

Electric dipole moments do not require the CP -violating phases of supersymmetry to be small

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We report the first fully general numerical calculation of the neutron and electron dipole moments, including the seven significant phases. We find that there are major regions in the parameter space where none of the phases are required to be small, contrary to the conventional wisdom. The electric dipole moments do provide useful constraints, allowing other regions of parameter space to be carved away. We keep all superpartner masses light, and so agreement with experimental limits arises purely from interesting relations among soft breaking parameters. [S0556-2821(99)02811-8]

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I. INTRODUCTION

The general parametric structure of the minimal supersymmetric standard model (MSSM) includes a large number of CP -violating phases. Their presence has largely been ignored in phenomenological analyses because of severe constraints imposed on individual phases by the experimental upper limits for electron and neutron electric dipole moments (EDM's) if other phases are set to zero. These limits would generally constrain the phases considered individually to be less than 10^{-2} unless the mass parameters are pushed beyond the TeV scale [1]. Recently, it has been emphasized [2], however, that cancellations between different terms contributing to the dipole moments can allow for values of the phases very different from zero even when the superpartner masses are relatively light. Since this can have important consequences [3] for extraction of the parameters in the supersymmetric (SUSY) Lagrangian from experimental data, for calculation of dark matter densities and scattering cross sections, for baryogenesis, for Higgs boson limits, and more, it is rather important to study the problem of constraints on the complex phases without making any unnecessary simplifying assumptions based on theoretical prejudice. To put it differently, the phases may or may not actually be small. We must find out from data, without making assumptions that lead to excluding regions of parameter space where parameters are large. If the phases are large, they affect many CP -conserving quantities throughout particle physics; so it is even more important to proceed carefully. The phases can only be large if certain approximate relations among soft breaking parameters hold; these relations will be checked in future experiments. The relevant relations are not fine-tuned, but are quite reasonable, with various soft breaking parameters related in size and sign to one another.

Some important results have already been reported in the literature. Ibrahim and Nath have presented [2] some of the formulas needed in the analysis and calculated the EDM's in the framework of minimal supergravity model. Falk *et al.* [4] have analyzed the case of two phases and confirmed the analysis of Ibrahim and Nath; they also applied it to the calculation of neutralino relic density and detection rate. Some similar results were reported in Ref. [3] and phenom-

ological consequences of CP -violating phases in the MSSM were studied in [5].

In this paper we want to address this issue in its entirety in order to establish a connection between the usual parameters of the MSSM Lagrangian and ranges of the phases allowed by experimental data on the electric dipoles. We work in the framework of the simplest possible model, neglecting the flavor mixing but avoiding any assumptions about unification of the soft breaking parameters. We use light superpartner masses; so apart from relations among soft breaking parameters the resulting EDM's would be very large.

II. PHASE STRUCTURE OF THE FULL MSSM

We define the MSSM to be the supersymmetric theory with the same particles as the standard model (SM) plus their superpartners, the SM gauge group, two Higgs doublets, and conserved R parity.

The MSSM Lagrangian [6,7] depends on a total of 126 parameters and it includes three well-known sources of CP -violating phases. The first is related to the two Higgs doublets present in the model since both the μ parameter in the superpotential and the soft breaking parameter b can be complex and their phases are denoted φ_μ and φ_b respectively. Three more phases, φ_1 , φ_2 , and φ_3 , enter through the complex masses of the gauginos associated with the standard gauge groups. Finally, most of the phases originate in the flavor sector of the Lagrangian, in either the scalar soft mass matrices $\mathbf{m}_{Q,\bar{u},\bar{d},L,\bar{e}}^2$ or the trilinear matrices $\mathbf{a}_{u,d,e}$. The mass matrices are Hermitian so only off-diagonal terms can be complex but the trilinear matrices are general 3×3 matrices, allowing for the diagonal entries to also be complex.

The impact of the phases associated with the off-diagonal terms on experimental observables is suppressed by the same mechanism which is required to suppress the existence of large flavor changing neutral effects and for the purposes of this study all these phases can be neglected; if some of them matter, it will only strengthen our results. We assume that all the scalar soft mass matrices and trilinear parameters are flavor diagonal and that the complex trilinear terms are proportional to the corresponding Yukawa couplings a_f

$=y_f A_f e^{i\varphi_{A_f}}$, where φ_{A_f} are the relevant phases.¹

It is important to realize that not all of the listed phases are physical. Above the electroweak symmetry breaking scale, the Lagrangian possesses two partial $U(1)$ symmetries which can be promoted to full symmetries by treating the dimensionful parameters as spurions charged under those symmetries [7]. Under an R symmetry the Grassmann variable θ ($\bar{\theta}$) is charged $+1$ (-1) and therefore this symmetry distinguishes between component fields of the superfields. If the charge of a chiral superfield is r , its scalar component field ϕ transforms under R symmetry with charge r , the fermionic field ψ has charge $r-1$ and the auxiliary F scalar field possesses charge $r-2$. In order to preserve the R invariance of the superpotential it is convenient to choose $r=1$ for the matter superfields and $r=0$ for the Higgs superfields. The advantage of this choice is also clear from the fact that R symmetry defined in this way is not broken in the process of electroweak symmetry breaking. The vector superfields are not charged under R symmetry, and so only the gaugino component field λ ($\bar{\lambda}$) obtains a charge of $+1$ (-1). It is clear that to preserve R symmetry as a full symmetry of the superpotential and also of the soft SUSY breaking terms in the Lagrangian, μ , A_f and the gaugino masses M_i have to be charged under the R symmetry.

The whole MSSM Lagrangian with the exception of the μ term in the superpotential and the b soft breaking term is also invariant under a Peccei-Quinn (PQ) symmetry. This symmetry transforms the Higgs fields with charge -2 and the matter fields $Q, \bar{u}, \bar{d}, L, \bar{e}$ with charge $+1$. Again, if μ and b are treated as spurions, full symmetry is restored above the electroweak scale. Below this scale, the PQ symmetry is broken as the Higgs fields acquire vacuum expectation values. Physical observables can only depend on such combinations of parameters which are invariant under all symmetries of the Lagrangian. For the unbroken theory we have two symmetries and therefore two conditions, allowing us to eliminate two phases. When electroweak symmetry is broken we are left with only one unbroken symmetry, but the phase of b , which is related to the phase of the Higgs vacuum expectation values (VEV's), can be absorbed into the physical Higgs fields by appropriate redefinition. It is therefore natural to take b (and the VEV's) to be real and set one more phase to be zero. We prefer in this paper to take $\varphi_2=0$, thus explicitly violating reparametrization invariance but one has to keep in mind that all other parameter choices are related to our choice by an R transformation. A fully reparametrization invariant approach to CP -violation in SUSY theories will be discussed elsewhere [8]. Our numerical results do not depend in any way on this simplification.

Taking into account our parametrization choice, the final set of phases considered in the discussion of the electron and neutron electric dipole moments includes three phases appearing in the chargino-neutralino-gluino sector, namely φ_1 ,

φ_3 and φ_μ , and four phases φ_{A_u} , φ_{A_d} , φ_{A_l} and φ_{A_e} corresponding to the trilinear soft breaking parameters relevant in the dipole moment calculation as discussed in the following section. As we will see below, even though a_u , a_d , a_e are proportional to small Yukawa couplings, their phases enter because contributions to the EDM's require a chirality flip leading to dipole moments' proportionality to the relevant mass.

III. ELECTRIC DIPOLE MOMENT CALCULATION

The electric dipole interaction of a spin-1/2 particle f with an electromagnetic field is described by an effective Lagrangian

$$\mathcal{L}_{EDM} = -\frac{i}{2} d_f \bar{f} \sigma^{\mu\nu} \gamma_5 f F_{\mu\nu}. \quad (3.1)$$

In theories with CP -violating interactions, the electric dipole d_f receives contributions from loop diagrams. The best way to account for such contributions is to use the effective theory approach in which the heavy particles are decoupled at some large scale Q and the full theory is matched with an effective theory including a full set of CP -violating operators [9–11]. If we restrict ourselves to dimension 5 and 6 operators, the effective Lagrangian takes the form

$$\mathcal{L}_{eff} = \sum_{i=1}^3 C_i(Q) O_i(Q) \quad (3.2)$$

where the $C_i(Q)$ are Wilson coefficients evaluated at scale Q , and the O_i are the three considered operators

$$O_1 = -\frac{i}{2} \bar{f} \sigma^{\mu\nu} \gamma_5 f F_{\mu\nu}, \quad (3.3)$$

$$O_2 = -\frac{i}{2} \bar{f} \sigma^{\mu\nu} \gamma_5 T^a f G_{\mu\nu}^a, \quad (3.4)$$

$$O_3 = -\frac{1}{6} f_{abc} G_{\mu\rho}^a G_{\nu\sigma}^{b\rho} G_{\lambda\sigma}^c \epsilon^{\mu\nu\lambda\sigma}. \quad (3.5)$$

It is obvious that all three operators contribute when the external fermionic particles are quarks, while in the case of the electron C_2^e and C_3^e are identically zero.

Supersymmetric models contribute to the Wilson coefficients at the one loop level and they include several types of graphs as shown in Fig. 1. Chargino (\tilde{C}), neutralino (\tilde{N}) and gluino (\tilde{g}) loops where the second particle in the loop is a scalar superpartner, either a slepton or a squark, contribute to C_1 and C_2 coefficients depending on whether a photon or a gluon is radiated. Charginos and neutralinos interact either through their gaugino (λ) or Higgsino (\tilde{H}) components and the sfermions are represented by their left or right (L, R) chiral components. The contributions can be calculated at the electroweak scale since a typical SUSY scale in most models

¹In further text we take (unless explicitly stated otherwise) all the dimensionful parameters to be real and positive; their phases are always written explicitly.

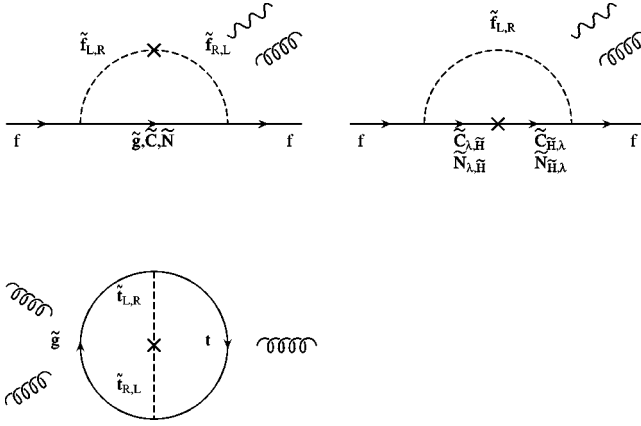


FIG. 1. One and two loop Feynman diagrams contributing to the calculation of the electric dipole moments in the MSSM. The graphs involve exchange of gluinos \tilde{g} , charginos \tilde{C} , neutralinos \tilde{N} and corresponding flavors of sfermions \tilde{f} . Charginos and neutralinos interact either through their gaugino (λ) or Higgsino (\tilde{H}) components and the sfermions are represented by their left or right (L, R) chiral components. The gluon and photon line can originate on any internal leg carrying corresponding charge.

is of the same order of magnitude. For the gluino loop contribution to the quark EDM the matching gives

$$C_1^{qk-\tilde{g}}(Q) = -\frac{2}{3} \frac{e\alpha_S}{\pi} \sum_{i=1}^6 \text{Im}(\Delta_i^{qk-\tilde{g}}) \frac{m_{\tilde{g}}}{m_i^2} B\left(\frac{m_{\tilde{g}}^2}{m_i^2}\right), \quad (3.6)$$

$$C_2^{qk-\tilde{g}}(Q) = \frac{g_S\alpha_S}{4\pi} \sum_{i=1}^6 \text{Im}(\Delta_i^{qk-\tilde{g}}) \frac{m_{\tilde{g}}}{m_i^2} C\left(\frac{m_{\tilde{g}}^2}{m_i^2}\right). \quad (3.7)$$

The neutralino and chargino loops contribute both to the electron and quark electric dipole moments and one finds

$$C_1^{fk-\tilde{N}}(Q) = \frac{e\alpha}{4\pi \sin^2\theta_W} Q_f \sum_{i=1}^6 \sum_{j=1}^4 \text{Im}(\Delta_{ij}^{fk-\tilde{N}}) \frac{m_{\tilde{N}_j}}{m_i^2} B\left(\frac{m_{\tilde{N}_j}^2}{m_i^2}\right), \quad (3.8)$$

$$C_2^{qk-\tilde{N}}(Q) = \frac{g_S g^2}{16\pi^2} \sum_{i=1}^6 \sum_{j=1}^4 \text{Im}(\Delta_{ij}^{qk-\tilde{N}}) \frac{m_{\tilde{N}_j}}{m_i^2} B\left(\frac{m_{\tilde{N}_j}^2}{m_i^2}\right) \quad (3.9)$$

and

$$C_1^{fk-\tilde{C}}(Q) = -\frac{e\alpha}{4\pi \sin^2\theta_W} \sum_{i=1}^6 \sum_{j=1}^2 \text{Im}(\Delta_{ij}^{fk-\tilde{C}}) \frac{m_{\tilde{C}_j}}{m_i^2} \times \left[Q_f' B\left(\frac{m_{\tilde{C}_j}^2}{m_i^2}\right) + (Q_f - Q_f') A\left(\frac{m_{\tilde{C}_j}^2}{m_i^2}\right) \right], \quad (3.10)$$

$$C_2^{qk-\tilde{C}}(Q) = -\frac{g_S g^2}{16\pi^2} \sum_{i=1}^6 \sum_{j=1}^2 \text{Im}(\Delta_{ij}^{qk-\tilde{C}}) \frac{m_{\tilde{C}_j}}{m_i^2} B\left(\frac{m_{\tilde{C}_j}^2}{m_i^2}\right). \quad (3.11)$$

In Eqs. (3.7)–(3.12), m_i are the masses of the corresponding scalar particle running in the loop and A , B and C are the loop functions obtained by integrating out the heavy particles in the loop. These functions, together with the vertex Δ functions calculated in our phase parametrization, can be found in the Appendix. Q_f denotes the electric charge of the external fermion and Q_f' is the charge of internal sfermion when different from Q_f .

The gluonic operator O_3 obtains a contribution from the top-quark–top-squark loop with a gluino exchange as shown in Fig. 1 and one has

$$C_3^f(Q) = -3\alpha_S m_t \left(\frac{g_S}{4\pi}\right)^2 \text{Im}(\Delta_2^{u_3-\tilde{g}}) \frac{m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2}{m_g^5} \times H\left(\frac{m_{\tilde{t}_1}^2}{m_g^2}, \frac{m_{\tilde{t}_2}^2}{m_g^2}, \frac{m_t^2}{m_g^2}\right), \quad (3.12)$$

where the loop function H can be found in the Appendix.

The Wilson coefficients then have to be evolved from the decoupling scale Q down below the chirality breaking scale Λ_χ using the renormalization group equations (RGE's) in order to account for resummation of the logarithmic corrections. So far, only QCD corrections for quark operator Wilson coefficients have been estimated [12] for RGE evolution down from the electroweak scale to Λ_χ , giving

$$C_i^q(\Lambda_\chi) = \eta_i C_i^q(Q), \quad (3.13)$$

where $\eta_1 \approx 1.53$ and $\eta_2 \approx \eta_3 \approx 3.4$. All other corrections are neglected in our calculation. At the low scale, the CP -violating operators O_i have to be projected on the electric dipole operator to evaluate their contribution to the numerical value of the electric dipole. This is a complicated task since the chirality breaking scale $\Lambda_\chi = 1.18$ GeV is very close to the QCD scale and perturbative methods are not reliable in this region. The best thing one can do at present is to use naive dimensional analysis [13] which yields

$$d_f = C_1^f(\Lambda_\chi) + \frac{e}{4\pi} C_2^f(\Lambda_\chi) + \frac{e\Lambda_\chi}{4\pi} C_3^f(\Lambda_\chi). \quad (3.14)$$

Finally, since the neutron is a composite particle, one has to use a phenomenological neutron model to calculate the neutron EDM from the moments of the constituting quarks. From the simple $SU(6)$ quark model one obtains

$$d_n = \frac{1}{3}(4d_d - d_u), \quad (3.15)$$

where d_d and d_u are the EDM's of the down and up quarks respectively.

One of the important features of the contributions to the EDM is the fact that the effective Lagrangian in Eq. (3.1) requires different chirality of the initial and final particles. In the supersymmetric diagrams this can happen in two ways — either the exchanged squark or slepton change chirality via L - R mixing terms in the sfermion mass² matrices and couple to the gaugino component of the intermediate spin-1/2 particle, or the L and R sfermions preserve their chirality and couple to the Higgsino components of charginos or neutralinos. As a result, all contributions are directly proportional to the mass of the external particle since both the L - R mixing sfermion mass term and the Higgsino-fermion-sfermion coupling are proportional to the relevant Yukawa coupling. Another consequence of the chirality flip is the explicit proportionality of the contributions to the mass of the intermediate spin-1/2 particle.

IV. CONSTRAINTS ON THE PHASES

As already mentioned, we present a numerical treatment of the electric dipole moment calculation, with the main emphasis on the cancellations between various contributions to the Wilson coefficients. This allows large values of the phases to give contributions consistent with the experimental bounds on the values of the electric dipole moment of both the electron and the neutron. Current experimental limits for the neutron require that [14]

$$|d_n| < 1.1 \times 10^{-25} \text{ e cm} \quad (4.1)$$

and, for the electron [15],

$$|d_e| < 4.3 \times 10^{-27} \text{ e cm} \quad (4.2)$$

at 95% confidence level.

We start our analysis by choosing a simple set of MSSM parameters which leads to a fairly light spectrum of the superpartners while still keeping the general set of the seven relevant CP -violating phases, φ_1 , φ_3 , φ_μ , φ_{A_u} , φ_{A_d} , φ_{A_t} and φ_{A_e} , which we consider. Since our results do not assume heavy spectrum suppression of the CP -violation effects, they are fairly general in the sense that increasing the masses of the superpartners can only broaden the effect of the cancellation between different contributions to the electric dipole moment. The resulting ranges of the phases for different spectra will differ quantitatively from our examples but the general observation that the phases indeed do not have to be small will still remain valid. To simplify the set of parameters we assume that the squark and slepton soft masses and the trilinear soft parameters are flavor diagonal. We also take the diagonal entries of these matrices to be universal for all three generations and neglect any splitting between up-type and down-type right-handed squark masses. Similarly, for the sneutrinos we consider a single universal soft mass for all three flavors. The trilinear parameter A is assumed to be not only flavor universal but also the same for both sleptons and squarks. As a result, we are left with the following set of soft parameters in the scalar flavor sector: $m_{\tilde{\nu}}$, $m_{\tilde{\chi}_L}$, $m_{\tilde{\chi}_R}$, $m_{\tilde{q}_L}$, $m_{\tilde{q}_R}$ and A . We do not assume any relation between the

TABLE I. Standard set of parameters (values at EW scale).

$M_1 = 75 \text{ GeV}$	$m_{\tilde{\nu}} = 185 \text{ GeV}$
$M_2 = 85 \text{ GeV}$	$m_{\tilde{\chi}_L} = 195 \text{ GeV}$
$M_3 = 250 \text{ GeV}$	$m_{\tilde{\chi}_R} = 225 \text{ GeV}$
$\mu = 450 \text{ GeV}$	$m_{\tilde{q}_L} = 340 \text{ GeV}$
$A = 250 \text{ GeV}$	$m_{\tilde{q}_R} = 360 \text{ GeV}$
$\tan \beta = 1.2$	$m_A = 300 \text{ GeV}$

gaugino masses other than taking $M_1 < M_2 < M_3$. Unless stated otherwise, all of our calculations consistently employ a common set of parameters shown in Table I

We have varied them sufficiently to show that our results are qualitatively unchanged for significant regions of these parameters.

A. Electron EDM

Let us now concentrate on discussing the cancellation mechanism in the two cases of the EDM calculation. The electron EDM limits are more constraining than the neutron EDM limits and are also simpler to study since there are only two large contributions. The electron EDM calculation involves only the chargino and neutralino contribution to the C_1 Wilson coefficient corresponding to the electric dipole moment operator and only three phases enter the calculation, namely φ_μ , φ_1 and φ_{A_e} . Since the neutralinos are mixtures of both $U(1)$ and $SU(2)$ gauginos and both neutral Higgsinos, the neutralino contribution includes both types of chirality flipping processes — from the gaugino exchange with L - R slepton mixing as well as from the process with gaugino-Higgsino mixing and requiring no chirality flip in the slepton sector. On the other hand, chargino exchange can only proceed through the latter channel since the chargino $SU(2)$ gaugino component only couples to left-handed fields. In order for a cancellation between these two contributions to occur, certain conditions have to be met.

The first condition requires that the two contributions have opposite sign over at least a subset of the phase parameter space. In fact, this requirement is automatically satisfied for contributions coming from the gaugino-Higgsino mixing diagrams. These contributions, involving both charginos and neutralinos, depend on φ_μ and have opposite sign over the whole range of φ_μ due to the fact that the μ parameter enters the neutralino and chargino mass matrices with opposite phase. This “fortunate” feature can be traced back to the antisymmetry of the $SU(2)$ metric ϵ appearing in the superpotential.

The neutralino contribution of this type could in principle also depend on φ_1 , which would upset the exact anticoincidence of the signs. In practice, the dominant part of the contribution comes from the $SU(2)$ gaugino-Higgsino mixing and the effect of φ_1 is constrained to a shift in the phase of the neutralino contribution from the gaugino-Higgsino interaction. The gaugino-gaugino diagram for the neutralino contribution, unlike the gaugino-Higgsino diagrams, involves L - R mixing in the selectron sector. The imaginary part of the relevant phase dependent term is

$$\text{Im}\left(\frac{m_{LR}^2}{m_{LL}^2}N_{1j}^{*2}\right) \simeq -\frac{1}{m_e^2}m_e[A_e\sin(\varphi_1+\varphi_{A_e}) + \mu\tan\beta\sin(\varphi_\mu-\varphi_1)], \quad (4.3)$$

where m_{LR}^2 and m_{LL}^2 are elements of the selectron mass matrix. In order for this expression to have the opposite sign to the chargino contribution, which is negative for $0 < \varphi_\mu < \pi$ and positive for $\pi < \varphi_\mu < 2\pi$, various possibilities occur depending on the relative sizes of A_e and $\mu \tan \beta$. For example, in the two limiting cases where $|\mu \tan \beta| \gg |A_e|$ and $|A_e| \gg |\mu \tan \beta|$, we get in terms of the phases that, modulo 2π , we have to impose $\varphi_\mu - \varphi_1 \sim -\varphi_\mu$ and $\varphi_1 + \varphi_{A_e} \sim -\varphi_\mu$ respectively. In general this contribution is opposite in sign to the chargino one over a significant part of parameter space.

The second condition of cancellation requires the chargino and neutralino contributions to be of the same magnitude. In the chargino sector, the gaugino-Higgsino mixing involves only one type of gaugino while in the neutralino sector there are two gaugino states and therefore the elements of the neutralino diagonalizing matrix N generally yield smaller imaginary parts than the elements of the chargino matrices U, V . Moreover, the chargino contribution is enhanced due to larger values of the $A(x)$ loop function as compared to the $B(x)$ function in the neutralino expression. This comes from the fact that the photon in the chargino loop diagram is emitted from the fermionic leg of the corresponding diagram as opposed to the scalar leg in the neutralino diagram. Both these effects decrease the relative magnitude of the neutralino contribution compared to the chargino contribution. On the other hand, in the neutralino case the gaugino-gaugino contribution can balance some of the difference between the two contributions. For that to happen, it is important that the relative size of the chargino-Higgsino contribution decrease and the relative size of the gaugino-gaugino contribution increase, which can be achieved by increasing μ . That brings us back to the first condition of opposite sign which can be satisfied for $|\mu| \tan \beta \gg |A_e|$ if $\varphi_1 \sim \pi$.

As a result, for suitable combinations of the dimensionful parameters an almost exact cancellation can occur for the whole range of φ_μ as exemplified in Fig. 2a. In this plot we chose $\mu = 700$ GeV in addition to our standard set of parameters and φ_1 was set to be equal to π . The values of φ_{A_e} were varied randomly, leading to the result that the values of the neutralino contribution and of the total dipole moment form bands of non-zero width, while the chargino contribution is independent of φ_{A_e} . It is clear from the plot, however, that virtually all values of φ_μ would be allowed for this particular set of parameters depending only on a suitable choice of the φ_{A_e} value range. This is also significant because φ_{A_e} is otherwise irrelevant not only in the neutron EDM calculation but also in most other phenomenological considerations. Later figures show effects of varying μ . Note that without cancellations one would have to have each contribution reduced by $\sim 10^{-2}$; i.e., each phase would have to be $\lesssim 10^{-2}$, as in the usual result.

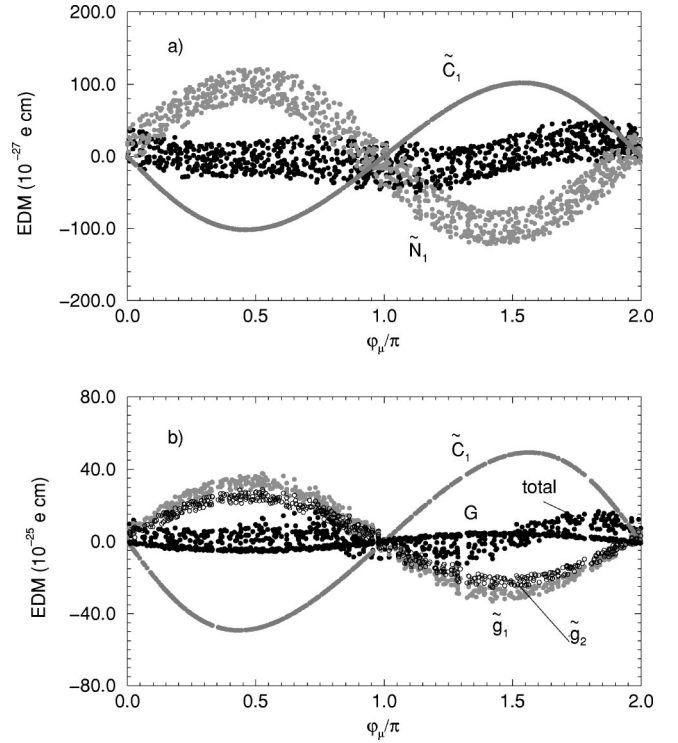


FIG. 2. Illustration of the cancellation mechanism in the EDM calculation. See the discussion in the text. Frame (a) includes the contributions to the electron dipole moment arising from neutralino and chargino loops contributions to the C_1 Wilson coefficient for varying φ_μ , $\varphi_1 \sim \pi$ and values of φ_{A_e} sampled randomly. A standard set of parameters (see text) with $\mu = 700$ GeV was used. Frame (b) shows the neutron EDM contribution from the gluino loop graph projection into C_1 and C_2 (\tilde{g}_1 , and \tilde{g}_2), from the chargino loop contribution to C_1 (\tilde{C}_1) and from the gluino-top-quark-top-squark graph contributing through the purely gluonic operator Wilson coefficient $C_3(G)$. In this case, the standard set of parameters is adopted with $\mu = 300$ GeV, $\varphi_3 \sim \pi$ and $\varphi_{A_q} \sim 0$ for $q = u, d, t$. In both cases the natural cancellations can give a total of order the experimental limits for most or all of φ_μ . If the cancellation effects were not included one would conclude that all phases would have to be of order 10^{-2} to not exceed the experimental limits.

Thus we see that the strong constraint from the electron EDM limit is naturally satisfied over a significant part of the parameter space, though not all of it. The neutralino and chargino contributions can automatically have opposite sign and the same magnitude for most of the φ_μ range when the mass parameters are in certain ratios depending on the other phases. Although not all the values of the total electron EDM in Fig. 2a satisfy the experimental constraint, they all fall within a band of values less than 12 times the 1σ upper bound on the electron EDM magnitude. Therefore the amount of tuning, which can be taken as the ratio of the actual bandwidth to the 2σ experimental margin, is about 6 in this case. That is significantly less than the tuning factors of 100 and more required by constraining the values of the phases to be less than 10^2 . While the cancellations do require related magnitudes of some parameters, and could thus be interpreted as a fine-tuning, we think that the required mass

relations are typically the kind of relations that might arise in a theory of the soft breaking parameters, and are likely to be a clue to the form of the theory. The resulting relations are predictions that can be tested in other experiments.

To put it in another way, there are two ways to satisfy the electron EDM constraints. One possibility is that the phases are small or zero as a result of some presently unknown mechanism. Alternatively, the phases could be large, and the masses could have certain approximate ranges of reasonable values. The relevant signs would automatically give the needed cancellation, which need not have happened. The two alternatives lead to very different predictions for many other observables. The naturalness of the cancellation that occurs leads us to consider the solution with large phases seriously enough to convince us to analyze the full parameter space and to study the resulting predictions, which we will report on later.

B. Neutron EDM

Next we turn to the neutron EDM, where cancellations are easier to obtain. First of all, all three operators in Eq. (3.2) receive contributions from the MSSM one loop diagrams involving quarks as incoming and outgoing particles. The gluino-squark diagram projects on both O_1 and O_2 operators, and the contribution of the relevant Wilson coefficients $C_1^{q_k-\tilde{g}}$ and $C_2^{q_k-\tilde{g}}$ to the EDM is numerically comparable. The contribution of $C_2^{q_k-\tilde{g}}$ is seemingly suppressed by the factor of $e/4\pi$ in Eq. (3.15) compared to $C_1^{q_k-\tilde{g}}$, but that is compensated by enhancements from the factor of g_S/e , and mainly from the loop function $C(x)$ in the matching conditions (3.7) and (3.8). This is again a consequence of the fact that the gluino leg in the diagram can emit gluons but not photons. The chargino loop $C_1^{q_k-\tilde{C}}$ contribution is typically of the same order as the gluino loop contributions while $C_2^{q_k-\tilde{C}}$ contributes negligibly since in this case the g_S/e enhancement alone does not overcome the suppression from $e/4\pi$. Both neutralino contributions from $C_1^{q_k-\tilde{N}}$ and $C_2^{q_k-\tilde{N}}$ can be safely neglected in the neutron dipole analysis. Reasons similar to those for the electron case lead to a suppression of $C_1^{q_k-\tilde{N}}$ compared to $C_1^{q_k-\tilde{C}}$, but in the quark case this effect is more pronounced since the squarks are typically heavier than the sleptons and they have fractional charges. Correspondingly, the contribution from $C_2^{q_k-\tilde{N}}$ is even smaller than that from $C_1^{q_k-\tilde{N}}$. This effectively reduces the number of phases by eliminating φ_1 as one of the parameters numerically relevant in the neutron EDM calculation.

As in the electron case, it is necessary that the chargino contribution be opposite in sign to the sum of the other three contributions for the cancellation to occur. The gluino contribution exhibits the same behavior as the gaugino part of the neutralino contribution in the electron case and Eq. (4.3) transforms into

$$\text{Im}\left(\frac{m_{LR}^2}{m_{LL}^2}G^{*2}\right) \simeq -\frac{1}{m_c^2}m_q[A_q\sin(\varphi_3+\varphi_{A_q}) + \mu f(\beta)\sin(\varphi_\mu-\varphi_3)], \quad (4.4)$$

where $A_q=A_u, A_d$ and $f(\beta)=\cot\beta, \tan\beta$ for up and down type quarks respectively. The contribution from the pure gluonic operator, on the other hand, depends only on φ_3 , φ_μ and φ_{A_i} as far as phases are concerned and the role of φ_μ and φ_{A_i} is again determined by the relative size of $\mu\cot\beta$ and A_i . This implies that φ_3 and φ_μ are the crucial phases in the EDM calculation. In order to demonstrate the cancellation on a practical example, in Fig. 2b we set φ_3 equal to π and take all three trilinear parameter phases to be consistent with zero. All these choices are enforced within a small variation around the central value, leading to a non-zero width of the gluino and pure gluonic contribution. In addition we choose $\mu=300$ GeV so that it is comparable in magnitude to $A_q=250$ GeV and the off diagonal squark mixing terms get a comparable contribution from both terms in Eq. (4.4). The resulting sum total of the neutron EDM is consistent with zero over a wide range of φ_μ . As φ_3 , φ_{A_u} , φ_{A_d} and φ_{A_t} are varied this situation will persist for large but correlated ranges of these phases. The variation with μ is shown in later figures. Similarly to the electron case, the amount of fine-tuning required to satisfy the experimental constraint can be estimated by taking the largest sizes in the resulting band to the 2σ experimental bound to be around 8 or so, again considerably less than in the case without cancellations.

C. Numerical results

The effects of the cancellation mechanism on the ranges of phases allowed by the EDM experimental limits can be explored by varying all phases randomly for a given set of mass parameters and plotting the allowed points projected on planes in the phase parameter space. In Figs. 3 and 3b we show the allowed regions for the standard parameter set with $\mu=450$ GeV in the φ_μ - φ_1 and φ_μ - φ_3 planes, respectively. The solid circles signify the points allowed by electron EDM constraints and the open circles stand for those allowed by the neutron EDM limits. The φ_1 dependence has little significance as far as the neutron constraints are concerned, while in the electron case φ_1 has to be correlated with φ_μ in order to satisfy the limits. Only a selected band of the values of φ_μ is allowed by the electron constraints and the neutron constraint imposes a correlation between the values of φ_3 and φ_μ within this band. Still, when these conditions are satisfied, values of φ_μ very different from 0 or π are allowed. All values of φ_1 and φ_3 can occur while the EDM limits are respected.

In Fig. 4 we display the same results as in Fig. 3 but we take $\mu=60$ GeV. The range of φ_μ is constrained by both electron and neutron limits in this case, and the interval allowed by both is significantly narrower than in the previous case. Nevertheless, all values of φ_1 and φ_3 are permitted again.

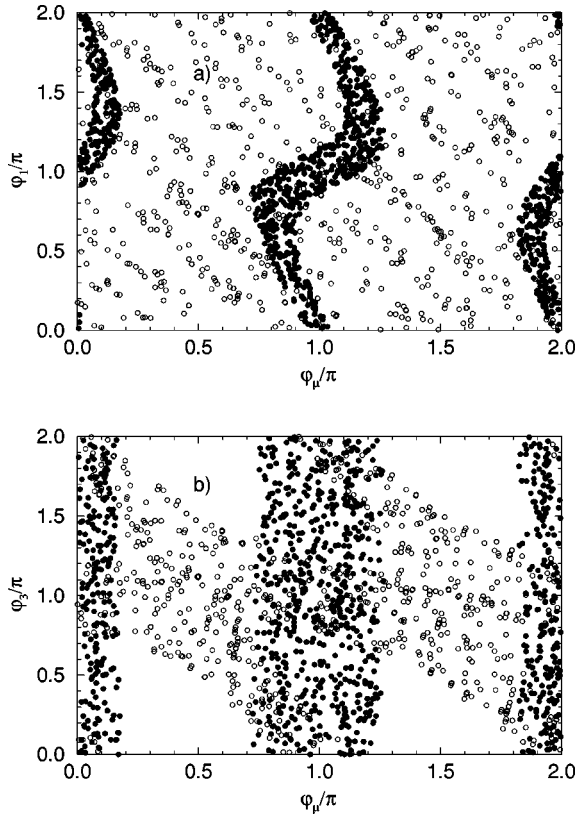


FIG. 3. Plots of regions allowed by the electron (solid circles) and neutron (open circles) EDM limits in the $\varphi_\mu - \varphi_1$ plane (a) and the $\varphi_\mu - \varphi_3$ plane (b). A value of $\mu = 450$ GeV was chosen together with the standard parameter set and all phases were sampled randomly. Regions satisfying both electron and neutron EDM limits are shown as overlaps of the regions allowed by the individual constraints.

It is important to see how the range of allowed values of φ_μ depends on μ because φ_μ plays a crucial role in the electron as well as in the neutron EDM calculation. Fig. 5a displays this range for both calculations with the standard parameter set and varying μ . The overall trend shows that for larger values of μ it is easier to satisfy the EDM limits. The *b* frame shows the effects of A_e variation on the electron EDM constraints when $\mu = 450$ GeV. Similarly, in Fig. 6 we examined the dependence of the allowed φ_μ range on an overall scaling parameter x which rescales all the dimensionful parameters in the standard set and $\mu = 450$ GeV according to the formula $M' = xM$. It is interesting to note that in order to allow the full range of φ_μ one has to go to fairly large parameters $x > 4$ while the same effect can be obtained by raising μ to be larger than 450 GeV.

Finally, in Figs. 7a and 7b we plot the lightest neutralino mass vs φ_μ and φ_1 respectively for the standard parameter set and μ varied from 50 GeV to 800 GeV. The neutralino masses can vary quite dramatically in the allowed regions and this fact substantially affects phenomenological observables at colliders and cosmological implications of the supersymmetric model.

We have chosen one particular value of $\tan\beta = 1.2$ for our presentation of the cancellation mechanism, but we have ex-

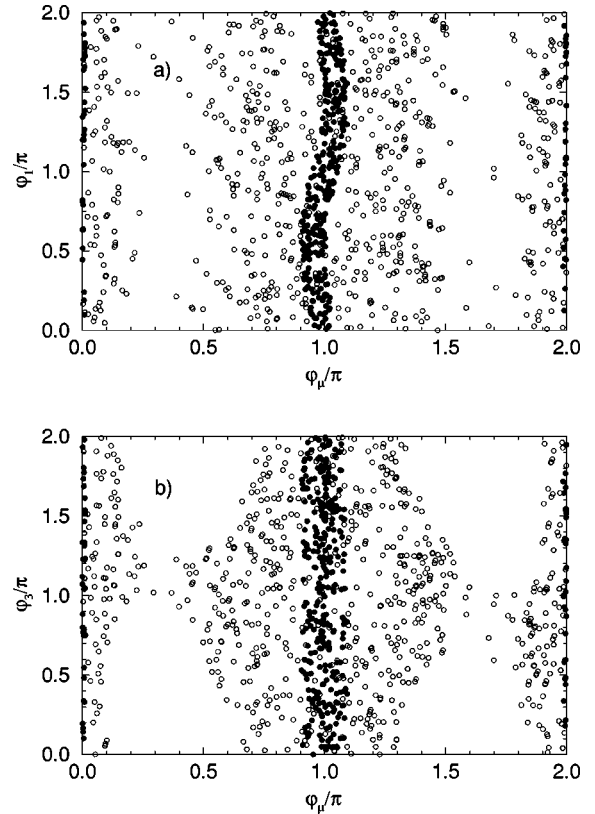


FIG. 4. Same as Fig. 3, but for $\mu = 60$ GeV and the standard set of parameters. Again, all phases were varied randomly.

amined the general case. If $\tan\beta$ is increased to larger values, the mechanism is still in place if other conditions discussed in this section are satisfied but the magnitude of individual contributions (e.g. the chargino and neutralino contribution in the electron case) increases. The effect of the cancellation remains qualitatively the same and large values of the phases can be achieved.

V. CONCLUSION

We have shown that the role of the cancellation mechanism in the calculation of the electron and neutron electric dipole moments within the general framework of the MSSM including a non-restricted set of *CP*-violating phases has crucial consequences for the range of individual phases. Even with a light sparticle spectrum, phases can have values very different from zero and still satisfy experimental bounds on the values of the electron and neutron EDM's.

A trivial but possible way to avoid constraints from the dipole moment measurements is the traditional one that all supersymmetric phases are equal to zero or unnaturally small. This would require the existence of some presently unknown mechanism which would ensure that there is negligible *CP* violation in the SUSY breaking sector of the MSSM Lagrangian. On the other hand, we have found that the phases may be large while certain approximate relations hold among the mass parameters and phases, resulting in cancellations in the calculation of the electron and neutron EDM. These relations could in principle also come from a

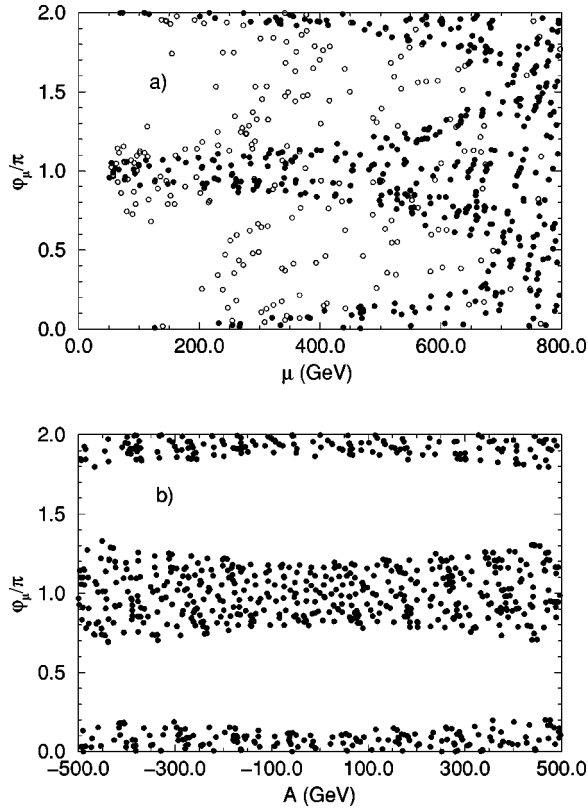


FIG. 5. Frame (a) shows variation of the φ_μ allowed region with μ for the standard set of parameters and other phases sampled randomly. The values of $A=A_e=A_u=A_d=A_t$ were also varied from -500 GeV to 500 GeV. Open (solid) circles denote points allowed by the neutron (electron) EDM limit. Frame (b) demonstrates variation of the φ_μ range allowed by the electron EDM limits with the values of A for $\mu=450$ GeV.

theory of SUSY breaking predicting the exact form of the soft SUSY breaking sector in the Lagrangian. We have shown in this paper that the latter possibility is legitimate and the ultimate decision between the two alternatives should be made based on experimental measurements.

We have presented a study of the constraints imposed on the phases by electron and neutron EDM data for some par-

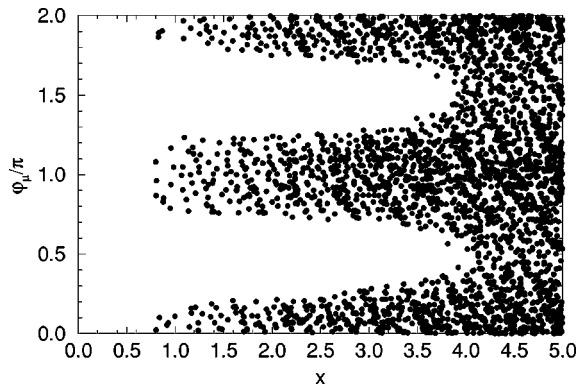


FIG. 6. We plot the points allowed by the electron EDM limits for parameter sets with all the mass parameters scaled by x with respect to the standard set. All phases are sampled randomly.

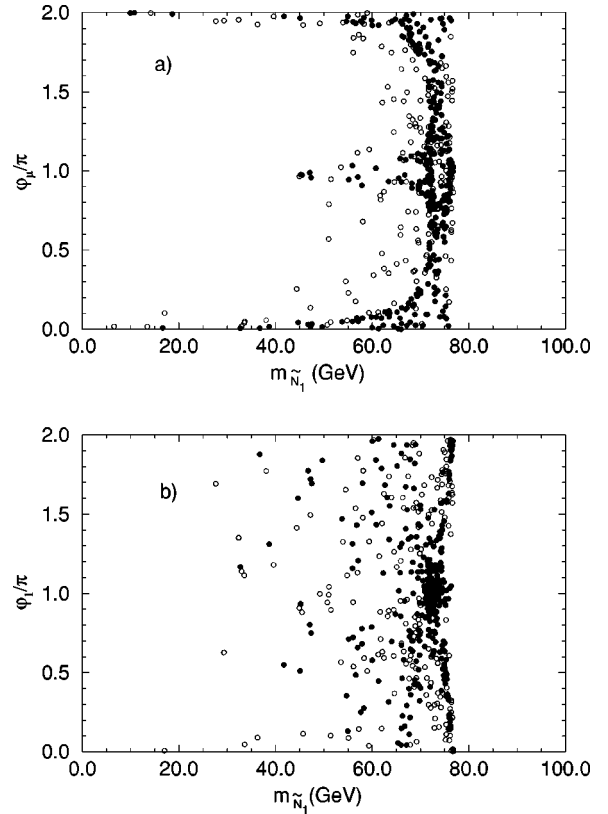


FIG. 7. Plots of the lightest neutralino masses allowed by the neutron (open circles) and electron (solid circles) EDM limits vs φ_μ in (a) and φ_1 in frame (b). In addition to all phases, the values of μ were also varied from 50 GeV to 800 GeV and all other parameters were standard.

ticular values of soft parameters with relatively light spectra. The results exhibit general features typical for similar choices and they show that all considered phases can have non-zero values. φ_μ is severely constrained while other phases can have any value as long as certain correlations with φ_μ are respected. The constraints on the phases relax as heavier spectra or large values of μ are considered.

The fact that phases can be non-vanishing is very important if one considers the general correspondence between the parameters in the supersymmetric Lagrangian and various observables which will possibly be measured at future collider experiments. For example, without a determination of the phases it is not possible to measure the value of $\tan\beta$. It is also important to realize that the presence of phases has a substantial impact on the neutralino relic density calculation and on the magnitude of the corresponding neutralino scattering cross section for dark matter detection. If progress in supersymmetric particle physics proceeds by the historical path, it will be essential to measure the phases to learn the form of the soft breaking Lagrangian, and thereby be led to recognize the mechanism of supersymmetry breaking.

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APPENDIX

In this appendix we summarize all the calculational details necessary for evaluation of the contributions to electric dipole moments of elementary particles in the MSSM at the one-loop level. The effect of CP -violating phases enters through the particular vertex Δ functions characteristic for each type of contributing diagrams. These functions depend on the matrices diagonalizing the generally complex mass matrices of the participating supersymmetric particles.

In our parametrization, the gluino mass is complex and can be diagonalized by a single complex number G defined by

$$G^* M_3 e^{i\varphi_3} G^{-1} = M_3, \quad (\text{A1})$$

resulting in $G = e^{i\varphi_3/2}$. Similarly, the chargino mass matrix

$$\mathcal{M}_C = \begin{pmatrix} M_2 & \sqrt{2} M_W \sin \beta \\ \sqrt{2} M_W \cos \beta & \mu e^{i\varphi_\mu} \end{pmatrix} \quad (\text{A2})$$

is diagonalized by two generally complex unitary matrices U and V so that

$$U^* \mathcal{M}_C V^{-1} = \mathcal{M}_C^{diag}. \quad (\text{A3})$$

The neutralino mass matrix contains two phases, φ_1 and φ_μ , in our parametrization

$$\mathcal{M}_N = \begin{pmatrix} M_1 e^{i\varphi_1} & 0 & -M_Z \sin \theta_W \cos \beta & M_Z \sin \theta_W \sin \beta \\ 0 & M_2 & M_Z \cos \theta_W \cos \beta & -M_Z \cos \theta_W \sin \beta \\ -M_Z \sin \theta_W \cos \beta & M_Z \cos \theta_W \cos \beta & 0 & -\mu e^{i\varphi_\mu} \\ M_Z \sin \theta_W \sin \beta & -M_Z \cos \theta_W \sin \beta & -\mu e^{i\varphi_\mu} & 0 \end{pmatrix}, \quad (\text{A4})$$

and the diagonalization matrix N satisfies

$$N^{-1} * \mathcal{M}_N N = \mathcal{M}_N^{diag}. \quad (\text{A5})$$

Finally, the scalar superpartners of the three families of fermions in the standard model obtain masses through a general mass matrix

$$\mathcal{M}_{\tilde{u}, \tilde{d}, \tilde{e}} = \begin{pmatrix} \mathbf{m}_{Q, Q, L}^2 + \mathbf{m}_{u, d, e} \mathbf{m}_{u, d, e}^\dagger + D_L \mathbf{1} & \mathbf{a}_{u, d, e}^\dagger v_{u, d, d} - \mu e^{i\varphi_\mu} v_{d, u, u} \mathbf{1} \\ \mathbf{a}_{u, d, e} v_{u, d, d} - \mu e^{-i\varphi_\mu} v_{d, u, u} \mathbf{1} & \mathbf{m}_{u, d, e}^2 + \mathbf{m}_{u, d, e}^\dagger \mathbf{m}_{u, d, e} + D_R \mathbf{1} \end{pmatrix} \quad (\text{A6})$$

where $D_L = M_Z^2 (T_3 - Q \sin^2 \theta_W) \cos 2\beta$ and $D_R = M_Z^2 Q \sin^2 \theta_W \cos 2\beta$, and v_u, v_d are the VEV's of the two neutral Higgs fields coupling to the up-type and down-type particles respectively. The parameters in bold print are 3×3 matrices, generally complex as discussed in the main text. The mass matrix in Eq. (A6) can be diagonalized by a pair of 3×6 matrices relating the interaction and mass eigenstates

$$\tilde{f}_i^L = \Gamma_{(f)ij}^L \tilde{f}_j^{diag} \quad (\text{A7})$$

$$\tilde{f}_i^R = \Gamma_{(f)ij}^R \tilde{f}_j^{diag} \quad (\text{A8})$$

for each type of fermion and all families $i=1,2,3$. Our notation distinguishes between the three types of sfermions, \tilde{u} , \tilde{d} and \tilde{e} , and individual flavor states are numbered according to the family number, so, for example, the \tilde{u}_3^L field corresponds to the left-handed top squark field.

The gluino vertex function reflects the fact that the gluino is a pure gaugino and the only possible way to produce a chirality changing effective vertex is to make use of L - R squark mixing and get

$$\Delta_i^{qk-\tilde{g}} = \Gamma_{(q)ki}^R \Gamma_{(q)ki}^{L*} G^{*2} \quad (\text{A9})$$

with no summation implied over i and $q=u,d$. The neutralino vertex function can be obtained in a similar way, giving

$$\begin{aligned} \Delta_i^{fk-\tilde{N}_j} = & \{ \sqrt{2} \tan \theta_W Q N_{1j}^* \Gamma_{(q)ki}^R - \lambda_f N_{hj}^* \Gamma_{(q)ki}^L \} \\ & \times \{ -\sqrt{2} [\tan \theta_W (Q - T_3) N_{1j}^* + T_3 N_{2j}^*] \Gamma_{(q)ki}^{L*} \\ & + N_{hj}^* \Gamma_{(q)ki}^{R*} \} \end{aligned} \quad (\text{A10})$$

where $\lambda_u = m_u / \sqrt{2} M_W \sin \beta$, $\lambda_{d,e} = m_{d,e} / \sqrt{2} M_W \cos \beta$, and $h=3$ for $h=d,e$ and $h=4$ for $f=u$. It is obvious from the structure of the function that the neutralino effective vertex

includes both gaugino and Higgsino interactions. Finally, the chargino vertex function for individual types of particles takes the form

$$\Delta_i^{u_k-\tilde{C}_j} = \lambda_u V_{j2}^* \Gamma_{(d)ki}^L (U_{j1}^* \Gamma_{(d)ki}^{L*} - \lambda_d U_{j2}^* \Gamma_{(d)ki}^{R*}) \quad (\text{A11})$$

$$\Delta_i^{d_k-\tilde{C}_j} = \lambda_d U_{j2}^* \Gamma_{(u)ki}^L (V_{j1}^* \Gamma_{(u)ki}^{L*} - \lambda_u V_{j2}^* \Gamma_{(u)ki}^{R*}) \quad (\text{A12})$$

$$\Delta_i^{e_k-\tilde{C}_j} = \lambda_e U_{j2}^* V_{j1}^*. \quad (\text{A13})$$

In order to make this paper self-contained, we also list the necessary loop functions coming from integrating out the supersymmetric particles in the one loop diagrams in the case of the electric and chromoelectric dipole operators [16]

$$A(x) = \frac{1}{2(1-x)^2} \left(3-x + \frac{2 \ln(x)}{1-x} \right) \quad (\text{A14})$$

$$B(x) = \frac{1}{2(1-x)^2} \left(1+x + \frac{2x \ln(x)}{1-x} \right) \quad (\text{A15})$$

$$C(x) = \frac{1}{6(1-x)^2} \times \left(10x - 26 + \frac{2x \ln(x)}{1-x} - \frac{18 \ln(x)}{1-x} \right) \quad (\text{A16})$$

and from the two loop calculation in the case of the purely gluonic operator [17]

$$H(z_1, z_2, z_3) = \frac{1}{2} \int_0^1 dx \int_0^1 du \int_0^1 dy x(1-x)u \frac{N_1 N_2}{D^4}, \quad (\text{A17})$$

where

$$N_1 = u(1-x) + z_3 x(1-x)(1-u) - 2ux[z_1 y + z_2(1-y)]$$

$$N_2 = (1-x)^2(1-u)^2 + u^2 - \frac{1}{9}x^2(1-u)^2$$

$$D = u(1-x) + z_3 x(1-x)(1-u) + ux[z_1 y + z_2(1-y)].$$

The integrals in the above definition of H can be simplified and evaluated numerically.

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- [1] A. Masiero and L. Silvestrini, in *Perspectives on Supersymmetry*, edited by G. L. Kane (World Scientific, Singapore, 1998).
- [2] T. Ibrahim and P. Nath, Phys. Lett. B **418**, 98 (1998); Phys. Rev. D **57**, 478 (1998); **58**, 019901(E) (1998); **58**, 111301 (1998).
- [3] M. Brhlik and G.L. Kane, Phys. Lett. B **437**, 331 (1998); G. L. Kane, presented at ‘‘Dark Matter in the Universe,’’ Proceedings, Santa Monica, CA, 1998, edited by D. Cline (unpublished).
- [4] T. Falk and K.A. Olive, Phys. Lett. B **439**, 71 (1998); T. Falk, A. Ferstl and K.A. Olive, Phys. Rev. D **59**, 055009 (1999); T. Falk and K.A. Olive, Phys. Lett. B **375**, 196 (1996); T. Falk, K.A. Olive and M. Srednicki, *ibid.* **354**, 99 (1995).
- [5] S. Bar-Shalom, D. Atwood and A. Soni, Phys. Rev. D **57**, 1495 (1998); A. Pilaftsis, Phys. Lett. B **435**, 88 (1998).
- [6] In our notation we follow S. P. Martin, in *Perspectives on Supersymmetry* [1].
- [7] S. Dimopoulos and D. Sutter, Nucl. Phys. **B452**, 496 (1995); see also S. Dimopoulos and S. Thomas, *ibid.* **B465**, 23 (1996).
- [8] R. Akhoury, M. Brhlik, G. L. Kane and A. Sinkovics (in preparation).
- [9] J. Ellis, S. Ferrara and D.V. Nanopoulos, Phys. Lett. **114B**, 231 (1982); W. Buchmuller and D. Wyler, *ibid.* **121B**, 321 (1983); J. Polchinski and M.B. Wise, *ibid.* **125B**, 393 (1983); F. del Aguila, M. Gavela, J. Grifols and A. Mendez, *ibid.* **126B**, 71 (1983); E. Franco and M. Mangano, *ibid.* **135B**, 445 (1984); J.M. Gerard, W. Grimus, A. Raychaudhuri and G. Zoupanos, *ibid.* **140B**, 349 (1984); J.M. Gerard, W. Grimus, A. Masiero, D.V. Nanopoulos and A. Raychaudhuri, Nucl. Phys. **B253**, 93 (1985); E. Gabrielli and G. Giudice, *ibid.* **B433**, 3 (1995).
- [10] M. Dugan B. Grinstein and L. Hall, Nucl. Phys. **B253**, 413 (1985); A.I. Sanda, Phys. Rev. D **32**, 2992 (1985); P. Nath, Phys. Rev. Lett. **66**, 2565 (1991); Y. Kizukuri and N. Oshimo, Phys. Rev. D **45**, 1806 (1992); **46**, 3025 (1992); R. Garisto, Nucl. Phys. **B419**, 279 (1994).
- [11] S. Weinberg, Phys. Rev. Lett. **63**, 2333 (1989); E. Braaten, C.S. Li and T.C. Yuan, *ibid.* **64**, 1709 (1990); J. Dai, H. Dykstra, S. Paban and D.A. Dicus, Phys. Lett. B **237**, 216 (1990).
- [12] R. Arnowitt, J. Lopez and D.V. Nanopoulos, Phys. Rev. D **42**, 2423 (1990); R. Arnowitt, M. Duff and K. Stelle, *ibid.* **43**, 3085 (1991).
- [13] A. Manohar and H. Georgi, Nucl. Phys. **B234**, 189 (1984).
- [14] I.S. Altarev *et al.*, Phys. Lett. B **276**, 242 (1992); K.F. Smith *et al.*, *ibid.* **234**, 191 (1990).
- [15] E.D. Commins, S.B. Ross, D. DeMille and B.S. Regan, Phys. Rev. A **50**, 2960 (1994); K. Abdullah *et al.*, Phys. Rev. Lett. **65**, 2347 (1990).
- [16] See, e.g., Kizukuri and Oshimo [10].
- [17] See, e.g., Dai *et al.* [11].