

Exclusive rare decays of heavy baryons to light baryons: $\Lambda_b \rightarrow \Lambda \gamma$ and $\Lambda_b \rightarrow \Lambda l^+ l^-$

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The rare decays $\Lambda_b \rightarrow \Lambda \gamma$ and $\Lambda_b \rightarrow \Lambda l^+ l^-$ ($l = e, \mu$) are examined. We use QCD sum rules to calculate the hadronic matrix elements governing the decays. The Λ polarization in the decays is analyzed and it is shown that the polarization parameter in $\Lambda_b \rightarrow \Lambda \gamma$ does not depend on the values of hadronic form factors. The energy spectrum of Λ in $\Lambda_b \rightarrow \Lambda l^+ l^-$ is given. [S0556-2821(99)02511-4]

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I. INTRODUCTION

Processes associated with the flavor-changing neutral current (FCNC) $b \rightarrow s$ transition have regained much attention since the measurement of FCNC decays of the type $b \rightarrow s \gamma$ by CLEO [1,2]. It is well known that these processes are forbidden at the tree level in the standard model (SM) and are strongly suppressed by the Glashow-Iliopoulos-Maiani (GIM) mechanism, particularly for up type quarks. On the other hand, it is very sensitive to possible higher mass scales and interactions predicated by supersymmetric theories, two Higgs doublet models, etc. Such interactions shape the $b \rightarrow s$ transition via operators and their Wilson coefficients appearing in the low energy $\Delta B = 1$ effective Hamiltonian. Hence the study of such a process is an important way to test the Cabibbo-Kobayashi-Maskawa (CKM) sector of the SM and possibly opens a window to physics beyond the SM.

For the experimental side, there is some data on the exclusive decay $B \rightarrow K^* \gamma$ and the inclusive decay $B \rightarrow X_s \gamma$ with the branching ratio [3]

$$\text{Br}(B^+ \rightarrow K^{*+} (892) \gamma) = (5.7 \pm 3.3) \times 10^{-5},$$

$$\text{Br}(B^0 \rightarrow K^{*0} (892) \gamma) = (4.0 \pm 1.9) \times 10^{-5},$$

$$\text{Br}(B \rightarrow X_s \gamma) = (2.3 \pm 0.7) \times 10^{-4}.$$

These data have prompted a number of studies aimed at restricting the parameter space of various extensions of the SM [9]. Similar analyses based on decays of B mesons have also been performed for the transition $b \rightarrow s l^+ l^-$, which has not been observed yet [3] and only upper limits on inclusive branching ratios have been given [4]:

$$\text{Br}(b \rightarrow s e^+ e^-) < 5.7 \times 10^{-5},$$

$$\text{Br}(b \rightarrow s \mu^+ \mu^-) < 5.8 \times 10^{-5}$$

$$\text{and } \text{Br}(b \rightarrow s e^\pm \mu^\mp) < 2.2 \times 10^{-5} \text{ (at 90\% C.L.)}.$$

However, to analyze the helicity structure of the effective Hamiltonian mediating the transition $b \rightarrow s$, such analyses are not enough since the information on the handedness of the quark is lost in the hadronization process. To access the helicity of the quarks, analyzing the decay of baryons is the only way. One experimental drawback of baryon decay compared with B meson decay is that the production rate of Λ_b

baryons in b quark hadronization is $(10.1^{+3.9}_{-3.1})\%$, which is significantly less than that of B meson [$\text{Br}(\bar{b} \rightarrow B^+) = \text{Br}(\bar{b} \rightarrow B^0) = (39.7^{+1.8}_{-2.2})\%$, $\text{Br}(\bar{b} \rightarrow B_s^0) = (10.5^{+1.8}_{-1.7})\%$] [3], hence the relevant analyses have to wait for more data on heavy quark decays from future colliders.

For exclusive heavy-to-light decays one has to calculate hadronic matrix elements of operators in the effective Hamiltonian between a heavy hadron and a light hadron, which is related to the nonperturbative aspect of QCD. There are a number of papers to calculate the hadronic matrix elements in exclusive rare decays $B \rightarrow K^{(*)} l^+ l^-$ [5,6]. For exclusive heavy-to-light decays $\Lambda_b \rightarrow \Lambda l^+ l^-$ there are a lot of form factors to describe the hadronic matrix elements. However, for Λ_b we may use the heavy quark effective theory (HQET). It is well known that HQET simplifies greatly the analysis of the decay of heavy hadrons in that heavy quark symmetries restrict the number of form factors, in particular, for the baryonic transition $\Lambda_Q \rightarrow$ light spin-1/2 baryon, there are only two independent form factors irrelative to Dirac matrix of relevant operators [12]; for heavy hadron to heavy hadron transition, we only have one form factor, which is known as Isgur-Wise function [10]. The computation of two form factors, F_1 and F_2 , in HQET is the main work in analyzing exclusive decays of Λ_b to light baryon. $\Lambda_b \rightarrow \Lambda \gamma$ has been investigated in detail by Mannel and Rocksiegel [2], where the simple pole model was adopted to compute F_1 and F_2 . As we know, $\Lambda_b \rightarrow \Lambda l^+ l^-$ has not been examined yet. The form factors F_1 and F_2 in $\Lambda_c \rightarrow \Lambda$ have been calculated in nonrelativistic and relativistic quark models [7,8]. In this paper we employ the widely applied approach of QCD sum rules, which is based on general features of QCD [11], to calculate the F_1 and F_2 . For our purpose, we can use some expressions of Ref. [13] due to the similarity. We analyze the Λ polarization. An interesting result is that the polarization parameter in $\Lambda_b \rightarrow \Lambda \gamma$ does not depend on the values of hadronic form factors, F_1 and F_2 .

This paper is organized as follows. In Sec. II we write down the SM effective Hamiltonian governing transitions $b \rightarrow s \gamma$ and $b \rightarrow s l^+ l^-$, and give some information on the corresponding Wilson coefficients. In Sec. III, we compute the form factors F_1 and F_2 , by using QCD sum rules. Sections IV and V contribute to the analysis of decay $\Lambda_b \rightarrow \Lambda \gamma$ and $\Lambda_b \rightarrow \Lambda l^+ l^-$, respectively. In Sec. VI, we discuss our numerical results along with some relevant points.

TABLE I. Wilson coefficients $C_i(\mu)$ for $\Lambda_{\overline{MS}}^{(5)}=225$ MeV, $\mu=5$ GeV, and $m_t=174$ GeV; NDR scheme.

C_1	-0.243
C_2	1.105
C_3	1.083×10^{-2}
C_4	-2.514×10^{-2}
C_5	7.266×10^{-3}
C_6	-3.063×10^{-2}
C_7	-0.312
C_9	4.193
C_{10}	-4.578

II. EFFECTIVE HAMILTONIAN

In the SM, the Hamiltonian relevant for $b \rightarrow s$ transition consists of ten operators $O_i, i=1, \dots, 10$. Neglecting terms proportional to $V_{ub}V_{us}^*$ (the ratio $|V_{ub}V_{us}^*/V_{tb}V_{ts}^*|$ is of order 10^{-2}), the effective Hamiltonian takes the form [14]

$$H_W = 4 \frac{G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_{i=1}^{10} C_i(\mu) O_i(\mu), \quad (1)$$

where

$$\begin{aligned} O_1 &= (\bar{s}_{L\alpha} \gamma^\mu b_{L\alpha}) (\bar{c}_{L\beta} \gamma_\mu c_{L\beta}), \\ O_2 &= (\bar{s}_{L\alpha} \gamma^\mu b_{L\beta}) (\bar{c}_{L\beta} \gamma_\mu c_{L\alpha}), \\ O_3 &= (\bar{s}_{L\alpha} \gamma^\mu b_{L\alpha}) [(\bar{u}_{L\beta} \gamma_\mu u_{L\beta}) + \dots + (\bar{b}_{L\beta} \gamma_\mu b_{L\beta})], \\ O_4 &= (\bar{s}_{L\alpha} \gamma^\mu b_{L\beta}) [(\bar{u}_{L\beta} \gamma_\mu u_{L\alpha}) + \dots + (\bar{b}_{L\beta} \gamma_\mu b_{L\alpha})], \\ O_5 &= (\bar{s}_{L\alpha} \gamma^\mu b_{L\alpha}) [(\bar{u}_{R\beta} \gamma_\mu u_{R\beta}) + \dots + (\bar{b}_{R\beta} \gamma_\mu b_{R\beta})], \\ O_6 &= (\bar{s}_{L\alpha} \gamma^\mu b_{L\beta}) [(\bar{u}_{R\beta} \gamma_\mu u_{R\alpha}) + \dots + (\bar{b}_{R\beta} \gamma_\mu b_{R\alpha})], \\ O_7 &= \frac{e}{16\pi^2} m_b (\bar{s}_{L\alpha} \sigma^{\mu\nu} b_{R\alpha}) F_{\mu\nu}, \\ O_8 &= \frac{g_s}{16\pi^2} m_b \left[\bar{s}_{L\alpha} \sigma^{\mu\nu} \left(\frac{\lambda^a}{2} \right)_{\alpha\beta} b_{R\beta} \right] G_{\mu\nu}^a, \\ O_9 &= \frac{e^2}{16\pi^2} (\bar{s}_{L\alpha} \gamma^\mu b_{L\alpha}) \bar{l} \gamma_\mu l, \\ O_{10} &= \frac{e^2}{16\pi^2} (\bar{s}_{L\alpha} \gamma^\mu b_{L\alpha}) \bar{l} \gamma_\mu \gamma_5 l. \end{aligned} \quad (2)$$

Here α, β are color indices, $b_{R,L} = [(1 \pm \gamma_5)/2]b$, and $\sigma^{\mu\nu} = (i/2)[\gamma^\mu, \gamma^\nu]$; e and g_s are the electromagnetic and the strong coupling constants, respectively. The Wilson coefficients are given in Table I [16,6], where NDR denotes the naive dimensional regularization scheme and \overline{MS} denotes the modified minimal subtraction scheme. The coefficients' dependence on regularization scheme must disappear in the decay amplitude if all corrections are taken into account. The

operator responsible for decay $b \rightarrow s \gamma$ is O_7 , while operators responsible for decay $b \rightarrow s l^+ l^-$ are O_7, O_9, O_{10} . Because O_7 does not include lepton fields, it has to be combined with γll vertex and lepton fields to contribute to $b \rightarrow s l^+ l^-$. The result operator is $O_7' = (e/16\pi^2) m_b \bar{s} \sigma_{\mu\nu} (1 + \gamma_5) q^\nu / q^2 b \bar{l} \gamma^\mu l$.

We would like to write O_9, O_{10} and O_7' in forms allowing for non-SM couplings (omitting the color indices):

$$\begin{aligned} O_9 &= \frac{e^2}{32\pi^2} (\bar{s} \gamma^\mu (h_V - h_A \gamma_5) b) \bar{l} \gamma_\mu l, \\ O_{10} &= \frac{e^2}{32\pi^2} (\bar{s} \gamma^\mu (h_V - h_A \gamma_5) b) \bar{l} \gamma_\mu \gamma_5 l, \\ O_7' &= \frac{e^2}{16\pi^2} m_b (\bar{s} \sigma_{\mu\nu} (g_V - g_A \gamma_5) q^\nu / q^2 b) \bar{l} \gamma^\mu l, \end{aligned} \quad (3)$$

where h_V, g_V and h_A, g_A are the vector and axial vector couplings respectively, in order to discuss possible effects of models beyond SM. In general the parameters h_V and h_A in O_{10} may be different from those in O_9 and for the sake of simplicity we consider the case of same parameters as shown in Eq. (3). In the SM $h_V=1, h_A=1, g_V=1, g_A=-1$ if we neglect the mass of strange quark in respect to b quark.

As mentioned in Sec. I, the heavy quark symmetries restrict the number of form factors to two [12]:

$$\begin{aligned} \langle \Lambda(p, s) | \bar{s} \Gamma b | \Lambda_b(v, s') \rangle &= \bar{u}_\Lambda(p, s) \{ F_1(p \cdot v) \\ &\quad + \not{v} F_2(p \cdot v) \} \Gamma u_{\Lambda_b}(v, s'). \end{aligned} \quad (4)$$

The following section contributes to the computation of F_1 and F_2 by QCD sum rules in HQET.

III. COMPUTATION OF F_1 AND F_2

In Ref. [13], using QCD sum rules, we computed the form factors of Λ_b to p transition. Here we follow the approach and use the same expressions obtained in the work.

To compute F_1 and F_2 within the QCD sum rule approach we need to consider the three-point correlator

$$\begin{aligned} \Pi(P', P, z) &= i^2 \int d^4x d^4y e^{ik \cdot x - iP \cdot y} \\ &\quad \times \langle 0 | T \tilde{j}^v(x) \bar{h}_v(0) \Gamma s(0) \bar{j}(y) | 0 \rangle, \end{aligned} \quad (5)$$

of the flavor-changing current $\bar{h}_v \Gamma s$ and of Λ_b current \tilde{j}^v and Λ current j , where $P' = m_b v + k$ and $z = P \cdot v$. The baryonic currents for Λ_b are

$$\tilde{j}^v = \epsilon^{abc} (q_1^{\text{Ta}} \tilde{C} \Gamma q_2^b) h_v^c,$$

where $\tilde{\Gamma}$ has two kinds of choice, γ_5 and $\gamma_5 \not{v}$. We choose γ_5 for the sake of simplicity since the numerical differences resulting from the different choices of \tilde{j}^v are not significant [15].

The three-quark *tensor currents* for proton and Λ are [17]

$$J_p^T(x_1, x_2, x_3) = \sigma^{\mu\nu} \gamma_5 d^a(x_1) u^b(x_2) C \sigma_{\mu\nu} u^c(x_3) \epsilon^{abc},$$

$$J_\Lambda^T(x_1, x_2, x_3) = \sigma^{\mu\nu} \gamma_5 d^a(x_1) u^b(x_2) C \sigma_{\mu\nu} s^c(x_3) \epsilon^{abc}.$$

After Fierz transformation, they can be written in $S + P$ form

$$\begin{aligned} J_p^T(x_1, x_2, x_3) &= 4[u^a(x_3)u^b(x_2)C\gamma_5 d^c(x_1) \\ &\quad + \gamma_5 u^a(x_3)u^b(x_2)Cd^c(x_1)]\epsilon^{abc}, \\ J_\Lambda^T(x_1, x_2, x_3) &= 4[s^a(x_3)u^b(x_2)C\gamma_5 d^c(x_1) \\ &\quad + \gamma_5 s^a(x_3)u^b(x_2)Cd^c(x_1)]\epsilon^{abc}. \end{aligned} \quad (6)$$

The similarity allows us to use directly the analytical expressions obtained in the sum rule analysis of p final state to compute the form factors of Λ final state if we neglect the mass of s quark.

After inserting a complete set of physical intermediate states, as the phenomenological consequence of Eq. (5), we have

$$\begin{aligned} \Pi(P', P, z) &= f_{\Lambda_b} f_\Lambda \frac{2}{(\omega - 2\bar{\Lambda})} \\ &\quad \times P_+ \Gamma[F_1(z) + F_2(z)\not{v}] \frac{\not{P} + m_\Lambda}{P^2 - m_\Lambda^2} + \text{res}, \end{aligned} \quad (7)$$

where $P_+ = (1 + \not{v})/2$, $\bar{\Lambda} = m_{\Lambda_b} - m_b$, $\omega = 2k \cdot v$ and f_{Λ_b} , f_Λ are the so-called “decay constants” which are defined by

$$\langle 0 | \tilde{j}^v | \Lambda_b \rangle = f_{\Lambda_b} u, \quad \langle p(P) | \tilde{j} | 0 \rangle = f_\Lambda \bar{u}(P). \quad (8)$$

They can be found in Refs. [15,18] and Refs. [17,19], respectively. To obtain Eq. (7), we have taken into account Eq. (4) and the heavy quark limit.

By introducing the assumption of quark-hadron duality, the contribution from higher resonant and continuum states can be treated as

$$\text{res} = \int_{D'} d\nu ds \frac{\rho_{\text{pert}}(\nu, s, z)}{(\nu - \omega)(s - P^2)}. \quad (9)$$

The region D' is characterized by one or two continuum thresholds ν_c , s_c . From the theoretical point of view, $\Pi(P', P, z)$ is combination of the perturbative contribution and the condensate contribution. In order to incorporate the above assumption, we should express the perturbative term in the form of dispersion relation

$$\Pi_{\text{pert}}^i(\omega, P^2, z) = \int d\nu ds \frac{\rho^i(\nu, s, z)}{(\nu - \omega)(s - P^2)} \quad (i = 1, 2), \quad (10)$$

where $i = 1, 2$ denote the different terms associated with F_1 and F_2 , respectively. In the standard way, we employ a double Borel transformation $\omega \rightarrow M$, $P^2 \rightarrow T$ in order to suppress the higher excited state and continuum state contributions. Because the SU_3 violation effects are small in QCD sum rule analyses of three point functions for mesons [20], we expect the effects are probably even smaller for baryons. Therefore, we neglect the s quark mass in calculations and quote from Ref. [13] the analytical expressions of $\rho^i(\nu, s, z)$, $i = 1, 2$ and the condensate contributions. Thus, the resulting Borel transformed sum rules for F_1 and F_2 can be written as

$$\begin{aligned} -2f_{\Lambda_b} f_\Lambda F_1 e^{-2\bar{\Lambda}/M - m_\Lambda^2/T} &= \int_0^{\nu_c} d\nu \int_0^{2\nu z} ds \rho_{\text{pert}}^1 e^{-s/T - \nu/M} - \frac{1}{3} \langle \bar{q}q \rangle^2 \\ &\quad - \frac{1}{32\pi^4} \langle \alpha_s GG \rangle \int_0^{T/4} \left(1 - \frac{4\beta}{T} \right) e^{-4\beta(1-4\beta/T)/M^2 - 8\beta z/(TM)} d\beta, \\ &\quad - 2f_{\Lambda_b} f_\Lambda m_\Lambda F_2 e^{-2\bar{\Lambda}/M - m_\Lambda^2/T} \\ &= \int_0^{\nu_c} d\nu \int_0^{2\nu z} ds \rho_{\text{pert}}^2 e^{-s/T - \nu/M} + \frac{1}{8\pi^4} \langle \alpha_s GG \rangle \int_0^{T/4} \left(1 - \frac{4\beta}{T} \right) \frac{\beta}{M} e^{-4\beta(1-4\beta/T)/M^2 - 8\beta z/(TM)} d\beta, \end{aligned} \quad (11)$$

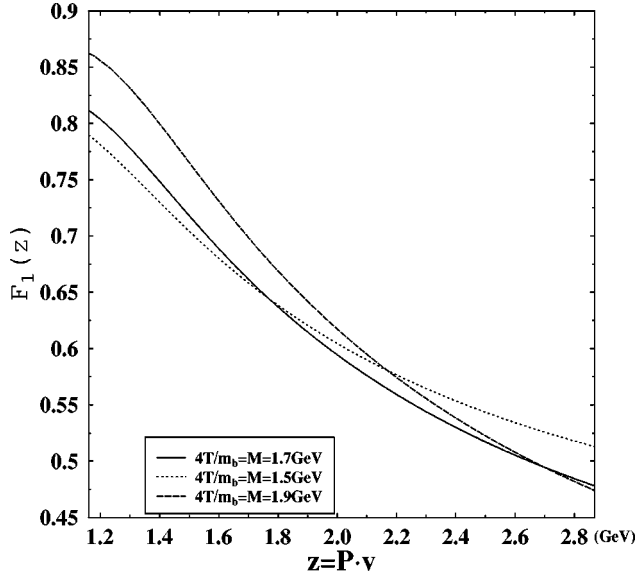


FIG. 1. Form factor $F_1(z)$ with different values of Borel parameter T and M .

and

$$\begin{aligned} \rho_{pert}^1 &= \frac{1}{32\pi^4\sigma^3} [-2z^3\sigma^3 - (-s + z(\nu + 2z))^3 \\ &\quad + 3z^2(-s + z(\nu + 2z))\sigma^2], \\ \rho_{pert}^2 &= \frac{-1}{64\pi^4\sigma^3} [s - 2z^2 + z(-\nu + \sigma)]^2 [\nu s + 8z^3 \\ &\quad - 4z^2(-2\nu + \sigma) - 2z(-\nu^2 + 5s + \nu\sigma)], \end{aligned} \quad (12)$$

with $\sigma = \sqrt{-4s + (\nu + 2z)^2}$. It should be mentioned here that we only retain the condensates with dimension lower than 7. In the numerical analysis, the “decay constants” and some other constants we used are [15,18,19,3]

$$m_{\Lambda_b} = 5.64 \text{ GeV}, \quad m_{\Lambda} = 1.116 \text{ GeV},$$

$$f_{\Lambda_b} = \sqrt{0.0003} \text{ GeV}^3,$$

$$f_{\Lambda} = 0.0208 \text{ GeV}^3, \quad \bar{\Lambda} = 0.79 \text{ GeV},$$

$$\langle \bar{q}q \rangle \approx -(0.23 \text{ GeV})^3, \quad (13)$$

$$\langle \alpha_s GG \rangle \approx 0.04 \text{ GeV}^4.$$

Again, owing to the small difference between p and Λ both in their mass and “decay constant,” the results here are similar to those of Ref. [13]. With the threshold $\nu_c = 3.5 \text{ GeV}$, we can have a reasonably good window for F_1 , where $1.5 \text{ GeV} < 4T/m_b = M < 1.9 \text{ GeV}$. The results are given in Figs. 1 and 2, respectively, where the different curves correspond to different choices of the Borel parameters. The comments made in Ref. [13] concerning about the sum rules, such as the condensates dominance property (for about

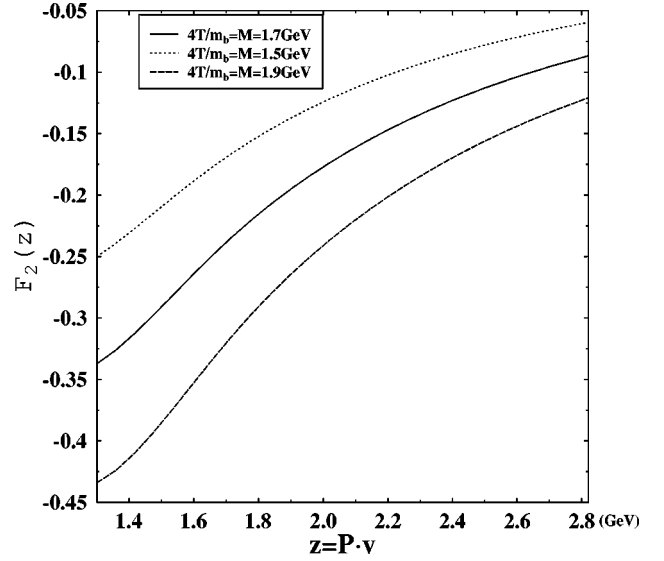


FIG. 2. Form factor $F_2(z)$ with different values of Borel parameter T and M .

55%), still hold here. One can see from Figs. 1 and 2 that $R = F_2/F_1 = -0.42$ at $q^2 = q_{max}^2$ which is almost the same as that of p final state.

IV. DECAY $\Lambda_b \rightarrow \Lambda \gamma$

As analyzed in Ref. [2], the long distance contribution to $\Lambda_b \rightarrow \Lambda \gamma$ is negligibly small, so we shall only consider the short distance contribution to the decay. The matrix element of the operator O_7 between the initial and final state is

$$\begin{aligned} &\langle \Lambda(p, s), \gamma(k, \varepsilon) | O_7 | \Lambda_b(v, s') \rangle \\ &= \frac{e}{32\pi^2} m_b \langle \Lambda(p, s) | \bar{s} \sigma_{\mu\nu} (g_V - g_A \gamma_5) b | \Lambda_b(v, s') \rangle \\ &\quad \times \langle \gamma(k, \varepsilon) | F^{\mu\nu} | 0 \rangle. \end{aligned} \quad (14)$$

In order to study the helicity of the final quark, we concentrated on the decay rate of unpolarized Λ_b baryons into light baryons with definite spin directions. We obtain the decay rate from Eqs. (4) and (14) as follows:

$$\begin{aligned} \Gamma &= \frac{C_7^2 G_F^2}{\pi} (V_{ts}^* V_{tb})^2 \left(\frac{e}{16\pi^2} \right)^2 m_b^2 m_{\Lambda_b}^3 (1 - x^2) \\ &\quad \times \left\{ \frac{g_V^2 + g_A^2}{2} [(1 - 2x^2 + x^4) |F_1|^2 + 2(x - 2x^3 + x^5) F_1 F_2 \right. \\ &\quad \left. + (x^2 - 2x^4 + x^6) |F_2|^2] + g_V g_A (v \cdot s) [(2x - 2x^3) |F_1|^2 \right. \\ &\quad \left. + 2(2x^2 - 2x^4) F_1 F_2 + (2x^3 - 2x^5) |F_2|^2] \right\}, \end{aligned} \quad (15)$$

where $x = m_{\Lambda}/m_{\Lambda_b}$. Our result is exactly the same as that of Ref. [2] with real F_1 and F_2 , as it should be. In order to compare the result with experiment we rewrite the rate in terms of the polarization variables as defined in Ref. [2]

$$\Gamma = \Gamma_0 [1 + \alpha \hat{\mathbf{p}} \cdot \mathbf{S}_\Lambda], \quad (16)$$

where $\hat{\mathbf{p}}$ is the unit momentum vector of the Λ and \mathbf{S}_Λ is its spin vector. From expression (15), it is straightforward to find that

$$\alpha = \frac{2x}{(1+x^2)} \frac{2g_V g_A}{(g_V^2 + g_A^2)}, \quad (17)$$

$$\Gamma_0 = \frac{C_7^2 G_F^2}{2\pi} (V_{ts}^* V_{tb})^2 \left(\frac{e}{16\pi^2} \right)^2 m_b^2 m_{\Lambda_b}^3 \times (1-x^2)^3 (F_1 + F_2 x)^2 (g_V^2 + g_A^2). \quad (18)$$

Equation (17) is different from that in Ref. [2] and means that the polarization parameter α is independent of the form factors F_1 and F_2 . The conclusion that α does not depend on the hadronic structure has also been obtained in Ref. [21]. However, in Ref. [21] $F_2 = 0$ has been assumed so that the conclusion is trivially obtained and the factor $2x/(1+x^2)$ in Eq. (17) is missed there. The QCD sum rules analyses give that $F_1 = 0.50 \pm 0.03$ and $F_2 = -0.10 \pm 0.03$ at the point $p^0 = (m_{\Lambda_b}^2 + m_\Lambda^2)/(2m_{\Lambda_b}) = 2.93$ GeV. Although this point is

to the disadvantage of HQET application, which justifies Eq. (4), for the recoil at this point is the largest, we still assume that the heavy quark symmetries are applicable to some extent at this point. Taking $x = 0.20$, we have

$$\alpha = 0.38 \times \frac{2g_V g_A}{(g_V^2 + g_A^2)}, \quad (19)$$

and

$$\Gamma_0 = (1.06 \pm 0.16) \times 10^{-17} (g_V^2 + g_A^2) \text{ GeV}, \quad (20)$$

where the uncertainty is rooted in the uncertainties of F_1 and F_2 . In the SM, we take $g_V = 1$, $g_A = -1$, the corresponding branching ratio is

$$\text{Br}(\Lambda_b \rightarrow \Lambda \gamma) = (3.7 \pm 0.5) \times 10^{-5} \quad (21)$$

which is within the range obtained in Ref. [2].

V. DECAY $\Lambda_b \rightarrow \Lambda l^+ l^-$

The decay of lepton final state is a little more involved than that of γ final state in the integration over phase space. The relevant matrix elements of the process are

$$\begin{aligned} \langle \Lambda(p, s), l^-(p_1, s_1), l^+(p_2, s_2) | O_7' | \Lambda_b(v, s') \rangle &= \frac{e^2}{16\pi^2} m_b \langle \Lambda(p, s) | \bar{s} \sigma_{\mu\nu} (g_V - g_A \gamma_5) \frac{q^\nu}{q^2} b | \Lambda_b(v, s') \rangle \\ &\times \langle l^-(p_1, s_1), l^+(p_2, s_2) | \bar{l} \gamma^\mu l | 0 \rangle, \\ \langle \Lambda(p, s), l^-(p_1, s_1), l^+(p_2, s_2) | O_9 | \Lambda_b(v, s') \rangle &= \frac{e^2}{32\pi^2} \langle \Lambda(p, s) | \bar{s} \gamma_\mu (h_V - h_A \gamma_5) b | \Lambda_b(v, s') \rangle \\ &\times \langle l^-(p_1, s_1), l^+(p_2, s_2) | \bar{l} \gamma^\mu l | 0 \rangle, \\ \langle \Lambda(p, s), l^-(p_1, s_1), l^+(p_2, s_2) | O_{10} | \Lambda_b(v, s') \rangle &= \frac{e^2}{32\pi^2} \langle \Lambda(p, s) | \bar{s} \gamma_\mu (h_V - h_A \gamma_5) b | \Lambda_b(v, s') \rangle \\ &\times \langle l^-(p_1, s_1), l^+(p_2, s_2) | \bar{l} \gamma^\mu \gamma^5 l | 0 \rangle. \end{aligned} \quad (22)$$

To obtain from the above expressions the differential width with respect to the energy E of Λ is a matter of algebra, we obtain

$$\frac{d\Gamma}{dy} = A(y) + s \cdot v B(y), \quad (23)$$

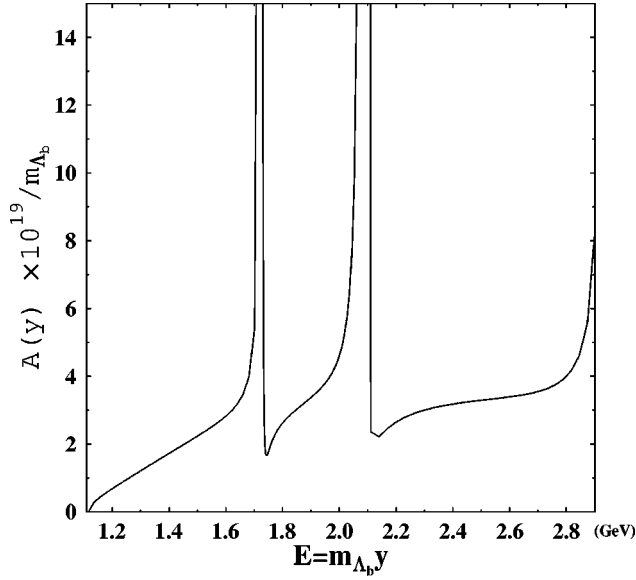
where $y = E/m_{\Lambda_b}$. The expressions of $A(y)$ and $B(y)$ are not presented here for they are too tedious. They can be found in eprint hep-ph/9811303. The numerical information associated with them can be found in Fig. 3 and Fig. 4. In the expressions, we have replaced C_9 with C_9^{eff} , which takes the main long distance contribution into consideration by the following formalism [14, 22, 23]:

$$\begin{aligned} C_9^{eff} &= C_9 + (3C_1 + C_2) \\ &\times \left[h(x, s) + k \sum_{i=1}^2 \frac{\pi \Gamma(\psi \rightarrow l^+ l^-) M_{\psi_i}}{q^2 - M_{\psi_i}^2 + i M_{\psi_i} \Gamma_{\psi_i}} \right], \end{aligned} \quad (24)$$

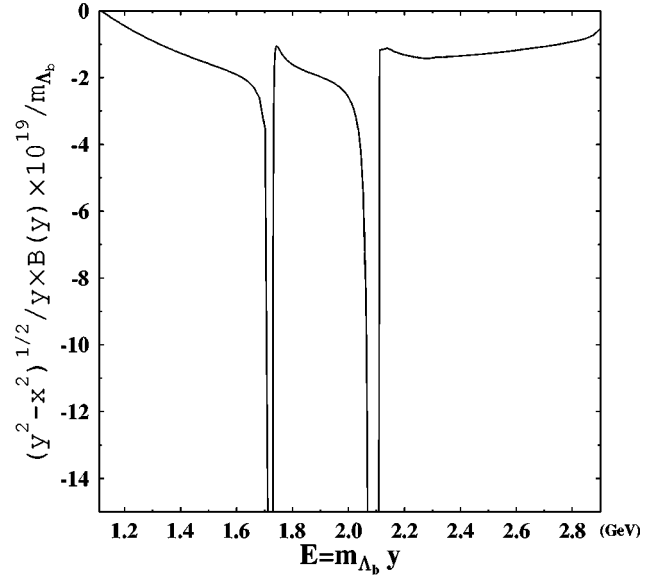
where

$$\begin{aligned} h(x, s) &= - \left[\frac{4}{9} \ln x^2 - \frac{8}{27} - \frac{16}{9} \frac{x^2}{s} + \frac{4}{9} \sqrt{\frac{4x^2}{s} - 1} \right. \\ &\times \left. \left(2 + \frac{4x^2}{s} \right) \arctan \left(\frac{4x^2}{s} - 1 \right) \right]^{-1/2} \end{aligned}$$

if $s < 4x^2$ and

FIG. 3. The spin-independent term of the spectrum: $A(y)$.

$$h(x, s) = - \left\{ \frac{4}{9} \ln x^2 - \frac{8}{27} - \frac{16}{9} \frac{x^2}{s} + \frac{2}{9} \sqrt{1 - \frac{4x^2}{s}} \left(2 + \frac{4x^2}{s} \right) \right. \\ \left. \times \left[\ln \left| \frac{1 + \sqrt{1 - 4x^2/s}}{1 - \sqrt{1 - 4x^2/s}} \right| - i\pi \right] \right\}$$

FIG. 4. The spin-dependent term of the spectrum: $[(\sqrt{y^2 - x^2})/y]B(y)$.

if $s > 4x^2$, with $x = m_c/m_b$ and $s = q^2/m_b^2 (q^2 = M_{l^+l^-}^2)$. As in Sec. V, we write the total width in the form of Eq. (16)

$$\Gamma' = \Gamma'_0 [1 + \alpha' \hat{\mathbf{p}} \cdot \mathbf{S}_\Lambda]. \quad (25)$$

Our calculation yields

$$\Gamma'_0 = (0.0045g_A^2 + 0.0045g_V^2 + 0.0075g_A h_A + 1.64h_A^2 - 0.0069g_V h_V + 1.64h_V^2)(1.00 \pm 0.24) \times 10^{-17} \text{ GeV} \\ \alpha' = \frac{-0.012g_V h_A + 0.044h_A^2 + 0.0036g_A g_V + 0.052g_A h_A - 0.020g_A h_V + 0.052g_V h_V - 1.80h_A h_V - 0.042h_V^2}{0.0045g_A^2 + 0.0045g_V^2 + 0.0075g_A h_A + 1.64h_A^2 - 0.0069g_V h_V + 1.64h_V^2} \\ \times (1.00 \pm 0.08), \quad (26)$$

where the uncertainties are mainly rooted in the uncertainties of F_1 and F_2 . In the SM, we take $h_V = 1, h_A = 1, g_V = 1, g_A = -1$, then $2\Gamma'_0$ gives the total decay width $(6.57 \pm 1.58) \times 10^{-17} \text{ GeV}$ and $\alpha' = -0.54 \pm 0.04$. The differential widths, $A(y)$ and $[(\sqrt{y^2 - x^2})/y]B(y)$ (the spin-dependent term of the energy spectrum in the rest frame of Λ_b), are given in Fig. 3 and Fig. 4, respectively.

VI. DISCUSSIONS

In this paper we have analyzed some features of the rare decays $\Lambda_b \rightarrow \Lambda \gamma$ and $\Lambda_b \rightarrow \Lambda l^+ l^-$, using an approach based on three-point function QCD sum rules to compute the relevant form factors.

We have considered only the short distance contribution for $\Lambda_b \rightarrow \Lambda \gamma$. There are some estimations on the long distance contribution in Ref. [2], the results turn out to be negligibly small, the decay $\Lambda_b \rightarrow \Lambda \gamma$ is dominated by the short

distance piece. For $\Lambda_b \rightarrow \Lambda l^+ l^-$ we have taken into account the main long distance contributions included in the coefficient C_9^{eff} .

As we have mentioned in Sec. IV, in applying QCD sum rules analysis for $\Lambda_b \rightarrow \Lambda \gamma$, we need to assume the heavy quark symmetry is applicable at its most disadvantage phase space corner. For the process $\Lambda_b \rightarrow \Lambda l^+ l^-$, the heavy quark symmetry works well in most of phase space. In particular, one can see from Fig. 1 that the form factor F_1 becomes larger when z approaches the zero recoil point so that its contributions to the decay are dominant.

Our results show that the total decay width $\Gamma(\Lambda_b \rightarrow \Lambda l^+ l^-)$ is larger than that for $\Lambda_b \rightarrow \gamma$, which is due to the dominance of the long distance (resonance) contributions to the decay $\Lambda_b \rightarrow \Lambda l^+ l^-$. From the expressions of Eq. (19) and Eq. (26), in addition to that the total width of lepton final state is greater than that of γ final state, its polarization effect is more remarkable than that of γ final state either. So the

measurements of polarization parameter in the decay $\Lambda_b \rightarrow \Lambda l^+ l^-$ is of a complement to those in $\Lambda_b \rightarrow \Lambda \gamma$ in order to discover new physics. From Eq. (26), we can also find that the contributions to decay $\Lambda_b \rightarrow \Lambda l^+ l^-$ mainly come from O_9 and O_{10} .

We emphasize that the polarization parameter in $\Lambda_b \rightarrow \Lambda \gamma$ is independent of the hadronic structure of Λ in the heavy quark limit so that it is a good quantity to probe new physics beyond SM. In addition to non-SM couplings in O_7 , we have also considered the non-SM couplings in O_9 and

O_{10} and examined their effects in rare decays of Λ_b . It is expected that the measurements of $\Lambda_b \rightarrow \Lambda \gamma$ and $\Lambda_b \rightarrow \Lambda l^+ l^-$ will give us some information on physics beyond the SM.

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