Orbital angular momentum in deep inelastic scattering

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In this work we address several issues associated with the orbital angular momentum relevant for leading twist polarized deep inelastic scattering. We present a detailed analysis of the light-front helicity operator (generator of rotations in the transverse plane) in QCD. We explicitly show that the operator constructed from the manifestly gauge invariant, symmetric energy momentum tensor in QCD, in the gauge $A^+=0$, after the elimination of constraint variables, is equal to the naive canonical form of the light-front helicity operator plus surface terms. Restricting to the topologically trivial sector, we eliminate the residual gauge degrees of freedom and surface terms. Having constructed the gauge fixed light-front helicity operator, we introduce quark and gluon orbital helicity distribution functions relevant for polarized deep inelastic scattering as a Fourier transform of the forward hadron matrix elements of appropriate bilocal operators. The utility of these definitions is illustrated with the calculation of anomalous dimensions in perturbation theory. We explicitly verify the helicity sum rule for dressed quark and gluon targets in light-front perturbation theory. We also consider the internal orbital helicity of a composite system in an arbitrary reference frame and contrast the results in the nonrelativistic situation versus the light-front (relativistic) case. [S0556-2821(99)00211-8]

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I. INTRODUCTION

The role of orbital angular momentum in deep inelastic scattering was first emphasized by Sehgal [1] and Ratcliff [2]. Recently, orbital angular momentum in QCD has attracted a lot of attention [3] in the context of the composition of nucleon helicity (in light-front quantization) in terms of quark and gluon degrees of freedom. The well-known polarized structure function g_1 measures (ignoring anomaly) the chirality of quarks and antiquarks in the nucleon in the deep inelastic region. On the light front, chirality coincides with helicity and thus g_1 constitutes a measurement of the intrinsic helicity of fermionic constituents in the nucleon. Since experimentally this contribution to the nucleon helicity is shown to be very small, great interest has arisen in the contributions of intrinsic gluon helicity and quark and gluon internal orbital helicities in understanding the nucleonic spin structure.

In this work, first, we study the generator of the rotations in the transverse plane (which we call the light-front helicity operator) in light-front QCD starting from the manifestly gauge invariant, symmetric, energy-momentum tensor. This operator appears to be interaction dependent. Further, it contains unphysical degrees of freedom (constraint variables in light-front theory) and it is even unclear whether this operator will generate the correct transformation laws pertaining to an angular momentum operator. To proceed, we pick the light-front gauge $A^+=0$ and use the constraint equations to eliminate the unphysical degrees of freedom. At this stage, we still have the residual gauge freedom in this gauge associated with x^- independent gauge transformations. We restrict our considerations to the *topologically trivial sector* and require that the physical field vanish at the boundary ($x^{-,\perp}\to\infty$). This *eliminates the residual gauge freedom* and also all the *surface terms*. The resulting *gauge fixed helicity operator* agrees with the naive canonical one (which is free of interactions and naturally separates into quark and gluon orbital and spin helicities) at the operator level.

Having constructed the gauge fixed orbital helicity operator in the topologically trivial sector of QCD, we proceed to define nonperturbative parton distributions as the Fourier transform along the light-front longitudinal direction of the forward hadron matrix element of appropriate bilocal operators for light-front internal orbital helicity. Apart from providing nonperturbative information on the distribution of nucleon helicity among its partonic constituents, they serve another useful purpose for the perturbative region. As we have recently shown [4], by replacing the hadron target by a dressed parton target in the definition of the distribution function, one can easily calculate the splitting functions and corresponding anomalous dimensions of leading twist operators. One can also investigate other issues of perturbative concern in the case of higher twist operators. The method uses intuitive light-front Fock space expansion for the operators in the bilocal expressions and also for the state. We proceed to explicitly evaluate the splitting functions and corresponding anomalous dimensions relevant for orbital helicity at one loop level in a physically transparent manner.

There have been recent studies of the quark orbital motion using nonrelativistic quark models [5]. It is of importance to study in what respect the physics of orbital helicity in the relativistic case differs from its nonrelativistic counterpart. In the context of *gauge-fixed light-front helicity operator* in QCD which is free from interactions we address this issue in the Appendix.

II. LIGHT-FRONT HELICITY OPERATOR J^3 FROM THE MANIFESTLY GAUGE INVARIANT ENERGY MOMENTUM TENSOR

It is well known that even though the *energy-momentum density* (which gives rise to Hamiltonian and three-momentum) and the *generalized angular momentum density* (which gives rise to angular momentum and boosts) can be expressed in a manifestly covariant, gauge invariant form, the explicit form of Poincare generators in quantum field theory depends on the frame of reference and may also depend up on the gauge choice. This of course does not imply that the theory has lost Lorenz and gauge symmetry. The symmetries are no longer manifest, but the physical observables in the theory still obey the consequences of the symmetries.

Poincare generators can be further classified as kinematical (which do not contain interactions and do not change the quantization surface) and dynamical (which contain interactions and change the quantization surface). Which operator is dynamical and which is kinematical of course depends on the choice of quantization surface. It is well known that in light-front field theory, on which our formalism of deep inelastic scattering is based, the generators of boosts and the rotation in the transverse plane (light-front helicity) are kinematical like three momenta whereas the generators of rotations about the two transverse axes are dynamical like the Hamiltonian. The operator in light-front field theory relevant to the ''proton spin crisis'' is the light-front helicity operator which belongs to the kinematical subgroup. In light-front literature, it is customary to construct this operator from the canonical symmetric energy momentum tensor and one explicitly finds that this operator is indeed free of interaction and has the same form as in free field theory [6].

In non-Abelian gauge theories such as QCD, one should be extra cautious since such theories are known to exhibit nontrivial topological effects. In this work, we restrict our attention to the topologically trivial sector of QCD. In this sector, interactions do not affect kinematical generators [7]. In view of the prevailing confusion in the literature (see Ref. [8] for a list of recent papers on the subject), we provide an explicit demonstration of this fact in this section in the case of the light-front helicity operator.

We start from the manifestly gauge invariant, symmetric energy momentum tensor in QCD:

$$\Theta^{\mu\nu}=\frac{i}{2}\overline{\psi}[\gamma^\mu D^\nu+\gamma^\nu D^\mu]\psi-F^{\mu\lambda a}F^{\nu a}_{\lambda}-g^{\mu\nu}\times\left\{-\frac{1}{4}(F_{\lambda\sigma a})^2+\overline{\psi}(i\gamma^\lambda D_\lambda-m)\psi\right\}, \tag{2.1}$$

where $iD^\mu=i\partial^\mu+gA^\mu$, $F^{\mu\lambda a}=\partial^\mu A^{\lambda a}-\partial^\lambda A^{\mu a}+gf^{abc}A^{\mu b}A^{\lambda c}$, $F^{\nu a}_{\lambda}=\partial^\nu A^a_\lambda-\partial_\lambda A^{\nu a}+gf^{abc}A^{\nu b}A^c_\lambda$.

We define the light-front helicity operator

$$\mathcal{J}^3=\frac{1}{2}\int dx^- d^2x^\perp[x^1\Theta^{+2}-x^2\Theta^{+1}]. \tag{2.2}$$

$\mathcal{J}^3$ is a manifestly gauge invariant operator by construction. However, it depends explicitly on the interaction and does not appear to be a kinematical operator at all. Furthermore, it is not apparent that $\mathcal{J}^3$ generates the correct transformations as an angular momentum operator. Thus at this stage, we are not justified to call it a helicity operator.

Explicitly, we have

$$\mathcal{J}^3=\frac{1}{2}\int dx^- d^2x^\perp\left\{x^1\left[\frac{i}{2}\overline{\psi}(\gamma^+D^2+\gamma^2D^+)\psi-F^{+\lambda a}F^{2a}_{\lambda}\right]-x^2\left[\frac{i}{2}\overline{\psi}(\gamma^+D^1+\gamma^1D^+)\psi-F^{+\lambda a}F^{1a}_{\lambda}\right]\right\}. \tag{2.3}$$

The fermion field can be decomposed as $\psi^\pm=\Lambda^\pm\psi$, with $\Lambda^\pm=\frac{1}{4}\gamma^\mp\gamma^\pm$. We shall work in the gauge $A^+=0$. In this gauge, we still have residual gauge freedom associated with x^--independent gauge transformations. Note that only ψ^+ and A^i are dynamical variables whereas ψ^- and A^- are constrained.

We have

$$\frac{i}{2}\overline{\psi}(\gamma^+D^2+\gamma^2D^+)\psi=\psi^{+\dagger}i\partial^2\psi^++g\psi^{+\dagger}T^a\psi^+A^2_a+\frac{i}{2}\overline{\psi}\gamma^2 i\partial^+\psi. \tag{2.4}$$

Using the constraint equation

$$i\partial^+\psi^-=[\alpha^\perp\cdot(i\partial^\perp+gA^\perp)+\gamma^0m]\psi^+, \tag{2.5}$$

to eliminate the constraint variable ψ^- we arrive at, after some algebra,

$$\frac{i}{2}\overline{\psi}\gamma^2\partial^+\psi=i\psi^{+\dagger}\partial^2\psi^++\frac{1}{2}\partial^1(\psi^{+\dagger}\Sigma^3\psi^+)+g\psi^{+\dagger}T^a\psi^+A^{2a}+\frac{i}{2}\partial^+(\psi^{-\dagger}\alpha_2\psi^+)-\frac{i}{2}\partial^2(\psi^{+\dagger}\psi^+). \tag{2.6}$$

Now we restrict ourselves to the topologically trivial sector by requiring that the dynamical fields (ψ^+ and A^i) vanish at $x^{-,i}\to\infty$. The residual gauge freedom and the surface terms are no longer present and so we drop total derivatives of ∂^+ and ∂^2. Note that the term involving ∂^1 is not a surface term since Θ^{+2} is multiplied by x^1.

Collecting the results together, we have

$$\frac{i}{2}\overline{\psi}(\gamma^+D^2+\gamma^2D^+)\psi=2i\psi^{+\dagger}\partial^2\psi^++\frac{1}{2}\partial^1(\psi^{+\dagger}\Sigma^3\psi^+)+2g\psi^{+\dagger}T^a\psi^+A^{2a}, \tag{2.7}$$

where $\Sigma^3=i\gamma^1\gamma^2$.

In the gauge $A^+=0$,

$$-F^{+\lambda a}F^{2a}_{\lambda}=-\frac{1}{2}(\partial^+)^2A^{-a}A^{2a}+\partial^+A^{ja}(\partial^2A^{ja}-\partial^jA^{2a})+gf^{abc}(\partial^+A^{ja})A^{2b}A^{jc}+\frac{1}{2}\partial^+(\partial^+A^{-a}A^{2a}). \tag{2.8}$$

We have the constraint equation for the elimination of the variable A^-,

$$\frac{1}{2}(\partial^+)^2 A^{-a} = \partial^+ \partial^i A^{ia} + g f^{abc} A^{ib} \partial^+ A^{ic} + 2g\psi^{+\dagger} T^a \psi^+. \tag{2.9}$$

Thus

$$\begin{aligned}
-F^{+\lambda a}F_{\lambda}^{2a} = {} & \partial^i A^{ia}\partial^+ A^{2a} + \partial^+ A^{ja}(\partial^2 A^{ja} - \partial^j A^{2a}) \\
& - 2g\psi^{+\dagger}T^a\psi^+ A^{2a} + \frac{1}{2}\partial^+(\partial^+ A^{-a} A^{2a}) \\
& - \partial^+(\partial^i A^{ia} A^{2a}) \\
= {} & \partial^+ A^{1a}\partial^2 A^{1a} + \partial^+ A^{2a}\partial^2 A^{2a} - 2g\psi^{+\dagger}T^a\psi^+ A^{2a} \\
& + \partial^1(A^{1a}\partial^+ A^{2a}) + \frac{1}{2}\partial^+(\partial^+ A^{-a} A^{2a}) \\
& - \partial^+(\partial^i A^{ia} A^{2a}) - \partial^+(A^{1a}\partial^1 A^{2a}).
\end{aligned} \tag{2.10}$$

Collecting the results together,

$$\begin{aligned}
\Theta^{+2} = {} & 2i\psi^{+\dagger}\partial^2\psi^+ + \frac{1}{2}\partial^1(\psi^{+\dagger}\Sigma^3\psi^+) + \partial^+ A^{1a}\partial^2 A^{1a} \\
& + \partial^+ A^{2a}\partial^2 A^{2a} + \partial^1(A^{1a}\partial^+ A^{2a}).
\end{aligned} \tag{2.11}$$

We have dropped the surface terms at $x^- = \pm\infty$. By a similar calculation,

$$\begin{aligned}
\Theta^{+1} = {} & 2i\psi^{+\dagger}\partial^1\psi^+ - \frac{1}{2}\partial^2(\psi^{+\dagger}\Sigma^3\psi^+) + \partial^+ A^{1a}\partial^1 A^{1a} \\
& + \partial^+ A^{2a}\partial^1 A^{2a} + \partial^2(A^{2a}\partial^+ A^{1a}).
\end{aligned} \tag{2.12}$$

From the above two equations it is clear that Θ^{+1} and Θ^{+2} agree with the free field theory form at the operator level. This shows that in light-front quantization, with $A^+=0$ gauge, $\mathcal{J}^3 = J^3$ (the naive canonical form independent of interactions) at the operator level, provided the fields vanish at the boundary. Explicitly,

$$J^3 = J^3_{f(o)} + J^3_{f(i)} + J^3_{g(o)} + J^3_{g(i)} \tag{2.13}$$

with

$$\begin{aligned}
J^3_{f(o)} &= \int dx^- d^2x^\perp \psi^{+\dagger} i(x^1\partial^2 - x^2\partial^1)\psi^+, \\
J^3_{f(i)} &= \frac{1}{2}\int dx^- d^2x^\perp \psi^{+\dagger}\Sigma^3\psi^+, \\
J^3_{g(o)} &= \frac{1}{2}\int dx^- d^2x^\perp \{x^1[\partial^+ A^1\partial^2 A^1 + \partial^+ A^2\partial^2 A^2] \\
&\quad - x^2[\partial^+ A^1\partial^1 A^1 + \partial^+ A^2\partial^1 A^2]\}, \\
J^3_{g(i)} &= \frac{1}{2}\int dx^- d^2x^\perp [A^1\partial^+ A^2 - A^2\partial^+ A^1].
\end{aligned} \tag{2.14}$$

The color indices are implicit in these equations.

Using canonical commutation relations, we explicitly find that

$$i[J^3_{f(o)}, \psi^+(x)] = (x^1\partial^2 - x^2\partial^1)\psi^+(x), \tag{2.15}$$

$$i[J^3_{f(i)}, \psi^+(x)] = \frac{1}{2}\gamma^1\gamma^2\psi^+(x), \tag{2.16}$$

$$i[J^3_{g(o)}, A^i(x)] = (x^1\partial^2 - x^2\partial^1)A^i(x), \tag{2.17}$$

$$i[J^3_{g(i)}, A^i(x)] = -\epsilon_{ij}A^j(x). \tag{2.18}$$

Thus these operators do qualify as angular momentum operators (generators of rotations in the transverse plane) in the theory [6].

To summarize, the helicity operator constructed from manifestly gauge invariant, symmetric, energy momentum tensor in QCD, in the gauge $A^+=0$, and after the elimination of constraint variables, is equal to the naive canonical form of the light-front helicity operator plus surface terms. In the topologically trivial sector, we can legitimately require the dynamical fields to vanish at the boundary. This eliminates the residual gauge degrees of freedom and removes the surface terms. Thus we have a gauge fixed Poincare generator which we consider in the following sections.

III. ORBITAL HELICITY DISTRIBUTION FUNCTIONS

We define the orbital helicity distribution for the fermion

$$\begin{aligned}
\Delta q_L(x,Q^2) = {} & \frac{1}{4\pi P^+}\int d\eta e^{-i\eta x} \\
& \times \langle PS|[\overline{\psi}(\xi^-)\gamma^+ i(x^1\partial^2 - x^2\partial^1)\psi(0) + \text{H.c.}]|PS\rangle
\end{aligned} \tag{3.1}$$

with $\eta = \frac{1}{2}P^+\xi^-$. Here $|PS\rangle$ denotes the hadron state with momentum P and helicity S.

We define the light-front orbital helicity distribution for the gluon as

$$\begin{aligned}
\Delta g_L(x,Q^2) = {} & \frac{-1}{4\pi P^+}\int d\eta e^{-i\eta x}\langle PS|[x^1 F^{+\alpha}(\xi^-)\partial^2 A_\alpha(0) \\
& - x^2 F^{+\alpha}(\xi^-)\partial^1 A_\alpha(0)]|PS\rangle.
\end{aligned} \tag{3.2}$$

These distributions are defined in analogy with the more familiar intrinsic helicity distributions for quarks and gluons given as follows.

For the fermion, the intrinsic light-front helicity distribution function is given by

$$\begin{aligned}
\Delta q(x,Q^2) = {} & \frac{1}{8\pi S^+}\int d\eta e^{-i\eta x} \\
& \times \langle PS|[\overline{\psi}(\xi^-)\gamma^+\Sigma^3\psi(0) + \text{H.c.}]|PS\rangle,
\end{aligned} \tag{3.3}$$

where $\Sigma^3 = i\gamma^1\gamma^2$. This is the same as the chirality distribution function g_1.

For the gluon, the intrinsic light-front helicity distribution is defined [9] as

$$\Delta g(x,Q^2)=-\frac{i}{4\pi(P^+)^2x}\int d\eta e^{-i\eta x}\times\langle PS|F^{+\alpha}(\xi^-)\tilde{F}^{+}_{\alpha}(0)|PS\rangle. \tag{3.4}$$

The dual tensor

$$\tilde{F}^{\mu\nu}=\frac{1}{2}\epsilon^{\mu\nu\rho\sigma}F_{\rho\sigma} \quad \text{with} \quad \epsilon^{+1-2}=2. \tag{3.5}$$

Note that the above distribution functions are defined in the light-front gauge $A^+=0$. In the two-component representation [10] we have the dynamical fermion field,

$$\psi_+(x)=\sum_\lambda \chi_\lambda\int\frac{dk^+d^2k_\perp}{2(2\pi)^3\sqrt{k^+}}(b_\lambda(k)e^{-ikx}+d^\dagger_{-\lambda}(k)e^{ikx}), \tag{3.6}$$

and the dynamical gauge field

$$A^i(x)=\sum_\lambda\int\frac{dk^+d^2k_\perp}{2(2\pi)^3k^+}(\varepsilon^i(\lambda)a_\lambda(k)e^{-ikx}+\text{H.c.}), \tag{3.7}$$

with

$$\{b_\lambda(k),b^\dagger_{\lambda'}(k')\}=\{d_\lambda(k),d^\dagger_{\lambda'}(k')\}=2(2\pi)^3k^+\delta(k^+-k'^+)\delta^2(k_\perp-k'_\perp)\delta_{\lambda\lambda'}, \tag{3.8}$$

$$[a_\lambda(k),a^\dagger_{\lambda'}(k')]=2(2\pi)^3k^+\delta(k^+-k'^+)\delta^2(k_\perp-k'_\perp)\delta_{\lambda\lambda'}, \tag{3.9}$$

and χ_λ is the eigenstate of σ_z in the two-component spinor of ψ_+ by the use of the following light-front γ matrix representation [11]:

$$\gamma^0=\begin{bmatrix}0 & -i\\ i & 0\end{bmatrix}, \quad \gamma^3=\begin{bmatrix}0 & i\\ i & 0\end{bmatrix}, \quad \gamma^i=\begin{bmatrix}-i\tilde{\sigma}^i & 0\\ 0 & i\tilde{\sigma}^i\end{bmatrix} \tag{3.10}$$

with $\tilde{\sigma}^1=\sigma^2,\tilde{\sigma}^2=-\sigma^1$ and $\varepsilon^i(\lambda)$ the polarization vector of transverse gauge field.

Note that integration of the above distribution functions over x is directly related to the expectation values of the corresponding helicity operators as follows:

$$\int_0^1 dx\Delta q(x,Q^2)=\frac{1}{\mathcal{N}}\langle PS|J^3_{q(i)}|PS\rangle,$$

$$\int_0^1 dx\Delta q_L(x,Q^2)=\frac{1}{\mathcal{N}}\langle PS|J^3_{q(o)}|PS\rangle,$$

$$\int_0^1 dx\Delta g(x,Q^2)=\frac{1}{\mathcal{N}}\langle PS|J^3_{g(i)}|PS\rangle,$$

$$\int_0^1 dx\Delta g_L(x,Q^2)=\frac{1}{\mathcal{N}}\langle PS|J^3_{g(o)}|PS\rangle, \tag{3.11}$$

where $\mathcal{N}=2(2\pi)^3P^+\delta^3(0)$.

IV. PERTURBATIVE CALCULATION OF ANOMALOUS DIMENSIONS

In this section, we evaluate the internal helicity distribution functions for a dressed quark in perturbative QCD by replacing the hadron target by a dressed quark target. We have provided the necessary details of the calculation which may serve as the stepping stone for more realistic calculation with meson target. From this simple calculation, we have illustrated how easily one can extract the relevant splitting functions and evaluate the corresponding anomalous dimensions. Note that since we are not interested in exhaustive calculation of various anomalous dimensions and the purpose of this section being illustrative, we can safely drop the derivative of delta function in the following calculations and work explicitly with forward matrix element.

The dressed quark state with fixed helicity can be expressed as

$$|k^+,k_\perp,\lambda\rangle=\Phi^\lambda(k)b^\dagger_\lambda(k)|0\rangle+\sum_{\lambda_1\lambda_2}\int\frac{dk_1^+d^2k_{\perp 1}}{\sqrt{2(2\pi)^3k_1^+}}\frac{dk_2^+d^2k_{\perp 2}}{\sqrt{2(2\pi)^3k_2^+}}\times\sqrt{2(2\pi)^3k^+}\delta^3(k-k_1-k_2)\times\Phi^\lambda_{\lambda_1\lambda_2}(k;k_1,k_2)b^\dagger_{\lambda_1}(k_1)a^\dagger_{\lambda_2}(k_2)|0\rangle+\cdots, \tag{4.1}$$

where the normalization of the state is determined by

$$\langle k'^+,k'_\perp,\lambda'|k^+,k_\perp,\lambda\rangle=2(2\pi)^3k^+\delta_{\lambda,\lambda'}\delta(k^+-k'^+)\times\delta^2(k_\perp-k'_\perp). \tag{4.2}$$

We introduce the boost invariant amplitudes ψ_1^λ and $\psi^\lambda_{\sigma_1,\lambda_2}(x,\kappa^\perp)$ respectively by $\Phi^\lambda(k)=\psi_1^\lambda$ and $\Phi^\lambda_{\lambda_1\lambda_2}(k;k_1,k_2)=(1/\sqrt{P^+})\psi^\lambda_{\sigma_1\lambda_2}(x,\kappa^\perp)$. From the light-front QCD Hamiltonian, to lowest order in perturbation theory, we have

$$\psi^\lambda_{\sigma_1\lambda_2}(x,\kappa_\perp)=-\frac{g}{\sqrt{2(2\pi)^3}}T^a\frac{x(1-x)}{\kappa_\perp^2+m_q^2(1-x)^2}\chi^\dagger_{\sigma_1}\times\frac{1}{\sqrt{1-x}}\left\{2\frac{\kappa_\perp^i}{1-x}+\frac{1}{x}(\tilde{\sigma}_\perp\cdot\kappa_\perp)\tilde{\sigma}^i-im_q\tilde{\sigma}^i\frac{1-x}{x}\right\}\chi_\lambda\varepsilon^{i*}(\lambda_2)\quad\psi_1^\lambda. \tag{4.3}$$

Here x is the longitudinal momentum fraction carried by the quark. We shall ignore the m_q dependence in the above wave function which can lead to higher twist effects in orbital helicity. In the following we take the helicity of the dressed quark to be $+\frac{1}{2}$. Due to transverse boost invariance, without loss of generality, we take the transverse momentum of the initial quark to be zero.

First we evaluate the gluon intrinsic helicity distribution function given in Eq. (3.4) in the dressed quark state.

The nonvanishing contribution comes from the quark-gluon state. We get

$$\Delta g(1-x,Q^2)=\sum_{\sigma_1,\lambda_2}\lambda_2\int d^2\kappa^\perp \psi^{\uparrow}_{\sigma_1\lambda_2}{}^*(x,\kappa^\perp)\psi^{\uparrow}_{\sigma_1\lambda_2}(x,\kappa^\perp)$$

$$=\frac{\alpha_s}{2\pi}C_f\ln\frac{Q^2}{\mu^2}x^2(1-x)^2\frac{1}{1-x}\times\left[\frac{1}{x^2(1-x)^2}-\frac{1}{(1-x)^2}\right]. \quad (4.4)$$

The first (second) term inside the square brackets arises from the state with gluon helicity $+1$ (-1). Thus we have the gluon intrinsic helicity contribution in the dressed quark state

$$\Delta g(1-x,Q^2)=\frac{\alpha_s}{2\pi}C_f\ln\frac{Q^2}{\mu^2}(1+x). \quad (4.5)$$

Note that the gluon distribution function has the argument $(1-x)$ since we have assigned x to the quark in the dressed quark state.

Next we evaluate the quark orbital helicity distribution function given in Eq. (3.1) in the dressed quark state. The nonvanishing contribution comes from the quark-gluon state. We get

$$\Delta q_L(x,Q^2)=\sum_{\sigma_1,\lambda_2}\int d^2\kappa^\perp(1-x)\psi^{\uparrow}_{\sigma_1\lambda_2}{}^*(x,\kappa^\perp)\times\left(-i\frac{\partial}{\partial\phi}\right)\psi^{\uparrow}_{\sigma_1\lambda_2}(x,\kappa^\perp)$$

$$=-\frac{\alpha_s}{2\pi}C_f\ln\frac{Q^2}{\mu^2}(1-x)x^2\times(1-x)^2\frac{1}{1-x}\left[\frac{1}{x^2(1-x)^2}-\frac{1}{(1-x)^2}\right]. \quad (4.6)$$

The first (second) term inside the square brackets arises from the state with gluon helicity $+1$ (-1). Thus we have the quark orbital helicity contribution in the dressed quark state

$$\Delta q_L(x,Q^2)=-\frac{\alpha_s}{2\pi}C_f\ln\frac{Q^2}{\mu^2}(1-x)(1+x). \quad (4.7)$$

Similarly we get the gluon orbital helicity distribution defined in Eq. (3.2) in the dressed quark state

$$\Delta g_L(1-x,Q^2)=\sum_{\sigma_1,\lambda_2}\int d^2\kappa^\perp x\psi^{\uparrow}_{\sigma_1\lambda_2}{}^*(x,\kappa^\perp)\times\left(-i\frac{\partial}{\partial\phi}\right)\psi^{\uparrow}_{\sigma_1\lambda_2}(x,\kappa^\perp)$$

$$=-\frac{\alpha_s}{2\pi}C_f\ln\frac{Q^2}{\mu^2}x(1+x). \quad (4.8)$$

We note that the helicity is conserved at the quark gluon vertex. For the initial quark of zero transverse momentum, total helicity of the initial state is the intrinsic helicity of the initial quark, namely, $+\frac{1}{2}$ in our case. Since we have neglected quark mass effects, the final quark also has intrinsic helicity $+\frac{1}{2}$. Thus total helicity conservation implies that the contributions from gluon intrinsic helicity and quark and gluon internal orbital helicities have to cancel. This is readily verified using Eqs. (4.5), (4.7), and (4.8).

From Eqs. (4.5), (4.7) and (4.8) we extract the relevant splitting functions. The splitting functions are

$$P_{SS(gq)}(1-x)=C_f(1+x),$$

$$P_{LS(qq)}(x)=-C_f(1-x^2),$$

$$P_{LS(gq)}(1-x)=-C_fx(1+x). \quad (4.9)$$

We define the anomalous dimension $A^n=\int_0^1 dx x^{n-1}P(x)$. The anomalous dimensions are given by

$$A^n_{SS(gq)}=C_f\frac{n+2}{n(n+1)},\quad A^n_{LS(qq)}=-C_f\frac{2}{n(n+2)},$$

$$A^n_{LS(gq)}=-C_f\frac{n+4}{n(n+1)(n+2)}. \quad (4.10)$$

These anomalous dimensions agree with those given in the recent work of Hägler and Schäfer [12].

V. VERIFICATION OF HELICITY SUM RULE

Helicity sum rule for the fermion target is given by

$$\frac{1}{\mathcal{N}}\langle PS|[J^3_{q(i)}+J^3_{q(o)}+J^3_{g(i)}+J^3_{g(o)}]|PS\rangle=\pm\frac{1}{2}. \quad (5.1)$$

For boson target the RHS of the above equation should be replaced by the corresponding helicity.

Here we verify the correctness of our definitions of distribution functions in the context of helicity sum rule for a dressed quark as well as a dressed gluon target perturbatively. For simplicity, we take the external transverse momenta of the target to be zero so that there is no net angular momentum associated with the center of mass of the target. Using the field expansions, given in Eqs. (3.6) and (3.7), we have

$$J^3_{f(o)}=i\sum_s \int \frac{dk^+ d^2k^\perp}{2(2\pi)^3 k^+}\left[b^\dagger(k,s)\left[k^2\frac{\partial}{\partial k^1}-k^1\frac{\partial}{\partial k^2}\right]b(k,s)+d^\dagger(k,s)\left[k^2\frac{\partial}{\partial k^1}-k^1\frac{\partial}{\partial k^2}\right]d(k,s)\right],$$

$$J^3_{f(i)}=\frac{1}{2}\sum_\lambda \lambda\int \frac{dk^+ d^2k^\perp}{2(2\pi)^3 k^+}[b^\dagger(k,\lambda)b(k,\lambda)+d^\dagger(k,\lambda)d(k,\lambda)],$$

$$J^3_{g(o)}=i\sum_\lambda \int \frac{dk^+ d^2k^\perp}{2(2\pi)^3 k^+}a^\dagger(k,\lambda)\left[k^2\frac{\partial}{\partial k^1}-k^1\frac{\partial}{\partial k^2}\right]a(k,\lambda),$$

$$j^3_{g(i)}=\sum_\lambda \lambda\int \frac{dk^+ d^2k^\perp}{2(2\pi)^3 k^+}a^\dagger(k,\lambda)a(k,\lambda). \tag{5.2}$$

For a dressed quark target having helicity $+\frac{1}{2}$ we get

$$\frac{1}{\mathcal{N}}\langle P,\uparrow|J^3_{f(i)}|P,\uparrow\rangle_q=\int dx\left[\frac{1}{2}\delta(1-x)+\frac{\alpha}{2\pi}C_f\ln\frac{Q^2}{\mu^2}\left[\frac{1+x^2}{(1-x)_+}+\frac{3}{2}\delta(1-x)\right]\right]=\frac{1}{2},$$

$$\frac{1}{\mathcal{N}}\langle P,\uparrow|J^3_{f(o)}|P,\uparrow\rangle_q=-\frac{\alpha}{2\pi}C_f\ln\frac{Q^2}{\mu^2}\int dx(1-x)(1+x),$$

$$\frac{1}{\mathcal{N}}\langle P,\uparrow|J^3_{g(i)}|P,\uparrow\rangle_q=\frac{\alpha}{2\pi}C_f\ln\frac{Q^2}{\mu^2}\int dx(1+x),$$

$$\frac{1}{\mathcal{N}}\langle P,\uparrow|J^3_{g(o)}|P,\uparrow\rangle_q=-\frac{\alpha}{2\pi}C_f\ln\frac{Q^2}{\mu^2}\int dx\ \ x(1+x). \tag{5.3}$$

Adding all the contributions, we get

$$\frac{1}{\mathcal{N}}\langle P,\uparrow|J^3_{f(i)}+J^3_{f(o)}+J^3_{g(i)}+J^3_{g(o)}|P,\uparrow\rangle_q=\frac{1}{2}. \tag{5.4}$$

For a dressed gluon having helicity $+1$, the corresponding expressions are worked out to be the following:

$$\frac{1}{\mathcal{N}}\langle P,\uparrow|J^3_{f(i)}|P,\uparrow\rangle_g=0,$$

$$\frac{1}{\mathcal{N}}\langle P,\uparrow|J^3_{f(o)}|P,\uparrow\rangle_g=\frac{\alpha}{2\pi}N_fT_f\ln\frac{Q^2}{\mu^2}\int dx[x^2+(1-x)^2],$$

$$\frac{1}{\mathcal{N}}\langle P,\uparrow|J^3_{g(i)}|P,\uparrow\rangle_g=\psi_1^*\psi_1=1-\frac{\alpha}{2\pi}N_fT_f\ln\frac{Q^2}{\mu^2}\int dx[x^2+(1-x)^2],$$

$$\frac{1}{\mathcal{N}}\langle P,\uparrow|J^3_{g(o)}|P,\uparrow\rangle_g=0. \tag{5.5}$$

Adding all the contributions, we get

$$\frac{1}{\mathcal{N}}\langle P,\uparrow|J^3_{f(i)}+J^3_{f(o)}+J^3_{g(i)}+J^3_{g(o)}|P,\uparrow\rangle_g=1. \tag{5.6}$$

Note that in evaluating the above expression, we have used the Fock expansion of the target states. For the dressed quark we have used Eq. (4.1), while for gluon we have used similar expansion but ignored two-gluon Fock sector for simplicity.

VI. SUMMARY, CONCLUSIONS AND DISCUSSION

We have presented a detailed analysis of the light-front helicity operator (generator of rotations in the transverse plane) in QCD. We have explicitly shown that the operator constructed from manifestly gauge invariant, symmetric energy momentum tensor in QCD, in the gauge $A^+=0$, and after the elimination of constraint variables, is equal to the naive canonical form of the light-front helicity operator plus surface terms. In the topologically trivial sector, we can legitimately require the dynamical fields to vanish at the boundary. This eliminates the residual gauge degrees of freedom and removes the surface terms.

Next, we have defined nonperturbative quark and gluon orbital helicity distribution functions as Fourier transform of forward hadron matrix elements of appropriate bilocal operators with bilocality only in the light-front longitudinal space. We have calculated these distribution functions by replacing the hadron target by a dressed parton providing all the necessary details. From these simple calculations we have illustrated the utility of the newly defined distribution functions in the calculation of splitting functions and hence anomalous dimensions in perturbation theory. We have also verified the helicity sum rule explicitly to the first nontrivial order in perturbation theory.

Lastly, in the Appendix, we have compared and contrasted the expressions for internal orbital helicity in nonrelativistic and light-front (relativistic) cases. Our calculation shows that the role played by particle masses in the internal orbital angular momentum in the nonrelativistic case is replaced by the longitudinal momentum fraction in the relativistic case. Although four terms appear in the expression of L_3 for individual particles in two body system, only the term proportional to the total internal L_3 contributes due to transverse boost invariance of the multiparton wave function in light-front dynamics. We also note the occurrence of the longitudinal momentum fraction $x_2(x_1)$ multiplied by the total internal L_3 in the expressions of L_3 for particle one(two). This explains why one needs to take first moment with respect to x as well as $(1-x)$ for the respective distributions in obtaining the helicity sum rule [3].

Our explicit demonstration that the operator constructed from manifestly gauge invariant, symmetric energy momentum tensor in QCD, in the gauge $A^+=0$, and after the elimination of constraint variables and residual gauge freedom, is equal to the naive canonical form of the light-front helicity operator is facilitated by the fact that in light-front theory only transverse gauge fields are dynamical degrees of freedom. The conjugate momenta (color electric fields) are constrained variables in the theory. Thus we were able to show explicitly that the resulting gauge fixed operator is free of interactions. The question naturally arises as to whether this result is valid in other gauges also. Several years ago, in the context of magnetic monopole solutions, it has been shown [13] that in Yang-Mills-Higgs system, quantized in the axial gauge $A_3=0$ using the Dirac procedure, the angular momentum operator constructed from manifestly gauge invariant symmetric energy momentum tensor differs from the canonical one only by surface terms. In the study of QCD in $A_3=0$ gauge, it has been shown [14] that in the presence of surface terms, Poincare algebra holds only in the physical subspace. The situation in $A^0=0$ gauge or in covariant gauges where unphysical degrees of freedom are present is to be investigated. Another interesting problem to be studied is the helicity conservation in the topologically nontrivial sector of QCD and its implications, if any, for deep inelastic scattering.

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APPENDIX: INTERNAL ORBITAL HELICITY: NONRELATIVISTIC VERSUS LIGHT-FRONT (RELATIVISTIC) CASE

Here we address the nonrelativistic versus the light-front case. We need to decompose the total orbital angular momentum of a composite system as a sum of the orbital angular momentum associated with internal motion and the orbital angular momentum associated with the center of mass motion. We are interested only in the former and not in the latter. For illustrative purposes, consider a two body system consisting of two particles with masses m_1 and m_2 and momenta $\mathbf{k}_1$ and $\mathbf{k}_2$. Let $\mathbf{P}$ denote the total momentum. In the nonrelativistic case, let $\mathbf{q}$ denote the relative momentum, i.e., $\mathbf{q}=(m_2\mathbf{k}_1-m_1\mathbf{k}_2)/(m_1+m_2)$. It is well known [5] that the contribution of particle one (two) to the third component of internal orbital angular momentum is given by

$$L^3_{1(2)}=i\frac{m_{2(1)}}{m_1+m_2}\left[q^2\frac{\partial}{\partial q^1}-q^1\frac{\partial}{\partial q^2}\right]. \tag{A1}$$

Next consider the light-front case. Let $k_1=(k_1^+,k_1^\perp)$ and $k_2=(k_2^+,k_2^\perp)$ denote the single particle momenta and $P=(P^+,P^\perp)$ denote the total momentum of the two particle system, i.e., $k_1^{+,i}+k_2^{+,i}=P^{+,i}$. Light-front kinematics allows us to introduce boost-invariant internal transverse momentum $q^\perp$ and longitudinal momentum fraction x_i by

$$k_1^\perp=q^\perp+x_1P^\perp,\quad k_1^+=x_1P^+,\quad k_2^\perp=-q^\perp+x_2P^\perp,$$
$$k_2^+=x_2P^+, \tag{A2}$$

we have $x_1+x_2=1$ and $q^\perp=x_2k_1^\perp-x_1k_2^\perp$. For the first particle, we have

$$\begin{aligned}L_1^3&=i\left[k_1^2\frac{\partial}{\partial k_1^1}-k_1^1\frac{\partial}{\partial k_1^2}\right]\\&=ix_2\left[q^2\frac{\partial}{\partial q^1}-q^1\frac{\partial}{\partial q^2}\right]+ix_1\left[P^2\frac{\partial}{\partial P^1}-P^1\frac{\partial}{\partial P^2}\right]\\&\quad+ix_1x_2\left[P^2\frac{\partial}{\partial q^1}-P^1\frac{\partial}{\partial q^2}\right]+i\left[q^2\frac{\partial}{\partial P^1}-q^1\frac{\partial}{\partial P^2}\right].\end{aligned} \tag{A3}$$

For the second particle, we have

$$\begin{aligned}L_2^3&=i\left[k_2^2\frac{\partial}{\partial k_2^1}-k_2^1\frac{\partial}{\partial k_2^2}\right]\\&=ix_1\left[q^2\frac{\partial}{\partial q^1}-q^1\frac{\partial}{\partial q^2}\right]+ix_2\left[P^2\frac{\partial}{\partial P^1}-P^1\frac{\partial}{\partial P^2}\right]\\&\quad-ix_1x_2\left[P^2\frac{\partial}{\partial q^1}-P^1\frac{\partial}{\partial q^2}\right]-i\left[q^2\frac{\partial}{\partial P^1}-q^1\frac{\partial}{\partial P^2}\right].\end{aligned} \tag{A4}$$

Total orbital helicity

$$L^3=L_1^3+L_2^3=i\left[q^2\frac{\partial}{\partial q^1}-q^1\frac{\partial}{\partial q^2}\right]+i\left[P^2\frac{\partial}{\partial P^1}-P^1\frac{\partial}{\partial P^2}\right]. \tag{A5}$$

Thus we have decomposed the total orbital helicity of a two particle system into internal orbital helicity and the orbital helicity associated with the ‘‘center of mass motion.’’

Note that the internal orbital helicity carried by particle one is the total internal helicity multiplied by the longitudinal momentum fraction carried by particle two and vice versa. This factor can be understood by comparison with the situation in nonrelativistic dynamics and recalling the close analogy between Galilean relativity and light-front dynamics in the transverse plane. In nonrelativistic two-body problem, the center of mass coordinate is defined by $\vec{R}=(m_1\vec{r}_1+m_2\vec{r}_2)/(m_1+m_2)$. The generator of Galilean boost is $\vec{B}=-\Sigma_i m_i\vec{r}_i$. Thus in nonrelativistic dynamics, $\vec{R}=-\vec{B}/M$ with $M=m_1+m_2$. In light-front dynamics, the variable analogous to $B^\perp$ is $E^\perp$, the generator of transverse boost and the variable analogous to M is P^+. Thus in light-front theory, the transverse center of mass coordinate $R^\perp=\Sigma_i k_i^+ r_i^\perp/\Sigma_i k_i^+=x_1r_1^\perp+x_2r_2^\perp$. Thus we recognize that instead of $m_2/(m_1+m_2)[m_1/(m_1+m_2)]$ in nonrelativistic theory, $x_2[x_1]$ appears in light-front theory.

By comparing light-front (relativistic) and nonrelativistic cases, we readily see that the role played by particle masses in individual contributions to the third component of internal orbital angular momentum in nonrelativistic dynamics is replaced by longitudinal momentum fractions in relativistic (light-front) theory. This also shows that the physical picture of the third component of internal orbital angular momentum is drastically different in non-relativistic and relativistic cases. We stress that it is only the latter, in which parton masses do not appear at all, that is of relevance to the nucleon helicity problem. Lastly, we emphasize that it is the transverse boost invariance in light front dynamics that makes possible the separation of dynamics associated with the center of mass and the internal dynamics. *In equal-time relativistic theory, this separation cannot be achieved at the kinematical level since boosts are dynamical*.

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