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Pseudoscalar and scalar meson masses at finite temperature

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The composite operator formalism is applied to QCD at finite temperature to calculate the masses of scalar and pseudoscalar mesons. In particular the ratio of the sigma mass to the pion mass is an interesting measure of the degree of chiral symmetry breaking at different temperatures. We calculate the temperature T^* at which $M_\sigma(T)\leqslant 2M_\pi(T)$, above which the sigma partial width into two pions vanishes. We find $T^*=0.95T_c$ (where T_c is the critical temperature for the chiral phase transition), within the full effective potential given by the formalism. We find that an expansion *à la* Landau of the effective potential around the critical point in the limit of small quark mass provides for a very good determination of T^*. [S0556-2821(99)05411-9]

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I. INTRODUCTION

The concept of chiral symmetry breaking in strong interactions goes back to the pioneering work of Nambu [1], and it has been widely explored and discussed since that time. Restoration of chiral symmetry at high densities was proposed by Lee and Wick [2] already in 1974. Theoretical procedures for studying chiral symmetry restoration at high temperatures were proposed in the same year by Dolan and Jackiw [3] and by Weinberg [4].

QCD at finite temperature and density has attracted much interest. For the latest studies of the QCD phase diagram see for instance [5]. The physical applications are to high energy heavy ion collisions [6] and to the physics of the early universe [7]. A review of lattice calculations can be found for instance in Ref. [8]. The Nambu–Jona-Lasinio model has been used under different approximations to obtain indications on the chiral transition (see for instance [9–14]).

A number of peculiar features may emerge due to the chiral phase transition and recently much attention has been attracted by the possibility of disoriented chiral condensate formation (see for instance [15–17]).

It is strongly suspected that a single transition occurs rather than separate transitions for deconfinement and chiral symmetry. We had suggested a heuristic argument indicating that, at least for zero density, the critical temperature for chiral transition, T_c, coincides with that for deconfinement, T_d [18]. The order parameters usually considered cover extreme and opposite ranges. The thermally averaged Polyakov loop is suitable in the limit of infinite quark masses to describe the transition from the confined to the nonconfined phase. The other extreme is the limit of vanishing quark masses, where the thermally averaged quark-antiquark bilinears are the typical order parameters for chiral symmetry transition.

We shall deal here with the chiral transition, concentrating on the thermally averaged quark bilinears at finite temperatures as order parameters for chirality. For light massive quarks, such as u and d, rigorously, chiral symmetry is already broken in the Lagrangian, but we can still retain the notion of phase transition, by looking at the region of T where the condensate has a rapid variation. The current quark mass plays a role analogous to that of an external magnetic field in the ferromagnetic transition, as it explicitly violates the chiral symmetry whose restoration characterizes the phase transition.

The analysis will be based on a composite operator formalism at finite temperature. The formalism makes use of an effective action for composite operators [19,20]. Actually the effective potential admits of a Landau expansion around the critical point and thus the behavior of the condensate is well reconstructed by knowing the coefficients (which are infrared safe) and the critical exponents. We shall compare with the results following from a Landau expansion and show the general agreement.

Within the composite operator formalism we had already discussed the T dependence of $f_\pi(T)$ [21], in the whole range of temperatures up to T_c. We shall here mainly deal with the ratio of the scalar to pseudoscalar mass M_σ/M_π at varying temperature. This ratio has a peculiar theoretical interest as a sensible indicator of the degree of chiral symmetry breaking.

Let us for the moment neglect the small u and d quark masses. When chiral symmetry is restored the two mesons are degenerate in mass. The ratio is then equal to unity. At zero temperature instead in the broken chiral phase the pion is a goldstone and has vanishing mass, while the sigma has a finite mass from the chiral condensation. Thus one expects the mass ratio to decrease from ∞ to one at the chiral transition. Quark masses will change this picture quantitatively leading to a decrease from a finite value to a value close to one.

Beyond some temperature, before approaching the critical value for the transition, the sigma will not have phase space

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left to decay into two pions. Its instability will decrease when increasing the temperature, and the channel into two pions will finally be suppressed at some temperature in the vicinity but lower than the critical temperature. The decay channel into two photons will still be available and become the dominant channel [13,22].

It is too early to say whether a possible experimental signature for the transition may be inferred from such behavior. An accurate determination of the mass ratio versus temperature is in itself of theoretical interest as it constitutes a significant parameter for the degree of chiral symmetry breaking at finite temperature. This we shall do in this paper in the composite operator formalism by making use of parameter values obtained from fits to zero temperature QCD.

In Sec. II we shall summarize the main results of the application of the composite operator formalism in QCD. In Sec. III we determine the relevant QCD parameters and discuss the hadronic masses at nonzero temperature. The temperature dependence of the observables is discussed in Sec. IV, where we also calculate the temperature at which the sigma can no longer decay via strong interaction.

II. EFFECTIVE ACTION FOR QCD COMPOSITE OPERATORS

Following Ref. [19] the zero temperature Euclidean effective action for an $SU(N)$ QCD-like gauge theory is

$$\Gamma(\mathbf{\Sigma}) = -\mathrm{Tr}\ln\left[\mathbf{S}_0^{-1} + \frac{\delta\Gamma_2}{\delta\mathbf{S}}\right] - \mathrm{Tr}[\mathbf{\Sigma}\mathbf{S}] - \mathbf{\Gamma}_2(\mathbf{S}) + \text{counterterms}, \quad (1)$$

where $\mathbf{S}_0^{-1} = (i\hat{p} - \mathbf{m})$, $\mathbf{m}$ is the bare quark mass matrix and $\mathbf{\Gamma}_2(\mathbf{S})$ is the sum of all the two-particle irreducible vacuum diagrams with fermionic propagator $\mathbf{S}$ and $\mathbf{\Sigma} = -\delta\mathbf{\Gamma}_2/\delta\mathbf{S}$. At two-loop level $\mathbf{\Gamma}_2 = \frac{1}{2}\mathrm{Tr}(\mathbf{S}\Delta\mathbf{S})$, where Δ is the gauge boson propagator, so that $\mathbf{\Sigma} = -\Delta\mathbf{S}$, $\mathrm{Tr}[\mathbf{S}\delta\mathbf{\Gamma}_2/\delta\mathbf{S}] = 2\mathbf{\Gamma}_2$ and one can rewrite Eq. (1) in terms of $\mathbf{\Sigma}$:

$$\Gamma(\mathbf{\Sigma}) = -\mathrm{Tr}\ln[\mathbf{S}_0^{-1} - \mathbf{\Sigma}] + \mathbf{\Gamma}_2(\mathbf{\Sigma}) + \text{counterterms}$$
$$= -\mathrm{Tr}\ln[\mathbf{S}_0^{-1} - \mathbf{\Sigma}] + \frac{1}{2}\mathrm{Tr}(\mathbf{\Sigma}\Delta^{-1}\mathbf{\Sigma}) + \text{counterterms}. \quad (2)$$

Here the variable $\mathbf{\Sigma}$ plays the role of a dynamical variable. At the minimum of the functional action, that is when the Schwinger-Dyson equation is satisfied, $\mathbf{\Sigma}$ is nothing but the fermion self-energy. A parametrization for $\mathbf{\Sigma}$, employed in [19] was

$$\mathbf{\Sigma} = (\mathbf{s} + i\gamma_5\mathbf{p})f(k) \equiv \mathbf{\Sigma}_s + i\gamma_5\mathbf{\Sigma}_p \quad (3)$$

with a suitable ansatz for $f(k)$, and with $\mathbf{s}$ and $\mathbf{p}$ scalar and pseudoscalar constant fields, respectively, to be taken as the variational parameters.

The effective potential one obtains from Eq. (2) (see Ref. [19]) is

$$V = \frac{\Gamma}{\Omega} = -\frac{8\pi^2 N}{3C_2 g^2}\int\frac{d^4k}{(2\pi)^4}\mathrm{tr}[\mathbf{\Sigma}_s\Box_k\mathbf{\Sigma}_s + \mathbf{\Sigma}_p\Box_k\mathbf{\Sigma}_p] - N\,\mathrm{Tr}\ln[i\hat{k} - (\mathbf{m} + \mathbf{\Sigma}_s) - i\gamma_5\mathbf{\Sigma}_p] + \delta Z\,\mathrm{tr}(\mathbf{ms}), \quad (4)$$

where C_2 is the quadratic Casimir of the fermion representation [for $SU(3)_c$ $C_2 = 4/3$]. Furthermore $\mathbf{\Sigma}_s \equiv \lambda_\alpha s_\alpha f(k)/\sqrt{2}$, $\mathbf{\Sigma}_p \equiv \lambda_\alpha p_\alpha f(k)/\sqrt{2}$, $\mathbf{m} \equiv \lambda_\alpha m_\alpha/\sqrt{2}$ ($\alpha = 0,\ldots,8$, $\lambda_0 = \sqrt{2/3I}$, $\lambda_i =$ Gell-Mann matrices, $i = 1,\ldots,8$), g is the gauge coupling constant and Ω is the four-dimensional volume. In Eq. (4) δZ has a divergent part to compensate the leading divergence proportional to $\mathrm{Tr}(\mathbf{ms})$ in the logarithmic term. We remark that both the current mass $\mathbf{m}$ and the self-energy $\mathbf{\Sigma}$ are, in general, matrices in flavor space. However, as discussed in Refs. [19,20] if we neglect the mixing between different flavors originating, for instance, from terms such as the 't Hooft determinant [23], it follows that only the flavor diagonal elements of the fermion self-energy and mass can be different from zero at the minimum. With vanishing off-diagonal terms, the effective potential decomposes into the sum of n_f contributions (n_f = number of flavors), one for each flavor. Therefore, to study the minima of the effective potential, it is formally sufficient to consider a single contribution. Of course, the choice of a given flavor number will reflect in the particular parameters assumed. In the present paper, as in Ref. [19], we will take $n_f = 3$ and a number of color $N = 3$. The value of the parameters will be specified later on.

As far as the choice for the function $f(k)$ is concerned, in QCD, the operator product expansion suggests (neglecting logarithmic corrections) to take for $f(k)$ a momentum behavior as $1/k^2$ for large k^2. We have chosen as a variational ansatz [24]

$$f(k) = \frac{M^3}{k^2 + M^2}, \quad (5)$$

where M is a momentum scale which is expected to be of the order of Λ_{QCD}.

To extend the zero-temperature theory to finite temperature we can still work with continuous energies by substituting for the sum over discrete energies $\omega_n = (2n+1)\pi/\beta$ (where $\beta = 1/T$) a sum of integrals over continuous energies by means of the Poisson's formula [25]

$$\int\frac{d^4k}{(2\pi)^4}f(k) \to \frac{1}{\beta}\sum_{n=-\infty}^{n=+\infty}\int\frac{d^3k}{(2\pi)^3}f(\omega_n,\mathbf{k}) = \sum_{n=-\infty}^{n=+\infty}(-)^n\int\frac{d^4k}{(2\pi)^4}f(k)e^{in\beta k_0}. \quad (6)$$

This substitution corresponds to the imaginary-time formalism after using the Poisson's formula and allows for an equivalent version af the Dolan-Jackiw finite temperature Feynman rules [3,4].

To explicitly evaluate the effective potential we have to choose how to parametrize the self-energy $\mathbf{\Sigma}$ at $T \neq 0$. Unfortunately the finite-temperature Schwinger-Dyson equations for the self-energy are very hard to treat and one may need to include a vector component too into its decomposition. We therefore limit ourselves to generalize the ansatz done at $T=0$ [see Eqs. (3), (5)], and so the final form for the effective potential, for a quark of mass *m*, is (see Ref. [24])

$$V=\frac{\Gamma}{\Omega}=-\frac{8\pi^2N}{3C_2g^2(T)}\int\frac{d^4k}{(2\pi)^4}[\Sigma_s\Box_k\Sigma_s+\Sigma_p\Box_k\Sigma_p] -2N\sum_{n=-\infty}^{n=+\infty}(-)^n\int\frac{d^4k}{(2\pi)^4}\ln[k^2+(m+\Sigma_s)^2+\Sigma_p^2] +\delta Zms, \tag{7}$$

where $\Sigma_s\equiv sf(k)$ and $\Sigma_p\equiv pf(k)$.

Let us now comment on the choice for the gauge coupling constant. As suggested by asymptotic freedom and renormalization-group considerations, we expect the strong forces to weaken at high temperature [26]. We shall then assume that in the UV region the coupling constant $g(T)$ depends logarithmically on the temperature *T*. We take into account this assumption by writing

$$\frac{g^2(T)}{2\pi^2}\equiv\frac{1}{c(T)}\equiv\frac{1}{c_0+c_1(T)}=\frac{1}{c_0+\frac{\pi^2}{b}\ln\left(1+\xi\frac{T^2}{M^2}\right)}, \tag{8}$$

where $b=24\pi^2/(11N-2n_f)$ and we will discuss the parameters c_0, *M*, and ξ later on.

At $T\neq 0$ the effective potential does not acquire any extra divergence with respect to the $T=0$ case. The renormalization at $T=0$ can be performed by adding a counterterm and requiring that the derivative of the effective potential with respect to the term which breaks explicitly the chiral symmetry, evaluated at the minimum, satisfies for each flavor the renormalization condition [19]

$$\lim_{m\to 0}\left.\frac{\delta V}{\delta(m\langle\bar\psi\psi\rangle_0)}\right|_{\min}=1. \tag{9}$$

The fermion condensate $\langle\bar\psi\psi\rangle_T$ is related to the minimum $\bar{s}(T)$ of the effective potential, renormalized at the scale *M* through the relation (see Ref. [19])

$$\langle\bar\psi\psi\rangle_T=\frac{3M^3}{g^2(T)}\bar{s}(T) \tag{10}$$

and $\langle\bar\psi\psi\rangle_0\equiv\langle\bar\psi\psi\rangle_{T=0}$.

At finite *T* we do not have any additional divergence with respect to the $T=0$ case; nevertheless, in order to satisfy the generalization of the normalization condition (9) at finite *T* we have to add a finite counterterm to that determined at $T=0$ [27]. Finally we recall that with the appropriate normalization for the pion field (see next section), Eq. (9) and its generalization at finite *T* is also equivalent to the Adler-Dashen formula

$$M_\pi^2(T)f_\pi^2(T)=-2m\langle\bar\psi\psi\rangle_T. \tag{11}$$

III. HADRONIC MASSES AT $T\neq 0$ AND DETERMINATION OF THE PARAMETERS

To compute the masses of the scalar and pseudoscalar mesons (σ and π, respectively) one has to take the second derivative of the effective potential (7) with respect to the scalar field *s* and the pseudoscalar field *p*, evaluated at the minimum [for the sake of simplicity we are assuming that the up and down quarks are degenerate in mass and $m=(m_u+m_d)/2$ in Eq. (7)].

The actual values of the masses will be obtained by multiplying the second derivative by the appropriate factor *a* that relates the physical fields φ_π (φ_σ) to *p* (*s*) according to $\varphi_\pi=ap,\varphi_\sigma=as$. This factor can be obtained in terms of the decay constant f_π through standard arguments of current algebra [19]. One gets

$$a=-\frac{f_\pi}{\sqrt{2}\bar{s}},$$

$$M_\sigma^2=\left.\frac{\partial^2V}{\partial\varphi_\sigma^2}\right|_{\min}=\frac{1}{a^2}\left.\frac{\partial^2V}{\partial s^2}\right|_{\min}=\frac{2\bar{s}^2}{f_\pi^2}\left.\frac{\partial^2V}{\partial s^2}\right|_{\min},$$

$$M_\pi^2=\left.\frac{\partial^2V}{\partial\varphi_\pi^2}\right|_{\min}=\frac{1}{a^2}\left.\frac{\partial^2V}{\partial p^2}\right|_{\min}=\frac{2\bar{s}^2}{f_\pi^2}\left.\frac{\partial^2V}{\partial p^2}\right|_{\min}, \tag{12}$$

where $\bar{s}$ is the extremum of the effective potential in the presence of a bare mass.

To derive a more physically transparent expression for the masses of the mesons as expressed by Eq. (12) through the second derivative of the effective potential, we will use the gap equation and the generalization at finite *T* of the normalization condition (9). The extremum condition is

$$\frac{\partial V}{\partial s}=0\to\frac{N}{s}\left[-2c(T)\int\frac{d^4k}{(2\pi)^4}\bar\Sigma_s(k)\Box_k\bar\Sigma_s(k) -4\sum_n(-)^n\int\frac{d^4k}{(2\pi)^4}\frac{(m+\bar\Sigma_s(k))\bar\Sigma_s(k)}{[k^2+(m+\bar\Sigma_s(k))^2]}e^{in\beta k_0}\right] +m\,\delta Z=0, \tag{13}$$

$$\frac{\partial V}{\partial p}=0\to\bar{p}=0, \tag{14}$$

where $\bar\Sigma_s(k)\equiv\bar{s}f(k)$.

As far as the normalization condition is concerned, by using the relation (10) between the scalar field at the minimum and the scalar condensate we shall write in general

$$\delta Z = N\left[\frac{M^3}{2\pi^2}c(T)+\frac{4}{s_0}\sum_n (-)^n \times\int\frac{d^4k}{(2\pi)^4}\frac{\Sigma_0(k)}{k^2+\Sigma_0^2(k)}e^{in\beta k_0}\right] \tag{15}$$

with $\Sigma_0(k)=s_0 f(k)$ and where s_0 is the minimum of the effective potential in the massless case.

Now, by using the gap equations (13), (14) and the normalization condition (15) in the explicit expressions (12) for the meson masses, we can eliminate δZ and $c(T)$ to finally obtain, for a quark of mass m:

$$M_\pi^2 = -2\frac{m}{f_\pi^2}\langle\bar\psi\psi\rangle_T+\frac{8Nm}{f_\pi^2}\sum_n (-)^n \times\int\frac{d^4k}{(2\pi)^4}\left[\frac{\bar\Sigma_s}{k^2+(m+\bar\Sigma_s)^2}-\frac{\bar s}{s_0}\frac{\Sigma_0}{k^2+\Sigma_0^2}\right]e^{in\beta k_0}, \tag{16}$$

$$M_\sigma^2 = M_\pi^2+\frac{16N}{f_\pi^2}\sum_n (-)^n \times\int\frac{d^4k}{(2\pi)^4}\frac{(m+\bar\Sigma_s)^2\bar\Sigma_s^2}{[k^2+(m+\bar\Sigma_s)^2]^2}e^{in\beta k_0}. \tag{17}$$

We notice that Eq. (16) reproduces, in the small mass limit, the Adler-Dashen formula (11) and differs from this formula only by terms of order m^2.

In order to fix the values of the parameters c_0 and $s_0(T=0)$ we follow the same procedure of Ref. [19]. In the massive case the effective potential is UV divergent and from the normalization condition (in the small-mass limit) and gap equation one is able to fix, through a self-consistency relation, the values $c_0=0.554$. By inserting this value in the gap equation for massless quarks, one finds that chiral symmetry is spontaneously broken (at $T=0$) via a minimum of V located at $s_0(T=0)=-4.06$ and the point $s_0(T=0)=0$ is a local maximum. To determine the mass scale M and the quark masses from the experimental data, one has to derive the explicit expressions for the masses and for the decay constants of the pseudoscalar octet mesons which are the pseudo-Goldstone bosons of chiral symmetry breaking. These expressions constitute a system of coupled equations which we have solved by an approximation method. The experimental inputs are the decay constant and mass of the charged pion, the charged kaon mass and the electromagnetic mass difference between the neutral and the charged kaon. The outputs of the numerical fit for the octet meson masses (agreement within 3%) are the masses of the u, d, and s quarks and the mass parameter M. The values we get are the following [19,20,28]:

$$M=280\ \text{MeV},$$
$$m_u=8\ \text{MeV},$$
$$m_d=11\ \text{MeV},$$
$$m_s=181\ \text{MeV},$$
$$m\equiv\frac{m_u+m_d}{2}=9.5\ \text{MeV}. \tag{18}$$

Finally, the parameter ξ, which appears in the expression (8) for the running coupling constant, has to be determined on phenomenological grounds. If we compare our model in the low-T regime with the results of Ref. [29] we find $\xi=0.44(n_f^2-1)/n_f$ for n_f flavors, which gives $\xi\simeq 1$ for $n_f=3$ [21].

With these values for the relevant parameters it is easy to determine the masses of the pseudoscalar and scalar mesons at $T=0$ (we are using the value $f_\pi=93$ MeV for the pion decay constant)

$$M_\pi\simeq 135\ \text{MeV},\quad M_\sigma\simeq 630\ \text{MeV} \tag{19}$$

and (again at $T=0$) the fermionic condensates at M for massless and massive quarks

$$\langle\bar\psi\psi\rangle_0\simeq(-197\ \text{MeV})^3\quad(\text{massless quarks}), \tag{20}$$

$$\langle\bar\psi\psi\rangle_0\simeq(-200\ \text{MeV})^3\quad(\text{massive quarks}). \tag{21}$$

The value we have obtained for the mass of the scalar meson, $M_\sigma\simeq 630$ MeV, accords very well with those obtained by other authors within different approaches to the study of dynamical chiral symmetry breaking in QCD [13,14].

IV. THE SCALAR-PSEUDOSCALAR $M_\sigma(T)/M_\pi(T)$ MASS RATIO

It is well known that as the temperature T increases we expect some changes in the hadronic properties of matter as a consequence of a possible phase transition.

For instance, a physical quantity as $f_\pi(T)$ will decrease as T increases [21]. This means that the pion tends to decouple from matter (quarks and leptons) as T increases. As far as the pion mass is concerned, $M_\pi(T)$ has a weak dependence on the temperature. In fact the value of $M_\pi(T)$ is dominated by the current quark mass for temperatures below the critical value, whereas it becomes independent of m at the critical temperature or above it. This should be expected because, for $T\simeq T_c$, the pion becomes an ordinary resonance and its mass is dominated by Λ_{QCD} and not by m. This is a clear sign that the pion loses its Goldstone nature once the approximate chiral symmetry is restored [20].

The scalar meson mass $M_\sigma(T)$ will decrease in association with partial chiral symmetry restoration in hot matter. In fact the scalar σ meson can be described as a $q\bar q$ quasibound state and its mass is about twice the constituent quark mass

[13,19,30]. Thus the σ-mass will decrease substantially with increasing temperature and this behavior is mainly determined by the T-dependence of the constituent quark mass. The decreasing of the σ-mass is expected from the restoration of chiral symmetry at $T=T_c$. In fact $M_\sigma \to 0$ for $m=0$ at $T=T_c$. As a consequence $M_\sigma(T)$ will become smaller than $2M_\pi(T)$ at some temperature $T^* < T_c$ and the σ-meson, which has a large width at $T=0$ due to the decay $\sigma \to 2\pi$, would appear as a sharp resonance at high temperature since the amplitude for $\sigma \to 2\pi$ would vanish above T^* (near the critical point of the chiral phase transition). Thus one can expect to have a better chance of seeing the σ-meson emerge as a clear, narrow state at nonzero temperature. In fact, since for $T > T^*$ the decay width of σ by strong interaction vanishes, only the electromagnetic width coming from $\sigma \to 2\gamma$ is left [13,22].

Within our model we are able to make some rough estimate of the temperature T^* at which this phenomenon takes place by simply studying the behavior of the meson masses (16), (17) as a function of the temperature. However, since we are, as already said, mainly interested in determining the temperature T^* at which $M_\sigma(T) \leqslant 2M_\pi(T)$, instead of evaluating separately $M_\sigma(T)$ and $M_\pi(T)$ it will be sufficient to study the ratio M_σ^2/M_π^2:

$$\frac{M_\sigma^2}{M_\pi^2} = 1 - \frac{24}{m} \frac{\sum_n (-)^n \int \frac{d^4k}{(2\pi)^4} \frac{(m+\overline{\Sigma}_s)^2}{[k^2+(m+\overline{\Sigma}_s)^2]^2} \overline{\Sigma}_s^2 e^{in\beta k_0}}{\langle \bar{\psi}\psi \rangle_T - 12 \sum_n (-)^n \int \frac{d^4k}{(2\pi)^4} \left[\frac{\overline{\Sigma}_s}{k^2+(m+\overline{\Sigma}_s)^2} - \frac{\bar{s}}{s_0} \frac{\Sigma_{0s}}{k^2+\Sigma_{0s}^2} \right] e^{in\beta k_0}}. \tag{22}$$

By straightforward calculations at the leading order in $\alpha = m/M$ we obtain

$$\frac{M_\sigma^2}{M_\pi^2} \simeq 1 - 16\pi^2 \frac{s_0^3(T)}{c(T)} \frac{M}{m} \sum_n (-)^n \times \int \frac{d^4y}{(2\pi)^4} \frac{e^{in\beta M y_0}}{[y^2(y^2+1)^2+s_0^2]^2}. \tag{23}$$

Furthermore, as we expect that the temperature T^* will be not far from the critical temperature of the chiral phase transition, we can start our investigation by performing a mean-field expansion *à la* Landau of the effective potential in the small quark-mass limit around the critical point. Following Ref. [20] we have

$$V = \frac{3M^4}{4\pi^2} [a_2(T)\varphi^2 + a_4(T)\varphi^4 + b_1(T)\alpha s + \cdots], \tag{24}$$

where $\varphi^2 = s^2 + p^2$.

If we now compute the masses of the scalar and pseudoscalar mesons from the expression (24) of the effective potential we get

$$\frac{M_\sigma^2}{M_\pi^2} \simeq 1 - 8\frac{M}{m} \frac{a_4(T)}{b_1(T)} s_0^3(T) + \cdots. \tag{25}$$

By noticing that the generalization at finite T of the normalization condition (9) on the effective potential reads now

$$b_1(T) = 2c(T) \tag{26}$$

and that the coefficient $a_4(T)$ is given by the expression (see Ref. [21])

$$a_4(T) = 4\pi^2 \sum_n (-)^n \int \frac{d^4y}{(2\pi)^4} \frac{1}{[y^2(y^2+1)^2+s_0^2]^2} e^{in\beta M y_0}. \tag{27}$$

It is easy to check that Eq. (25) is nothing but Eq. (23).

Just to have a rough idea of the value of the temperature T^* we can push our approximation further by evaluating $a_4(T)$ and $b_1(T)$ at T_c and using for $s_0(T)$ the expression for the massless minimum around T_c [20]

$$s_0(T) = a_s(T_c) \left(1 - \frac{T}{T_c} \right)^{1/2}, \quad T \leqslant T_c \tag{28}$$

in Eq. (25).

The numerical values for the coefficients relevant for the evaluation of the mass ratio are

$$a_4(T_c) = 0.0033, \quad b_1(T_c) = 1.25, \quad a_s(T_c) = -7.28 \tag{29}$$

and $T_c \simeq 103$ MeV [20].

By now imposing the condition $M_\sigma/2M_\pi \leqslant 1$ we have a constraint on the temperature ratio T/T_c

$$\frac{M_\sigma}{2M_\pi} \leqslant 1 \Rightarrow \frac{T}{T_c} \geqslant 0.95 \tag{30}$$

and this constraint fixes $T^* = 0.95T_c \simeq 98$ MeV.

The approximations we have made up to now [we have evaluated $a_4(T)$ and $b_1(T)$ at T_c, and we have chosen for $s_0(T)$ the behavior as given by Eq. (28)] allow us to study M_σ/M_π only for T very close to T_c. To study the mass ratio M_σ/M_π as a function of T for all the values of the temperature from $T=0$ up to T very close to T_c we have to go back to Eq. (23) which is valid for any T at the leading order in the

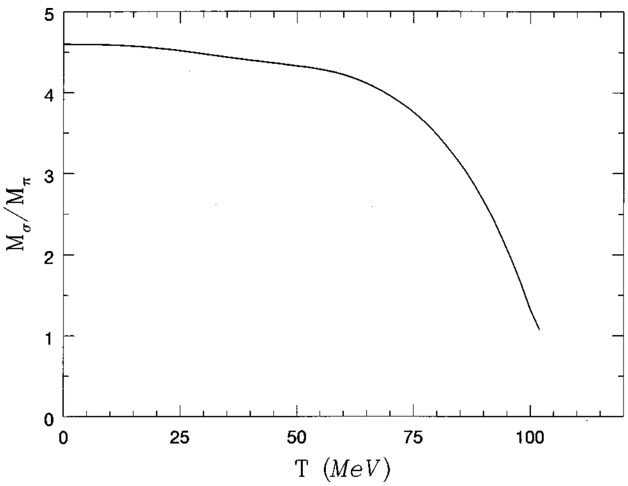

FIG. 1. Plot of M_σ/M_π versus T at the leading order in m, for $m=9.5$ MeV.

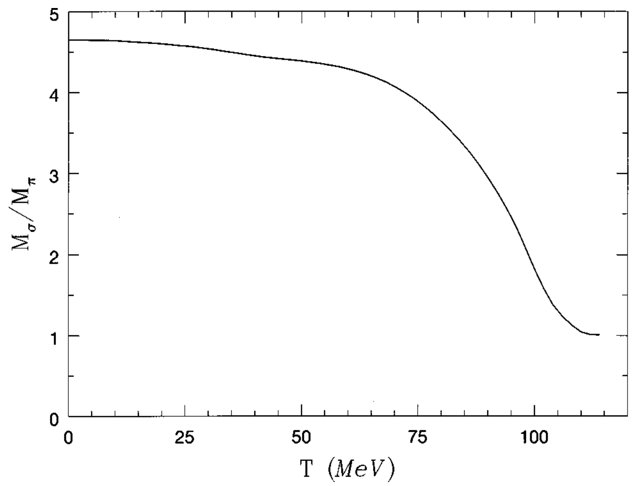

FIG. 2. Plot of M_σ/M_π versus T at all orders in m, for $m=9.5$ MeV.

quark mass m. To evaluate M_σ^2/M_π^2 we can perform the integration in y_0 by using the Cauchy theorem and summing up the series; then, through a numerical calculation we can finally perform the integration left. In Fig. 1 we plot the mass ratio M_σ/M_π as a function of T for a mass $m=(m_u+m_d)/2=9.5$ MeV; from the numerical analysis we can evaluate again the temperature T^*

$$\frac{M_\sigma}{2M_\pi}\leqslant 1 \Rightarrow \frac{T}{T_c}\geqslant 0.93 \tag{31}$$

and this constraint now fixes $T^*=0.93T_c\simeq 95$ MeV.

We remark that this result for T^* is in complete agreement with that previously derived by studying the Landau expansion of the effective potential for T very close to T_c. Finally, if one wants to study M_σ/M_π including all the orders in the quark mass m and for any temperature it is necessary to go back to the general formula (22). In Fig. 2 it is plotted the behavior of the mass ratio as a function of T as derived by the explicit evaluation of Eq. (22). In this case the temperature T^* we are looking for is determined again by the condition

$$\frac{M_\sigma}{2M_\pi}\leqslant 1 \Rightarrow \frac{T}{T_c}\geqslant 0.95 \tag{32}$$

which fixes $T^*=0.95T_c\simeq 98$ MeV. Curiously this result exactly coincides with that obtained through the Landau expansion and the evaluation of the coefficients $a_4(T)$, $b_1(T)$ at T_c and by putting for s_0 the expression (28) as suggested by the mean-field theory.

V. CONCLUSIONS

We have applied the composite operator formalism to QCD at finite temperature. After carrying out consistently the renormalization in the effective potential, we have calculated the masses of the scalar and pseudoscalar mesons. The relevant QCD parameters have been derived from the expressions for the masses and decay constants of the pseudoscalar multiplet, by inserting as inputs the mass and decay constant of the pion, the charged kaon mass and the electromagnetic mass difference between neutral and charged kaon.

A main purpose of the calculation was to determine the temperature T^* at which $M_\sigma(T)\leqslant 2M_\pi(T)$. Such temperature will be reached before the critical point, since the scalar meson mass $M_\sigma(T)$ is expected to decrease along with partial chiral symmetry restoration in hot matter, and at the same time the pion will lose its Goldstone nature. The temperature T^* is also expected not to lie far from the critical temperature of the chiral transition. In principle the σ-meson would appear as a narrow state at high enough temperature. For $T>T^*$ its partial width by strong interaction has to vanish.

The result of our calculation gives $T^*=0.95T_c$, in an approximation including current quark masses and with the full effective potential given by the formalism. We find that this result almost coincides with a much simpler treatment using a mean-field expansion of the effective potential *à la* Landau around the critical point in the limit of small quark mass.

It is perhaps premature to discuss whether a possible experimental signature for the QCD phase transition may be inferred from the behavior of the sigma width. The mass ratio itself is in any case a significant parameter to describe the degree of chiral symmetry breaking at different temperatures and therefore it has to be considered as a quantity of intrinsic theoretical interest.

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