming.

[1] J. Franklin, Phys. Rev. D **12**, 2077 (1975).
[2] J. Franklin, Phys. Rev. D **53**, 564 (1996).
[3] J. Franklin, Phys. Rev. D **55**, 423 (1997).
[4] J. Franklin, Phys. Rev. **172**, 1807 (1968).
[5] Particle Data Group, L. Montanet *et al.*, Phys. Rev. D **50**, 1173 (1994).
[6] E791 Collaboration, E. M. Aitala *et al.*, Phys. Lett. **96B**, 292 (1991).
[7] E687 Collaboration, P. Frabetti *et al.*, Phys. Lett. B **365**, 461 (1996).
[8] CLEO II Collaboration, G. Brandenburg *et al.*, Phys. Rev. Lett. **78**, 2304 (1997).
[9] CLEO II Collaboration, K. Edwards *et al.*, Phys. Lett. B **373**, 261 (1996).
[10] CLEO II Collaboration, C. P. Jessup *et al.*, Phys. Rev. Lett. **82**, 492 (1999).

[11] CLEO II Collaboration, P. Avery *et al.*, Phys. Rev. Lett. **75**, 4364 (1996).

[12] CLEO II Collaboration, G. Crawford *et al.*, Phys. Rev. Lett. **71**, 3259 (1993).

[13] CLEO II Collaboration, L. Gibbons *et al.*, Phys. Rev. Lett. **77**, 810 (1995).

[14] C. Itoh *et al.*, Prog. Theor. Phys. **54**, 908 (1975); K. Lane and S. Weinberg, Phys. Rev. Lett. **37**, 717 (1976); S. Ono, Phys. Rev. D **15**, 3492 (1977); N. G. Deshpande *et al.*, *ibid.* **15**, 1885 (1977); L.-H. Chan, *ibid.* **15**, 2478 (1977); **31**, 204 (1985); D. B. Lichtenberg, *ibid.* **16**, 231 (1977); C. S. Kalman and G. Jakimow, Lett. Nuovo Cimento **19**, 403 (1997); A. C. D. Wright, Phys. Rev. D **17**, 3130 (1978); N. Isgur, *ibid.* **21**, 779 (1980); J. M. Richard and P. Taxil, Z. Phys. C **26**, 421 (1984); W.-Y. P. Wang and D. B. Lichtenberg, Phys. Rev. D **35**, 3526 (1987); S. Capstick, *ibid.* **36**, 2800 (1987); S. Sinha *et al.*, Phys. Lett. B **218**, 333 (1989); M. Genovese *et al.*, Phys. Rev. D **59**, 014012 (1999).

[15] Reference [12] is the only experiment to simultaneously measure both Σ_c mass differences. For this single experiment, the Σ_c sum rule combination gives $\Sigma_c^{++}+\Sigma_c^0-2\Sigma_c^+=-1.7\pm1.0$.