Broken R parity contributions to flavor changing rates and CP asymmetries in fermion pair production at leptonic colliders

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We examine the effects of the R parity odd renormalizable interactions on flavor changing rates and CP asymmetries in the production of fermion-antifermion pairs at leptonic (electron and muon) colliders. In the reactions $l^- + l^+ \to f_J + \overline{f_{J'}}$ $(l=e,\ \mu;\ J \neq J')$ the produced fermions may be leptons, down quarks, or up quarks, and the center of mass energies may range from the Z-boson pole up to 1000 GeV. Off the Z-boson pole, the flavor changing rates are controlled by tree level amplitudes and the CP asymmetries by interference terms between tree and loop level amplitudes. At the Z-boson pole, both observables involve loop amplitudes. The lepton number violating interactions, associated with the coupling constants λ_{ijk}, λ'_{ijk}, are only taken into account. The consideration of loop amplitudes is restricted to the photon and Z-boson vertex corrections. We briefly review flavor violation physics at colliders. We present numerical results using a single, species and family independent, mass parameter $\tilde{m}$ for all the scalar superpartners and considering simple assumptions for the family dependence of the R parity odd coupling constants. Finite nondiagonal rates (CP asymmetries) entail nonvanishing products of two (four) different coupling constants in different family configurations. For lepton pair production, the Z-boson decays branching ratios $B_{JJ'} = B(Z \to l_J^- + l_{J'}^+)$ scale in order of magnitude as $B_{JJ'} \approx (\lambda/0.1)^4 (100\ \mathrm{GeV}/\tilde{m})^{2.5} 10^{-9}$, with coupling constants $\lambda = \lambda_{ijk}$ or λ'_{ijk} in appropriate family configurations. The corresponding results for d- and u quarks are larger, due to an extra color factor $N_c = 3$. The flavor nondiagonal rates, at energies well above the Z-boson pole, slowly decrease with the center of mass energy and scale with the mass parameter approximately as $\sigma_{JJ'} \approx (\lambda/0.1)^4 (100\ \mathrm{GeV}/\tilde{m})^{2-3} (1-10)$ fbarn. Including the contributions from an sneutrino s-channel exchange could raise the rates for leptons or d quarks by one order of magnitude. The CP-odd asymmetries at the Z-boson pole, $\mathcal{A}_{JJ'} = (B_{JJ'} - B_{J'J})/(B_{JJ'} + B_{J'J})$, vary inside the range $(10^{-1} - 10^{-3}) \sin\psi$, where ψ is the CP-odd phase. At energies higher than the Z-boson pole, CP-odd asymmetries for leptons, d-quark and u-quark pair production lie approximately at $(10^{-2} - 10^{-3}) \sin\psi$, irrespective of whether one deals with light or heavy flavors. [S0556-2821(99)07409-3]

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I. INTRODUCTION

An approximate R parity symmetry could greatly enhance our insight into the supersymmetric flavor problem. As is known, the dimension-4 R parity odd superpotential trilinear in the quarks and leptons superfields,

$$W_{R\ odd} = \sum_{i,j,k} \left(\frac{1}{2} \lambda_{ijk} L_i L_j E_k^c + \lambda'_{ijk} Q_i L_j D_k^c + \frac{1}{2} \lambda''_{ijk} U_i^c D_j^c D_k^c \right), \tag{1}$$

adds new dimensionless couplings to the family spaces of quarks and leptons and their superpartners. Comparing with the analogous situation for the Higgs-meson-matter Yukawa interactions, one naturally expects the set of 45 dimensionless coupling constants $\lambda_{ijk} = -\lambda_{jik}$, λ'_{ijk}, $\lambda''_{ijk} = -\lambda''_{ikj}$ to exhibit a nontrivial hierarchical structure in the family spaces. Our goal in this work will be to examine a particular class of tests at high energy colliders by which one could access direct information on the family structure of these coupling constants.

The R parity symmetry has inspired a vast literature since the pioneering period of the early 1980s [1–8] and the maturation period of the late 1980s and early 1990s [9–14]. This subject is currently witnessing a renewed interest [15,16]. As is well known, the R parity odd interactions can contribute at the tree level, by exchange of the scalar superpartners, to processes which violate the baryon and lepton numbers as well as the leptons and quarks flavors. The major part of the existing experimental constraints on coupling constants is formed from the indirect bounds gathered from the low energy phenomenology. Most often, these have been derived on the basis of the so-called single coupling hypothesis, where a single one of the coupling constants is assumed to dominate over all the others, so that each of the coupling constants contributes once at a time. Apart from a few isolated cases, the typical bounds derived under this assumption, assuming a linear dependence on the superpartner masses, are of order $[\lambda,\ \lambda',\ \lambda''] < (10^{-1} - 10^{-2}) \tilde{m}/100\ \mathrm{GeV}$.

One important variant of the single coupling hypothesis can be defined by assuming that the dominance of single operators applies at the level of the gauge (current) basis fields rather than the mass eigenstate fields, as was implicit in the above original version. This appears as a more natural assumption in models where the presumed hierarchies in coupling constants originate from physics at higher scales (gauge, flavor, or strings). Flavor changing contributions may then be induced even when a single R parity odd coupling constant is assumed to dominate [17]. While the redefined mass basis superpotential may then depend on the various unitary transformation matrices $V^{u,d}_{L,R}$, [18], two distinguished predictive choices are those where the generation mixing is represented solely in terms of the Cabibbo-Kobayashi-Maskawa (CKM) matrix, with flavor changing effects appearing in either up-quark or down-quark flavors [17]. A similar situation holds for leptons with respect to the couplings λ_{ijk} and transformations $V^{l,\nu}_{L,R}$.

A large set of constraints has also been obtained by applying an extended hypothesis of dominance of coupling constants by pairs (or more). Several analyses dealing with hadron flavor changing effects (mixing parameters for the neutral light and heavy flavored mesons, rare mesons decays such as $K\to\pi+\nu+\overline{\nu}, \ldots$) [17], lepton flavor changing effects (leptons decays, $l^{\pm}_l\to l^{\pm}_k+l^-_n+l^+_p$ [19], $\mu^-\to e^-$ conversion processes [20], neutrinos Majorana mass [21], etc.), lepton number violating effects (neutrinoless double beta decay [22–24]), or baryon number violating effects (proton decay partial branchings [25], rare nonleptonic decays of heavy mesons [26], nuclei desintegration [27], etc.) have led to strong bounds on a large number of quadratic products of the coupling constants. All of the above low energy works, however, suffer from one or other form of model dependence, whether they rely on the consideration of loop diagrams [25], on additional assumptions concerning the flavor mixing [17,19,20], or on hadronic wave functions inputs [26,27].

Proceeding further with a linkage of R parity with physics beyond the standard model, our main observation in this work is that the R-parity-odd coupling constants could by themselves be an independent source of CP violation. Of course, the idea that the R parity violating (RPV) interactions could act as a source of superweak CP violation is not a new one in the supersymmetry literature. The principal motivation is that, whether the RPV interactions operate by themselves or in association with the gauge interactions, by exploiting the absence of strong constraints on violations with respect to the flavors of quarks, leptons, and the scalar superpartners by RPV interactions, one could greatly enhance the potential for observability of CP violation. To our knowledge, one of the earliest discussions of this possibility is contained in Ref. [8], where the role of a relative complex phase in a pair of λ''_{ijk} coupling constants was analyzed in connection with neutral K- and $\overline{K}$-meson mixing and decays and also with the neutron electric dipole moment. This subject has attracted increased interest in the recent literature [28–37]. Thus, the role of complex λ'_{ijk} coupling constants was considered in an analysis of the muon polarization in the decay $K^+\to\mu^++\nu+\gamma$ [33] and also of the neutral B- and $\overline{B}$-meson CP-odd decays asymmetries [29,31,32], that of complex λ_{ijk} interactions was considered in a study of the spin-dependent asymmetries of sneutrino-antisneutrino resonant production of τ-lepton pairs, $l^-l^+\to\tilde{\nu},\bar{\tilde{\nu}}\to\tau^+\tau^-$ [35], and that of complex λ''_{ijk} interactions was considered as a possible explanation for the cosmological baryon asymmetry [34], as well as in the neutral B, $\overline{B}$ decay asymmetries [32]. An interesting alternative proposal [30] is to embed the CP-odd phase in the scalar superpartner interactions corresponding to interactions of $A'_{ijk}\lambda'_{ijk}$ type. Furthermore, even if one assumes that the R-parity-odd interactions are CP conserving, these could still lead, in combination with other possible sources of complex phases in the minimal supersymmetric standard model, to new tests of CP violation. Thus, in the hypothesis of a pair of dominant coupling constants new contributions involving the coupling constants λ'_{ijk} and the CKM complex phase can arise for CP-odd observables associated with the neutral meson mixing parameters and decays [29,31,32]. Also, through the interference with the extra CP-odd phases present in the soft supersymmetry parameters A, the interactions λ_{ijk} and λ'_{ijk} may induce new contributions to electric dipole moments [38].

We propose in this work to examine the effect that R-parity-odd CP violating interactions could have on flavor nondiagonal rates and CP asymmetries in the production at high energy colliders of fermion-antifermion pairs of different families. We consider the two-body reactions $l^-(k)+l^+(k')\to f_J(p)+\overline{f_{J'}}(p')$ $(J\neq J')$, where l stands for electron or muon, the produced fermions are leptons, down quarks, or up quarks, and the center of mass energies span the relevant range of existing and planned leptonic (electron or muon) colliders, namely, from the Z-boson pole up to 1000 GeV. High energy colliders tests of the RPV contributions to the flavor diagonal reactions were recently examined in [39–42] and for flavor nondiagonal reactions in [43].

The physics of CP nonconservation at high energy colliders has motivated a wide variety of proposals in the past [44] and is currently the focus of important activity. In this work we shall limit ourselves to the simplest kind of observable, namely, the spin-independent observable involving differences in rates between a given flavor nondiagonal process and its CP-conjugated process. While the R-parity-odd interactions contribute to flavor changing amplitudes already at the tree level, their contribution to spin-independent CP-odd observables entails the consideration of loop diagrams. Thus, the CP asymmetries in the Z-boson pole branching fractions $B(Z\to f_J+\overline{f_{J'}})$ are controlled by a complex phase interference between nondiagonal flavor contributions to loop amplitudes, whereas the off Z-boson pole asymmetries are controlled instead by a complex phase interference between tree and loop amplitudes. Finite contributions at tree level order can arise for spin-dependent CP-odd observables, as discussed in Refs. [35,36].

It is useful to recall at this point that contributions in the standard model to the flavor changing rates and/or CP asymmetries can only appear through loop diagrams involving the quark–gauge-bosons interactions. Corresponding contributions involving squark-gaugino or slepton-gaugino interactions also arise in the minimal supersymmetric standard model. In studies performed some time ago within the standard model, the flavor nondiagonal vector bosons (Z-boson and/or W-boson) decay rates asymmetries [45–47] and CP-odd asymmetries [48,49] were found to be exceedingly small. (Similar conclusions were reached in top-quark phenomenology [50].) On the other hand, in most proposals of physics beyond the standard model, the prospects for observing flavor changing effects in rates [45–49,51] or in CP asymmetries [44,52] are on the optimistic side. Large effects

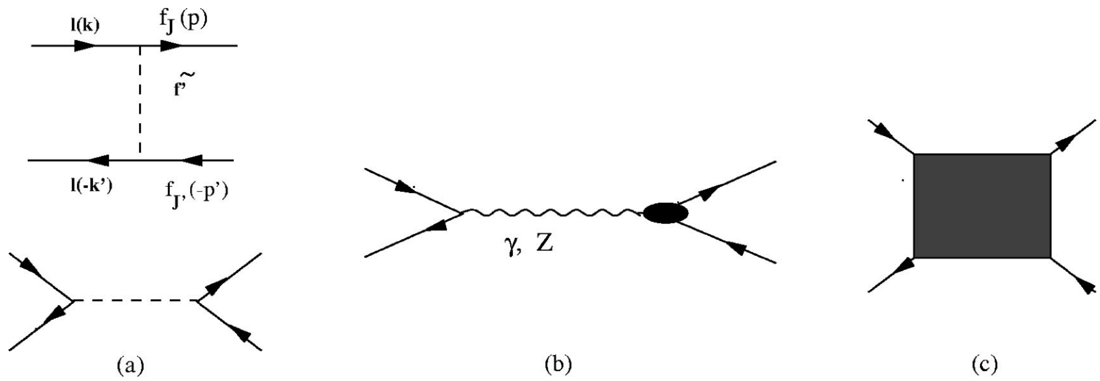

FIG. 1. Flavor nondiagonal process of $l^- l^+$ production of a fermion-antifermion pair, $l^-(k)+l^+(k')\to f_J(p)+\overline{f}_{J'}(p')$. The tree level diagrams in (a) represent *t*- and *s*-channel exchange amplitudes. The loop level diagrams represent γ- and *Z*-gauge-boson exchange amplitudes with dressed vertices in (b) and box amplitudes in (c).

were reported for the supersymmetric corrections in flavor changing *Z*-boson decay rates arising from squarks flavor mixings [53], but the conclusions from this initial work have been challenged in a subsequent work [54] involving a more complete calculation.

The possibility that the *R*-parity-odd interactions could contribute to the CP asymmetries at observable levels depends in the first place on the accompanying mechanisms responsible for the flavor changing rates. Our working assumption in this work will be that the *R*-parity-odd interactions are the dominant contributors to flavor nondiagonal amplitudes.

The contents of this paper are organized into four sections. In Sec. II, we develop the basic formalism for describing the scattering amplitudes at tree and one-loop levels. We discuss the case of leptons, down quarks, and up quarks successively in Secs. II A, II B, and II C. The evaluation of the one-loop diagrams is based on the standard formalism of [56]. Our calculations here closely parallel similar ones developed [57,58] in connection with corrections to the *Z*-boson partial widths. In Sec. III, we first briefly review the physics of flavor violation and next present our numerical results for the integrated cross sections (rates) and CP asymmetries for fermion pair production at and off the *Z*-boson pole. In Sec. IV, we state our main conclusions and discuss the impact of our results on possible experimental measurements.

II. PRODUCTION OF FERMION PAIRS OF DIFFERENT FLAVORS

In this section we shall examine the contributions induced by the RPV couplings on the flavor changing processes $l^-(k)+l^+(k')\to f_J(p)+\overline{f}_{J'}(p')$ ($l=e,\ \mu$; $J\neq J'$), where f stands for leptons or quarks and J, J' are family indices. The relevant tree and one-loop level diagrams are shown schematically in Fig. 1. At one-loop order, there arise γ- and *Z*-boson exchange triangle diagrams as well as box diagrams. In the sequel, for clarity, we shall present the formalism for the one-loop contributions only for the dressed $Zf\overline{f}$ vertex in the *Z*-boson exchange amplitude. The dressed γ exchange amplitude has a similar structure and will be added in together with the *Z*-boson exchange amplitude at the level of the numerical results. Since we shall repeatedly refer in the text to the *R*-parity-odd effective Lagrangian for the fermion-sfermion Yukawa interactions, we quote below its full expression:

$$L=\sum_{ijk}\Bigg\{\frac{1}{2}\lambda_{ijk}[\tilde{\nu}_{iL}\overline{e_{kR}}e_{jL}+\tilde{e}_{jL}\overline{e_{kR}}\nu_{iL}+\tilde{e}^\star_{kR}\overline{\nu^c_{iR}}e_{jL}-(i\to j)]$$
$$+\lambda'_{ijk}[\tilde{\nu}_{iL}\overline{d_{kR}}d_{jL}+\tilde{d}_{jL}\overline{d_{kR}}\nu_{iL}+\tilde{d}^\star_{kR}\overline{\nu^c_{iR}}d_{jL}$$
$$-\tilde{e}_{iL}\overline{d_{kR}}u_{jL}-\tilde{u}_{jL}\overline{d_{kR}}e_{iL}-\tilde{d}^\star_{kR}\overline{e^c_{iR}}u_{jL}]$$
$$+\frac{1}{2}\lambda''_{ijk}\epsilon_{\alpha\beta\gamma}[\tilde{u}^\star_{i\alpha R}\overline{d_{j\beta R}}d^c_{k\gamma L}+\tilde{d}^\star_{j\beta R}\overline{u_{i\alpha R}}d^c_{k\gamma L}$$
$$+\tilde{d}^\star_{k\gamma R}\overline{u_{i\alpha R}}d^c_{j\beta L}-(j\to k)]\Bigg\}+\text{H.c.},\qquad(2)$$

noting that the summation runs over the (quarks and leptons) families indices $i,j,k=[(e,\mu,\tau),\ (d,s,b),\ (u,c,t)]$, subject to the antisymmetry properties $\lambda_{ijk}=-\lambda_{jik}$, $\lambda''_{ijk}=-\lambda''_{ikj}$. We use conventions for the ordering of the operations on Dirac spinors such that charge conjugation acts first, chirality projection second, and Dirac bar third, so that $\overline{\psi^c_{L,R}}=\overline{(\psi^c)_{L,R}}$.

A. Charged lepton-antilepton pairs

1. General formalism

The process $l^-(k)+l^+(k')\to e^-_J(p)+e^+_{J'}(p')$, for $l=e$, μ; $J\neq J'$, can pick up a finite contribution at the tree level from the *R*-parity-odd couplings λ_{ijk} only. For clarity, we treat in the following the case of electron colliders, noting that the case of muon colliders is easily deduced by replacing all occurrences in the RPV coupling constants of the index 1 by the index 2. There occur both *t*-channel and *s*-channel $\tilde{\nu}_{iL}$ exchange contributions, of the type shown by the Feynman diagrams in (a) of Fig. 1. The scattering amplitude at the tree level, M_t, reads

$$M_t^{JJ'} = -\frac{1}{2(t-m^2_{\tilde{\nu}_{iL}})} \times[\lambda_{i1J}\lambda^{\star}_{i1J'}\overline{u}_R(p)\gamma_\mu v_R(p')\overline{v}_L(k')\gamma_\mu u_L(k) + \lambda^{\star}_{iJ1}\lambda_{iJ'1}\overline{u}_L(p)\gamma_\mu v_L(p')\overline{v}_R(k')\gamma_\mu u_R(k)] - \frac{1}{s-m^2_{\tilde{\nu}_{iL}}}[\lambda_{i11}\lambda^{\star}_{iJJ'}\overline{v}_R(k')u_L(k)\overline{u}_L(p)v_R(p') + \lambda^{\star}_{i11}\lambda_{iJJ'}\overline{v}_L(k')u_R(k)\overline{u}_R(p)v_L(p')], \quad (3)$$

where to obtain the saturation structure in the Dirac spinors indices for the t-channel terms we have applied the Fierz rearrangement formula $\overline{u}_R(p)u_L(k)\overline{v}_L(k')v_R(p') = \frac{1}{2}\overline{u}_R(p)\gamma_\mu v_R(p')\overline{v}_L(k')\gamma_\mu u_L(k)$. The t-channel (s-channel) exchange terms on the right-hand side of Eq. (3) include two terms each, called $\mathcal{R}$ and $\mathcal{L}$ type, respectively. These two terms differ by a chirality flip $L \leftrightarrow R$ and correspond to distinct diagrams where the exchanged sneutrino is emitted or absorbed at the upper (right-handed) vertex.

The Z-boson exchange amplitude [diagram (b) in Fig. 1] at the loop level, M_l, reads

$$M_l^{JJ'} = \left(\frac{g}{2\cos\theta_W}\right)^2 \overline{v}(k')\gamma_\mu[a(e_L)P_L + a(e_R)P_R]u(k)\frac{1}{s-m_Z^2+im_Z\Gamma_Z}\Gamma^Z_\mu(p,p'), \quad (4)$$

where the Z-boson current amplitude vertex function $\Gamma^Z_\mu(p,p')$ is defined through the effective Lagrangian density

$$L = -\frac{g}{2\cos\theta_W}Z^\mu\Gamma^Z_\mu(p,p').$$

For later convenience, we record for the processes $Z(P=p+p') \to f(p) + \overline{f'}(p')$ and $Z(P) \to \tilde{f}_H(p) + \tilde{f}^{\star}_H(p')$ the familiar definitions of the Z-boson bare vertex functions,

$$\Gamma^Z_\mu(p,p') = \{\overline{f}(p)\gamma_\mu[a(f_L)P_L + a(f_R)P_R]f'(p') + (p-p')_\mu\tilde{f}^{\star}_H(p')a(\tilde{f}_H)\tilde{f}_H(p)\}, \quad (5)$$

where the quantities denoted, $a(f_H) \equiv a_H(f)$ and $a(\tilde{f}_H)$, taking equal values for both fermions and sfermions, are defined by $a(f_H) = a(\tilde{f}_H) = 2T_3^H(f) - 2Qx_W$, where $H=(L,R)$, $x_W = \sin^2\theta_W$, T_3^H are $SU(2)_H$ Cartan subalgebra generators, and $Q = T_3^L + Y$, Y are electric charge and weak hypercharge. These parameters satisfy the useful relations $a(\tilde{f}^{\star}_H) = -a(\tilde{f}_H)$, $a_L(f^c) = -a_R(f)$, $a_R(f^c) = -a_L(f)$. Throughout this paper we shall use the conventions in the Haber-Kane review [59] [metric signature $(+---)$, $P_{(L/R)} = (1 \mp \gamma_5)/2$, etc.] and adopt the familiar summation convention on dummy indices.

The Lorentz covariant structure of the dressed Z-boson current amplitude in the process $Z(P) \to f_J(p) + \overline{f}_{J'}(p')$, for a generic value of the Z-boson invariant mass $s=P^2$, involves three pairs of vectorial and tensorial vertex functions, which are defined in terms of the general decomposition

$$\Gamma^Z_\mu(p,p') = \overline{u}(p)\Bigg[\gamma_\mu[\tilde{A}_L^{JJ'}(f)P_L + \tilde{A}_R^{JJ'}(f)P_R] + \frac{1}{m_J+m_{J'}}\sigma_{\mu\nu}\{(p+p')^\nu[ia^{JJ'} + \gamma_5 d^{JJ'}] + (p-p')^\nu[ib^{JJ'} + \gamma_5 e^{JJ'}]\}\Bigg]v(p'), \quad (6)$$

where $\sigma_{\mu\nu} = (i/2)[\gamma_\mu, \gamma_\nu]$. The vector vertex functions separate additively into the classical (bare) and loop contributions, $\tilde{A}_H^{JJ'}(f) = a_H(f)\delta_{JJ'} + A_H^{JJ'}(f)$ $(H=L,R)$. The tensor vertex functions, associated with $\sigma^{\mu\nu}(p+p')_\nu$, include the familiar magnetic and electric $Zf\overline{f}$ couplings, such that the flavor diagonal vertex functions

$$-\frac{g}{2\cos\theta_W}\frac{1}{2m_J}[a^{JJ}, d^{JJ}],$$

identify, in the small momentum transfer limit, with the fermion–Z-boson current magnetic and (P- and CP-odd) electric dipole moments, respectively. In working with the spinor matrix elements, it is helpful to recall the mass shell relations $\overline{u}(p)\not{p} = m_J\overline{u}(p)$, $\not{p}'v(p') = -m_{J'}v(p')$, and the Gordon-type identities, appropriate to the saturation of the Dirac spinor indices, $\overline{u}(p)\cdots v(p')$,

$$\left[(p \pm p')_\mu\begin{pmatrix}\gamma_5\\1\end{pmatrix} + i\sigma_{\mu\nu}(p \mp p')^\nu\begin{pmatrix}\gamma_5\\1\end{pmatrix}\right] = (m_J + m_{J'})\gamma_\mu\begin{pmatrix}\gamma_5\\1\end{pmatrix},$$

$$\left[(p \mp p')_\mu\begin{pmatrix}\gamma_5\\1\end{pmatrix} + i\sigma_{\mu\nu}(p \pm p')^\nu\begin{pmatrix}\gamma_5\\1\end{pmatrix}\right] = (m_J - m_{J'})\gamma_\mu\begin{pmatrix}\gamma_5\\1\end{pmatrix}.$$

Based on these identities, one also checks that the additional vertex functions $[b^{JJ'}, e^{JJ'}]$, associated with the Lorentz covariants $\sigma^{\mu\nu}(p-p')_\nu[1,\gamma_5]$, can be expressed as linear combinations of the vector or axial covariants γ_μ $[1,\gamma_5]$ and the total momentum covariants $(p+p')_\mu$ $[1,\gamma_5]$. The latter will yield, upon contraction with the initial state Zl^-l^+ vertex function, to negligible mass terms in the initial leptons.

Let us now perform the summation over the initial and final states polarizations for the summed tree and loop amplitudes $M^{JJ'} = M_t^{JJ'} + M_l^{JJ'}$, where the lower suffices t,l stand for tree and loop, respectively. (We shall not be interested in this work in spin observables.) A straightforward calculation, carried out for the squared sum of the tree and loop amplitudes, yields the result (a useful textbook to consult here is Ref. [60])

$$\sum_{pol} |M_t^{JJ'}+M_l^{JJ'}|^2=\left|-\frac{\lambda_{i1J}\lambda^\star_{i1J'}}{2(t-m^2_{\tilde\nu_{iL}})}+\left(\frac{g}{2\cos\theta_W}\right)^2\frac{a(e_L)A_R^{JJ'}(e,s+i\epsilon)}{s-m_Z^2+im_Z\Gamma_Z}\right|^2 16(k\cdot p)(k'\cdot p')+8m_Jm_{J'}(k\cdot k')\varphi_{LL}(\mathcal{R})$$
$$+8m_e^2(p\cdot p')\varphi_{RR}(\mathcal{R})+\left|-\frac{\lambda_{iJ1}\lambda^\star_{iJ'1}}{2(t-m^2_{\tilde\nu_{iL}})}+\left(\frac{g}{2\cos\theta_W}\right)^2\frac{a(e_R)A_L^{JJ'}(e,s+i\epsilon)}{s-m_Z^2+im_Z\Gamma_Z}\right|^2 16(k\cdot p)(k'\cdot p')$$
$$+8m_Jm_{J'}(k\cdot k')\varphi_{RR}(\mathcal{L})+8m_e^2(p\cdot p')\varphi_{LL}(\mathcal{L})+8\left|\frac{\lambda_{i11}\lambda^\star_{iJJ'}}{s-m^2_{\tilde\nu_{iL}}}\right|^2(k\cdot k')(p\cdot p'), \tag{7}$$

where we have introduced the following functions, associated with the $\mathcal{R}$- and $\mathcal{L}$-type contributions:

$$\varphi_{HH'}(\mathcal{R})=-\left(\frac{g}{2\cos\theta_W}\right)^2\left(\frac{a(e_H)A_{H'}^{JJ'}(e,s+i\epsilon)}{s-m_Z^2+im_Z\Gamma_Z}\right)^\star \times\left(\frac{\lambda_{i1J}\lambda^\star_{i1J'}}{2(t-m^2_{\tilde\nu_{iL}})}\right)+\text{c.c.},$$

$$\varphi_{HH'}(\mathcal{L})=-\left(\frac{g}{2\cos\theta_W}\right)^2\left(\frac{a(e_H)A_{H'}^{JJ'}(e,s+i\epsilon)}{s-m_Z^2+im_Z\Gamma_Z}\right)^\star \times\left(\frac{\lambda_{iJ1}\lambda^\star_{iJ'1}}{2(t-m^2_{\tilde\nu_{iL}})}\right)+\text{c.c.}\,. \tag{8}$$

The two sets of terms in Eqs. (8) and (7), labelled by the letters $\mathcal{R},\mathcal{L}$, are associated with the two *t*-channel exchange contributions in the tree amplitude, Eq. (3), which differ by the spinor chirality structure and the substitutions $\lambda_{i1J}\lambda^\star_{i1J'}\to\lambda^\star_{iJ1}\lambda_{iJ'1}$. The terminology $\mathcal{L}$, R is motivated by the fact that these contributions are controlled by the *Z*-boson left and right chirality vertex functions A_L and A_R, respectively, in the massless limit.

The imaginary shift in the argument $s+i\epsilon$ (representing the upper lip of the cut real axis in the complex *s* plane) of the vertex functions, $A_H^{JJ'}(f,s+i\epsilon)$, has been appended to remind us that the one-loop vertex functions are complex functions in the complex plane of the virtual *Z*-boson mass squared, $s=(p+p')^2$, with branch cuts starting at the physical thresholds where production processes, such as $Z\to f+\bar f$ or $Z\to\tilde f+\tilde f^\star$, are raised on shell. For notational simplicity, we have omitted writing several terms proportional to the initial leptons masses and also some of the small subleading terms arising from the loop amplitude squared. At the energies of interest, whose scale is set by the initial center of mass energy or by the *Z*-boson mass, the terms involving factors of the initial leptons masses m_e are entirely negligible, of course. Thus, the contributions associated with $\varphi_{RR}(\mathcal{R})$, $\varphi_{LL}(\mathcal{L})$ can safely be dropped. Also, the contribution from $\varphi_{LL}(\mathcal{R})$ and $\varphi_{RR}(\mathcal{L})$, which are proportional to the final state leptons masses m_J, and $m_{J'}$, can to a good approximation be neglected for lepton production. Always in the same approximation, we find also that interference terms are absent between the *s*-channel exchange and the *t*-channel amplitudes and between the *s*-channel tree and *Z*-boson exchange loop amplitudes. Similarly, because of the opposite chirality structure of the first two terms in $M_t^{JJ'}$, their cross-product contributions give negligibly small mass terms.

2. *CP* asymmetries

Our main concern in this work bears on the comparison of the pair of *CP* conjugate reactions $l^-(k)+l^+(k')\to e_J^-(p)+e_{J'}^+(p')$ and $l^-(k)+l^+(k')\to e_{J'}^-(p)+e_J^+(p')$. Denoting the summed tree and one-loop probability amplitudes for these reactions as $M^{JJ'}=M_t^{JJ'}+M_l^{JJ'}$, $\bar M^{JJ'}=M_t^{J'J}+M_l^{J'J}=M^{J'J}$, we observe that these amplitudes are simply related to one another by means of a specific complex conjugation operation. The general structure of this relationship can be expressed schematically as

$$M^{JJ'}=a_0^{JJ'}+\sum_\alpha a_\alpha^{JJ'}F_\alpha^{JJ'}(s+i\epsilon),$$

$$\bar M^{JJ'}=a_0^{JJ'\star}+\sum_\alpha a_\alpha^{JJ'\star}F_\alpha^{J'J}(s+i\epsilon), \tag{9}$$

where for each of the equations above, referring to amplitudes for pairs of *CP* conjugate processes, the first and second terms correspond to the tree and loop level contributions, with $a_0^{JJ'}$, $a_0^{J'J}=a_0^{JJ'\star}$, representing the tree amplitudes and $a_\alpha^{JJ'}$, $a_\alpha^{J'J}=a_\alpha^{JJ'\star}$ and $F_\alpha^{JJ'}$, $F_\alpha^{J'J}=F_\alpha^{JJ'}$ representing the complex-valued coupling constants products and momentum integrals in the loop amplitudes. The functions $F^{JJ'}$ must be symmetric under the interchange $J\leftrightarrow J'$. The summation index α labels the family configurations for the intermediate fermions and sfermions which can run inside the loops. Defining the *CP* asymmetries by the normalized differences

$$\mathcal{A}_{JJ'}=\frac{|M^{JJ'}|^2-|\bar{M}^{JJ'}|^2}{|M^{JJ'}|^2+|\bar{M}^{JJ'}|^2}$$

and inserting the decompositions in Eq. (9), the result separates additively into two types of terms:

$$\mathcal{A}_{JJ'}=\frac{2}{|a_0|^2}\Bigg[\sum_{\alpha}\mathrm{Im}(a_0a_\alpha^\star)\mathrm{Im}[F_\alpha(s+i\epsilon)] - \sum_{\alpha<\alpha'}\mathrm{Im}(a_\alpha a_{\alpha'}^\star)\mathrm{Im}[F_\alpha(s+i\epsilon)F_{\alpha'}^\star(s+i\epsilon)]\Bigg], \tag{10}$$

where, for notational simplicity, we have suppressed the fixed external family indices on $a_0^{JJ'}$, $a_\alpha^{JJ'}$, and $F_\alpha^{JJ'}$, and replaced the full denominator by the tree level amplitude, since this is expected to dominate over the loop amplitude. The first term in Eq. (10) is associated with an interference between tree and loop amplitudes and the second with an interference between terms arising from different family contributions in the loop amplitude. In the second term of Eq. (10), the two imaginary part factors are antisymmetric under the interchange of indices α and α', so that their product is symmetric and allows one to write $\Sigma_{\alpha<\alpha'}=\frac{1}{2}\Sigma_{\alpha\neq\alpha'}$. To obtain a more explicit formula, let us specialize to the specific case where the Z-boson vertex functions decompose as $A_H^{JJ'}(f,s+i\epsilon)=\Sigma_\alpha b_{JJ'}^{H\alpha}I_{H\alpha}^{JJ'}(s+i\epsilon)$. The first factors $b_{JJ'}^{H\alpha}=\lambda_{ijJ}\lambda_{ijJ'}^\star$ [using $\alpha=(ij)$ and notation for the one-loop contributions to be described in the next subsection] include the CP-odd phase from the R-parity-odd coupling constants. The second factors $I_{H\alpha}^{JJ'}$ include the CP-even phase from the unitarity cuts associated with the physical on-shell intermediate states. In the notation of Eq. (9),

$$a_\alpha^{JJ'}=\left(\frac{g}{2\cos\theta_W}\right)^2 a(e_{H'})b_{JJ'}^{H\alpha},$$

$$F_\alpha^{JJ'}=I_{H\alpha}^{JJ'}(s+i\epsilon)/(s-m_Z^2+im_Z\Gamma_Z),$$

where the right-hand sides incorporate appropriate sums over the chirality indices H', H of the initial and final fermions, respectively.

Applying Eq. (9) to the asymmetry integrated with respect to the scattering angle, one derives, for the corresponding integrated tree loop interference contribution,

$$\mathcal{A}_{JJ'}=-4\left(\frac{g}{2\cos\theta_W}\right)^2 a(e_L)\mathrm{Im}[\lambda_{i1J}\lambda_{i1J'}^\star a_{JJ'}^{\alpha\star}(f_R)] \times\mathrm{Im}\left(\frac{I_\alpha^R(s+i\epsilon)}{s-m_Z^2+im_Z\Gamma_Z}\right)\int_{-1}^{1}dx\ \frac{(1-x)^2}{2(t-m_{\tilde{\nu}_{iL}}^2)} \times\left[\sum_i\left|\lambda_{i1J}\lambda_{i1J'}^\star\right|^2\int_{-1}^{1}dx\ \frac{(1-x)^2}{4(t-m_{\tilde{\nu}_{iL}}^2)^2}\right]^{-1}, \tag{11}$$

where θ, ($x=\cos\theta$) denotes the scattering angle variable in the center of mass frame and the Mandelstam variables in the case of massless final state fermions take the simplified expressions $s\equiv(k+k')^2$, $t\equiv(k-p)^2=-\frac{1}{2}s(1-x)$, $u\equiv(k-p')^2=-\frac{1}{2}s(1+x)$. Useful kinematical relations in the general case with final fermions masses m_J, $m_{J'}$ are $\sqrt{s}=2k=E_p+E_{p'}$, $t=m_J^2-sE_p(1-\beta x)$, $u=m_{J'}^2-sE_{p'}(1+\beta' x)$, where $E_p=(s+m_J^2-m_{J'}^2)/(2\sqrt{s})$, $E_{p'}=(s+m_{J'}^2-m_J^2)/(2\sqrt{s})$, $\beta=p/E_p$, $\beta'=p/E_{p'}$, with k, p denoting the center of mass momenta of the two-body initial and final states, respectively. The unpolarized differential cross section reads then $d\sigma/dx=(|p|/128\pi s|k|)\Sigma_{pol}|M|^2$.

For the Z-boson pole observables, the flavor nondiagonal branching ratios and CP asymmetries (where one sets $s=m_Z^2$) are defined in terms of the notation specified in the preceding paragraph by the equations

$$B_{JJ'}\equiv\frac{\Gamma(Z\to f_J+\bar{f}_{J'})+\Gamma(Z\to f_{J'}+\bar{f}_J)}{\Gamma(Z\to\mathrm{all})}=2\frac{|A_L^{JJ'}(f)|^2+|A_R^{JJ'}(f)|^2}{\sum_f|a_L(f)|^2+|a_R(f)|^2},$$

$$\mathcal{A}_{JJ'}\equiv\frac{\Gamma(Z\to f_J+\bar{f}_{J'})-\Gamma(Z\to f_{J'}+\bar{f}_J)}{\Gamma(Z\to f_J+\bar{f}_{J'})+\Gamma(Z\to f_{J'}+\bar{f}_J)}=-2\frac{\sum_{H=L,R}\sum_{\alpha<\alpha'}\mathrm{Im}(b_{JJ'}^{H\alpha}b_{JJ'}^{H\alpha'\star})\mathrm{Im}[I_{H\alpha}^{JJ'}(s+i\epsilon)I_{H\alpha'}^{JJ'\star}(s+i\epsilon)]}{\sum_{H=L,R}|\sum_\alpha b_{JJ'}^{H\alpha}(f)F_H^\alpha(s+i\epsilon)|^2}. \tag{12}$$

For completeness, we recall the formula for the Z-boson decay width in fermion pairs (massless limit),

$$\Gamma(Z\to f_J+\bar{f}_{J'})=\frac{G_Fm_Z^3c_f}{12\sqrt{2}\pi}[|A_L^{JJ'}(f)|^2+|A_R^{JJ'}(f)|^2],$$

where $c_f=[1,N_c]$, for $[f=l,q]$ [$N_c=3$ is the number of colors in the $SU(3)_c$ color group] and the experimental value for the total width, $\Gamma(Z\to\mathrm{all})_{\mathrm{expt}}=2.497$ GeV.

The expressions in Eqs. (11) and (12) for the CP asymmetries explicitly incorporate the property of these observ-

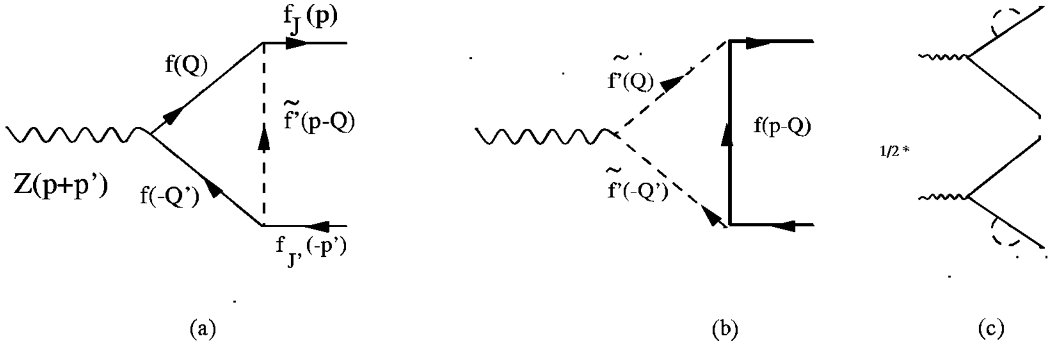

FIG. 2. One-loop diagrams for the dressed $Z(P)\ \ f(p)\overline{f}(p')$ vertex. The flow of four-momenta for the intermediate fermions in (a) is denoted as $Z(P)\to f(Q)+\overline{f}(Q')\to f_J(p)+\overline{f}_{J'}(p')$. A similar notation is used for the sfermion diagram in (b) where $Z(P)\to\tilde{f}'(Q)+\tilde{f}'^{\star}(Q')$ and for the self-energy diagrams in (c).

ables of depending on combinations of the RPV coupling constants, such as, $\mathrm{Arg}(\lambda_{i1J}\lambda^{\star}_{i1J'}\lambda^{\star}_{i'jJ}\lambda_{i'jJ'})$, or $\mathrm{Arg}(\lambda_{ijJ}\lambda^{\star}_{ijJ'}\lambda^{\star}_{i'j'J}\lambda_{i'j'J'})$, which are invariant under complex phase redefinitions of the fields. This freedom under rephasings of the quarks and leptons superfields actually removes 21 complex phases from the complete general set of 45 complex RPV coupling constants.

3. One-loop amplitudes

The relevant triangle Feynman diagrams, which contribute to the dressed Z-boson leptonic vertex $Z(P)l^-(p)l^+(p')$, appear in three types, fermionic, scalar, and self-energy, as shown in Fig. 2. We consider first the contributions induced by the R-parity-odd couplings λ'_{ijk}. The intermediate lines can assume two distinct configurations which both contribute, in the limit of vanishing external fermions masses to the left-chirality vertex functions only. We shall refer to such contributions by the name $\mathcal{L}$-type contributions, reserving the name $\mathcal{R}$-type to contributions to the right-chirality vector couplings. The two allowed configurations for the internal fermions and sfermions are $f=(\begin{smallmatrix} d_k \\ u^c_j\end{smallmatrix});\tilde{f}'=(\begin{smallmatrix}\tilde{u}^{\star}_{jL}\\ \tilde{d}_{kR}\end{smallmatrix})$. Our calculations of the triangle diagrams employ the kinematical conventions for the flow of electric charge and momenta indicated in Fig. 2, where $P=p+p'=Q+Q'=k+k'$. The summed fermion and scalar Z-boson current contributions are given by

$$\Gamma^Z_\mu(\mathcal{L})=+iN_c\lambda'^{\star}_{Jjk}\lambda'_{J'jk}\left[\int_Q\frac{\overline{u}(p)\{P_R(\not{Q}+m_f)\gamma_\mu[a(f_L)P_L+a(f_R)P_R](-\not{Q}'+m_f)P_L\}v(p')}{(Q^2-m_f^2)[(Q-p-p')^2-m_f^2][(Q-p)^2-m^2_{\tilde{f}'}]}\right.$$
$$\left.+\int_Q\frac{a(\tilde{f}'_L)(Q-Q')_\mu\overline{u}(p)[P_R(\not{p}-\not{Q}+m_f)P_L]v(p')}{(Q^2-m^2_{\tilde{f}'})[(Q-p-p')^2-m^2_{\tilde{f}'}][(Q-p)^2-m^2_f]}\right]. \tag{13}$$

The integration measure is defined as $\int_Q=[1/(2\pi)^4]\int d^4Q$. For a convenient derivation of the self-energy diagrams, one may invoke the on-shell renormalization condition which relates these to the fields renormalization constants. Defining schematically the self-energy vertex functions for a Dirac fermion field ψ by the Lagrangian density $L=i\overline{\psi}[\not{p}-m+\Sigma(p)]\psi$, $\Sigma(p)=m\sigma_0+\not{p}(\sigma^LP_L+\sigma^RP_R)$, the transition from bare to renormalized fields and mass terms may be effected by the replacements

$$\psi_H\to\frac{\psi_H}{(1+\sigma^H)^{1/2}}=\psi_HZ_H^{1/2},$$

$$m\to m\frac{(1+\sigma^L)^{1/2}(1+\sigma^R)^{1/2}}{(1-\sigma_0)}\quad (H=L,R).$$

By a straightforward generalization to the case of several fields, labelled by a family index J, the fields renormalization constants become matrices, $Z^H_{JJ'}=(1+\sigma^H)^{-1}_{JJ'}$. The self-energy contributions to the dressed Z-boson vertex function is then described as

$$\begin{aligned}\Gamma^Z_\mu(p,p')_{SE}&=\sum_{H=L,R}[(Z^H_{JJ'}Z^{H\star}_{J'J})^{1/2}-1]\\ &\quad\times\overline{u}(p)\gamma_\mu a(f_H)P_Hv(p')\\ &=-\sum_{H=L,R}\frac{1}{2}[\sigma^H_{JJ'}(p)\\ &\quad+\sigma^{H\star}_{J'J}(p')]\overline{u}(p)\gamma_\mu a(f_H)P_Hv(p'),\end{aligned} \tag{14}$$

where, for the case at hand,

$$\Sigma_{JJ'}(p) = -iN_c \lambda'^{\star}_{Jjk}\lambda'_{J'jk} \times \int_Q \frac{P_R(\not{Q}+m_f)P_L}{(-Q^2+m_f^2)[-(Q-p)^2+m_{\tilde{f}'}^2]} \quad (15)$$

gives the self-energy vertex function, implying that $\sigma^R_{JJ'} = 0$ and $\sigma_0 = 0$. Similar Feynman graphs to those of Fig. 2, and similar formulas to those of Eqs. (13) and (14), obtain, for the dressed photon current case, $\gamma(P) l^-(p) l^+(p')$.

We organize our one-loop calculations in line with the approach developed by 't Hooft and Veltman [55] and Passarino and Veltman [56], keeping in mind that our spacetime metric has an opposite signature to theirs $(-+++)$. For definiteness, we recall the conventional notations for the two-point and three-point integrals,

$$\frac{i\pi^2}{(2\pi)^4}[B_0, -p_\mu B_1] = \int_Q \frac{[1, Q_\mu]}{(-Q^2+m_1^2)[-(Q-p)^2+m_2^2]}, \quad (16)$$

$$\frac{i\pi^2}{(2\pi)^4}[C_0,\ -p_\mu C_{11} - p'_\mu C_{12},\ p_\mu p_\nu C_{21} + p'_\nu p'_\mu C_{22} + (p_\mu p'_\nu + p_\nu p'_\mu) C_{23}\ -g_{\mu\nu} C_{24}]$$
$$= \int_Q \frac{[1, Q_\mu, Q_\mu Q_\nu]}{(-Q^2+m_1^2)[-(Q-p)^2+m_2^2][-(Q-p-p')^2+m_3^2]}, \quad (17)$$

where the arguments for the B and C functions are defined as $B_A(-p, m_1, m_2)$ where $(A=0,1)$ and $C_B(-p,-p',m_1,m_2,m_3)$ where $(B=0,11,12,21,22,23,24)$. In the algebraic derivation of the one-loop amplitudes, we find it convenient to introduce the definitions $p_\mu = \frac{1}{2}P_\mu + \rho_\mu$, $p'_\mu = \frac{1}{2}P_\mu - \rho_\mu$, where $P = p + p'$, $\rho = \frac{1}{2}(p - p')$. The terms proportional to the Lorentz covariant $P^\mu = (p+p')^\mu$ will then reduce, for the full Z-boson exchange amplitude in Eq. (4), to negligible mass terms in the initial leptons.

Dropping mass terms for all external fermions, the tensorial couplings cancel out and we need keep track of the vector couplings only, with the result

$$A_L^{JJ'}(\mathcal{L}) = N_c \frac{\lambda'^{\star}_{Jjk}\lambda'_{J'jk}}{(4\pi)^2}[a(f_L) m_f^2 C_0 + a(f_R) \times (B_0^{(1)} - 2C_{24} - m_{\tilde{f}'}^2 C_0) + 2a(\tilde{f}'_L)\tilde{C}_{24} + a(f_L) B_1^{(2)}],$$

$$A_R^{JJ'}(\mathcal{L}) = 0. \quad (18)$$

The cancellation of the right-chirality vertex function in this case is the reason behind our naming these contributions as $\mathcal{L}$ type. The two-point and three-point integrals functions without a tilde arise through the fermion current triangle contribution and the self-energy contribution (represented by the term proportional to $B_1^{(2)}$). These involve the argument variables according to the following conventions: $B_A^{(1)} = B_A(-p-p', m_f, m_f)$, $B_A^{(2)} = B_A(-p, m_f, m_{\tilde{f}'})$, $B_A^{(3)} = B_A(-p', m_{\tilde{f}'}, m_f)$, and $C_B = C_A(-p,-p',m_f,m_{\tilde{f}'},m_f)$. The integral functions with a tilde arise in the sfermion current diagram and are described by the argument variables $\tilde{C}_A = C_A(-p,-p',m_{\tilde{f}'},m_f,m_{\tilde{f}'})$.

A very useful check on the above results concerns the cancellation of ultraviolet divergences. This is indeed expected on the basis of the general rule that those interaction terms which are absent from the classical action, as is the case for the flavor changing currents, cannot undergo renormalization. A detailed discussion of this property is developed in [61]. The logarithmically divergent terms in Eq. (18), proportional to the quantity, $\Delta = -2/(D-4) + \gamma - \ln\pi$, as defined in [56], arise from the two- and three-point integrals as $B_0 \to \Delta$, $B_1 \to -\frac{1}{2}\Delta$, $C_{24} \to \frac{1}{4}\Delta$, all other integrals being finite. Performing these substitutions, we indeed find that Δ comes accompanied by the overall factors $[-a(e_L) + a(\tilde{u}_L^\star) + a(d_R)]$ or $[-a(e_L) + a(\tilde{d}_R) + a_R(u^c)]$, which both vanish in the relevant configurations for $f, \tilde{f}'$.

Let us now consider the R-parity-odd Yukawa interactions involving the λ_{ijk}. These contribute through the same triangle diagrams as in Fig. 2. There arise contributions of $\mathcal{L}$ type, in the single configuration, $f = e_k$, $\tilde{f}' = \tilde{\nu}^\star_{iL}$, and of $\mathcal{R}$ type in the two configurations $f = \binom{e_j}{\nu_i}$, $\tilde{f}' = \binom{\tilde{\nu}_{iL}}{\tilde{e}_{jL}}$. Following the same derivation as above, and neglecting all of the external mass terms, we obtain the following results for the one-loop vector coupling vertex functions:

$$A_L^{JJ'}(\mathcal{L}) = \frac{\lambda^{\star}_{iJk}\lambda_{iJ'k}}{(4\pi)^2}[a(f_L) m_f^2 C_0 + a(f_R)(B_0^{(1)} - 2C_{24} - m_{\tilde{f}'}^2 C_0) + 2a(\tilde{f}'_L)\tilde{C}_{24} + a(f_L) B_1^{(2)}],$$

$$A_R^{JJ'}(\mathcal{R}) = \frac{\lambda_{ijJ}\lambda^{\star}_{ijJ'}}{(4\pi)^2}[a(f_R)m_f^2 C_0 + a(f_L)(B_0^{(1)} - 2C_{24} - m_{\tilde f'}^2 C_0) + 2a(\tilde f'_L)\tilde C_{24} + a(f_R)B_1^{(2)}], \tag{19}$$

with $A_R^{JJ'}(\mathcal{L})=0$, $A_L^{JJ'}(\mathcal{R})=0$. We note that the $\mathcal{L}$, R contributions are related by a mere chirality flip transformation and that the color factor N_c is absent in the present case.

B. Down-quark–antiquark pairs

The processes involving flavor nondiagonal final down-quark–antiquark pairs, $l^-(k)+l^+(k')\rightarrow d_J(p)+\overline{d}_{J'}(p')$, pick up nonvanishing contributions only from the λ'_{ijk} interactions. Our discussion here will be brief since this case is formally similar to the leptonic case treated in Sec. II A. In particular, the external fermions masses, for all three families, can be neglected to a good approximation at the energy scales of interest. The tree level amplitude comprises an $\mathcal{R}$-type single t-channel $\tilde u$-squark exchange diagram and two s-channel diagrams involving $\tilde\nu$ and $\tilde{\bar\nu}$ sneutrinos of the type shown in (a) of Fig. 1,

$$M_t^{JJ'} = -\frac{\lambda'_{1jJ}\lambda'^{\star}_{1jJ'}}{2(t-m^2_{\tilde u_{jL}})}\overline{u_R}(p)\gamma_\mu v_R(p')\overline{v_L}(k')\gamma_\mu u_L(k) - \frac{1}{s-m^2_{\tilde\nu_{iL}}}[\lambda_{i11}\lambda'^{\star}_{iJJ'}\overline{v_R}(k')u_L(k)\overline{u_L}(p)v_R(p') + \lambda^{\star}_{i11}\lambda'_{iJJ'}\overline{v_L}(k')u_R(k)\overline{u_R}(p)v_L(p')], \tag{20}$$

where a Kronecker symbol factor δ_{ab}, expressing the dependence on the final state quark color indices $d^a\overline{d}_b$, has been suppressed. This dependence will induce in the analogue of the formula in Eq. (7), expressing the rates, an extra color factor N_c.

At the one-loop level, the dressed $Z\, d_J\, \overline{d}_{J'}$ vertex functions in the Z-boson s-channel exchange amplitude can be described by the same type of triangle diagrams as in Fig. 2. The field configurations circulating in the loop correspond now to quarks and sleptons of $\mathcal{L}$ type, $f=d_k$, $\tilde l' = \tilde\nu^{\star}_{iL}$, and of $\mathcal{R}$ type, $f=\binom{d_j}{u_j}$, $\tilde l' = \binom{\tilde\nu_{iL}}{\tilde e_{iL}}$. There also occurs corresponding lepton-squark field configurations of $\mathcal{L}$ type, $l=\nu_i^c$, $\tilde f' = \tilde d_{kR}$, and $\mathcal{R}$ type, $l=\binom{\nu_i}{e_i}$; $\tilde f' = \binom{\tilde d_{jL}}{\tilde u_{jL}}$. The $\mathcal{L}$- and $\mathcal{R}$-type contributions differ by a chirality flip, the first contributing to $A_L^{JJ'}$ and the second to $A_R^{JJ'}$. The calculations are formally similar to those in Sec. II A and the final results have a nearly identical structure to those given in Eqs. (18). For clarity, we quote the final formulas for the one-loop vector coupling vertex functions:

$$A_L^{JJ'}(\mathcal{L}) = \frac{\lambda'^{\star}_{iJk}\lambda'_{iJ'k}}{(4\pi)^2}[a(f_L)m_f^2 C_0 + a(f_R)(B_0^{(1)} - 2C_{24} - m_{\tilde f'}^2 C_0) + 2a(\tilde f'_L)\tilde C_{24} + a(d_L)B_1^{(2)}],$$

$$A_R^{JJ'}(\mathcal{R}) = \frac{\lambda'^{\star}_{ijJ'}\lambda'_{ijJ}}{(4\pi)^2}[a(f_R)m_f^2 C_0 + a(f_L)(B_0^{(1)} - 2C_{24} - m_{\tilde f'}^2 C_0) + 2a(\tilde f'_L)\tilde C_{24} + a(d_R)B_1^{(2)}], \tag{21}$$

where the intermediate fermion-sfermion fields are labelled by the indices f, $\tilde f'$. There are implicit sums in Eq. (21) over the above quoted lepton-squark and quark-slepton configurations. The attendant ultraviolet divergences are accompanied again with vanishing factors $a(\tilde d_R) - a(d_L) + a(\nu_R^c) = 0, a(\tilde d_L) - a(d_R) + a(\nu_L) = 0$.

C. Up-quark–antiquark pairs

The production processes of up-quark–antiquark pairs of different families, $l^-(k)+l^+(k')\rightarrow u_J(p)+\overline{u}_{J'}(p')$, may be controlled by the λ'_{ijk} interactions only. The tree amplitude is associated with a u-channel $\tilde d$-squark exchange, of type similar to that shown by (a) in Fig. 1, and can be expressed as

$$M_t^{JJ'} = -\frac{\lambda'^{\star}_{1Jk}\lambda'_{1J'k}}{2(u-m^2_{\tilde d_{kR}})}\overline{v_L}(k')\gamma^\mu u_L(k)\overline{u_L}(p)\gamma_\mu v_L(p') \tag{22}$$

after using the Fierz reordering identity appropriate to commuting Dirac (rather than anticommuting field) spinors,

$$\overline{u^c}(k)P_L v(p')\overline{u(p)}P_R v^c(k') = +\tfrac{1}{2}\overline{v_L}(k')\gamma^\mu u_L(k)\overline{u_L}(p)\gamma_\mu v_L(p').$$

We have omitted the Kronecker symbol δ_{ab} on the $u^a\overline{u_b}$ color indices, which will result in an extra color factor $N_c = 3$ for the rates, as shown explicitly in Eq. (23) below. The present case is formally similar to the leptonic case treated in Sec. II A, except for a chirality flip in the final fermions. We are especially interested here in final states containing a top quark, such as $t\overline{c}$ or $t\overline{u}$, for which external particle mass terms cannot obviously be ignored. The equation, analogous to Eq. (7), which expresses the summations over the initial and final polarizations in the total (tree and loop) amplitude, takes now the form

$$\sum_{pol}|M_t^{JJ'}+M_l^{JJ'}|^2$$
$$=N_c\left[\left|-\frac{\lambda'_{1J'k}\lambda'^{\star}_{1Jk}}{2(u-m^2_{\tilde d_{kR}})}+\left(\frac{g}{2\cos\theta_W}\right)^2\times\frac{a(e_L)A_L^{JJ'}(u,s+i\epsilon)}{s-m_Z^2+im_Z\Gamma_Z}\right|^2 16(k\cdot p')(k'\cdot p)+8m_Jm_{J'}(k\cdot k')\varphi_{LR}(\mathcal{L})\right], \tag{23}$$

where $O(m_e^2)$ terms were ignored and we have denoted

$$\varphi_{LR}(\mathcal{L})=+\left(\frac{g}{2\cos\theta_W}\right)^2\left(\frac{a(e_L)A_R^{JJ'}(i\epsilon)}{s-m_Z^2+im_Z\Gamma_Z}\right)^{\star}\times\left(\frac{\lambda'_{1J'k}\lambda'^{\star}_{1Jk}}{2(u-m^2_{\tilde d_{kR}})}\right)+\text{c.c.} \tag{24}$$

The modified structure for the kinematical factors in the above up-quarks case, Eq. (23), in comparison with the leptons and *d*-quark case, Eq. (7), reflects the difference in chiral structure for the RPV tree level amplitude.

In the massless limit for both the initial and final fermions [where helicity $h=(-1,+1)$ and chirality, $H=(L,R)$ coincide] the RPV interactions contribute to the helicity amplitudes for the process $l^-+l^+\to f_J+\overline{f_{J'}}$ in the mixed-type helicity configurations $h_{l^-}=-h_{l^+}$, $h_{f_J}=-h_{\overline{f}_{J'}}$ (the same as for the *R* parity conserving (RPC) gauge interactions) which are further restricted by the conditions $h_{l^-}=-h_{f_J}$ for lepton and *d*-quark production and $h_{l^-}=h_{f_J}$ for up-quark production. The dependence of the RPV scattering amplitudes on scattering angle has a kinematical factor in the numerator of the form $[1+h_{l^-}h_{f_J}\cos\theta]^2$. [The parts in our formulas in Eqs. (23) and (7), containing the interference terms between RPV and RPC contributions, partially agree with the published results [40,41]. We disagree with [40,41] on the relative signs of RPV and RPC contributions and with [41] on the helicity structure for the up-quark case. Concerning the latter up-quark case, our results concur with those reported in a recent study [43].]

The states in the internal loops of the triangle diagrams occur in two distinct $\mathcal{L}$-type configurations $f=\binom{d_k}{e_i}$, $\tilde f'=\binom{\tilde e^{\star}_{iL}}{\tilde d_{kR}}$. The calculations involved in keeping track of the mass terms are rather tedious. They were performed by means of the MATHEMATICA software package TRACER [62] whose results were checked against those obtained by means of FEYNCALC [63]. The relevant formulas for the vertex functions read

$$A_L^{JJ'}(\mathcal{L})=\frac{\lambda'_{iJ'k}\lambda'^{\star}_{iJk}}{(4\pi)^2}\{a_L(u)\,B_1^{(2)}+a(f_L)\,m_f^2C_0+a(\tilde f')[2\tilde C_{24}+2m_J^2(\tilde C_{12}-\tilde C_{21}+\tilde C_{23}-\tilde C_{11})]+a(f_R)\,[B_0^{(1)}-2\,C_{24}-m_{\tilde f'}^{\,2}C_0+m_J^2(C_0+3C_{11}-2C_{12}+2C_{21}-2C_{23})-m_{J'}^2C_{12}]\},$$

$$A_R^{JJ'}(\mathcal{L})=\frac{\lambda'_{iJ'k}\lambda'^{\star}_{iJk}}{(4\pi)^2}m_Jm_{J'}[2a(\tilde f')\,(-\tilde C_{23}+\tilde C_{22})+a(f_R)\,(-C_{11}+C_{12}-2C_{23}+2C_{22})]. \tag{25}$$

The above formulas include an implicit sum over the two allowed configurations for the internal fermions and sfermions, namely, $a(d_{kH})$, $a(\tilde e^{\star}_{iL})$ and $a(e^c_{iH})$, $a(\tilde d_{kR})$. For completeness, we also display the formula expressing the tensorial covariants

$$\Gamma^Z_\mu(p,p')_{tensorial}=\frac{\lambda'_{iJ'k}\lambda'^{\star}_{iJk}}{(4\pi)^2}i\sigma_{\mu\nu}p^\nu\times\{m_JP_L[a(f_R)(C_{11}-C_{12}+C_{21}-C_{23})-a(\tilde f')(\tilde C_{11}+\tilde C_{21}-\tilde C_{12}-\tilde C_{23})]+m_{J'}P_R[+a(f_R)(C_{22}-C_{23})+a(\tilde f')(\tilde C_{23}-\tilde C_{22})]\}. \tag{26}$$

The complete $Zf_J\overline{f_{J'}}$ vertex function $\Gamma^\mu=\Gamma^\mu_{vectorial}+\Gamma^\mu_{tensorial}$ should (after extracting the external Dirac spinors and the RPV coupling constant factors) be symmetrical under the interchange $J\leftrightarrow J'$ or, more specifically, under the interchange $m_J\leftrightarrow m_{J'}$. This property is not explicit on the expressions in Eqs. (25) and (26), but can be established by reexpressing the Lorentz covariants by means of the Gordon identity. The naive use of Eq. (12) to compute CP-odd asymmetries would seem to yield finite contributions (even in the absence of a CP-odd phase) from the mass terms in the vectorial vertex functions $A_L^{JJ'}$ owing to their lack of symmetry under $m_J\leftrightarrow m_{J'}$. Clearly, this cannot hold true and is an artifact of restricting ourselves to the vectorial couplings. Including the tensorial couplings is necessary for a consistent treatment of the contributions depending on the external fermions masses. Nevertheless, we emphasize that the tensorial vertex contributions will not be included in our numerical results.

Finally, we add a general comment concerning the photon vertex functions $A_{L,R}^{\gamma JJ'}$ and the way to incorporate the γ exchange contributions in the total amplitudes, Eqs. (7) and (23). One needs to add terms obtained by substituting $g/2\cos\theta_W\to e/2=g\sin\theta_W/2$, $a_{L,R}(f)\to 2Q(f)$, $(s-m_Z^2+im_Z\Gamma_Z)^{-1}\to s^{-1}$, along with the substitution of Z bosons by photon vertex functions $A_{L,R}^{JJ'}(\tilde e,s+i\epsilon)\to A_{L,R}^{\gamma JJ'}(\tilde e,s+i\epsilon)$.

The substitution which adds in both Z-boson and photon exchange contributions reads explicitly

$$[a_{R,L}(e)A_{L,R}^{JJ'}]\rightarrow\Bigg[a_{R,L}(e)\sum_f a(f)C_f^Z + 2Q(e)\sin^2\theta_W\cos^2\theta_W[(s-m_Z^2 + im_Z\Gamma_Z)/s]\sum_f 2Q(f)C_f^\gamma\Bigg],$$

where we have used the schematic representation $A_{L,R}^{JJ'} = \Sigma_f a(f)C_f$.

III. BASIC ASSUMPTIONS AND RESULTS

A. General context of flavor changing physics

To place the discussion of the RPV effects in perspective, we briefly review the current situation of flavor changing physics. In the standard model, nondiagonal effects with respect to the quark flavor arise through loop diagrams. The typical structure of one-loop contributions to, say, the $Zf\bar{f}$ vertex function $\Sigma_i V_{iJ}^\star V_{iJ'} I(m_i^{f2}/m_Z^2)$ involves a summation over quark families of CKM matrices factors times a loop integral. This schematic formula shows explicitly how the CKM matrix unitarity, along with the near quark masses degeneracies relative to the Z-boson mass scale (valid for all quarks with the exception of the top quark), strongly suppresses flavor changing effects. Indeed, for the down-quark–antiquark case, the Z-boson decay branching fractions $B_{JJ'}$ were estimated at the values 10^{-7} for $(\bar{b}s+\bar{s}b)$, 10^{-9} for $(\bar{b}d+\bar{d}b)$, 10^{-11} for $(\bar{s}d+\bar{d}s)$, and the corresponding CP asymmetries $\mathcal{A}_{JJ'}$ at the values $[10^{-5},\ 10^{-3},\ 10^{-1}]\sin\delta_{CKM}$ [48,49], respectively.

By contrast, flavor changing effects are expected to attain observable levels in several extensions of the standard model. Thus, one to three orders of magnitudes can be gained on rates $B_{JJ'}$ in models accommodating a fourth quark family [48,49]. For the two-Higgs-doublet extended standard model, a recent comprehensive study of fermion-antifermion pair production at leptonic colliders [51] quotes, for the flavor changing rates, $B_{JJ'}\approx 10^{-6}-10^{-8}$ for $Z\rightarrow(\bar{b}+s)+(\bar{s}+b)$ and $\sigma_{JJ'}\approx(10^{-5}-10^{-6})R$, where $R=\sigma(e^++e^-\rightarrow\mu^++\mu^-)=4\pi\alpha^2/(3s)=86.8/(\sqrt{s})^2\,\mathrm{fb}(\mathrm{TeV})^{-2}$. Large CP violation signals are also found in the reaction $p\bar{p}\rightarrow t\bar{b}X$ in the two-Higgs-doublet and supersymmetric models [52].

For the minimal supersymmetric standard model, due to the expected nearness of superpartner masses to m_Z, flavor changing loop corrections can become threateningly large, unless their contributions are bounded by postulating either a degeneracy of the soft supersymmetry breaking scalars masses parameters or an alignment of the fermion and scalar superpartner current-mass basis transformation matrices. An early calculation of the contribution to Z-boson decay flavor changing rates $Z\rightarrow q_J\bar{q}_{J'}$, induced by radiative corrections from gluino-squark triangle diagrams of squark flavor mixing, found [53] $B_{JJ'}\approx 10^{-5}$. This result is suspect since a more complete calculation of the effect performed subsequently [54] obtained considerably smaller contributions. Both calculations rely on unrealistic inputs, including a wrong mass for the top quark and too low values for the superpartner mass parameters. It is hoped that a complete updated study could be soon performed. In fact, during the last few years, the study of loop corrections in extended versions of the standard model has evolved into a streamlined activity. For instance, calculations of loop contributions to the magnetic moment of the τ lepton or of the heavy quarks, such as those reported in [64] (two-Higgs-doublet model) or in [65] (minimal supersymmetric standard model), could be usefully transposed to the case of fermion pair production observables.

The information from experimental searches on flavor changing physics at high energy colliders is rather meager [66]. Upper bounds for the leptonic Z-boson branching ratios $B_{JJ'}$ are reported [67] at 1.7×10^{-6} for $(\bar{e}\mu+\bar{\mu}e)$, 9.8×10^{-6} for $(\bar{e}\tau+\bar{\tau}e)$, and 1.7×10^{-5} for $(\bar{\mu}\tau+\bar{\tau}\mu)$. No results have been quoted so far for d- or u- quark pair production, reflecting the hard experimental problems faced in identifying quark flavors at high energies. The prospect for experimental measurements at future leptonic colliders is brightest for cases involving one top quark owing to the easier kinematical identification offered by the large mass disparity in the final state jets. For leptonic colliders at energies above those of the CERN e^+e^- collider LEP, the reactions involving the production of Higgs or heavy Z'-gauge bosons which subsequently decay to fermion pairs could be effective sources of flavor nondiagonal effects, especially when a top quark is produced. At still higher energies, in the TeV regime, the production subprocesses involving collisions of gauge boson pairs radiated by the incident leptons, as in $l^-+l^+\rightarrow W^-+W^++\nu+\bar{\nu}$, could lead to flavor nondiagonal final states, such as $\nu+\bar{\nu}+t+\bar{c}$ with rates of order a few fb [68].

B. Choices of parameters and models

Our main assumption in this work is that no other sources besides the R-parity-odd interactions contribute significantly to the flavor changing rates and CP asymmetries. However, to infer useful information from possible future experimental results, we must deal with two main types of uncertainties. The first concerns the family structure of the coupling constants. On this issue, one can only postulate specific hypotheses or make model-dependent statements. At this point, we may note that the experimental indirect upper bounds on single coupling constants are typically $\lambda<0.05$ or $\lambda'<0.05$ times $\tilde{m}/100$ GeV, except for three special cases where strong bounds exist: $\lambda'_{111}<3.9\times10^{-4}(\tilde{m}_q/100\ \mathrm{GeV})^2(\tilde{m}_g/100\ \mathrm{GeV})^{1/2}$ ($0\nu\beta\beta$ decay [22]), $\lambda'_{133}<2\times10^{-3}$ (ν_e mass [21]), and $\lambda'_{imk}<2\times10^{-2}(m_{\tilde{d}_{kR}}/100\ \mathrm{GeV})$ ($i,k=1,2,3;\ m=1,2$)($K\rightarrow\pi\nu\bar{\nu}$ [17]). Strong bounds have been derived for products of coupling constants pairs in specific family configurations. For instance, a valuable source for the λ_{ijk} coupling constants is provided by the rare decays $e_l^- \to e_m^- + e_n^- + e_p^+$ [19], which probe the combinations of coupling constants $F_{abcd} = \Sigma_i (100\ \mathrm{GeV}/m_{\tilde{\nu}_{iL}})^2 \lambda_{iab}\lambda^\star_{icd}$. Except for the strong bound $F^2_{1112} + F^2_{2111} < 4.3\times 10^{-13}$, $(\mu \to 3e)$, the other combinations of coupling constants involving the third generation are less strongly bound, as, for instance, $F^2_{1113} + F^2_{3111} < 3.1\times 10^{-5}$ $(\tau \to 3e)$ [19]. Another useful source is provided by the neutrinoless double beta decay process [23,22,24]. The strongest bounds occur for the following configurations of flavor indices (using the reference value $\tilde{m} = 100$ GeV): $\lambda'_{113}\lambda'_{131} < 7.9\times 10^{-8}$, $\lambda'_{112}\lambda'_{121} < 2.3\times 10^{-6}$, $\lambda'^2_{111} < 4.6\times 10^{-5}$, quoting from [24] where the initial analysis of [23] was updated. Finally, the strongest bounds deduced from neutral mesons ($B\bar{B}$, $K\bar{K}$) mixing parameters are $F'_{1311} < 2\times 10^{-5}$, $F'_{1331} < 3.3\times 10^{-8}$, $F'_{1221} < 4.5\times 10^{-9}$ [19], where $F'_{abcd} = \Sigma_i (100\ \mathrm{GeV}/m_{\tilde{\nu}_{iL}})^2 \lambda'_{iab}\lambda'^\star_{icd}$.

The second type of uncertainties concerns the spectrum of scalar superpartners. At one extreme are the experimental lower bounds, which reach, for sleptons, 40–65 GeV and, for squarks, 90 – 200 GeV, and at the other extreme, the theoretical naturalness requirement which sets an upper bound at 1 TeV.

In order to estimate the uncertainties in predictions emanating from the above two sources, it is necessary to delineate the dependence of amplitudes on sfermion masses. Examining the structure of the relevant contributions to flavor changing rates for, say, the lepton case, we note that the t-channel exchange tree amplitudes are given by a onefold summation over sfermion families, $\Sigma_i |t^i_{JJ'}|/\tilde{m}_i^2$, involving the combination of coupling constants, $t^i_{JJ'} = \lambda_{i1J}\lambda^\star_{i1J'}$. The typical structure for the leptonic loop amplitudes is a twofold summation over fermion and sfermion families, $\Sigma_{ij} l^{ij}_{JJ'} F^{ij}_{JJ'}(s + i\epsilon)$, where $l^{ij}_{JJ'} = \lambda_{ijJ}\lambda^\star_{ijJ'}$, and the loop integrals $F^{ij}_{JJ'}$ have a nontrivial dependence on the fermion and sfermion masses, as exhibited in the formulas derived in Secs. II A, II B, and II C [see, e.g., Eq. (21)].

The effective dependence on the superparticle masses involves ratios $m_f^2/\tilde{m}^2$ or $s/\tilde{m}^2$ in such a way that the dependence is suppressed for large $\tilde{m}$. (Obviously, $s = m_Z^2$ for Z-boson pole observables.) In applications such as ours where $s \geqslant m_Z^2$, all the fermions, with the exception of the top quark, can be regarded as being massless. In particular, the first two light families (for either l, d, u) should have comparable contributions, the third family behaving most distinctly in the top-quark case. A quick analysis, taking the explicit mass factors into account, indicates that loop amplitudes should scale with sfermion masses as $(s/\tilde{m}^2)^n$, with a variable exponent ranging in the interval $1 < n < 2$. Any possible enhancement effect from the explicit sfermion mass factors in Eq. (21) is moderated in the full result by the fact that the accompanying loop integral factor has itself a power decrease with increasing $\tilde{m}^2$. Thus, the Z-boson pole rates should depend on the masses $\tilde{m}$ roughly as $(1/\tilde{m}^2)^{2n}$, while the off Z-boson pole rates, being determined by the tree amplitudes, should behave more nearly as $(1/\tilde{m}^2)^2$. As for the asymmetries, since these are given by ratios of squared amplitudes, one expects them to have a weak sensitivity to the sfermion masses.

To infer the physical implications on the RPV coupling constants, we avoid making too detailed model-dependent assumptions on the scalar superpartner spectrum. Thus, we shall neglect mass splittings between all the sfermions and set uniformly all slepton, sneutrino, and squark masses at a unique family (species) independent value, $\tilde{m}$, chosen to vary in the wide variation interval $100 < \tilde{m} < 1000$ GeV. This prescription should suffice for the kind of semirealistic predictions at which we are aiming. This approximation makes more transparent the dependence on the RPV coupling constants, which then involves the quadratic products designated by $t^i_{JJ'}$ (tree) and $l^{ij}_{JJ'}$ (loop), where the dummy family indices refer to sfermions (tree) and fermions and sfermions (loop). For off Z-boson-pole observables, flavor nondiagonal rates are controlled by products of two different couplings, $|t^i_{JJ'}|^2$, and asymmetries by normalized products of four different couplings, $\mathrm{Im}(t^{i'\star}_{JJ'} l^{ij}_{JJ'})/|t^{i''}_{JJ'}|^2$. For Z-boson pole observables, rates and asymmetries are again controlled by products of two and four different coupling constants, $|l^{ij}_{JJ'}|^2$ and $\mathrm{Im}(l^{i'j'\star}_{JJ'} l^{ij}_{JJ'})/|l^{i''j''}_{JJ'}|^2$, respectively. Let us note that if the off-diagonal rates were dominated by some alternative mechanism, the asymmetries would then involve products of four different coupling constants rather than the above ratio.

It is useful here to set up a catalog of the species and families configurations for the sfermions (tree) or fermions and sfermions (loop) involved in the various cases. In the tree level amplitudes, these configurations are, for leptons, $t^i_{JJ'} = \lambda_{iJ1}\lambda^\star_{iJ'1}$, $\tilde{\nu}_{iL}$ (L type), $t^i_{JJ'} = \lambda_{i1J}\lambda^\star_{i1J'}$, $\tilde{\nu}_{iL}$ ($\mathcal{R}$ type); for d quarks, $t^j_{JJ'} = \lambda'_{1jJ}\lambda'^\star_{1jJ'}$, $\tilde{u}_{jL}$ ($\mathcal{R}$ type); for u quarks, $t^k_{JJ'} = \lambda'^\star_{1Jk}\lambda'_{1J'k}$, $\tilde{d}_{kR}$ ($\mathcal{L}$ type). In the loop level amplitudes, the coupling constants and internal fermion-sfermion configurations are, for leptons,

$$l^{jk}_{JJ'} = \lambda'^\star_{Jjk}\lambda'_{J'jk}, \quad \left[\begin{pmatrix} d_k \\ \tilde{u}^\star_{jL} \end{pmatrix}, \begin{pmatrix} u^c_j \\ \tilde{d}_{kR} \end{pmatrix} \right],$$

$$l^{ik}_{JJ'} = \lambda^\star_{iJk}\lambda_{iJ'k}, \quad \begin{pmatrix} e_k \\ \tilde{\nu}^\star_{iL} \end{pmatrix} \quad (\mathcal{L}\ \text{type}),$$

$$l^{ij}_{JJ'} = \lambda_{ijJ}\lambda^\star_{ijJ'}, \quad \left[\begin{pmatrix} e_j \\ \tilde{\nu}_{iL} \end{pmatrix}, \begin{pmatrix} \nu_i \\ \tilde{e}_{jL} \end{pmatrix} \right] \quad (\mathcal{R}\ \text{type});$$

for d quarks,

$$l^{ik}_{JJ'} = \lambda'^\star_{iJk}\lambda'_{iJ'k}, \quad \left[\begin{pmatrix} d_k \\ \tilde{\nu}^\star_{iL} \end{pmatrix}, \begin{pmatrix} \nu^c_i \\ \tilde{d}_{kR} \end{pmatrix} \right] \quad (\mathcal{L}\ \text{type}),$$

$$l^{ij}_{JJ'} = \lambda'^\star_{ijJ'}\lambda'_{ijJ}, \quad \left[\begin{pmatrix} d_j \\ \tilde{\nu}_{iL} \end{pmatrix}, \begin{pmatrix} u_j \\ \tilde{e}_{iL} \end{pmatrix}; \quad \begin{pmatrix} \nu_i \\ \tilde{d}_{jL} \end{pmatrix}, \begin{pmatrix} e_i \\ \tilde{u}_{jL} \end{pmatrix} \right] \quad (\mathcal{R}\ \text{type});$$

and for u quarks,

$$l^{ik}_{JJ'} = \lambda'^{\star}_{iJk}\lambda'_{iJ'k}\,, \quad \left[\begin{pmatrix} d_k \\ \tilde{e}^{\star}_{iL} \end{pmatrix}, \ \begin{pmatrix} e^c_i \\ \tilde{d}_{kR} \end{pmatrix} \right] \quad (\mathcal{L} \ \text{type}).$$

We shall present numerical results for a subset of the above list of cases. For leptons and d quarks, we shall restrict consideration to the $\mathcal{R}$-type terms which contribute to the Z-boson vertex function A_R. We also retain the slepton-quark internal states for d-quark production (involving $\lambda'^{\star}_{ijJ},\lambda'_{ijJ}$) and the slepton-lepton states for lepton production (involving $\lambda^{\star}_{ijJ'},\lambda_{ijJ}$) For the up-quark production, we consider the $\mathcal{L}$-type terms (involving $\lambda'^{\star}_{iJk}\lambda'_{iJ'k}$) and, for the off Z-boson pole case, omit the term φ_{LR} in Eq. (23) in the numerical results.

Since the running family index in the parameters relevant to tree level amplitudes refers to sfermions, consistently with the approximation of a uniform family-independent mass spectrum, we may as well consider that index as being fixed. Accordingly, we shall set these parameters at the reference value $t^i_{JJ'} = 10^{-2}$. In contrast to the off Z-boson pole rates, the asymmetries depend nontrivially on the fermion mass spectrum through one of the two family indices in $l^{ij}_{JJ'}$(i or j) associated with fermions. To discuss our predictions, rather than going through the list of four distinct coupling constants, we shall make certain general hypotheses regarding the generation dependence of the RPV interactions for the fermionic index. At one extreme is the case where all three generations are treated alike, the other extreme being the case where only one generation dominates. We shall consider three different cases which are distinguished by the interval over which the fermion family indices are allowed to range in the quantities $l^{ij}_{JJ'}$. We define case A by the prescription of equal values for all three families of fermions $(i=1,2,3)$, case B for the second and third families ($i=2,3$), and case C for the third family only ($i=3$). For all three cases, we set the relevant parameters uniformly at the reference values $l^{ij}_{JJ'} = 10^{-2}$. While the results in case C reflect directly on the situation associated with the hypothesis of dominant third family configurations, the corresponding results in situations where the first or second family is assumed dominant can be deduced by taking the differences between the results in cases A and B and cases B and C, respectively.

In order to obtain nonvanishing CP asymmetries, we still need to specify a prescription for introducing a relative CP-odd complex phase, denoted ψ, between the various RPV coupling constants. We shall set this at the reference value $\psi = \pi/2$. Since the CP asymmetries are proportional to the imaginary part of the phase factor the requisite dependence is simply reinstated by inserting the overall factor, $\sin \psi$. Different prescriptions must be implemented depending on whether one considers observables at or off the Z-boson pole. The Z-boson pole asymmetries are controlled by a relative complex phase between the combinations of coupling constants denoted $l^{ij}_{JJ'}$ only. For definiteness, we choose here to assign a nonvanishing complex phase only to the third fermion family, namely, $\arg(l^{ij}_{JJ'}) = [0,0,\pi/2]$, for ($i$ or $j=1,2,3$). In fact, a relative phase between light families only would contribute insignificantly to the Z-boson pole asymmetries, because of the antisymmetry in $\alpha \to \alpha'$ in Eq. (10) and the fact that $F^{JJ'}_{\alpha}(m^2_Z)$ are approximately equal when the fermion index in $\alpha=(i,j)$ belongs to the two first families. The off Z-boson pole asymmetries are controlled by a relative complex phase between the tree and loop level amplitudes. For definiteness, we choose here to assign a vanishing argument to the coupling constants combination $t^i_{JJ'}$ appearing at the tree level and nonvanishing arguments to the full set of loop amplitude combinations, namely, $\arg(l^{ij}_{JJ'}) = \pi/2$, where the fermion index (i or j as the case may be) varies over the ranges relevant to each of the three cases A, B, C.

C. Numerical results and discussion

1. Z-boson decay observables

We start by presenting the numerical results for the integrated rates associated with Z-boson decays into fermion pairs. These are given for the d-quark, lepton, and u-quark cases in Table I. We observe a fast decrease of rates with increasing values of the mass parameter $\tilde{m}$. Our results can be approximately fitted by a power law dependence which is intermediate between $\tilde{m}^{-2}$ and $\tilde{m}^{-3}$. Explicitly, the Z-boson flavor nondiagonal decay rates (for an average of cases A, B, C) are found to scale approximately as $B_{JJ'} \approx (\lambda'_{ijJ}\lambda'_{ijJ'}/0.01)^2 (100\ \text{GeV}/\tilde{m})^2 \times 5 \times 10^{-9}$, for d quarks, $B_{JJ'} \approx (\lambda_{ijJ}\lambda_{ijJ'}/0.01)^2 (100\ \text{GeV}/\tilde{m})^{2.9} \times 2 \times 10^{-9}$, for leptons, and $B_{JJ'} \approx (\lambda'_{iJk}\lambda'_{iJ'k}/0.01)^2 (100\ \text{GeV}/\tilde{m})^{2.8} \times 3 \times 10^{-9}$, for up quarks. When a top-quark intermediate state is allowed in the loop amplitude, this dominates over the contributions from the light families. This is clearly seen on the d-quark results which are somewhat larger than those for up quarks and significantly larger than those for leptons, the more so for larger $\tilde{m}$. This result is explained partly by the color factor and partly by the presence of the top-quark contribution only for the down-quark case. For contributions involving other intermediate states than up quarks, whether the internal fermion generation index in the RPV coupling constants λ_{ijk} runs over all three generations (case A), the second and third generations (case B), or the third generation only (case C), we find that rates get reduced by factors roughly less than 2 in each of these stages. Therefore, this comparison indicates a certain degree of family independence for the Z-boson branching fractions for the cases where either leptons or d quarks propagate inside the loops.

Proceeding next to the CP-odd asymmetries, since these are proportional to ratios of the RPV coupling constants, it follows in our prescription of using uniform values for these that asymmetries must be independent of the specific reference value chosen. As for their dependence on $\tilde{m}$, we see in Table I that this is rather strong and that the sense of variation with increasing $\tilde{m}$ corresponds (for absolute values of $\mathcal{A}_{JJ'}$) to a decrease for d quarks and an increase for u quarks

TABLE I. Flavor changing rates and CP asymmetries for d-quark, lepton, and u-quark pair production in the three cases, appearing in line entries as cases A, B, and C, which correspond to internal lines belonging to all three families, the second and third families, and the third family, respectively. The results for d quarks and leptons, unlike those for up quarks, are obtained in the approximation where one neglects the final fermions masses. The first four column fields (Z pole column entry) show results for the Z-boson pole branching fractions $B_{JJ'}$ and asymmetries $\mathcal{A}_{JJ'}$. The last four column fields (off Z pole column entry) show results for the flavor nondiagonal cross sections $\sigma_{JJ'}$ in fb and for the asymmetries $\mathcal{A}_{JJ'}$ with photon and Z-boson exchanges added in. The results in the two lines for the off Z-boson pole are associated with the two values for the center of mass energy, $s^{1/2}=200$, 500 GeV. The column subentries indicated by $\tilde{m}$ correspond to the sfermion mass parameter $\tilde{m}=100$, 1000 GeV. The notation $d-n$ stands for 10^{-n}.

	Z pole				Off Z pole			
	$\tilde{m}=100$		$\tilde{m}=1000$		$\tilde{m}=100$		$\tilde{m}=1000$	
	$B_{JJ'}$	$\mathcal{A}_{JJ'}$	$B_{JJ'}$	$\mathcal{A}_{JJ'}$	$\sigma_{JJ'}$	$\mathcal{A}_{JJ'}$	$\sigma_{JJ'}$	$\mathcal{A}_{JJ'}$
$d_J\bar{d}_{J'}$								
A	$5.6d-9$	0.38	$4.2d-11$	0.068	11.5	$-5.50d-3$	$1.46d-2$	$-5.72d-3$
					3.62	$-2.90d-3$	$6.90d-2$	$+3.17d-3$
B	$4.68d-9$	0.20	$4.12d-11$	0.034	11.5	$-6.80d-3$	$1.46d-2$	$-7.14d-3$
					3.62	$-3.47d-4$	$6.90d-2$	$+1.32d-3$
C	$3.8d-9$	0.0	$3.98d-11$	0.0	11.5	$-8.11d-3$	$1.46d-2$	$-8.55d-3$
					3.62	$+2.20d-3$	$6.90d-2$	$-5.18d-4$
$l_J^- l_{J'}^+$								
A	$3.2d-9$	$-0.44d-3$	$3.6d-12$	0.049	38.2	$-1.18d-3$	$3.84d-2$	$-1.61d-3$
					4.57	$-6.90d-3$	$3.67d-1$	$-1.04d-3$
B	$1.3d-9$	$-0.55d-3$	$1.5d-12$	$-0.54d-3$	38.2	$-7.90d-4$	$3.84d-2$	$-1.08d-3$
					4.57	$-4.60d-3$	$3.67d-1$	$-6.93d-3$
C	$6.53d-10$	0.0	$7.5d-13$	0.0	38.2	$-3.95d-4$	$3.84d-2$	$-5.38d-4$
					4.57	$-2.30d-3$	$3.67d-1$	$-3.46d-4$
$u\bar{c}$								
A	$6.5d-9$	$-0.69d-3$	$8.9d-12$	-0.12	11.5	$2.63d-3$	$1.46d-2$	$2.96d-3$
					3.62	$1.04d-2$	$6.90d-2$	$6.62d-3$
B	$2.56d-9$	$-0.89d-3$	$3.8d-12$	-0.11	11.5	$1.76d-3$	$1.46d-2$	$1.98d-3$
					3.62	$6.93d-3$	$6.90d-2$	$4.42d-3$
C	$1.26d-9$	0.0	$1.95d-12$	0.0	11.5	$8.90d-4$	$1.46d-2$	$1.00d-3$
					3.62	$3.47d-3$	$6.90d-2$	$2.21d-3$
$t\bar{c}$								
A					5.66	$2.15d-3$	$1.60d-3$	$3.31d-3$
					4.29	$7.02d-3$	$5.95d-2$	$6.56d-3$
B					5.66	$1.43d-3$	$1.60d-3$	$2.22d-3$
					4.29	$4.68d-3$	$5.95d-2$	$4.38d-3$
C					5.66	$7.22d-4$	$1.60d-3$	$1.13d-3$
					4.29	$2.34d-3$	$5.95d-2$	$2.19d-3$

and leptons. The comparison of different production cases shows that the CP asymmetries are largest, $O(10^{-1})$, for d quarks at small $\tilde{m}\simeq 100$ GeV and for u quarks at large $\tilde{m}\simeq 1000$ GeV. For leptons, the asymmetries are systematically small, $O(10^{-3}-10^{-4})$. The above features are explained by the occurrence for d-quark production of an intermediate top-quark contribution and also by the larger values of the rates at large $\tilde{m}$ in this case. The comparison of results in cases A and B indicates that the first two light families give roughly equal contributions in all cases.

For case C, the CP-odd asymmetries are vanishingly small, as expected from our prescription of assigning the CP-odd phase, since case C corresponds then to a situation where only single pairs of coupling constants dominate. Recall that for the specific cases considered in the numerical applications, namely, $\mathcal{R}$ type for d quarks and leptons and $\mathcal{L}$ type for u quarks, the relevant products of RPV coupling constants are $\lambda'^{\star}_{ijJ'}\lambda'_{ijJ}$, $\lambda^{\star}_{ijJ'}\lambda_{ijJ}$, $\lambda'^{\star}_{iJk}\lambda'_{iJ'k}$, respectively, where the fermion generation index among the dummy indices pairs (ij), (ik) refers to the third family. Nonvanishing contributions to $\mathcal{A}_{JJ'}$ could arise in case C if one assumed that two pairs of the above coupling constant products with different sfermion indices dominate, and further requiring that these sfermions be not mass degenerate. Another interesting possibility is by assuming that the hypothesis of a single pair of RPV coupling constants dominance applies for the fields current basis. Applying then to the quark superfields the transformation matrices relating these to mass basis fields, say, in the distinguished choice [17] where the flavor changing effects bear on u quarks, amounts to performing the substitution $\lambda'_{ijk}\to\lambda'^{B}_{ink}V^{\dagger}_{nj}$, where V is the CKM matrix. The CP-odd factor, for the d-quark case, say, acquires then the form $\mathrm{Im}(l^{ij\star}_{JJ'}l^{ij'}_{JJ'})$ $\to|\lambda'^{B}_{inJ'}\lambda'^{B\star}_{imJ}|^2\mathrm{Im}[(V^{\dagger})_{nj}(V^{\dagger})^{\star}_{mj}(V^{\dagger})^{\star}_{nj'}(V^{\dagger})_{mj'}]$, where the second factor on the right-hand side is recognized as the familiar plaquette term, proportional to the products of sines of all the CKM rotation angles times that of the CP-odd phase.

It may be useful to examine the bounds on the RPV coupling constants implied by the current experimental limits on the flavor nondiagonal leptonic widths [67], $B^{\mathrm{expt}}_{JJ'}<[1.7,\ 9.8,\ 17.0]\times10^{-6}$ for the family couples ($JJ'=12$, 23, 13). The contributions associated with the λ interactions can be directly deduced from the results in Table I. Choosing the value $\tilde{m}=100$ GeV and writing our numerical result as $B_{JJ'}\approx(\lambda_{ijJ}\lambda^{\star}_{ijJ'}/0.01)^2\times2\times2\times10^{-9}$, then under the hypothesis of a pair of dominant coupling constants, one deduces $\lambda_{ijJ}\lambda^{\star}_{ijJ'}<[0.46,\ 1.1,\ 1.4]$ for all fixed choices of the family couples i, j. (An extra factor of 2 in $B_{JJ'}$ has been included to account for the antisymmetry property of λ_{ijk} and the fact that two distinct configurations i,j and j,i for the internal lines are present.) For the λ' interactions, stronger bounds obtain because of the extra color factor and of the internal top-quark contributions. A numerical calculation (not reported in Table I) performed with the choice $\tilde{m}=100$ GeV for case C gives us $B_{JJ'}\approx(\lambda^{\star}_{Jjk}\lambda'_{J'jk}/0.01)^2\times1.17\times10^{-7}$, which, by comparison with the experimental limits, yields the bounds $\lambda'^{\star}_{Jjk}\lambda'_{J'jk}<[0.38,\ 0.91,\ 1.2]\times10^{-1}$, for the same family configurations ($J\,J'=12$, 23, 13) as above. These results agree in size to within a factor of 2 with results reported in a recently published work [69]. Finally, we note that the numerical results for the $\mathcal{L}$-type interactions for d quarks are qualitatively similar to those for u quarks. However, for leptons, the loop amplitudes from the λ' interactions pick up an extra color factor N_c which translates into overall factors N_c^2 in $\mathcal{A}_{JJ'}$ and $\mathcal{B}_{JJ'}$.

2. Fermion-antifermion pair production rates

Let us now proceed to the off Z-boson pole observables. The numerical results for the flavor nondiagonal integrated cross sections and CP asymmetries are shown in Table I for two selected values of the center of mass energy, $\sqrt{s}=200$ and 500 GeV. The numerical results displaying the variation of these observables with the center of mass of energy (fixed $\tilde{m}$) and with the superpartner mass parameter (fixed $\sqrt{s}$) are given in Figs. 3 and 4, respectively. All the results presented in this work include both photon and Z-boson exchange contributions. We observe here that the predictions for asymmetries are sensitive to the interference effects between photon and Z-boson exchange contributions.

We discuss first the predictions for flavor nondiagonal rates. We observe a strong decrease with increasing values of $\tilde{m}$ and a slow decrease with increasing values of $\sqrt{s}$. Following a rapid initial rise at threshold, the rates settle at values ranging between 10 and 10^{-1} fb for a wide interval of $\tilde{m}$ values. The dependence on $\tilde{m}$ can be approximately represented as, $\sigma_{JJ'}/[|t^{i}_{JJ'}/0.01|^2(100/\tilde{m})^{2-3}]\approx(1-10)\mathrm{fb}$ $\approx R[\sqrt{s}/(1\ \mathrm{TeV})]^2(10^{-1}-1)$. The rate of decrease of $\sigma_{JJ'}$ with $\tilde{m}$ slows down with increasing s. It is interesting to note that if we had considered here constant values of the product λ $(\tilde{m}/100$ GeV), rather than constant values of λ, the power dependence of rates on $\tilde{m}$ would be such as to lead to interestingly enhanced rates at large $\tilde{m}$.

The marked differences exhibited by the results for lepton pair production, apparent on windows (c) and (d) in Figs. 3 and 4, are due to our deliberate choice of adding the s-channel $\tilde{\nu}$ pole term for the lepton case while omitting it for the d-quark case. The larger rates found for leptons as compared to d quarks, in spite of the extra color factor present for d quarks (recall that the $l^+l^-\to f_J\overline{f}_{J'}$ reactions rates for down quarks and up quarks pick up an extra color factor N_c with respect to those for leptons), are thus explained by the strong enhancement induced by adding in the sneutrino exchange contribution. This choice was made here for illustrative purposes, setting for orientation the relevant coupling constant at the value $\lambda_{1JJ'}=0.1$. The $\tilde{\nu}$ propagator pole was smoothed out by employing the familiar shifted propagator prescription $(s-m^2_{\tilde{\nu}}+im_{\tilde{\nu}}\Gamma_{\tilde{\nu}})^{-1}$, while describing approximately the sneutrino decay width in terms of the RPV contributions alone, namely, $\Gamma(\tilde{\nu}_i\to l^-_k+l^+_j)$ $=\lambda^2_{ijk}\tilde{m}_i/16\pi$ and $\Gamma(\tilde{\nu}_i\to d_k+\overline{d}_j)=N_c\lambda'^2_{ijk}\tilde{m}_i/16\pi$.

Proceeding next to the CP-odd asymmetries, we note that since these scale as a function of the RPV coupling constants as $\mathrm{Im}(l^{ij}_{JJ'}l^{i'j'\star}_{JJ'})/|t^{i''}_{JJ'}|^2$, our present predictions are independent of the uniform reference value assigned to these coupling constants. If the generational dependence of the RPV coupling constants were to exhibit strong hierarchies, this peculiar rational dependence could induce strong suppression or enhancement factors.

The cusps in the dependence of $\mathcal{A}_{JJ'}$ on $\sqrt{s}$ (Fig. 3) occur at values of the center of mass energy where one crosses thresholds for fermion-antifermion (for the energies under

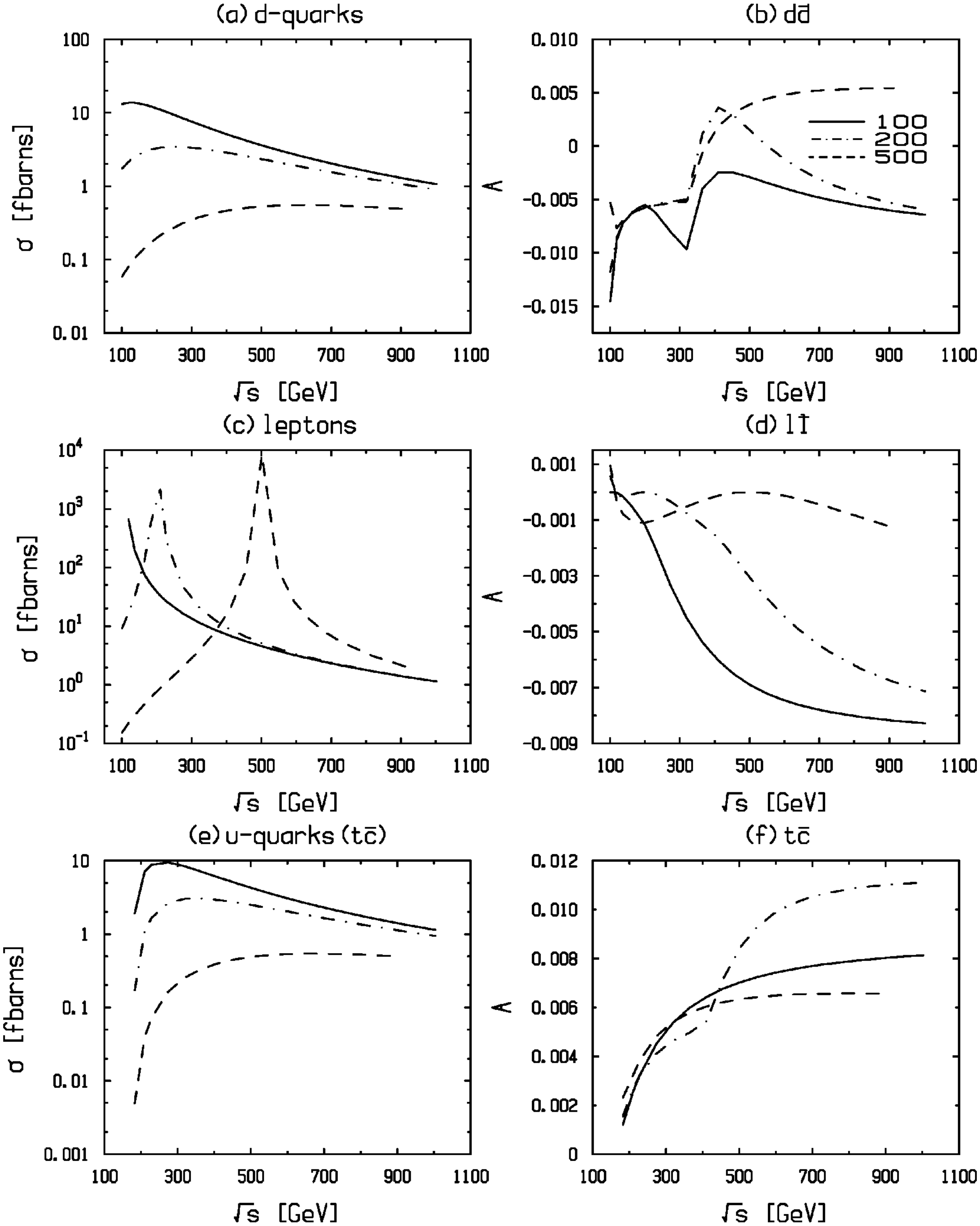

FIG. 3. Integrated flavor nondiagonal cross sections (left-hand side windows) and asymmetries (right-hand side windows) as functions of the center of mass energy in the production of down-quark–antiquark pairs [two upper figures (a) and (b)], lepton-antilepton pairs [two middle figures (c) and (d)], and up-quark–antiquark pairs of type $\bar{t}c+\bar{c}t$ [two lower figures (e) and (f)]. The tree level amplitude includes only the t-channel contribution for the d-quark case, both t- and s-channel exchange contributions for the lepton case, and the u-channel exchange for the up-quark case. The one-loop amplitudes, with both Z-boson and photon exchange contributions, include all three internal fermions generations, corresponding to case A. Three choices for the superpartner uniform mass parameter $\tilde{m}$ are considered: 100 GeV (solid lines), 200 GeV (dash-dotted lines), and 500 GeV (dashed lines).

consideration, $t\bar{t})$ pair production, $\sqrt{s}=2m_f$, and scalar superpartner pair production, $\sqrt{s}=2\tilde{m}$. These are the thresholds for the processes $l^-+l^+\rightarrow f\bar{f}$ or $l^-+l^+\rightarrow \tilde{f}'\tilde{f}'^\star$ at which the associated loop amplitudes acquire finite imaginary parts. Correspondingly, in the dependence of $\mathcal{A}_{JJ'}$ on $\tilde{m}$ (Fig. 4) the cusps appear at $\tilde{m}=\sqrt{s}/2$. We note in the results that the $t\bar{t}$ contributions act to suppress the asymmetries whereas the $\tilde{f}\tilde{f}^\star$ contributions rather act to enhance them. Sufficiently beyond these two-particle thresholds, the asymmetries vary weakly with $\tilde{m}$. A more rapid variation as a function of energy occurs in the lepton production case due to the addition there of the sneutrino pole contribution.

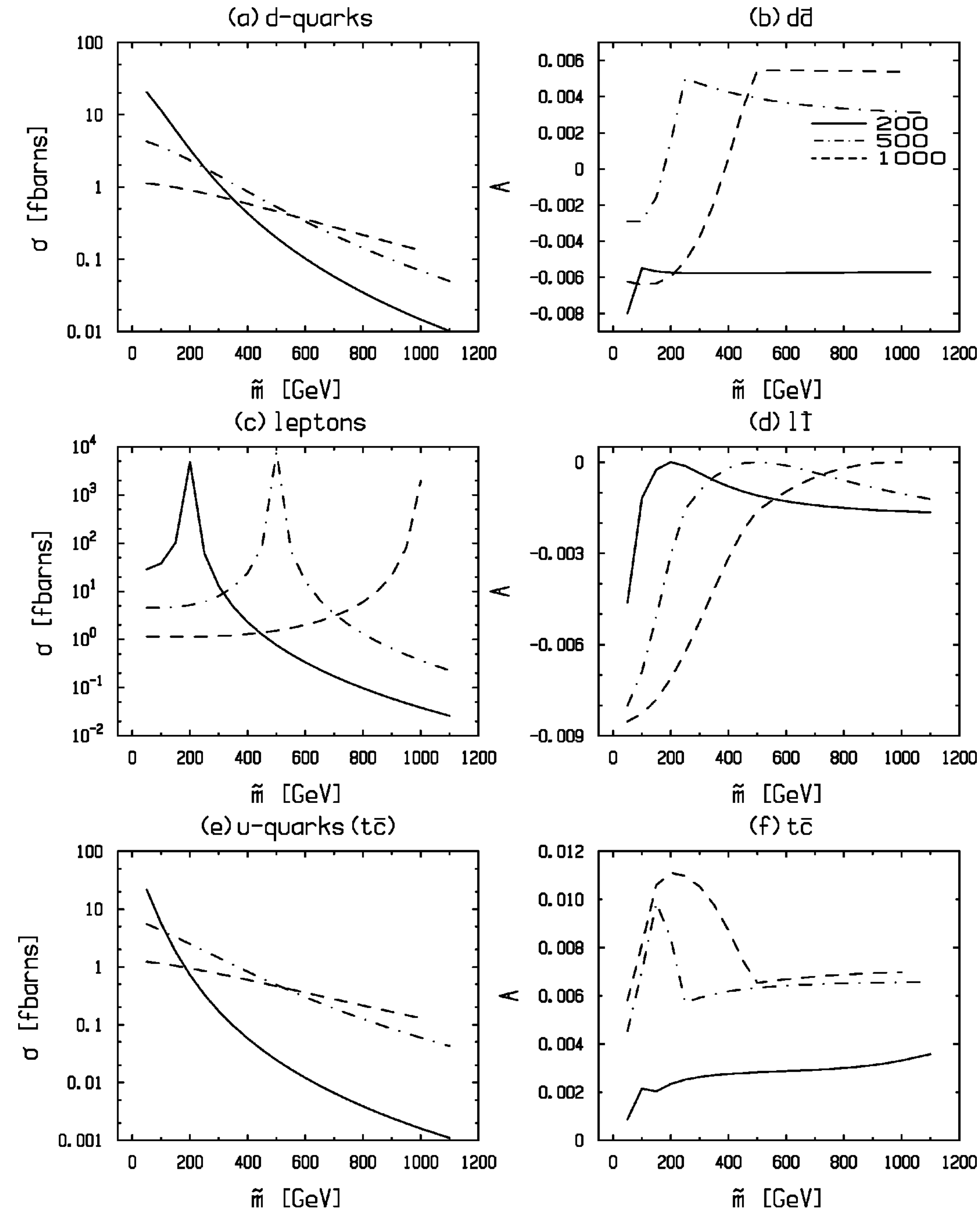

FIG. 4. Integrated flavor nondiagonal cross sections (left-hand side windows) and CP-odd asymmetries (right-hand side windows) as functions of the scalar superpartner mass parameter $\tilde{m}$ in the production of down-quark–antiquark pairs [two upper figures (a) and (b)], lepton-antilepton pairs [two middle figures (c) and (d)], and up-quark–antiquark pairs of type $t\bar{c}$ or $c\bar{t}$ [two lower figures (e) and (f)]. The tree level amplitude includes only the t-channel contribution for the d-quark case, both t- and s-channel exchange contributions for the lepton case, and the u-channel exchange for the up-quark case. The one-loop amplitudes, with both photon and Z-boson exchange contributions, include all three internal fermions generations, corresponding to case A, with three families running inside loops. Three choices for the center of mass energy $s^{1/2}$ are considered: 200 GeV (solid lines), 500 GeV (dash-dotted lines), and 1000 GeV (dashed lines).

The comparison of results for asymmetries in cases A, B, and C reflects on the dependence of loop integrals with respect to the internal fermion masses. An examination of Table I reveals that for leptons and up quarks, where intermediate states involve leptons or d quarks, all three families have nearly equal contributions. The results for down-quark production are enhanced because of the intermediate top-quark contribution, which dominates over that of lighter families. However, this effect is depleted when the finite imaginary part from $t\bar{t}$ sets in. The asymmetries for up-quark production assume values in the range $10^{-2}-10^{-3}$, irrespective of the fact that the final fermions belong to light or heavy families. Finally, we recall again that for the lepton production case, the asymmetries induced by the $\mathcal{L}$-type λ' interactions are enhanced by a color factor N_c.

IV. CONCLUSIONS

The two-body production at high energy leptonic colliders of fermion pairs of different families could provide valuable information on the flavor structure of *R*-parity-odd Yukawa interactions. One can only wish that an experimental identification of lepton and quark flavors at high energies becomes accessible in the future. Although the supersymmetric loop corrections to these processes may not be as strongly suppressed as their standard model counterparts, one expects that the degeneracy or alignment constraints on the scalar superpartner masses and flavor mixing should severely bound their contributions. Systematic studies of the supersymmetry corrections to the flavor changing rates and CP asymmetries in fermion pair production should be strongly encouraged.

An important characteristic of the *R*-parity-odd interactions is that they can contribute to integrated rates at the tree level and to CP asymmetries through interference terms between the tree and loop amplitudes. While we have restricted ourselves to a subset of loop contributions associated with *Z*-boson exchange, a large number of contributions, involving quark-slepton or lepton-squark intermediate states in various families configurations, could still occur. The contributions to rates and asymmetries depend strongly on the values of the *R*-parity-odd coupling constants. Only the rates are directly sensitive to the supersymmetry breaking scale. To circumvent the uncertainties from the sparticle spectrum, we have resorted to the simplifying assumption that the scalar superpartner mass differences and mixings can be neglected. We have set the RPV coupling constants at a uniform value while sampling a set of cases from which one might reconstruct the family dependence of the RPV coupling constants. We have also embedded a CP complex phase in the RPV coupling constants in a specific way, meant to serve mainly as an illustrative example. Although the representative cases that we have considered represent a small fraction of the host of possible variations, they give a fair idea of the sizes to expect. Since these processes cover a wide range of family configurations, one optimitistic possibility could be that one specific entry for the family configurations would enter with a sizable RPV coupling constant.

The contributions to the flavor changing rates have a strong sensitivity to the RPV coupling constants and the superpartner mass, involving high powers of these parameters. We find a generic dependence for the flavor changing *Z*-boson decay branching ratios of form $(\lambda\lambda/0.01)^2(100/\tilde{m})^{2.5}\times 10^{-9}$. For the typical bounds on the RPV coupling constants, it appears that these branchings are three orders of magnitude below the current experimental sensitivity. At higher energies, the flavor changing rates are of order of magnitude $(\lambda\lambda/0.01)^2(100/\tilde{m})^{2-3}(1-10)$ fb. Given the size for the typical integrated luminosity, $\mathcal{L}=50$ fb^{-1}/year, anticipated at the future leptonic machines, one can be moderately optimistic on the observation of clear signals.

The *Z*-boson pole CP-odd asymmetries are of the order of $(10^{-1}-10^{-3})\sin\psi$. For the off *Z*-boson pole reactions, a CP-odd phase ψ embedded in the RPV coupling constants shows up in asymmetries with reduced strength $(10^{-2}-10^{-3})\sin\psi$ for leptons, *d* quarks, and *u* quarks. The largely unknown structure of the RPV coupling constants in flavor space leaves room for good or bad surprises, since the peculiar rational dependence on the coupling constants, $\mathrm{Im}\,(\lambda\lambda^\star\lambda\lambda^\star)/\lambda^4$, and similarly with $\lambda\rightarrow\lambda'$, may lead to strong enhancement or suppression factors.

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[1] C. Aulakh and R. Mohapatra, Phys. Lett. **119B**, 136 (1983).
[2] F. Zwirner, Phys. Lett. **132B**, 103 (1983).
[3] L. Hall and M. Suzuki, Nucl. Phys. **B231**, 419 (1984).
[4] I. H. Lee, Nucl. Phys. **B246**, 120 (1984).
[5] J. Ellis, G. Gelmini, C. Jarlskog, G. G. Ross, and J. W. F. Valle, Phys. Lett. **150B**, 142 (1985).
[6] G. G. Ross and J. W. F. Valle, Phys. Lett. **151B**, 375 (1985).
[7] S. Dawson, Nucl. Phys. **B261**, 297 (1985).
[8] R. Barbieri and A. Masiero, Nucl. Phys. **B267**, 679 (1986).
[9] S. Dimopoulos and L. J. Hall, Phys. Lett. B **207**, 210 (1987).
[10] V. Barger, G.F. Giudice, and T. Han, Phys. Rev. D **40**, 2987 (1989).
[11] H. Dreiner and G. G. Ross, Nucl. Phys. **B365**, 591 (1991).
[12] L. Ibáñez and G. G. Ross, Phys. Lett. B **260**, 291 (1991); Nucl. Phys. **B368**, 3 (1992).
[13] I. Hinchliffe and T. Kaeding, Phys. Rev. D **47**, 279 (1993).
[14] R. Mohapatra, Prog. Part. Nucl. Phys. **31**, 39 (1993).
[15] G. Bhattacharyya, in *Supersymmetry '96 Theoretical Perspectives and Experimental Outlook*, Proceedings of the Conference, College Park, Maryland, edited by R. Mohapatra and A. Rasin [Nucl. Phys. B (Proc. Suppl.) **52A**, 83 (1997)].
[16] H. Dreiner, in *Perspectives on Supersymmetry*, edited by G. Kane (World Scientific, Singapore, 1998), hep-ph/9707435.
[17] K. Agashe and M. Graesser, Phys. Rev. D **54**, 4445 (1996).
[18] J. Ellis, S. Lola, and G. G. Ross, Nucl. Phys. **B526**, 115 (1998).
[19] D. Choudhury and P. Roy, Phys. Lett. B **378**, 153 (1996).
[20] J. E. Kim, P. Ko, and D.-G. Lee, Phys. Rev. D **56**, 100 (1997).
[21] R. M. Godbole, R. P. Roy, and X. Tata, Nucl. Phys. **B401**, 67 (1993).
[22] M. Hirsch, H. V. Klapdor-Kleingrothaus, and S. G. Kovalenko, Phys. Rev. Lett. **75**, 17 (1995); Phys. Rev. D **53**, 1329 (1996); in *Supersymmetry '96 Theoretical Perspectives and Experimental Outlook* [15], p. 257.

[23] K. S. Babu and R. N. Mohapatra, Phys. Rev. Lett. **75**, 2276 (1995).
[24] M. Hirsch and H. V. Klapdor-Kleingrothaus, in *Susy '97* [Nucl. Phys. B (Proc. Suppl.) **62**, 224 (1998)].
[25] A. Yu. Smirnov and F. Vissani, Phys. Lett. B **380**, 317 (1996).
[26] C. E. Carlson, P. Roy, and M. Sher, Phys. Lett. B **357**, 95 (1995).
[27] J. L. Goity and M. Sher, Phys. Lett. B **346**, 69 (1995).
[28] Y. Grossman and M. P. Worah, Phys. Lett. B **395**, 241 (1997).
[29] D. E. Kaplan, ''Violating R parity at the B factory,'' hep-ph/9703347.
[30] S. Abel, Phys. Lett. B **410**, 173 (1997).
[31] D. Guetta, Phys. Rev. D **58**, 116008 (1998).
[32] Ji-Ho Jang and Jae Sik Lee, Report No. KAIST-TH 98/12, hep-ph/9808406.
[33] C. H. Chen, C. Q. Geng, and C. C. Lih, Phys. Rev. D **56**, 6856 (1997).
[34] R. Adhikari and U. Sarkar, Phys. Lett. B **427**, 59 (1998).
[35] S. Bar-Shalom, G. Eilam, and A. Soni, Phys. Rev. Lett. **80**, 4629 (1998).
[36] S. Bar-Shalom, G. Eilam, and A. Soni, Phys. Rev. D **59**, 055012 (1999).
[37] T. Handoko and J. Hashida, Phys. Rev. D **58**, 094008 (1998).
[38] M. Frank and H. Hamidian, J. Phys. G **24**, 2203 (1998).
[39] D. Choudhury, Phys. Lett. B **376**, 201 (1996).
[40] J. Kalinowski, R. Rückl, H. Spiesberger, and P. M. Zerwas, Phys. Lett. B **406**, 314 (1997).
[41] J. Kalinowski, R. Rückl, H. Spiesberger, and P. M. Zerwas, Phys. Lett. B **414**, 297 (1997).
[42] J. Kalinowski, Acta Phys. Pol. B **28**, 2423 (1997).
[43] V. Mahanta and A. Ghosal, Phys. Rev. D **57**, 1735 (1998).
[44] J. Bernabéu and M. B. Gavela, in *CP Violation*, edited by C. Jarlskog (World Scientific, Singapore, 1988); D. Chang, Workshop on physics at current accelerators and supercolliders, Argonne National Laboratory, 1993; A. Soni, Nucl. Phys. B (Proc. Suppl.) **51A**, 32 (1996).
[45] A. Axelrod, Nucl. Phys. **B209**, 349 (1982); E. Ma and A. Pramudita, Phys. Rev. D **22**, 214 (1980).
[46] M. Clements, C. Footman, A. Kronfeld, S. Narasimhan, and D. Fotiadis, Phys. Rev. D **27**, 570 (1983).
[47] V. Ganapathi, T. Weiler, E. Laermann, I. S. Schmidt, and P. M. Zerwas, Phys. Rev. D **27**, 579 (1983).
[48] J. Bernabéu, A. Santamaria, and M. B. Gavela, Phys. Rev. Lett. **57**, 1514 (1986).
[49] W.-S. Hou, N. G. Deshpande, G. Eilam, and A. Soni, Phys. Rev. Lett. **57**, 1406 (1986).
[50] G. Eilam, J. L. Hewett, and A. Soni, Phys. Rev. Lett. **67**, 1979 (1991); B. Grzadkowski, J. F. Gunion, and P. Krawczyk, Phys. Lett. B **268**, 106 (1991).
[51] D. Atwood, L. Reina, and A. Soni, Phys. Rev. D **53**, 1199 (1996); **55**, 3156 (1997).
[52] D. Atwood, S. Bar-Shalom, G. Eilam, and A. Soni, Phys. Rev. D **54**, 5412 (1996).
[53] M. J. Duncan, Phys. Rev. D **31**, 1139 (1985).
[54] B. Mukhopadhyaya and A. Raychaudhuri, Phys. Rev. D **39**, 280 (1989).
[55] G. 't Hooft and M. Veltman, Nucl. Phys. **B153**, 365 (1979).
[56] G. Passarino and M. Veltman, Nucl. Phys. **B160**, 151 (1979).
[57] G. Bhattacharyya, J. Ellis, and K. Sridhar, Mod. Phys. Lett. A **10**, 1583 (1995).
[58] G. Bhattacharyya, D. Choudhury, and K. Sridhar, Phys. Lett. B **355**, 193 (1995).
[59] H. E. Haber and G. L. Kane, Phys. Rep. **117**, 175 (1985).
[60] M. E. Peskin and D. V. Schroeder, *An Introduction to Quantum Field Theory* (Addison-Wesley, Reading, MA, 1995).
[61] J. M. Soares and A. Barroso, Phys. Rev. D **39**, 1973 (1989).
[62] M. Jamin and M. E. Lauterbacher, Comput. Phys. Commun. **74**, 265 (1993).
[63] R. Mertig, M. Böhm, and A. Denner, Comput. Phys. Commun. **64**, 345 (1991).
[64] J. Bernabéu, D. Comelli, L. Lavoura, and J. P. Silva, Phys. Rev. D **53**, 5222 (1996).
[65] W. Hollik, J. I. Illana, S. Rigolin, and D. Stöckinger, Phys. Lett. B **416**, 345 (1998).
[66] OPAL Collaboration, M. Z. Akrawy *et al.*, Phys. Lett. B **254**, 293 (1991); M. Davier, in *Joint International Lepton-Photon Symposium and Europhysics Conference on High Energy Physics*, Geneva, Switzerland, 1991, edited by S. Hegarty, K. Potter, and E. Quercigh (World Scientific, Singapore, 1992).
[67] Particle Data Group, L. Wolfenstein and T. G. Trippe, Eur. Phys. J. C **3**, 62 (1998).
[68] S. Bar-Shalom, G. Eilam, A. Soni, and J. Wudka, Phys. Rev. Lett. **79**, 1217 (1997); W.-S. Hou, G.-L. Lin, and C.-Y. Ma, Phys. Rev. D **56**, 7434 (1997).
[69] M. Anwar Mughal, M. Sadiq, and K. Ahmed, Phys. Lett. B **417**, 87 (1998).