

## Effect of an $RRRR$ dimension 5 operator on proton decay in the minimal $SU(5)$ SUGRA GUT model

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We reanalyze the proton decay in the minimal  $SU(5)$  SUGRA GUT model. Unlike previous analyses, we take into account a Higgsino dressing diagram of a dimension 5 operator with right-handed matter fields ( $RRRR$  operator). It is shown that this diagram gives a dominant contribution for  $p \rightarrow K^+ \bar{\nu}_\tau$  over that from the  $LLLL$  operator, and the decay rate of this mode can be comparable with that of  $p \rightarrow K^+ \bar{\nu}_\mu$  which is dominated by the  $LLLL$  contribution. It is found that we cannot reduce both the decay rate of  $p \rightarrow K^+ \bar{\nu}_\tau$  and that of  $p \rightarrow K^+ \bar{\nu}_\mu$  simultaneously by adjusting relative phases between Yukawa couplings at colored Higgs interactions. Constraints on the colored Higgs boson mass  $M_C$  and a typical squark and slepton mass  $m_{\bar{f}}$  from the super-Kamiokande limit become considerably stronger due to the Higgsino dressing diagram of the  $RRRR$  operator:  $M_C > 6.5 \times 10^{16}$  GeV for  $m_{\bar{f}} < 1$  TeV and  $m_{\bar{f}} > 2.5$  TeV for  $M_C < 2.5 \times 10^{16}$  GeV. [S0556-2821(99)03111-2]

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### I. INTRODUCTION

The gauge coupling unification around  $M_X \sim 2 \times 10^{16}$  GeV [1] strongly suggests supersymmetric (SUSY) grand unified theory (GUT) [2]. In this model, the gauge hierarchy problem is naturally solved by supersymmetry. Also, this model makes successful predictions for the charge quantization and the bottom-tau mass ratio. Proton decay is one of the direct consequences of grand unification. The main decay mode  $p \rightarrow K^+ \bar{\nu}$  [3,4] in the minimal  $SU(5)$  supergravity (SUGRA) GUT model [5] has been searched for with underground experiments [6,7], and the previous results have already imposed severe constraints on this model. Recently new results of the proton decay search at super-Kamiokande have been reported [8]. The bound on the partial lifetime of the  $K^+ \bar{\nu}$  mode is  $\tau(p \rightarrow K^+ \bar{\nu}) > 5.5 \times 10^{32}$  yr (90% C.L.), where three neutrinos are not distinguished.

There are a number of detailed analyses on the nucleon decay in the minimal  $SU(5)$  SUGRA GUT model [9,3,4,10–13]. In previous analyses, it was believed that the contribution from the dimension 5 operator with left-handed matter fields ( $LLLL$  operator) was dominant for  $p \rightarrow K^+ \bar{\nu}$  [4]. In particular a Higgsino dressing diagram of the  $RRRR$  operator has been ignored in these analyses. It has been concluded that the main decay mode is  $p \rightarrow K^+ \bar{\nu}_\mu$  [3], and the decay rate of this mode can be suppressed sufficiently by adjusting relative phases between Yukawa couplings at colored Higgs interactions [10]. Recently it has been pointed out that the Higgsino dressing diagram of the  $RRRR$  operator gives a significant contribution to  $p \rightarrow K^+ \bar{\nu}_\tau$  in a large  $\tan \beta$  region in the context of a SUSY  $SO(10)$  GUT model [14].

In this paper, we reanalyze the proton decay including the  $RRRR$  operator in the minimal  $SU(5)$  SUGRA GUT model. We calculate all the dressing diagrams [10] (exchanging the charginos, the neutralinos, and the gluino) of the  $LLLL$  and  $RRRR$  operators, taking account of various mixing effects among the SUSY particles, such as flavor mixing of quarks

and squarks, left-right mixing of squarks and sleptons, and gaugino-Higgsino mixing of charginos and neutralinos. For this purpose we diagonalize mass matrices numerically to obtain the mixing factors at ‘‘ino’’ vertices and the dimension 5 couplings. We examine the effect of the relative phases between the Yukawa couplings at the colored Higgs interactions. We show that the Higgsino dressing diagram of the  $RRRR$  operator gives a dominant contribution for  $p \rightarrow K^+ \bar{\nu}_\tau$ , and the decay rate of this mode can be comparable with that of  $p \rightarrow K^+ \bar{\nu}_\mu$  which is dominated by the  $LLLL$  contribution. We find that we cannot reduce both the decay rate of  $p \rightarrow K^+ \bar{\nu}_\tau$  and that of  $p \rightarrow K^+ \bar{\nu}_\mu$  simultaneously by adjusting the relative phases. We obtain constraints on the colored Higgs mass and the typical mass scale of squarks and sleptons under the updated super-Kamiokande bound, and find that these constraints are much stronger than those derived from the analysis neglecting the  $RRRR$  effect.

This paper is organized as follows. In Sec. II, we describe the dimension 5 operators in the minimal  $SU(5)$  SUGRA GUT and briefly sketch our scheme to calculate the proton decay rates. We give a qualitative discussion on the  $RRRR$  contribution in Sec. III. We present results of our numerical calculation and discuss constraints on this model in Sec. IV. Formulas used in the calculation of the nucleon decay rates are summarized in the Appendix.

### II. DIMENSION 5 OPERATORS IN THE MINIMAL $SU(5)$ SUGRA GUT

Nucleon decay in the minimal  $SU(5)$  SUGRA GUT model is dominantly caused by dimension 5 operators [9], which are generated by the exchange of the colored Higgs multiplet. The dimension 5 operators relevant to the nucleon decay are described by the following superpotential:

$$W_5 = -\frac{1}{M_C} \left\{ \frac{1}{2} C_{5L}^{ijkl} Q_k Q_l Q_i L_j + C_{5R}^{ijkl} E_k^c U_l^c U_i^c D_j^c \right\}. \quad (1)$$

Here  $Q, U^c$ , and  $E^c$  are chiral superfields which contain a left-handed quark doublet, a charge conjugation of a right-handed up-type quark, and a charge conjugation of a right-handed charged lepton, respectively, and are embedded in the 10 representation of SU(5). The chiral superfields  $L$  and  $D^c$  contain a left-handed lepton doublet and a charge conjugation of a right-handed down-type quark, respectively, and are embedded in the  $\bar{5}$  representation. A mass of the colored Higgs superfields is denoted by  $M_C$ . The indices  $i, j, k, l = 1, 2, 3$  are generation labels. The first term in Eq. (1) represents the  $LLLL$  operator [4] which contains only left-handed SU(2) doublets. The second term in Eq. (1) represents the  $RRRR$  operator which contains only right-handed SU(2) singlets. The coefficients  $C_{5L}$  and  $C_{5R}$  in Eq. (1) are determined by Yukawa coupling matrices [10]. Approximately these are written as

$$\begin{aligned} C_{5L}^{ijkl}|_X &\approx (Y_D)_{ij}(V^T P Y_U V)_{kl}, \\ C_{5R}^{ijkl}|_X &\approx (P^* V^* Y_D)_{ij}(V^T Y_U)_{kl}, \end{aligned} \quad (2)$$

where  $Y_U$  and  $Y_D$  are diagonalized Yukawa coupling matrices for  $10 \times 10 \times 5_H$  and  $10 \times \bar{5} \times \bar{5}_H$  interactions, respectively. More precise expressions for  $C_{5L}$  and  $C_{5R}$  are given in the Appendix. The unitary matrix  $V$  is the Cabibbo-Kobayashi-Maskawa (CKM) matrix at the GUT scale. The matrix  $P = \text{diag}(P_1, P_2, P_3)$  is a diagonal unimodular phase matrix with  $|P_i| = 1$  and  $\det P = 1$ . We parametrize  $P$  as

$$P_1/P_3 = e^{i\phi_{13}}, \quad P_2/P_3 = e^{i\phi_{23}}. \quad (3)$$

The parameters  $\phi_{13}$  and  $\phi_{23}$  are relative phases between the Yukawa couplings at the colored Higgs interactions, and cannot be removed by field redefinitions [15]. The expressions for  $C_{5L}$  and  $C_{5R}$  in Eq. (2) are written in the flavor basis where the Yukawa coupling matrix for the  $10 \times \bar{5} \times \bar{5}_H$  interaction is diagonalized at the GUT scale. Numerical values of  $Y_U, Y_D$ , and  $V$  at the GUT scale are calculated from the quark masses and the CKM matrix at the electroweak scale using renormalization group equations (RGEs).

In the minimal SU(5) SUGRA GUT, soft SUSY breaking parameters at the Planck scale are described by  $m_0$ ,  $M_{gX}$ , and  $A_X$  which denote universal scalar mass, universal gaugino mass, and universal coefficient of the trilinear scalar couplings, respectively. Low energy values of the soft breaking parameters are determined by solving the one-loop RGEs [16]. The electroweak symmetry is broken radiatively [17] due to the effect of a large Yukawa coupling of the top quark, and we require that the correct vacuum expectation values of the Higgs fields at the electroweak scale are reproduced. We ignore RGE running effects between the Planck scale and the GUT scale for simplicity. In this approximation the phase matrix  $P$  decouples from the RGEs of the soft SUSY breaking parameters. Thus we have all the values of the parameters at the electroweak scale. The masses and the mixings are obtained by diagonalizing the mass matrices numerically. We evaluate hadronic matrix elements using the chiral Lagrangian method [18]. The parameters  $\alpha_p$  and  $\beta_p$

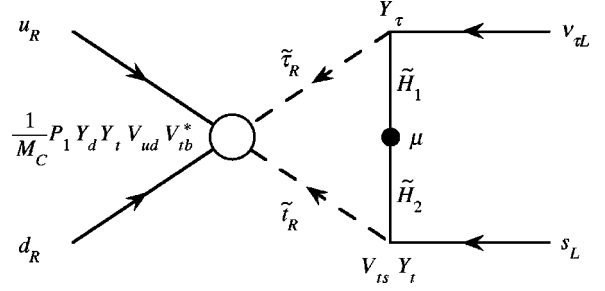


FIG. 1. Higgsino dressing diagram which gives a dominant contribution to the  $p \rightarrow K^+ \bar{\nu}_\tau$  mode. The circle represents the  $RRRR$  dimension 5 operator. We also have a similar diagram for  $(u_R s_R) \times (d_L \nu_{\tau L})$ .

defined by  $\langle 0 | \epsilon_{\hat{a}\hat{b}\hat{c}} (d_R^{\hat{a}} u_R^{\hat{b}}) u_L^{\hat{c}} | p \rangle = \alpha_p N_L$  and  $\langle 0 | \epsilon_{\hat{a}\hat{b}\hat{c}} (d_L^{\hat{a}} u_L^{\hat{b}}) u_L^{\hat{c}} | p \rangle = \beta_p N_L$  ( $N_L$  is a left-handed proton's wave function) are evaluated as  $0.003 \text{ GeV}^3 \leq \beta_p \leq 0.03 \text{ GeV}^3$  and  $\alpha_p = -\beta_p$  by various methods [19]. We use the smallest value  $\beta_p = -\alpha_p = 0.003 \text{ GeV}^3$  in our analysis to obtain conservative bounds. For the details of the methods of our analysis, see Refs. [13,14]. Formulas for relevant interactions and the nucleon decay rates are given in the Appendix.

### III. $RRRR$ CONTRIBUTION TO THE PROTON DECAY

The dimension 5 operators consist of two fermions and two bosons. Eliminating the two scalar bosons by gaugino or Higgsino exchange (dressing), we obtain the four-fermion interactions which cause the nucleon decay [4,10]. In the one-loop calculations of the dressing diagrams, we include all the dressing diagrams exchanging the charginos, the neutralinos, and the gluino of the  $LLLL$  and  $RRRR$  dimension 5 operators. In addition to the contributions from the dimension 5 operators, we include the contributions from dimension 6 operators mediated by the heavy gauge boson and the colored Higgs boson. Though the effects of the dimension 6 operators ( $\sim 1/M_X^2$ ) are negligibly small for  $p \rightarrow K^+ \bar{\nu}$ , these could be important for other decay modes such as  $p \rightarrow \pi^0 e^+$ . The major contribution of the  $LLLL$  operator comes from an ordinary diagram with wino dressing. The major contribution of the  $RRRR$  operator arises from a Higgsino dressing diagram depicted in Fig. 1. The circle in this figure represents the complex conjugation of  $C_{5R}^{ijkl}$  in Eq. (2) with  $i=j=1$  and  $k=l=3$ . This diagram contains the Yukawa couplings of the top quark and the tau lepton. The importance of this diagram has already been pointed out in Ref. [14] in the context of a SUSY SO(10) GUT model. This diagram has been ignored in previous analyses in the minimal SU(5) SUSY GUT [9,3,4,10–13], though the contributions from gaugino dressing of the  $RRRR$  operator were included in Ref. [10]. We show that this diagram indeed gives a significant contribution in the case of the minimal SU(5) SUGRA GUT model also.

Before we present the results of our numerical calculations, we give a rough estimation for the decay amplitudes for a qualitative understanding of the results. In the actual

calculations, however, we make full numerical analyses including contributions from all the dressing diagrams as well as those from dimension 6 operators. We also take account of various effects such as mixings between the SUSY particles. In addition to the soft breaking parameter dependence arising from the loop calculations, relative magnitudes between various contributions can be roughly understood by the form of the dimension 5 operator in Eq. (2). Counting the CKM suppression factors and the Yukawa coupling factors, it is easily shown that the  $RRRR$  contribution to the four-fermion operators  $(u_R d_R)(s_L \nu_{\tau L})$  and  $(u_R s_R)(d_L \nu_{\tau L})$  is dominated by a single (Higgsino dressing) diagram exchanging  $\tilde{t}_R$  (the right-handed scalar top quark) and  $\tilde{\tau}_R$  (the right-handed scalar tau lepton). For  $K^+ \bar{\nu}_\mu$  and  $K^+ \bar{\nu}_e$ , the  $RRRR$  contribution is negligible, since it is impossible to get a large Yukawa coupling of the third generation without small CKM suppression factors in this case. The  $LLLL$  contribution to  $(u_L d_L)(s_L \nu_{iL})$  and  $(u_L s_L)(d_L \nu_{iL})$  consists of two classes of (wino dressing) diagrams; they are  $\tilde{c}_L$  exchange diagrams and  $\tilde{t}_L$  exchange diagrams [10]. Neglecting all of various subleading effects, we can write the amplitudes (the coefficients of the four-fermion operators) for  $p \rightarrow K^+ \bar{\nu}_i$  as

$$\begin{aligned} \text{Amp}(p \rightarrow K^+ \bar{\nu}_e) &\sim [P_2 A_e(\tilde{c}_L) + P_3 A_e(\tilde{t}_L)]_{LLLL}, \\ \text{Amp}(p \rightarrow K^+ \bar{\nu}_\mu) &\sim [P_2 A_\mu(\tilde{c}_L) + P_3 A_\mu(\tilde{t}_L)]_{LLLL}, \\ \text{Amp}(p \rightarrow K^+ \bar{\nu}_\tau) &\sim [P_2 A_\tau(\tilde{c}_L) + P_3 A_\tau(\tilde{t}_L)]_{LLLL} \\ &\quad + [P_1 A_\tau(\tilde{t}_R)]_{RRRR}, \end{aligned} \quad (4)$$

where the subscript  $LLLL$  ( $RRRR$ ) represents the contribution from the  $LLLL$  ( $RRRR$ ) operator. We estimate  $A_i$  by only the  $(ud)(s\nu)$  type contributions here for simplicity, ignoring the  $(us)(d\nu)$  type contributions. The  $LLLL$  contributions for  $A_\tau$  can be written in a rough approximation as  $A_\tau(\tilde{c}_L) \sim g_2^2 Y_c Y_b V_{ub}^* V_{cd} V_{cs} M_2 / (M_C m_{\tilde{f}}^2)$  and  $A_\tau(\tilde{t}_L) \sim g_2^2 Y_t Y_b V_{ub}^* V_{td} V_{ts} M_2 / (M_C m_{\tilde{f}}^2)$ , where  $g_2$  is the weak SU(2) gauge coupling, and  $M_2$  is a mass of the wino. A typical mass scale of the squarks and the sleptons is denoted by  $m_{\tilde{f}}$ . For  $A_\mu$  and  $A_e$ , we just replace  $Y_b V_{ub}^*$  in the expressions for  $A_\tau$  by  $Y_s V_{us}^*$  and  $Y_d V_{ud}^*$ , respectively. The  $RRRR$  contribution is also evaluated as  $A_\tau(\tilde{t}_R) \sim Y_d Y_t^2 Y_\tau V_{ib}^* V_{ud} V_{ts} \mu / (M_C m_{\tilde{f}}^2)$ , where  $\mu$  is the supersymmetric Higgsino mass. The magnitude of  $\mu$  is determined from the radiative electroweak symmetry breaking condition, and satisfies  $|\mu| > |M_2|$  in the present scenario.

Relative magnitudes between these contributions are evaluated as follows. The magnitude of the  $\tilde{c}_L$  contribution is comparable with that of the  $\tilde{t}_L$  contribution for each generation mode  $|A_i(\tilde{c}_L)| \sim |A_i(\tilde{t}_L)|$ . Therefore, cancellations between the  $LLLL$  contributions  $P_2 A_i(\tilde{c}_L)$  and  $P_3 A_i(\tilde{t}_L)$  can occur simultaneously for three modes  $p \rightarrow K^+ \bar{\nu}_i$  ( $i=e, \mu$  and  $\tau$ ) by adjusting the relative phase  $\phi_{23}$  between  $P_2$  and  $P_3$  [10]. The magnitudes of the  $LLLL$  contributions

satisfy  $|P_2 A_\mu(\tilde{c}_L) + P_3 A_\mu(\tilde{t}_L)| > |P_2 A_\tau(\tilde{c}_L) + P_3 A_\tau(\tilde{t}_L)| > |P_2 A_e(\tilde{c}_L) + P_3 A_e(\tilde{t}_L)|$  independent of  $\phi_{23}$ . On the other hand, the magnitude of  $A_\tau(\tilde{t}_R)$  is larger than those of  $A_i(\tilde{c}_L)$  and  $A_i(\tilde{t}_L)$ , and the phase dependence of  $P_1 A_\tau(\tilde{t}_R)$  is different from those of  $P_2 A_i(\tilde{c}_L)$  and  $P_3 A_i(\tilde{t}_L)$ . Note that  $A_i(\tilde{c}_L)$  and  $A_i(\tilde{t}_L)$  are proportional to  $\sim 1/(\sin \beta \cos \beta) = \tan \beta + 1/\tan \beta$ , while  $A_\tau(\tilde{t}_R)$  is proportional to  $\sim (\tan \beta + 1/\tan \beta)^2$ , where  $\tan \beta$  is the ratio of the vacuum expectation values of the two Higgs bosons. Hence the  $RRRR$  contribution is more enhanced than the  $LLLL$  contributions for large  $\tan \beta$  [14].

#### IV. NUMERICAL RESULTS

Now we present the results of our numerical calculations. For the CKM matrix we adopt the standard parametrization [20], and we fix the parameters as  $V_{us} = 0.2196$ ,  $V_{cb} = 0.0395$ ,  $|V_{ub}/V_{cb}| = 0.08$ , and  $\delta_{13} = 90^\circ$  in the whole analysis, where  $\delta_{13}$  is a complex phase in the CKM matrix. The top quark mass is taken to be 175 GeV [21]. The colored Higgs mass  $M_C$  and the heavy gauge boson mass  $M_V$  are assumed as  $M_C = M_V = 2 \times 10^{16}$  GeV. We require constraint on the  $b \rightarrow s \gamma$  branching ratio from CLEO [22] and bounds on SUSY particle masses obtained from direct searches at the CERN  $e^+ e^-$  collider LEP [23], LEP II [24], and Fermilab Tevatron [25]. We also impose condition to avoid color and charge breaking vacua which is given in Ref. [26] at the electroweak scale.

We mainly discuss the main decay mode  $p \rightarrow K^+ \bar{\nu}$  in this paper. We first discuss the effects of the phases  $\phi_{13}$  and  $\phi_{23}$  parametrizing the matrix  $P$  in Eq. (3). In Fig. 2 we present the dependence of the decay rates  $\Gamma(p \rightarrow K^+ \bar{\nu}_i)$  on the phase  $\phi_{23}$ . As an illustration we fix the other phase  $\phi_{13}$  at  $210^\circ$ , and later we consider the whole parameter space of  $\phi_{13}$  and  $\phi_{23}$ . The soft SUSY breaking parameters are also fixed as  $m_0 = 1$  TeV,  $M_{gX} = 125$  GeV, and  $A_X = 0$  here. The sign of the Higgsino mass  $\mu$  is taken to be positive. With these parameters, all the masses of the scalar fermions other than the lighter  $\tilde{t}$  are around 1 TeV, and the mass of the lighter  $\tilde{t}$  is about 400 GeV. The lighter chargino is wino-like with a mass about 100 GeV. This figure shows that there is no region for the total decay rate  $\Gamma(p \rightarrow K^+ \bar{\nu})$  to be strongly suppressed, thus the whole region of  $\phi_{23}$  in Fig. 2 is excluded by the super-Kamiokande limit. The phase dependence of  $\Gamma(p \rightarrow K^+ \bar{\nu}_\tau)$  is quite different from those of  $\Gamma(p \rightarrow K^+ \bar{\nu}_\mu)$  and  $\Gamma(p \rightarrow K^+ \bar{\nu}_e)$ . Though  $\Gamma(p \rightarrow K^+ \bar{\nu}_\mu)$  and  $\Gamma(p \rightarrow K^+ \bar{\nu}_e)$  are highly suppressed around  $\phi_{23} \sim 160^\circ$ ,  $\Gamma(p \rightarrow K^+ \bar{\nu}_\tau)$  is not so in this region. There exists also the region  $\phi_{23} \sim 300^\circ$  where  $\Gamma(p \rightarrow K^+ \bar{\nu}_\tau)$  is reduced. In this region, however,  $\Gamma(p \rightarrow K^+ \bar{\nu}_\mu)$  and  $\Gamma(p \rightarrow K^+ \bar{\nu}_e)$  are not suppressed in turn. Note also that the  $K^+ \bar{\nu}_\tau$  mode can give the largest contribution.

This behavior can be understood as follows. For  $\bar{\nu}_\mu$  and  $\bar{\nu}_e$ , the effect of the  $RRRR$  operator is negligible, and the cancellation between the  $LLLL$  contributions directly leads

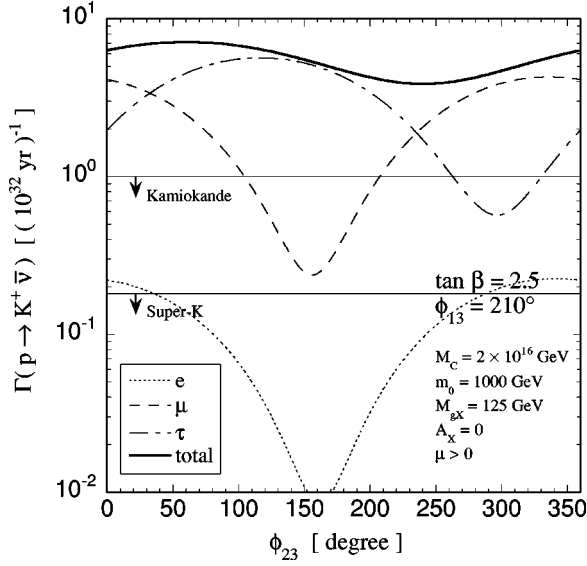


FIG. 2. Decay rates  $\Gamma(p \rightarrow K^+ \bar{\nu}_i)$  ( $i = e, \mu$ , and  $\tau$ ) as functions of the phase  $\phi_{23}$  for  $\tan \beta = 2.5$ . The other phase  $\phi_{13}$  is fixed at  $210^\circ$ . The CKM phase is taken as  $\delta_{13} = 90^\circ$ . We fix the soft SUSY breaking parameters as  $m_0 = 1$  TeV,  $M_{gX} = 125$  GeV, and  $A_X = 0$ . The sign of the supersymmetric Higgsino mass  $\mu$  is taken to be positive. The colored Higgs mass  $M_C$  and the heavy gauge boson mass  $M_V$  are assumed as  $M_C = M_V = 2 \times 10^{16}$  GeV. The horizontal lower line corresponds to the super-Kamiokande limit  $\tau(p \rightarrow K^+ \bar{\nu}) > 5.5 \times 10^{32}$  yr, and the horizontal upper line corresponds to the Kamiokande limit  $\tau(p \rightarrow K^+ \bar{\nu}) > 1.0 \times 10^{32}$  yr.

to the suppression of the decay rates. This cancellation indeed occurs around  $\phi_{23} \sim 160^\circ$  for both  $\bar{\nu}_\mu$  and  $\bar{\nu}_e$  simultaneously in Fig. 2. For  $\bar{\nu}_\tau$ , the situation is quite different. The similar cancellation between  $P_2 A_\tau(\tilde{c}_L)$  and  $P_3 A_\tau(\tilde{t}_L)$  takes place around  $\phi_{23} \sim 160^\circ$  for  $\bar{\nu}_\tau$  also. However, the  $RRRR$  operator gives a significant contribution for  $\bar{\nu}_\tau$ . Therefore,  $\Gamma(p \rightarrow K^+ \bar{\nu}_\tau)$  is not suppressed by the cancellation between the  $LLLL$  contributions in the presence of the large  $RRRR$  operator effect. Notice that it is possible to reduce  $\Gamma(p \rightarrow K^+ \bar{\nu}_\tau)$  by another cancellation between the  $LLLL$  contributions and the  $RRRR$  contribution. This reduction of  $\Gamma(p \rightarrow K^+ \bar{\nu}_\tau)$  indeed appears around  $\phi_{23} \sim 300^\circ$  in Fig. 2. The decay rate  $\Gamma(p \rightarrow K^+ \bar{\nu}_\mu)$  is rather large in this region. The reason is that  $P_2 A_\tau(\tilde{c}_L)$  and  $P_3 A_\tau(\tilde{t}_L)$  are constructive in this region in order to cooperate with each other to cancel the large  $RRRR$  contribution  $P_1 A_\tau(\tilde{t}_R)$ , hence  $P_2 A_\mu(\tilde{c}_L)$  and  $P_3 A_\mu(\tilde{t}_L)$  are also constructive in this region. Thus we cannot reduce both  $\Gamma(p \rightarrow K^+ \bar{\nu}_\tau)$  and  $\Gamma(p \rightarrow K^+ \bar{\nu}_\mu)$  simultaneously. Consequently, there is no region for the total decay rate  $\Gamma(p \rightarrow K^+ \bar{\nu})$  to be strongly suppressed. In the previous analysis [12] the region  $\phi_{23} \sim 160^\circ$  has been considered to be allowed by the Kamiokande limit  $\tau(p \rightarrow K^+ \bar{\nu}) > 1.0 \times 10^{32}$  yr (90% C.L.) [6]. However, the inclusion of the Higgsino dressing of the  $RRRR$  operator excludes this region. In Fig. 3 we show a contour plot of the partial lifetime  $\tau(p \rightarrow K^+ \bar{\nu})$  in the  $\phi_{13}$ - $\phi_{23}$  plane. It is found that there is no

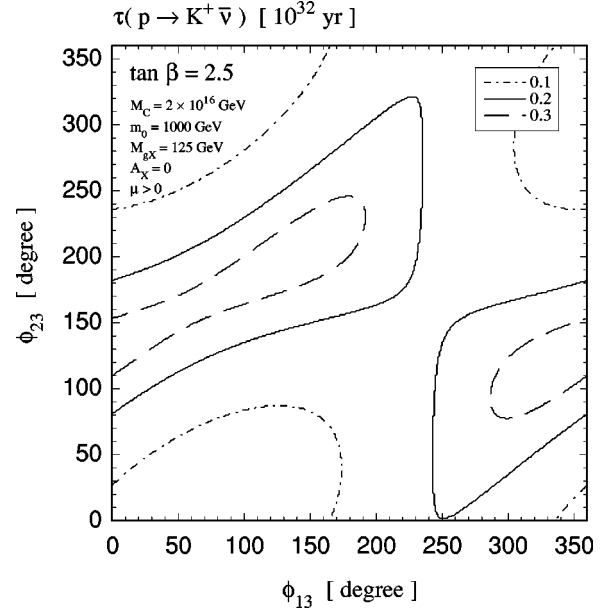


FIG. 3. Contour plot for the partial lifetime  $\tau(p \rightarrow K^+ \bar{\nu})$  in the  $\phi_{13}$ - $\phi_{23}$  plane. The contributions of three modes  $K^+ \bar{\nu}_e$ ,  $K^+ \bar{\nu}_\mu$ , and  $K^+ \bar{\nu}_\tau$  are included. We use the same parameters as in Fig. 2. The maximum value of the contour is less than  $0.5 \times 10^{32}$  yr.

region to make  $\tau(p \rightarrow K^+ \bar{\nu})$  longer than  $0.5 \times 10^{32}$  yr. This implies that we cannot reduce both  $\Gamma(p \rightarrow K^+ \bar{\nu}_\tau)$  and  $\Gamma(p \rightarrow K^+ \bar{\nu}_\mu)$  simultaneously, even if we adjust the two phases  $\phi_{13}$  and  $\phi_{23}$  anywhere. Consequently, the whole parameter region in this plane is excluded by the super-Kamiokande result.

Next we would like to consider the case where we vary the parameters we have fixed so far. The relevant parameters are the colored Higgs boson mass  $M_C$ , the soft SUSY breaking parameters, and  $\tan \beta$ . As for the constants  $\alpha_p$  and  $\beta_p$  in the hadronic matrix elements, we have chosen the smallest value [19]. Hence other choices of these constants lead to enhancement of the proton decay rate which corresponds to severer constraints on this model. The partial lifetime  $\tau(p \rightarrow K^+ \bar{\nu})$  is proportional to  $M_C^2$  in a very good approximation, since this mode is dominated by the dimension 5 operators. Using this fact and the calculated value of  $\tau(p \rightarrow K^+ \bar{\nu})$  for the fixed  $M_C = 2 \times 10^{16}$  GeV, we can obtain the lower bound on  $M_C$  from the experimental lower limit on  $\tau(p \rightarrow K^+ \bar{\nu})$ . In Fig. 4, we present the lower bound on  $M_C$  obtained from the super-Kamiokande limit as a function of the left-handed scalar up-quark mass  $m_{u_L}^-$ . Masses of the scalar fermions other than the lighter  $\tilde{t}$  are almost degenerate with  $m_{u_L}^-$ . The soft breaking parameters  $m_0$ ,  $M_{gX}$ , and  $A_X$  are scanned within the range of  $0 < m_0 < 3$  TeV,  $0 < M_{gX} < 1$  TeV, and  $-5 < A_X < 5$ , and  $\tan \beta$  is fixed at 2.5. Both signs of  $\mu$  are considered. The whole parameter region of the two phases  $\phi_{13}$  and  $\phi_{23}$  is examined. The solid curve in this figure represents the result with all the  $LLLL$  and  $RRRR$  contributions. It is shown that the lower bound on  $M_C$  decreases as  $\sim 1/m_{u_L}^-$  as  $m_{u_L}^-$  increases. This indicates that the

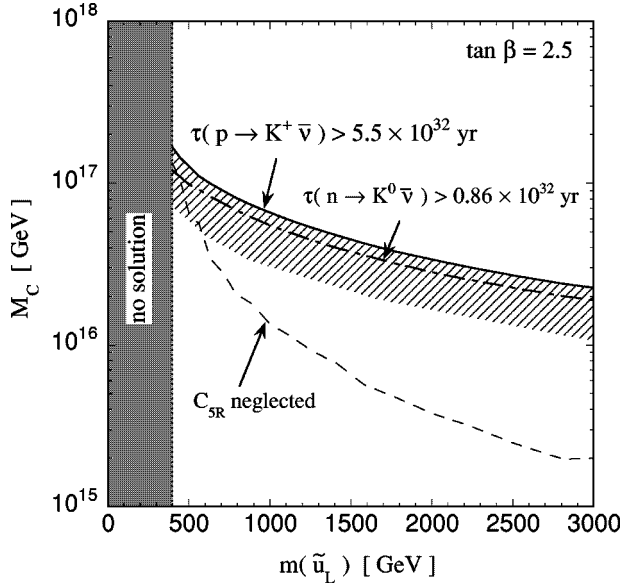


FIG. 4. Lower bound on the colored Higgs mass  $M_C$  as a function of the left-handed scalar up-quark mass  $m_{\tilde{u}_L}$ . The soft breaking parameters  $m_0$ ,  $M_{gX}$ , and  $A_X$  are scanned within the range of  $0 < m_0 < 3$  TeV,  $0 < M_{gX} < 1$  TeV, and  $-5 < A_X < 5$ , and  $\tan \beta$  is fixed at 2.5. Both signs of  $\mu$  are considered. The whole parameter region of the two phases  $\phi_{13}$  and  $\phi_{23}$  is examined. The solid curve represents the bound derived from the super-Kamiokande limit  $\tau(p \rightarrow K^+ \bar{\nu}) > 5.5 \times 10^{32}$  yr, and the dashed curve represents the corresponding result without the  $RRRR$  effect. The left-hand side of the vertical dotted line is excluded by other experimental constraints. The dash-dotted curve represents the bound derived from the Kamiokande limit on the neutron partial lifetime  $\tau(n \rightarrow K^0 \bar{\nu}) > 0.86 \times 10^{32}$  yr.

$RRRR$  effect is indeed relevant, since the decay amplitude from the  $RRRR$  operator is roughly proportional to  $\mu/(M_C m_{\tilde{f}}^2) \sim 1/(M_C m_{\tilde{f}})$ , where we use the fact that the magnitude of  $\mu$  is determined from the radiative electroweak symmetry breaking condition and scales as  $\mu \sim m_{\tilde{f}}$ . The dashed curve in Fig. 4 represents the result in the case where we ignore the  $RRRR$  effect. In this case the lower bound on  $M_C$  behaves as  $\sim 1/m_{\tilde{u}_L}^2$ , since the  $LLLL$  contribution is proportional to  $M_2/(M_C m_{\tilde{f}}^2)$ .

It is found from the solid curve in Fig. 4 that the colored Higgs mass  $M_C$  must be larger than  $6.5 \times 10^{16}$  GeV for  $\tan \beta = 2.5$  when the typical sfermion mass is less than 1 TeV. On the other hand, it has been pointed out that there exists an upper bound on  $M_C$  given by  $M_C \leq 2.5 \times 10^{16}$  GeV (90% C.L.) if we require the gauge coupling unification in the minimal contents of GUT superfields [12]. This upper bound is smaller than the lower bound derived from our proton decay analysis. Therefore it turns out that the minimal SU(5) SUGRA GUT model with the sfermion masses less than 1 TeV is excluded for  $\tan \beta = 2.5$ . Note that the inclusion of the  $RRRR$  effect is essential here. If we ignored the  $RRRR$  effect, we could find an allowed region around  $1.2 \times 10^{16}$  GeV  $\leq M_C \leq 2.5 \times 10^{16}$  GeV. We can also see from Fig. 4 that the typical sfermion mass  $m_{\tilde{f}}$  must be

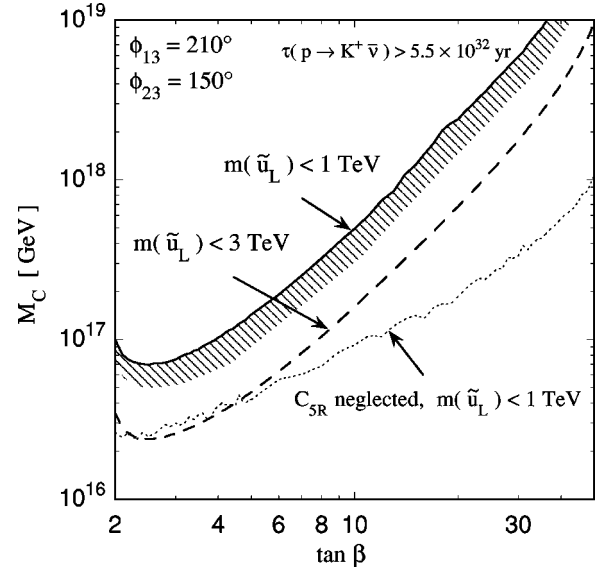


FIG. 5. The lower bound on the colored Higgs mass  $M_C$  obtained from the super-Kamiokande limit as a function of  $\tan \beta$ . The phase matrix  $P$  is fixed by  $\phi_{13} = 210^\circ$  and  $\phi_{23} = 150^\circ$ . The region below the solid curve is excluded if the left-handed scalar up-quark mass  $m_{\tilde{u}_L}$  is less than 1 TeV. The lower bound reduces to the dashed curve if we allow  $m_{\tilde{u}_L}$  up to 3 TeV. The result in the case where we ignore the  $RRRR$  effect is shown by the dotted curve for  $m_{\tilde{u}_L} < 1$  TeV.

larger than about 2.5 TeV when  $M_C$  is less than  $2.5 \times 10^{16}$  GeV in the  $\tan \beta = 2.5$  case. The  $RRRR$  effect plays an essential role again, since the lower bound on  $m_{\tilde{f}}$  would be 700 GeV if the  $RRRR$  effect were ignored. We also find that the Kamiokande limit on the neutron partial lifetime  $\tau(n \rightarrow K^0 \bar{\nu}) > 0.86 \times 10^{32}$  yr (90% C.L.) [6] already gives a comparable bound with that derived here from the super-Kamiokande limit on  $\tau(p \rightarrow K^+ \bar{\nu})$ , as shown by the dash-dotted curve in Fig. 4. If the super-Kamiokande updates the neutron limit from the Kamiokande, for example, by a factor of 5, then the lower bound on  $M_C$  will become  $\sqrt{5}$  times larger than that derived from the Kamiokande limit.

Let us discuss the  $\tan \beta$  dependence. Figure 5 shows the lower bound on the colored Higgs mass  $M_C$  obtained from the super-Kamiokande limit as a function of  $\tan \beta$ . Here we vary  $m_0$ ,  $M_{gX}$ ,  $A_X$ , and  $\text{sgn}(\mu)$  as in Fig. 4. The phases  $\phi_{13}$  and  $\phi_{23}$  are fixed as  $\phi_{13} = 210^\circ$  and  $\phi_{23} = 150^\circ$ . The result does not change much even if we take other values of  $\phi_{13}$  and  $\phi_{23}$ . The region below the solid curve is excluded if  $m_{\tilde{u}_L}$  is less than 1 TeV. The lower bound reduces to the dashed curve if we allow  $m_{\tilde{u}_L}$  up to 3 TeV. It is shown that the lower bound on  $M_C$  behaves as  $\sim \tan^2 \beta$  in a large  $\tan \beta$  region, as expected from the fact that the amplitude of  $p \rightarrow K^+ \bar{\nu}$  from the  $RRRR$  operator is roughly proportional to  $\sim \tan^2 \beta / M_C$ . On the other hand, the  $LLLL$  contribution is proportional to  $\sim \tan \beta / M_C$ , as shown by the dotted curve in Fig. 5. Thus the  $RRRR$  operator is dominant for large  $\tan \beta$  [14]. Note that the lower bound on  $M_C$  has the minimum at  $\tan \beta \approx 2.5$ . Thus we can conclude that for other value of

tan $\beta$  the constraints on  $M_C$  and  $m_{\tilde{f}}$  become severer than those shown in Fig. 4.

Finally, we comment on the other decay modes. For  $p \rightarrow \pi^+ \bar{\nu}$ , we obtain a similar result as that for the  $p \rightarrow K^+ \bar{\nu}$  mode: the third-generation mode  $p \rightarrow \pi^+ \bar{\nu}_\tau$  is dominated by the  $RRRR$  effect, while the  $RRR$  effect is negligible for the first and the second generation modes. Let us define  $r_i = \Gamma(p \rightarrow \pi^+ \bar{\nu}_i) / \Gamma(p \rightarrow K^+ \bar{\nu}_i)$  for  $i = e, \mu$ , and  $\tau$ . We see that  $r_\mu > 1$  is realized in a part of the  $\phi_{13}$ - $\phi_{23}$  parameter region where  $p \rightarrow K^+ \bar{\nu}_\mu$  mode is suppressed due to the cancellation between the  $LLLL$  contributions. This result is consistent with that given in the previous analysis [10]. As for the  $\bar{\nu}_\tau$  mode,  $r_\tau > 1$  is also possible in a different region where  $\Gamma(p \rightarrow K^+ \bar{\nu}_\tau)$  is reduced. Consequently the ratio  $r = \{\sum_i \Gamma(p \rightarrow \pi^+ \bar{\nu}_i)\} / \{\sum_i \Gamma(p \rightarrow K^+ \bar{\nu}_i)\}$  is smaller than 1 in the whole region of  $\phi_{13}$ - $\phi_{23}$  space. Moreover it has been reported that the lattice calculation of the hadronic matrix elements [27] gives a smaller value of the ratio  $\langle \pi | \mathcal{O}_b | p \rangle / \langle K | \mathcal{O}_b | p \rangle$  than the chiral Lagrangian estimation, where  $\mathcal{O}_b$  denotes the baryon number violating operators. Hence it follows that the ratio  $r$  is expected to be smaller when we use the lattice result for the hadronic matrix element. For the charged lepton mode  $p \rightarrow M \ell^+ (M = K^0, \pi^0, \eta \text{ and } \ell = e, \mu)$ , the effect of the  $RRRR$  operator is quite small, since we cannot have the tau lepton in the final state.

## V. CONCLUSIONS

We have reanalyzed the proton decay including the  $RRRR$  dimension 5 operator in the minimal SU(5) SUGRA GUT model. We have shown that the Higgsino dressing diagram of the  $RRRR$  operator gives a dominant contribution for  $p \rightarrow K^+ \bar{\nu}_\tau$ , and the decay rate of this mode can be comparable with that of  $p \rightarrow K^+ \bar{\nu}_\mu$ . We have found that we cannot reduce both the decay rate of  $p \rightarrow K^+ \bar{\nu}_\tau$  and that of  $p \rightarrow K^+ \bar{\nu}_\mu$  simultaneously by adjusting the relative phases  $\phi_{13}$  and  $\phi_{23}$  between the Yukawa couplings at the colored Higgs interactions. We have obtained the bounds on the colored Higgs mass  $M_C$  and the typical sfermion mass  $m_{\tilde{f}}$  from the new limit on  $\tau(p \rightarrow K^+ \bar{\nu})$  given by the super-Kamiokande. The colored Higgs boson mass  $M_C$  must be larger than  $6.5 \times 10^{16}$  GeV when  $m_{\tilde{f}}$  is less than 1 TeV. The typical sfermion mass  $m_{\tilde{f}}$  must be larger than 2.5 TeV when  $M_C$  is less than  $2.5 \times 10^{16}$  GeV.

## ACKNOWLEDGMENTS

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## APPENDIX: FORMULAS FOR THE CALCULATION OF THE NUCLEON DECAY

In this appendix, we summarize the formulas used in the calculation of the partial decay widths of the nucleon in the minimal SU(5) SUGRA GUT in order to clarify our notations and conventions. In subsection 1, generic formulas for the MSSM are summarized. The formulas specific to the calculation of the nucleon decay are given in subsection 2.

### 1. MSSM part

*Superpotential.* Yukawa couplings of the Higgs doublets and matter fields and the supersymmetric Higgs mass terms are given in the superpotential for the MSSM which is written as

$$\begin{aligned} W_{\text{MSSM}} = & f_D^{ij} Q_i^\alpha D_j^c H_{1\alpha} + f_U^{ij} \epsilon_{\alpha\beta} Q_i^\alpha U_j^c H_2^\beta + f_L^{ij} \epsilon^{\alpha\beta} E_i L_{j\alpha} H_{1\beta} \\ & + \mu H_{1\alpha} H_2^\alpha \\ = & f_D^{ij} (Q_i^u D_j^c H_1^- + Q_i^d D_j^c H_1^0) + f_U^{ij} (Q_i^u U_j^c H_2^0 \\ & - Q_i^d U_j^c H_2^+) + f_L^{ij} (E_i L_j^e H_1^0 - E_i L_j^\nu H_1^-) \\ & + \mu (H_1^0 H_2^0 + H_1^- H_2^+), \end{aligned} \quad (\text{A1})$$

where  $i, j$  and  $\alpha, \beta$  are generation and SU(2) suffices, respectively. Color indices are suppressed for simplicity. Components of the SU(2) doublets are denoted as

$$\begin{aligned} Q_i^\alpha = & \begin{pmatrix} Q_i^u \\ Q_i^d \end{pmatrix}, \quad L_{i\alpha} = (L_i^e \quad L_i^\nu), \\ H_{1\alpha} = & (H_1^- \quad H_1^0), \quad H_2^\alpha = \begin{pmatrix} H_2^+ \\ H_2^0 \end{pmatrix}. \end{aligned} \quad (\text{A2})$$

We take the generation basis for the superfields so that the Yukawa coupling matrices (equivalently the mass matrices) for the up-type quarks ( $f_U$ ) and the leptons ( $f_L$ ) should be diagonal (with real positive diagonal elements) at the electroweak scale. In this basis, the Yukawa coupling matrix for the down-type quarks  $f_D$  is written as

$$f_D(m_Z) = V_{\text{KM}}^* \hat{f}_D, \quad (\text{A3})$$

where  $\hat{f}_D$  is diagonal (real positive) and  $V_{\text{KM}}$  is the CKM matrix. We take the PDG's "standard" phase convention for  $V_{\text{KM}}$  [20]. The SUSY Higgs mass parameter  $\mu$  is taken as real in order to automatically avoid too-large electric dipole moments (EDMs) of the neutron and the electron. The sign of  $\mu$  is taken as a free "parameter."

*Soft SUSY breaking terms.* Soft SUSY breaking terms of the MSSM are given as

$$\begin{aligned} -\mathcal{L}_{\text{soft}} = & (m_Q^2)_i \tilde{q}_i^\alpha \tilde{q}_\alpha^{\dagger j} + (m_U^2)_i \tilde{u}^{\dagger i} \tilde{u}_j + (m_D^2)_i \tilde{d}^{\dagger i} \tilde{d}_j \\ & + (m_L^2)_i \tilde{l}^{\dagger i \alpha} \tilde{l}_{j\alpha} + (m_E^2)_j \tilde{e}_j^\alpha \tilde{e}^{\dagger j} + \Delta_1^2 h_1^\dagger h_{1\alpha} \\ & + \Delta_2^2 h_2^\dagger h_2^\alpha - (B \mu h_{1\alpha} h_2^\alpha + \text{H.c.}) + (A_U^{ij} \epsilon_{\alpha\beta} \tilde{q}_i^\alpha \tilde{u}_j h_2^\beta \end{aligned}$$

$$+A_D^{ij}\tilde{q}_i^\alpha\tilde{d}_j h_{1\alpha}+A_L^{ij}\epsilon^{\alpha\beta}\tilde{e}_i^\alpha\tilde{l}_j^\beta h_{1\beta}+\text{H.c.}) \\ +\left(\frac{M_1}{2}\tilde{B}\tilde{B}+\frac{M_2}{2}\tilde{W}\tilde{W}+\frac{M_3}{2}\tilde{G}\tilde{G}+\text{H.c.}\right). \quad (\text{A4})$$

where  $\tilde{q}$ ,  $\tilde{d}$ ,  $\tilde{u}$ ,  $\tilde{e}$ ,  $\tilde{l}$ ,  $h_1$ , and  $h_2$  are scalar components of  $Q$ ,  $D^c$ ,  $U^c$ ,  $E^c$ ,  $L$ ,  $H_1$ , and  $H_2$ , respectively, and  $\tilde{G}$ ,  $\tilde{W}$ , and  $\tilde{B}$  are SU(3), SU(2), and U(1) gaugino fields, respectively. The gaugino masses  $M_1$ ,  $M_2$ , and  $M_3$  are taken as real positive.

In the minimal SUGRA GUT model, the soft SUSY breaking parameters at the GUT scale  $M_X$  are written in terms of the universal soft SUSY breaking parameters  $m_0$  (universal scalar mass),  $M_{gX}$  (unified gaugino mass), and  $A_X$  (dimensionless universal trilinear coupling parameter):

$$m_Q^2(M_X)=m_U^2(M_X)=m_D^2(M_X)=m_0^2\mathbf{1}, \quad (\text{A5a})$$

$$m_L^2(M_X)=m_E^2(M_X)=m_0^2\mathbf{1}, \quad (\text{A5b})$$

$$\Delta_1^2(M_X)=\Delta_2^2(M_X)=m_0^2, \quad (\text{A5c})$$

$$M_1(M_X)=M_2(M_X)=M_3(M_X)=M_{gX}, \quad (\text{A5d})$$

$$A_U(M_X)=A_X m_0 f_U, \quad A_D(M_X)=A_X m_0 f_D, \quad (\text{A5e})$$

$$A_L(M_X)=A_X m_0 f_L, \quad (\text{A5f})$$

where  $\mathbf{1}$  is a  $3\times 3$  unit matrix in the generation space. We take  $A_X$  as real (with either sign) to avoid large EDMs.

*Mass matrices.* Mass matrices for squarks and sleptons are given as follows.

Up-type squark:

$$\mathcal{M}_u^2=\begin{pmatrix} m_{LL}^2(\tilde{u}) & m_{LR}^2(\tilde{u}) \\ m_{RL}^2(\tilde{u}) & m_{RR}^2(\tilde{u}) \end{pmatrix}, \quad (\text{A6a})$$

$$m_{LL}^2(\tilde{u})=v^2 s_\beta^2 f_U f_U^\dagger+m_Q^2+m_Z^2 c_{2\beta}\left(\frac{1}{2}-\frac{2}{3}s_W^2\right)\mathbf{1}, \quad (\text{A6b})$$

$$m_{RR}^2(\tilde{u})=v^2 s_\beta^2 f_U^\dagger f_U+m_U^2+m_Z^2 c_{2\beta}\left(\frac{2}{3}s_W^2\right)\mathbf{1}, \quad (\text{A6c})$$

$$m_{LR}^2(\tilde{u})=\mu^* f_U v c_\beta+A_U v s_\beta, \quad (\text{A6d})$$

$$m_{RL}^2(\tilde{u})=m_{LR}^{2\dagger}(\tilde{u}), \quad (\text{A6e})$$

down-type squark:

$$\mathcal{M}_d^2=\begin{pmatrix} m_{LL}^2(\tilde{d}) & m_{LR}^2(\tilde{d}) \\ m_{RL}^2(\tilde{d}) & m_{RR}^2(\tilde{d}) \end{pmatrix}, \quad (\text{A7a})$$

$$m_{LL}^2(\tilde{d})=v^2 c_\beta^2 f_D f_D^\dagger+m_Q^2+m_Z^2 c_{2\beta}\left(-\frac{1}{2}+\frac{1}{3}s_W^2\right)\mathbf{1}, \quad (\text{A7b})$$

$$m_{RR}^2(\tilde{d})=v^2 c_\beta^2 f_D^\dagger f_D+m_D^2+m_Z^2 c_{2\beta}\left(-\frac{1}{3}s_W^2\right)\mathbf{1}, \quad (\text{A7c})$$

$$m_{LR}^2(\tilde{d})=\mu^* f_D v s_\beta+A_D v c_\beta, \quad (\text{A7d})$$

$$m_{RL}^2(\tilde{d})=m_{LR}^{2\dagger}(\tilde{d}), \quad (\text{A7e})$$

charged slepton:

$$\mathcal{M}_l^2=\begin{pmatrix} m_{LL}^2(\tilde{l}) & m_{LR}^2(\tilde{l}) \\ m_{RL}^2(\tilde{l}) & m_{RR}^2(\tilde{l}) \end{pmatrix}, \quad (\text{A8a})$$

$$m_{LL}^2(\tilde{l})=v^2 c_\beta^2 f_L^\dagger f_L+m_L^2+m_Z^2 c_{2\beta}\left(-\frac{1}{2}+s_W^2\right)\mathbf{1}, \quad (\text{A8b})$$

$$m_{RR}^2(\tilde{l})=v^2 c_\beta^2 f_L f_L^\dagger+m_E^2+m_Z^2 c_{2\beta}(-s_W^2)\mathbf{1}, \quad (\text{A8c})$$

$$m_{RL}^2(\tilde{l})=\mu^* f_L v s_\beta+A_L v c_\beta, \quad (\text{A8d})$$

$$m_{LR}^2(\tilde{l})=m_{RL}^{2\dagger}(\tilde{l}), \quad (\text{A8e})$$

sneutrino:

$$\mathcal{M}_\nu^2=m_L^2+m_Z^2 c_\beta^2\left(\frac{1}{2}\right)\mathbf{1}, \quad (\text{A9})$$

where  $c_\beta=\cos\beta>0$ ,  $s_\beta=\sin\beta>0$ ,  $c_{2\beta}=\cos 2\beta$ ,  $s_W=\sin\theta_W$ , and  $v^2=\langle h_1\rangle^2+\langle h_2\rangle^2$  ( $v\approx 174$  GeV). The above mass matrices are diagonalized with use of  $6\times 6$  unitary matrices  $\tilde{U}_U$ ,  $\tilde{U}_D$ , and  $\tilde{U}_L$ , and a  $3\times 3$  unitary matrix  $\tilde{U}_N$ , which are defined as

$$\tilde{U}_U \mathcal{M}_u^2 \tilde{U}_U^\dagger = \text{diagonal}(m_{u_i}^2), \quad (\text{A10a})$$

$$\tilde{U}_D \mathcal{M}_d^2 \tilde{U}_D^\dagger = \text{diagonal}(m_{d_i}^2), \quad (\text{A10b})$$

$$\tilde{U}_L^\dagger \mathcal{M}_l^2 \tilde{U}_L = \text{diagonal}(m_{l_i}^2), \quad (\text{A10c})$$

$$\tilde{U}_N^\dagger \mathcal{M}_\nu^2 \tilde{U}_N = \text{diagonal}(m_{\nu_i}^2), \quad (\text{A10d})$$

where the superscript **T** stands for the transpose.

Mass matrices for charginos ( $\mathcal{M}_C$ ) and neutralinos ( $\mathcal{M}_N$ ) are given as follows:

$$\mathcal{M}_C = \begin{pmatrix} M_2 & \sqrt{2}m_{WS}\beta \\ -\sqrt{2}m_{WC}\beta & -\mu \end{pmatrix}, \quad (\text{A11a})$$

$\mathcal{M}_N$ 

$$= \begin{pmatrix} -M_1 & 0 & -m_{Z^S W^C \beta} & m_{Z^S W^S \beta} \\ 0 & -M_2 & m_{Z^C W^C \beta} & -m_{Z^C W^S \beta} \\ -m_{Z^S W^C \beta} & m_{Z^C W^C \beta} & 0 & \mu \\ m_{Z^S W^S \beta} & -m_{Z^C W^S \beta} & \mu & 0 \end{pmatrix}. \quad (\text{A11b})$$

$\mathcal{M}_C$  and  $\mathcal{M}_N$  are diagonalized with  $2 \times 2$  unitary matrices  $U_{\pm}$  and a  $4 \times 4$  unitary matrix  $U_N$ , respectively, which are defined as

$$-U_{-}^{\dagger} \mathcal{M}_C U_{+} = \text{diagonal}(M_C^{\alpha}), \quad (\text{A12a})$$

$$U_N^{\text{T}} \mathcal{M}_N U_N = \text{diagonal}(M_N^{\bar{\alpha}}), \quad (\text{A12b})$$

where all mass eigenvalues  $M_C^{\alpha} (\alpha=1,2)$  and  $M_N^{\bar{\alpha}} (\bar{\alpha}=1,2,3,4)$  are taken as real positive.

*Interaction Lagrangian in mass basis.* The quark (lepton) – squark (slepton) – ino (gluino, chargino, neutralino) interaction terms are given as follows:

$$\mathcal{L}_{\text{int}} = \mathcal{L}_{\text{int}}(\tilde{G}) + \mathcal{L}_{\text{int}}(\chi^{\pm}) + \mathcal{L}_{\text{int}}(\chi^0),$$

$$\begin{aligned} \mathcal{L}_{\text{int}}(\tilde{G}) = & -i\sqrt{2}g_3 \tilde{d}^{*I} \tilde{G} [(\Gamma_{GL}^{(d)})_I^j \mathbf{L} + (\Gamma_{GR}^{(d)})_I^j \mathbf{R}] d_j \\ & -i\sqrt{2}g_3 \tilde{u}^{*I} \tilde{G} [(\Gamma_{GL}^{(u)})_I^j \mathbf{L} + (\Gamma_{GR}^{(u)})_I^j \mathbf{R}] u_j + \text{H.c.}, \end{aligned} \quad (\text{A13a})$$

$$\begin{aligned} \mathcal{L}_{\text{int}}(\chi^{\pm}) = & g_2 \bar{\chi}_{\alpha}^{-} [(\Gamma_{CL}^{(d)})_I^{\alpha j} \mathbf{L} + (\Gamma_{CR}^{(d)})_I^{\alpha j} \mathbf{R}] d_j \tilde{u}^{*I} \\ & + g_2 \bar{\chi}_{\alpha}^{+} [(\Gamma_{CL}^{(u)})_I^{\alpha j} \mathbf{L} + (\Gamma_{CR}^{(u)})_I^{\alpha j} \mathbf{R}] u_j \tilde{d}^{*I} \\ & + g_2 \bar{\chi}_{\alpha}^{-} [(\Gamma_{CL}^{(l)})_I^{\alpha j} \mathbf{L} + (\Gamma_{CR}^{(l)})_I^{\alpha j} \mathbf{R}] l_j \tilde{\nu}^{*i} \\ & + g_2 \bar{\chi}_{\alpha}^{+} [(\Gamma_{CL}^{(\nu)})_I^{\alpha j} \mathbf{L} \nu_j \tilde{l}^{*I} + \text{H.c.}], \end{aligned} \quad (\text{A13b})$$

$$\begin{aligned} \mathcal{L}_{\text{int}}(\chi^0) = & g_2 \bar{\chi}_{\alpha}^0 [(\Gamma_{NL}^{(d)})_I^{\bar{\alpha} j} \mathbf{L} + (\Gamma_{NR}^{(d)})_I^{\bar{\alpha} j} \mathbf{R}] d_j \tilde{d}^{*I} \\ & + g_2 \bar{\chi}_{\alpha}^0 [(\Gamma_{NL}^{(u)})_I^{\bar{\alpha} j} \mathbf{L} + (\Gamma_{NR}^{(u)})_I^{\bar{\alpha} j} \mathbf{R}] u_j \tilde{u}^{*I} \\ & + g_2 \bar{\chi}_{\alpha}^0 [(\Gamma_{NL}^{(l)})_I^{\bar{\alpha} j} \mathbf{L} + (\Gamma_{NR}^{(l)})_I^{\bar{\alpha} j} \mathbf{R}] l_j \tilde{l}^{*I} \\ & + g_2 \bar{\chi}_{\alpha}^0 [(\Gamma_{NL}^{(\nu)})_I^{\bar{\alpha} j} \mathbf{L} \nu_j \tilde{\nu}^{*i} + \text{H.c.}], \end{aligned} \quad (\text{A13c})$$

where  $\mathbf{L} = \frac{1}{2}(1 - \gamma_5)$  and  $\mathbf{R} = \frac{1}{2}(1 + \gamma_5)$ ,  $g_2$  and  $g_3$  are SU(2) and SU(3) gauge coupling constants, respectively. Here and hereafter,  $\tilde{G}$ ,  $\chi_{\alpha}^{\pm}$ ,  $\chi_{\alpha}^0$ ,  $\tilde{u}_I$ ,  $\tilde{d}_I$ ,  $\tilde{l}_I$ ,  $\tilde{\nu}_i$ ,  $u_i$ ,  $d_i$ ,  $l_i$ , and  $\nu_i$  denote gluino, chargino, neutralino, up-type squark, down-type squark, charged slepton, sneutrino, up-type quark, down-type quark, charged lepton, and neutrino fields in mass basis, respectively. Ranges of the suffices are  $I=1,2,\dots,6$  (squarks and charged sleptons),  $i,j,k=1,2,3$  (quarks, leptons and sneutrinos),  $\alpha=1,2$  (charginos), and  $\bar{\alpha}=1,2,3,4$  (neutralinos). Mixing factors at each vertex are written in terms

of the mass-diagonalizing matrices  $\tilde{U}_U$ ,  $\tilde{U}_D$ ,  $\tilde{U}_L$ ,  $\tilde{U}_N$ ,  $U_{\pm}$ , and  $U_N$  as follows.

Gluino:

$$(\Gamma_{GL}^{(d)})_I^j = \sum_{k=1}^3 (\tilde{U}_D)_I^k (V_{\text{KM}})_k^j, \quad (\text{A14a})$$

$$(\Gamma_{GR}^{(d)})_I^j = (\tilde{U}_D)_I^{j+3}, \quad (\text{A14b})$$

$$(\Gamma_{GL}^{(u)})_I^j = (\tilde{U}_U)_I^j, \quad (\text{A14c})$$

$$(\Gamma_{GR}^{(u)})_I^j = (\tilde{U}_U)_I^{j+3}, \quad (\text{A14d})$$

chargino:

$$\begin{aligned} (\Gamma_{CL}^{(d)})_I^{\alpha j} = & \sum_{k=1}^3 \left\{ (\tilde{U}_U)_I^k (U_{+})_1^{\alpha} + (\tilde{U}_U)_I^{k+3} \frac{m_k^{(u)}}{\sqrt{2}m_{W^S \beta}} (U_{+})_2^{\alpha} \right\} \\ & \times (V_{\text{KM}})_k^j, \end{aligned} \quad (\text{A15a})$$

$$(\Gamma_{CR}^{(d)})_I^{\alpha j} = -\sum_{k=1}^3 (\tilde{U}_U)_I^k (V_{\text{KM}})_k^j \frac{m_j^{(d)}}{\sqrt{2}m_{W^C \beta}} (U_{-})_2^{\alpha}, \quad (\text{A15b})$$

$$\begin{aligned} (\Gamma_{CL}^{(u)})_I^{\alpha j} = & (\tilde{U}_D)_I^j (U_{-}^{\dagger})_1^{\alpha} \\ & - \sum_{k=1}^3 (\tilde{U}_D)_I^{k+3} \frac{m_k^{(d)}}{\sqrt{2}m_{W^C \beta}} (V_{\text{KM}})_k^j (U_{-}^{\dagger})_2^{\alpha}, \end{aligned} \quad (\text{A15c})$$

$$(\Gamma_{CR}^{(u)})_I^{\alpha j} = (\tilde{U}_D)_I^j \frac{m_j^{(u)}}{\sqrt{2}m_{W^S \beta}} (U_{+}^{\dagger})_2^{\alpha}, \quad (\text{A15d})$$

$$(\Gamma_{CL}^{(l)})_i^{\alpha j} = -(\tilde{U}_N^{\dagger})_i^j (U_{+})_1^{\alpha}, \quad (\text{A15e})$$

$$(\Gamma_{CR}^{(l)})_i^{\alpha j} = \frac{m_j^{(l)}}{\sqrt{2}m_{W^C \beta}} (\tilde{U}_N^{\dagger})_i^j (U_{-})_2^{\alpha}, \quad (\text{A15f})$$

$$(\Gamma_{CL}^{(\nu)})_I^{\alpha j} = -(\tilde{U}_L^{\dagger})_I^j (U_{-}^{\dagger})_1^{\alpha} + \frac{m_j^{(l)}}{\sqrt{2}m_{W^C \beta}} (\tilde{U}_L^{\dagger})_I^{j+3} (U_{-}^{\dagger})_2^{\alpha}, \quad (\text{A15g})$$

neutralino:

$$\begin{aligned} (\Gamma_{NL}^{(d)})_I^{\bar{\alpha} j} = & \sqrt{2} \left[ +\frac{1}{2} (U_N)_{\bar{\alpha}}^{\bar{\alpha}} - \frac{1}{6} t_w (U_N)_{\bar{\alpha}}^{\bar{\alpha}} \right] \sum_{k=1}^3 (\tilde{U}_D)_I^k (V_{\text{KM}})_k^j \\ & - \frac{m_j^{(d)}}{\sqrt{2}m_{W^C \beta}} (U_N)_{\bar{\alpha}}^{\bar{\alpha}} (\tilde{U}_D)_I^{j+3}, \end{aligned} \quad (\text{A16a})$$



$$\begin{aligned}
(\Gamma_{NR}^{(d)})_{I\bar{j}}^{\bar{\alpha}} &= \sqrt{2} \left[ -\frac{1}{3} t_W (U_N^\dagger)_\alpha^1 \right] (\tilde{U}_D)_I^{j+3} \\
&\quad - \frac{m_j^{(d)}}{\sqrt{2} m_W c_\beta} (U_N^\dagger)_\alpha^3 \sum_{k=1}^3 (\tilde{U}_D)_I^k (V_{KM})_k^j,
\end{aligned} \tag{A16b}$$

$$\begin{aligned}
(\Gamma_{NL}^{(u)})_{I\bar{j}}^{\bar{\alpha}} &= \sqrt{2} \left[ -\frac{1}{2} (U_N)_2^{\bar{\alpha}} - \frac{1}{6} t_W (U_N)_1^{\bar{\alpha}} \right] (\tilde{U}_U)_I^j \\
&\quad - \frac{m_j^{(u)}}{\sqrt{2} m_W s_\beta} (U_N)_4^{\bar{\alpha}} (\tilde{U}_U)_I^{j+3},
\end{aligned} \tag{A16c}$$

$$\begin{aligned}
(\Gamma_{NR}^{(u)})_{I\bar{j}}^{\bar{\alpha}} &= \sqrt{2} \left[ +\frac{2}{3} t_W (U_N^\dagger)_\alpha^2 \right] (\tilde{U}_U)_I^{j+3} \\
&\quad - \frac{m_j^{(u)}}{\sqrt{2} m_W s_\beta} (U_N^\dagger)_\alpha^4 (\tilde{U}_U)_I^j,
\end{aligned} \tag{A16d}$$

$$\begin{aligned}
(\Gamma_{NL}^{(l)})_{I\bar{j}}^{\bar{\alpha}} &= \sqrt{2} \left[ \frac{1}{2} (U_N)_2^{\bar{\alpha}} + \frac{1}{2} t_W (U_N)_1^{\bar{\alpha}} \right] (\tilde{U}_L^\dagger)_I^j \\
&\quad - \frac{m_j^{(l)}}{\sqrt{2} m_W c_\beta} (U_N)_3^{\bar{\alpha}} (\tilde{U}_L^\dagger)_I^{j+3},
\end{aligned} \tag{A16e}$$

$$\begin{aligned}
(\Gamma_{NR}^{(l)})_{I\bar{j}}^{\bar{\alpha}} &= \sqrt{2} \left[ -t_W (U_N^\dagger)_\alpha^1 \right] (\tilde{U}_L^\dagger)_I^{j+3} \\
&\quad - \frac{m_j^{(l)}}{\sqrt{2} m_W c_\beta} (U_N^\dagger)_\alpha^3 (\tilde{U}_L^\dagger)_I^j,
\end{aligned} \tag{A16f}$$

$$(\Gamma_{NL}^{(v)})_{I\bar{j}}^{\bar{\alpha}} = \sqrt{2} \left[ -\frac{1}{2} (U_N)_2^{\bar{\alpha}} + \frac{1}{2} t_W (U_N)_1^{\bar{\alpha}} \right] (\tilde{U}_N^\dagger)_I^j, \tag{A16g}$$

where  $t_W = \tan \theta_W$  and  $m_i^{(u)}$ ,  $m_i^{(d)}$ , and  $m_i^{(l)}$  are masses (real positive) of up-type quarks, down-type quarks, and charged leptons, respectively.

## 2. Formulas specific to the nucleon decay

*Dimension five operators.* Dimension five operators relevant to the nucleon decay are described by the following superpotential:

$$\begin{aligned}
W_5 = -\frac{1}{M_C} \left\{ C_{5L}^{ijkl} \frac{1}{2} \epsilon_{\hat{a}\hat{b}\hat{c}} \epsilon_{\alpha\beta} \hat{Q}_k^{\hat{a}\alpha} \hat{Q}_l^{\hat{b}\beta} \hat{Q}_i^{\hat{c}\gamma} L_{j\gamma} \right. \\
\left. + C_{5R}^{ijkl} \epsilon^{\hat{a}\hat{b}\hat{c}} E_k^c U_{l\hat{a}}^c U_{i\hat{b}}^c D_{j\hat{c}}^c \right\},
\end{aligned} \tag{A17}$$

where the suffices  $\hat{a}, \hat{b}, \hat{c}$  are color indices. The coefficients  $C_{5L}$  and  $C_{5R}$  are given at the GUT scale in terms of the Yukawa coupling matrices

$$C_{5L}^{ijkl}(M_X) = f_D^{im}(M_X) (V_{DL})_m^j f_U^{kn}(M_X) (V_{QU}^\dagger)_n^l, \tag{A18a}$$

$$C_{5R}^{ijkl}(M_X) = f_D^{mj}(M_X) (V_{QU})_m^i f_U^{nl}(M_X) (V_{QE})_n^k, \tag{A18b}$$

where  $V_{QU}, V_{QE}$ , and  $V_{DL}$  are  $3 \times 3$  unitary matrices which parametrize the differences between generation bases of the MSSM superfields embedded in  $SU(5)$  superfields  $\Psi(10)$  and  $\Phi(\bar{5})$ ; i.e., the MSSM multiplets are accommodated into  $\Psi$  and  $\Phi$  as

$$\Psi_i \Leftarrow \{Q_i, (V_{QU})_i^k U_k^c, (V_{QE})_i^k E_{kj}^c\}, \tag{A19a}$$

$$\Phi_i \Leftarrow \{D_i^c, (V_{DL})_i^k L_k\}. \tag{A19b}$$

$V_{QU}, V_{QE}$ , and  $V_{DL}$  are determined by the unitary matrices which diagonalize the Yukawa coupling matrices at  $M_X$  and the phase matrix  $P$ :

$$V_{QU} = U_Q^{(u)\dagger} P^\dagger U_U, \tag{A20a}$$

$$V_{QE} = U_Q^{(d)\dagger} U_E, \tag{A20b}$$

$$V_{DL} = U_D^\dagger U_L, \tag{A20c}$$

where the Yukawa coupling matrices are diagonalized with  $U$ 's as

$$U_Q^{(u)*} f_U(M_X) U_U^\dagger = Y_U, \tag{A21a}$$

$$U_Q^{(d)*} f_D(M_X) U_D^\dagger = Y_D, \tag{A21b}$$

$$U_E^* f_L(M_X) U_L^\dagger = Y_L. \tag{A21c}$$

$Y_U, Y_D$ , and  $Y_L$  are diagonal matrices with real positive diagonal elements. The CKM matrix at the GUT scale  $V \equiv V_{KM}(M_X)$  is also written in terms of  $U$ 's as

$$V = U_Q^{(u)} U_Q^{(d)\dagger}. \tag{A22}$$

In the present generation basis described in Subsection 1,  $U_Q^{(u)}, U_U, U_D \approx \mathbf{1}$ ,  $U_Q^{(d)} \approx V_{KM}^\dagger$ , and  $U_E = U_L = \mathbf{1}$ . Consequently,

$$V_{QU} \approx P^\dagger, V_{QE} \approx V_{KM} \approx V, V_{DL} \approx \mathbf{1}. \tag{A23}$$

The expressions for  $C_{5L,R}$  in Eq. (2) are obtained from Eq. (A18) in this approximation.

In the component form, the dimension five operators at the electroweak scale are written as

$$\begin{aligned}
\mathcal{L}_5 = & \frac{1}{M_C} \epsilon^{\hat{a}\hat{b}\hat{c}} \left\{ C(\tilde{u}\tilde{d}u_L)^{MNij} \tilde{u}_M^{\hat{a}} \tilde{d}_N^{\hat{b}} (u_{Li}^{\hat{c}} l_{Lj}) + C(\tilde{u}\tilde{u}d_L)^{MNij} \frac{1}{2} \tilde{u}_M^{\hat{a}} \tilde{u}_N^{\hat{b}} (d_{Li}^{\hat{c}} l_{Lj}) + C(\tilde{u}\tilde{d}u_R)^{MNij} \tilde{u}_M^{\hat{a}} \tilde{d}_N^{\hat{b}} (u_{Ri}^{\hat{c}} l_{Rj}) \right. \\
& + C(\tilde{u}\tilde{u}d_R)^{MNij} \frac{1}{2} \tilde{u}_M^{\hat{a}} \tilde{u}_N^{\hat{b}} (d_{Ri}^{\hat{c}} l_{Rj}) + C(\tilde{u}\tilde{d}d\nu_L)^{MNij} \tilde{u}_M^{\hat{a}} \tilde{d}_N^{\hat{b}} (d_{Li}^{\hat{c}} \nu_{Lj}) + C(\tilde{d}\tilde{d}u\nu_L)^{MNij} \frac{1}{2} \tilde{d}_M^{\hat{a}} \tilde{d}_N^{\hat{b}} (u_{Li}^{\hat{c}} \nu_{Lj}) \\
& + C(\tilde{u}\tilde{l}u_L)^{IJKl} \tilde{u}_I^{\hat{a}} \tilde{l}_J(u_{Lk}^{\hat{b}} d_{Li}^{\hat{c}}) + C(\tilde{d}\tilde{l}uu_L)^{IJKl} \frac{1}{2} \tilde{d}_I^{\hat{a}} \tilde{l}_J(u_{Lk}^{\hat{b}} u_{Li}^{\hat{c}}) + C(\tilde{u}\tilde{l}u_R)^{IJKl} \tilde{u}_I^{\hat{a}} \tilde{l}_J(u_{Rk}^{\hat{b}} d_{Rl}^{\hat{c}}) \\
& \left. + C(\tilde{d}\tilde{l}uu_R)^{IJKl} \frac{1}{2} \tilde{d}_I^{\hat{a}} \tilde{l}_J(u_{Rk}^{\hat{b}} u_{Rl}^{\hat{c}}) + C(\tilde{d}\tilde{v}ud_L)^{IJKl} \tilde{d}_I^{\hat{a}} \tilde{v}_j(u_{Lk}^{\hat{b}} d_{Li}^{\hat{c}}) + C(\tilde{u}\tilde{v}dd_L)^{IJKl} \frac{1}{2} \tilde{u}_I^{\hat{a}} \tilde{v}_j(d_{Lk}^{\hat{b}} d_{Li}^{\hat{c}}) \right\}, \quad (\text{A24})
\end{aligned}$$

where the suffices  $L, R$  of the quark/lepton fields denote the chirality. The coefficients  $C$ 's are written in terms of  $C_{5L, R}$  as follows:

$$C(\tilde{u}\tilde{d}u_L)^{MNij} = (C_{5L}^{ijkl} - C_{5L}^{kjil})(\tilde{U}_U^\dagger)_k^M (\tilde{U}_D^\dagger)_i^N, \quad (\text{A25a})$$

$$C(\tilde{u}\tilde{u}d_L)^{MNij} = (C_{5L}^{kjlm} - C_{5L}^{ljk m})(\tilde{U}_U^\dagger)_k^M (\tilde{U}_U^\dagger)_i^N (V_{\text{KM}})_m^j, \quad (\text{A25b})$$

$$C(\tilde{u}\tilde{d}u_R)^{MNij} = (C_{5R}^{*klji} - C_{5R}^{*iljk})(\tilde{U}_U^\dagger)_{k+3}^M (\tilde{U}_D^\dagger)_{i+3}^N, \quad (\text{A25c})$$

$$C(\tilde{u}\tilde{u}d_R)^{MNij} = (C_{5R}^{*lij k} - C_{5R}^{*kijl})(\tilde{U}_U^\dagger)_{k+3}^M (\tilde{U}_U^\dagger)_{i+3}^N, \quad (\text{A25d})$$

$$C(\tilde{u}\tilde{d}d\nu_L)^{MNij} = (C_{5L}^{mjkl} - C_{5L}^{ljk m})(\tilde{U}_U^\dagger)_k^M (\tilde{U}_D^\dagger)_i^N (V_{\text{KM}})_m^j, \quad (\text{A25e})$$

$$C(\tilde{d}\tilde{d}u\nu_L)^{MNij} = (C_{5L}^{ljik} - C_{5L}^{kjil})(\tilde{U}_D^\dagger)_k^M (\tilde{U}_D^\dagger)_i^N, \quad (\text{A25f})$$

$$C(\tilde{u}\tilde{l}u_L)^{IJKl} = (C_{5L}^{ijk m} - C_{5L}^{kjim})(\tilde{U}_U^\dagger)_i^I (\tilde{U}_L^\dagger)_j^J (V_{\text{KM}})_m^l, \quad (\text{A25g})$$

$$C(\tilde{d}\tilde{l}uu_L)^{IJKl} = (C_{5L}^{kjli} - C_{5L}^{ljk i})(\tilde{U}_D^\dagger)_i^I (\tilde{U}_L^\dagger)_j^J, \quad (\text{A25h})$$

$$C(\tilde{u}\tilde{l}u_R)^{IJKl} = (C_{5R}^{*klji} - C_{5R}^{*iljk})(\tilde{U}_U^\dagger)_{i+3}^I (\tilde{U}_L^\dagger)_{j+3}^J, \quad (\text{A25i})$$

$$C(\tilde{d}\tilde{l}uu_R)^{IJKl} = (C_{5R}^{*lij k} - C_{5R}^{*kijl})(\tilde{U}_D^\dagger)_{i+3}^I (\tilde{U}_L^\dagger)_{j+3}^J, \quad (\text{A25j})$$

$$C(\tilde{d}\tilde{v}ud_L)^{IJKl} = (C_{5L}^{ink m} - C_{5L}^{mnki})(\tilde{U}_D^\dagger)_i^I (\tilde{U}_N^\dagger)_n^j (V_{\text{KM}})_m^l, \quad (\text{A25k})$$

$$C(\tilde{u}\tilde{v}dd_L)^{IJKl} = (C_{5L}^{qnip} - C_{5L}^{pn iq})(\tilde{U}_U^\dagger)_i^I (\tilde{U}_N^\dagger)_n^j (V_{\text{KM}})_p^k (V_{\text{KM}})_q^l, \quad (\text{A25l})$$

$C_{5L}$  and  $C_{5R}$  at the electroweak scale are evaluated by solving the renormalization group equations

$$\begin{aligned}
(4\pi)^2 \Lambda \frac{d}{d\Lambda} C_{5L}^{ijkl} = & \left( -8g_3^2 - 6g_2^2 - \frac{2}{3}g_1^2 \right) C_{5L}^{ijkl} \\
& + C_{5L}^{mjkl} (f_{Df_D}^\dagger + f_{Uf_U}^\dagger)_m^i + C_{5L}^{imkl} (f_{Ll}^\dagger f_L)_m^j \\
& + C_{5L}^{ijml} (f_{Df_D}^\dagger + f_{Uf_U}^\dagger)_m^k \\
& + C_{5L}^{ijk m} (f_{Df_D}^\dagger + f_{Uf_U}^\dagger)_m^l, \quad (\text{A26a})
\end{aligned}$$

$$\begin{aligned}
(4\pi)^2 \Lambda \frac{d}{d\Lambda} C_{5R}^{ijkl} = & (-8g_3^2 - 4g_1^2) C_{5R}^{ijkl} + C_{5R}^{mjkl} (2f_{Uf_U}^\dagger)_m^i \\
& + C_{5R}^{imkl} (2f_{Df_D}^\dagger)_m^j + C_{5R}^{ijml} (2f_{Ll}^\dagger f_L)_m^k \\
& + C_{5R}^{ijk m} (2f_{Uf_U}^\dagger)_m^l, \quad (\text{A26b})
\end{aligned}$$

where  $\Lambda$  is the renormalization point.

*Effective interactions.* After the calculation of the one-loop (gluino-, chargino-, and neutralino-) dressing diagrams, effective four-Fermi interaction terms relevant to the nucleon decay are obtained as follows:

$$\begin{aligned}
\mathcal{L}_b = & \frac{1}{(4\pi)^2 M_C} \epsilon^{\hat{a}\hat{b}\hat{c}} \left\{ \tilde{C}_{LL}(udul)^{ik} (u_L^{\hat{a}} d_{Li}^{\hat{b}}) (u_L^{\hat{c}} l_{Lk}) \right. \\
& + \tilde{C}_{RL}(udul)^{ik} (u_R^{\hat{a}} d_{Ri}^{\hat{b}}) (u_L^{\hat{c}} l_{Lk}) + \tilde{C}_{LR}(udul)^{ik} (u_L^{\hat{a}} d_{Li}^{\hat{b}}) \\
& \times (u_R^{\hat{c}} l_{Rk}) + \tilde{C}_{RR}(udul)^{ik} (u_R^{\hat{a}} d_{Ri}^{\hat{b}}) (u_R^{\hat{c}} l_{Rk}) \\
& + \tilde{C}_{LL}(udd\nu)^{ijk} (u_L^{\hat{a}} d_{Li}^{\hat{b}}) (d_{Lj}^{\hat{c}} \nu_{Lk}) \\
& + \tilde{C}_{RL}(udd\nu)^{ijk} (u_R^{\hat{a}} d_{Ri}^{\hat{b}}) (d_{Lj}^{\hat{c}} \nu_{Lk}) \\
& \left. + \tilde{C}_{RL}(ddu\nu)^{ijk} \frac{1}{2} (d_{Ri}^{\hat{a}} d_{Rj}^{\hat{b}}) (u_L^{\hat{c}} \nu_{Lk}) \right\}, \quad (\text{A27})
\end{aligned}$$

$$\tilde{C}_{LL}(udul)^{ik} = \tilde{C}_{LL}(udul)_{\tilde{G}}^{ik} + \tilde{C}_{LL}(udul)_{\chi^\pm}^{ik} + \tilde{C}_{LL}(udul)_{\chi^0}^{ik}, \quad (\text{A28a})$$

$$\begin{aligned}\tilde{C}_{LL}(udul)_{\tilde{G}}^{ik} &= \frac{4}{3} \frac{g_3^2}{M_{\tilde{G}}} C(\tilde{u}\tilde{d}ul_L)^{MN1k} \\ &\times (\Gamma_{GL}^{(u)})_M^1 (\Gamma_{GL}^{(d)})_N^i H(u_M^{\tilde{G}}, x_N^{\tilde{G}}),\end{aligned}\quad (\text{A28b})$$

$$\begin{aligned}\tilde{C}_{LL}(udul)_{\chi^\pm}^{ik} &= \frac{g_2^2}{M_C^\alpha} [-C(\tilde{u}\tilde{d}ul_L)^{MN1k} (\Gamma_{CL}^{(u)})_N^{\alpha 1} (\Gamma_{CL}^{(d)})_M^{\alpha i} \\ &\times H(x_M^\alpha, u_N^\alpha) + C(\tilde{d}\tilde{v}ud_L)^{Nm1i} (\Gamma_{CL}^{(u)})_N^{\alpha 1} \\ &\times (\Gamma_{CL}^{(l)})_m^{\alpha k} H(u_N^\alpha, z_m^\alpha)],\end{aligned}\quad (\text{A28c})$$

$$\begin{aligned}\tilde{C}_{LL}(udul)_{\chi^0}^{ik} &= \frac{g_2^2}{M_N^\alpha} [C(\tilde{u}\tilde{d}ul_L)^{MN1k} (\Gamma_{NL}^{(u)})_M^{\bar{\alpha} 1} \\ &\times (\Gamma_{NL}^{(d)})_N^{\bar{\alpha} i} H(v_M^{\bar{\alpha}}, y_N^{\bar{\alpha}}) + C(\tilde{u}\tilde{l}ud_L)^{MN1i} \\ &\times (\Gamma_{NL}^{(u)})_M^{\bar{\alpha} 1} (\Gamma_{NL}^{(l)})_N^{\bar{\alpha} k} H(v_M^{\bar{\alpha}}, z_N^{\bar{\alpha}})],\end{aligned}\quad (\text{A28d})$$

$$\begin{aligned}\tilde{C}_{RL}(udul)^{ik} &= \tilde{C}_{RL}^{(6)}(udul)^{ik} + \tilde{C}_{RL}(udul)_{\tilde{G}}^{ik} \\ &+ \tilde{C}_{RL}(udul)_{\chi^\pm}^{ik} + \tilde{C}_{RL}(udul)_{\chi^0}^{ik},\end{aligned}\quad (\text{A29a})$$

$$\tilde{C}_{RL}(udul)_{\tilde{G}}^{ik} = \frac{4}{3} \frac{g_3^2}{M_{\tilde{G}}} C(\tilde{u}\tilde{d}ul_L)^{MN1k} (\Gamma_{GR}^{(u)})_M^1 (\Gamma_{GR}^{(d)})_N^i H(u_M^{\tilde{G}}, x_N^{\tilde{G}}), \quad (\text{A29b})$$

$$\tilde{C}_{RL}(udul)_{\chi^\pm}^{ik} = -\frac{g_2^2}{M_C^\alpha} C(\tilde{u}\tilde{d}ul_L)^{MN1k} (\Gamma_{CR}^{(u)})_N^{\alpha 1} (\Gamma_{CR}^{(d)})_M^{\alpha i} H(x_M^\alpha, u_N^\alpha), \quad (\text{A29c})$$

$$\tilde{C}_{RL}(udul)_{\chi^0}^{ik} = \frac{g_2^2}{M_N^\alpha} [C(\tilde{u}\tilde{d}ul_L)^{MN1k} (\Gamma_{NR}^{(u)})_M^{\bar{\alpha} 1} (\Gamma_{NR}^{(d)})_N^{\bar{\alpha} i} H(v_M^{\bar{\alpha}}, y_N^{\bar{\alpha}}) + C(\tilde{u}\tilde{l}ud_R)^{MN1i} (\Gamma_{NL}^{(u)})_M^{\bar{\alpha} 1} (\Gamma_{NL}^{(l)})_N^{\bar{\alpha} k} H(v_M^{\bar{\alpha}}, z_N^{\bar{\alpha}})], \quad (\text{A29d})$$

$$\tilde{C}_{LR}(udul)^{ik} = \tilde{C}_{LR}^{(6)}(udul)^{ik} + \tilde{C}_{LL}(udul)_{\tilde{G}}^{ik} + \tilde{C}_{LL}(udul)_{\chi^\pm}^{ik} + \tilde{C}_{LL}(udul)_{\chi^0}^{ik}, \quad (\text{A30a})$$

$$\tilde{C}_{LR}(udul)_{\tilde{G}}^{ik} = \frac{4}{3} \frac{g_3^2}{M_{\tilde{G}}} C(\tilde{u}\tilde{d}ul_R)^{MN1k} (\Gamma_{GL}^{(u)})_M^1 (\Gamma_{GL}^{(d)})_N^i H(u_M^{\tilde{G}}, x_N^{\tilde{G}}), \quad (\text{A30b})$$

$$\tilde{C}_{LR}(udul)_{\chi^\pm}^{ik} = \frac{g_2^2}{M_C^\alpha} [-C(\tilde{u}\tilde{d}ul_R)^{MN1k} (\Gamma_{CL}^{(u)})_N^{\alpha 1} (\Gamma_{CL}^{(d)})_M^{\alpha i} H(x_M^\alpha, u_N^\alpha) + C(\tilde{d}\tilde{v}ud_L)^{Nm1i} (\Gamma_{CR}^{(u)})_N^{\alpha 1} (\Gamma_{CR}^{(l)})_m^{\alpha k} H(u_N^\alpha, z_m^\alpha)], \quad (\text{A30c})$$

$$\tilde{C}_{LR}(udul)_{\chi^0}^{ik} = \frac{g_2^2}{M_N^\alpha} [C(\tilde{u}\tilde{d}ul_R)^{MN1k} (\Gamma_{NL}^{(u)})_M^{\bar{\alpha} 1} (\Gamma_{NL}^{(d)})_N^{\bar{\alpha} i} H(v_M^{\bar{\alpha}}, y_N^{\bar{\alpha}}) + C(\tilde{u}\tilde{l}ud_L)^{MN1i} (\Gamma_{NR}^{(u)})_M^{\bar{\alpha} 1} (\Gamma_{NR}^{(l)})_N^{\bar{\alpha} k} H(v_M^{\bar{\alpha}}, z_N^{\bar{\alpha}})], \quad (\text{A30d})$$

$$\tilde{C}_{RR}(udul)^{ik} = \tilde{C}_{RR}(udul)_{\tilde{G}}^{ik} + \tilde{C}_{RR}(udul)_{\chi^\pm}^{ik} + \tilde{C}_{RR}(udul)_{\chi^0}^{ik}, \quad (\text{A31a})$$

$$\tilde{C}_{RR}(udul)_{\tilde{G}}^{ik} = \frac{4}{3} \frac{g_3^2}{M_{\tilde{G}}} C(\tilde{u}\tilde{d}ul_R)^{MN1k} (\Gamma_{GR}^{(u)})_M^1 (\Gamma_{GR}^{(d)})_N^i H(u_M^{\tilde{G}}, x_N^{\tilde{G}}), \quad (\text{A31b})$$

$$\tilde{C}_{RR}(udul)_{\chi^\pm}^{ik} = -\frac{g_2^2}{M_C^\alpha} C(\tilde{u}\tilde{d}ul_R)^{MN1k} (\Gamma_{CR}^{(u)})_N^{\alpha 1} (\Gamma_{CR}^{(d)})_M^{\alpha i} H(x_M^\alpha, u_N^\alpha), \quad (\text{A31c})$$

$$\tilde{C}_{RR}(udul)_{\chi^0}^{ik} = \frac{g_2^2}{M_N^\alpha} [C(\tilde{u}\tilde{d}ul_R)^{MN1k} (\Gamma_{NR}^{(u)})_M^{\bar{\alpha} 1} (\Gamma_{NR}^{(d)})_N^{\bar{\alpha} i} H(v_M^{\bar{\alpha}}, y_N^{\bar{\alpha}}) + C(\tilde{u}\tilde{l}ud_R)^{MN1i} (\Gamma_{NR}^{(u)})_M^{\bar{\alpha} 1} (\Gamma_{NR}^{(l)})_N^{\bar{\alpha} k} H(v_M^{\bar{\alpha}}, z_N^{\bar{\alpha}})], \quad (\text{A31d})$$

$$\tilde{C}_{LL}(uddv)^{ijk} = \tilde{C}_{LL}(uddv)_{\tilde{G}}^{ijk} + \tilde{C}_{LL}(uddv)_{\chi^\pm}^{ijk} + \tilde{C}_{LL}(uddv)_{\chi^0}^{ijk}, \quad (\text{A32a})$$

TABLE I.  $A_{L,R}^{ijk}$  in Eq. (A37) for each nucleon decay mode.  $m_N$  is the nucleon mass  $m_N \approx m_p \approx m_n$  and  $m_{B'}$  is an averaged baryon mass  $m_{B'} \approx m_\Sigma \approx m_\Lambda$ .  $F \approx 0.48$  and  $D \approx 0.76$  are coupling constants for the interaction between baryons and mesons [14,18].

$B_i$	$l_k$	$M_j$		$A_L^{ijk}, A_R^{ijk}$
$p$	$l_k^+$	$\pi^0$	$L$	$\frac{1}{\sqrt{2}}(1+F+D) [\alpha_p \tilde{C}_{RL}(udul)^{1k} + \beta_p \tilde{C}_{LL}(udul)^{1k}]$
			$R$	$-\frac{1}{\sqrt{2}}(1+F+D) [\alpha_p \tilde{C}_{LR}(udul)^{1k} + \beta_p \tilde{C}_{RR}(udul)^{1k}]$
	$\eta^0$	$L$	$\sqrt{\frac{3}{2}}(-\frac{1}{3}+F-\frac{1}{3}D) \alpha_p \tilde{C}_{RL}(udul)^{1k} + \sqrt{\frac{3}{2}}(1+F-\frac{1}{3}D) \beta_p \tilde{C}_{LL}(udul)^{1k}$	
		$R$	$-\sqrt{\frac{3}{2}}(-\frac{1}{3}+F-\frac{1}{3}D) \alpha_p \tilde{C}_{LR}(udul)^{1k} - \sqrt{\frac{3}{2}}(1+F-\frac{1}{3}D) \beta_p \tilde{C}_{RR}(udul)^{1k}$	
	$K^0$	$L$	$(-1 + \frac{m_N}{m_{B'}}(F-D)) \alpha_p \tilde{C}_{RL}(udul)^{2k} + (1 + \frac{m_N}{m_{B'}}(F-D)) \beta_p \tilde{C}_{LL}(udul)^{2k}$	
		$R$	$-(-1 + \frac{m_N}{m_{B'}}(F-D)) \alpha_p \tilde{C}_{LR}(udul)^{2k} - (1 + \frac{m_N}{m_{B'}}(F-D)) \beta_p \tilde{C}_{RR}(udul)^{2k}$	
$\bar{\nu}_k$	$\pi^+$	$L$	$(1+F+D) [\alpha_p \tilde{C}_{RL}(uddv)^{11k} + \beta_p \tilde{C}_{LL}(uddv)^{11k}]$	
	$K^+$	$L$	$(1 - \frac{m_N}{m_{B'}}(F - \frac{1}{3}D)) \alpha_p \tilde{C}_{RL}(dduv)^{12k} + (1 + \frac{m_N}{m_{B'}}(F + \frac{1}{3}D)) \alpha_p \tilde{C}_{RL}(uddv)^{12k}$ $+ (\frac{m_N}{m_{B'}} \frac{2}{3}D) \alpha_p \tilde{C}_{RL}(uddv)^{21k} + (1 + \frac{m_N}{m_{B'}}(F + \frac{1}{3}D)) \beta_p \tilde{C}_{LL}(uddv)^{12k}$ $+ (\frac{m_N}{m_{B'}} \frac{2}{3}D) \beta_p \tilde{C}_{LL}(uddv)^{21k}$	
$n$	$l_k^+$	$\pi^-$	$L$	$(1+F+D) [\alpha_p \tilde{C}_{RL}(udul)^{1k} + \beta_p \tilde{C}_{LL}(udul)^{1k}]$
			$R$	$-(1+F+D) [\alpha_p \tilde{C}_{LR}(udul)^{1k} + \beta_p \tilde{C}_{RR}(udul)^{1k}]$
	$\bar{\nu}_k$	$\pi^0$	$L$	$-\frac{1}{\sqrt{2}}(1+F+D) [\alpha_p \tilde{C}_{RL}(uddv)^{11k} + \beta_p \tilde{C}_{LL}(uddv)^{11k}]$
		$\eta^0$	$L$	$\sqrt{\frac{3}{2}}(-\frac{1}{3}+F-\frac{1}{3}D) \alpha_p \tilde{C}_{RL}(uddv)^{11k} + \sqrt{\frac{3}{2}}(1+F-\frac{1}{3}D) \beta_p \tilde{C}_{LL}(uddv)^{11k}$
	$K^0$	$L$	$(-\frac{m_N}{m_{B'}} \frac{2}{3}D) \alpha_p \tilde{C}_{RL}(dduv)^{12k} + (1 + \frac{m_N}{m_{B'}}(F + \frac{1}{3}D)) \alpha_p \tilde{C}_{RL}(uddv)^{12k}$ $+ (-1 + \frac{m_N}{m_{B'}}(F - \frac{1}{3}D)) \alpha_p \tilde{C}_{RL}(uddv)^{21k} + (1 + \frac{m_N}{m_{B'}}(F + \frac{1}{3}D)) \beta_p \tilde{C}_{LL}(uddv)^{12k}$ $+ (1 + \frac{m_N}{m_{B'}}(F - \frac{1}{3}D)) \beta_p \tilde{C}_{LL}(uddv)^{21k}$	

$$\tilde{C}_{LL}(uddv)_{\tilde{G}}^{ijk} = \frac{4}{3} \frac{g_3^2}{M_{\tilde{G}}} [C(\tilde{u}\tilde{d}d\nu_L)^{MNjk}(\Gamma_{GL}^{(u)})_M^1(\Gamma_{GL}^{(d)})_N^i H(u_M^{\tilde{G}}, x_N^{\tilde{G}}) + C(\tilde{d}\tilde{u}d\nu_L)^{MN1k}(\Gamma_{GL}^{(d)})_M^j(\Gamma_{GL}^{(u)})_N^i H(x_M^{\tilde{G}}, x_N^{\tilde{G}})], \quad (\text{A32b})$$

$$\tilde{C}_{LL}(uddv)_{\chi^\pm}^{ijk} = \frac{g_2^2}{M_\chi^\alpha} [-C(\tilde{u}\tilde{d}d\nu_L)^{MNjk}(\Gamma_{CL}^{(u)})_N^{\alpha 1}(\Gamma_{CL}^{(d)})_M^{\alpha i} H(x_M^\alpha, u_N^\alpha) + C(\tilde{u}\tilde{l}ud_L)^{MN1i}(\Gamma_{CL}^{(d)})_M^{\alpha j}(\Gamma_{CL}^{(u)})_N^{\alpha k} H(x_M^\alpha, w_N^\alpha)], \quad (\text{A32c})$$

$$\tilde{C}_{LL}(uddv)_{\chi^0}^{ijk} = \frac{g_2^2}{M_N^\alpha} [C(\tilde{u}\tilde{d}d\nu_L)^{MNjk}(\Gamma_{NL}^{(u)})_M^{\bar{\alpha} 1}(\Gamma_{NL}^{(d)})_N^{\bar{\alpha} i} H(v_M^{\bar{\alpha}}, y_N^{\bar{\alpha}}) + C(\tilde{d}\tilde{u}d\nu_L)^{MN1k}(\Gamma_{NL}^{(d)})_M^{\bar{\alpha} j}(\Gamma_{NL}^{(u)})_N^{\bar{\alpha} i} H(y_M^{\bar{\alpha}}, y_N^{\bar{\alpha}})$$

$$+ C(\tilde{d}\tilde{v}ud_L)^{Mn1i}(\Gamma_{NL}^{(d)})_M^{\bar{\alpha} j}(\Gamma_{NL}^{(u)})_n^{\bar{\alpha} k} H(y_M^{\bar{\alpha}}, w_n^{\bar{\alpha}}) + C(\tilde{u}\tilde{v}dd_L)^{Mnji}(\Gamma_{NL}^{(u)})_M^{\bar{\alpha} 1}(\Gamma_{NL}^{(d)})_n^{\bar{\alpha} k} H(v_M^{\bar{\alpha}}, w_n^{\bar{\alpha}})], \quad (\text{A32d})$$

$$\tilde{C}_{RL}(uddv)^{ijk} = \tilde{C}_{RL}^{(6)}(uddv)^{ijk} + \tilde{C}_{RL}(uddv)_{\tilde{G}}^{ijk} + \tilde{C}_{RL}(uddv)_{\chi^\pm}^{ijk} + \tilde{C}_{RL}(uddv)_{\chi^0}^{ijk}, \quad (\text{A33a})$$

$$\tilde{C}_{RL}(uddv)_{\tilde{G}}^{ijk} = \frac{4}{3} \frac{g_3^2}{M_{\tilde{G}}} C(\tilde{u}\tilde{d}d\nu_L)^{MNjk}(\Gamma_{GR}^{(u)})_M^1(\Gamma_{GR}^{(d)})_N^i H(u_M^{\tilde{G}}, x_N^{\tilde{G}}), \quad (\text{A33b})$$

$$\tilde{C}_{RL}(udd\nu)_{\chi^\pm}^{ijk} = \frac{g_2^2}{M_C^\alpha} [-C(\tilde{u}\tilde{d}\nu_L)^{MNjk}(\Gamma_{CR}^{(u)})_{N1}^{\alpha 1}(\Gamma_{CR}^{(d)})_{M}^{\alpha i}H(x_M^\alpha, u_N^\alpha) + C(\tilde{u}\tilde{l}ud_R)^{MN1i}(\Gamma_{CL}^{(d)})_{M}^{\alpha j}(\Gamma_{CL}^{(\nu)})_{N}^{\alpha k}H(x_M^\alpha, w_N^\alpha)], \quad (\text{A33c})$$

$$\tilde{C}_{RL}(udd\nu)_{\chi^0}^{ijk} = \frac{g_2^2}{M_N^\alpha} C(\tilde{u}\tilde{d}\nu_L)^{MNjk}(\Gamma_{NR}^{(u)})_{M}^{\bar{\alpha} 1}(\Gamma_{NR}^{(d)})_{N}^{\bar{\alpha} i}H(y_M^{\bar{\alpha}}, y_N^{\bar{\alpha}}), \quad (\text{A33d})$$

$$\tilde{C}_{RL}(ddu\nu)^{ijk} = \tilde{C}_{RL}(ddu\nu)_{\tilde{G}}^{ijk} + \tilde{C}_{RL}(ddu\nu)_{\chi^0}^{ijk}, \quad (\text{A34a})$$

$$\tilde{C}_{RL}(ddu\nu)_{\tilde{G}}^{ijk} = \frac{4}{3} \frac{g_3^2}{M_{\tilde{G}}^\alpha} C(\tilde{d}\tilde{d}u\nu_L)^{MN1k}(\Gamma_{GR}^{(d)})_{M}^i(\Gamma_{GR}^{(d)})_{N}^jH(x_M^{\tilde{G}}, x_N^{\tilde{G}}), \quad (\text{A34b})$$

$$\tilde{C}_{RL}(ddu\nu)_{\chi^0}^{ijk} = \frac{g_2^2}{M_N^\alpha} C(\tilde{d}\tilde{d}u\nu_L)^{MN1k}(\Gamma_{NR}^{(d)})_{M}^{\bar{\alpha} i}(\Gamma_{NR}^{(d)})_{N}^{\bar{\alpha} j}H(y_M^{\bar{\alpha}}, y_N^{\bar{\alpha}}). \quad (\text{A34c})$$

Here,  $\tilde{C}_{RL,LR}^{(6)}$  are contributions from dimension six operators, whose magnitudes are quite small compared to the dimension five contributions for  $B \rightarrow M\bar{\nu}$  decay modes ( $B=p$  or  $n, M=K, \pi$  or  $\eta$ ). Notice that  $C(\tilde{u}\tilde{d}l_{L,R})$  and  $C(\tilde{d}\tilde{l}uu_{L,R})$  in Eq. (A24) do not contribute to the nucleon decay amplitude. The function  $H$  is defined as

$$H(x, y) = \frac{1}{x-y} \left( \frac{x \log x}{x-1} - \frac{y \log y}{y-1} \right), \quad (\text{A35})$$

and the arguments of  $H$  are ratios of SUSY particles' masses (squared):

$$x_{\tilde{G}} = \frac{m_{\tilde{d}_M}^2}{M_{\tilde{G}}^2}, \quad u_{\tilde{G}} = \frac{m_{\tilde{u}_M}^2}{M_{\tilde{G}}^2}, \quad (\text{A36a})$$

$$x_M^\alpha = \frac{m_{\tilde{u}_M}^2}{M_C^{\alpha 2}}, \quad u_M^\alpha = \frac{m_{\tilde{d}_M}^2}{M_C^{\alpha 2}}, \quad z_m^\alpha = \frac{m_{\tilde{\nu}_m}^2}{M_C^{\alpha 2}}, \quad w_M^\alpha = \frac{m_{\tilde{l}_M}^2}{M_C^{\alpha 2}}, \quad (\text{A36b})$$

$$y_M^{\bar{\alpha}} = \frac{m_{\tilde{u}_M}^2}{M_N^{\bar{\alpha} 2}}, \quad y_M^{\bar{\alpha}} = \frac{m_{\tilde{d}_M}^2}{M_N^{\bar{\alpha} 2}}, \quad z_M^{\bar{\alpha}} = \frac{m_{\tilde{l}_M}^2}{M_N^{\bar{\alpha} 2}}, \quad w_m^{\bar{\alpha}} = \frac{m_{\tilde{\nu}_m}^2}{M_N^{\bar{\alpha} 2}}. \quad (\text{A36c})$$

*Nucleon partial decay widths.* The effective quark Lagrangian Eq. (A27) is converted to an effective hadronic Lagrangian with use of the chiral Lagrangian technique (perturbative QCD corrections between the electroweak scale and  $\sim 1$  GeV scale are also taken into account), then partial decay widths of the nucleon are calculated as

$$\Gamma(B_i \rightarrow M_j l_k) = \frac{m_i}{32\pi} \left( 1 - \frac{m_j^2}{m_i^2} \right)^2 \frac{1}{f_\pi^2} (|A_L^{ijk}|^2 + |A_R^{ijk}|^2), \quad (\text{A37})$$

where the lepton mass is neglected only for the kinematics. The expressions for  $A_{L,R}^{ijk}$  are listed in Table I.

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