Determination of the HQET parameter λ_1 from an inclusive semileptonic B meson decay spectrum

K. K. Jeong*
Department of Physics, Yonsei University, Seoul 120-749, Korea

C. S. Kim†
Department of Physics, Yonsei University, Seoul 120-749, Korea
and School of Physics, Korea Institute for Advanced Study, Seoul 130-012, Korea

(Received 30 November 1998; published 7 May 1999)

We estimate the heavy quark effective theory parameter λ_1 from the inclusive semileptonic B-meson decay spectrum. By using recent CLEO double lepton tagging data of $B\rightarrow Xe\nu$, which show the lepton momentum as low as 0.6 GeV, we extracted $\lambda_1 \sim -0.58\ \mathrm{GeV}^2$. We also derived $\bar{\Lambda}\sim 0.46\ \mathrm{GeV}$ and $|V_{cb}|=0.041\pm 0.002$. [S0556-2821(99)00411-7]

PACS number(s): 12.39.Hg, 13.20.He

I. INTRODUCTION

As is well known, the heavy quark symmetry breaking parameters λ_1 and λ_2 can affect the shape of the B meson semileptonic decay spectrum substantially. While it is easy to obtain the value of λ_2, the hyperfine splitting term, from the mass difference between B and B^* mesons, it is very difficult to determine the value of parameter λ_1, which corresponds to the kinetic energy of a heavy quark inside a heavy meson. So finding the precise value of λ_1 is very important in understanding heavy meson decay.

The CLEO Collaboration measured the lepton spectrum in the inclusive $B\rightarrow Xl\bar{\nu}$ decay both by one lepton tagging [1], and by double lepton tagging [2]. In single lepton tagging data, leptons from secondary charm decay ($b\rightarrow c\rightarrow sl\nu$) dominate the low lepton energy region. These secondary leptons have typically lower energy than the primary ones, because they are from c quark decay. To obtain the $B\rightarrow Xl\nu$ lepton spectrum in the low E_l region from the single lepton tagging data, these secondary leptons must be separated by fitting the spectrum with some assumptions and models.

In Ref. [3], the parameter values of $\bar{\Lambda}$ and λ_1 were estimated (with fixed value of $\lambda_2=0.12\ \mathrm{GeV}^2$) by using lepton energy distribution of $E_l>1.5\ \mathrm{GeV}$ from CLEO data [4] of semileptonic decay $B\rightarrow Xl\bar{\nu}$ with single lepton tagging. The advantage of using single lepton tagging data is small statistical error, though we cannot use the low lepton energy part of the data ($E_l<1.5\ \mathrm{GeV}$).

In Ref. [2], the CLEO Collaboration separated $B\rightarrow Xl\bar{\nu}$ from cascade decays of $b\rightarrow c\rightarrow sl\nu$. They selected events with tagging leptons of momentum greater than 1.4 GeV, which are predominantly from the semileptonic decay of one of the two B mesons in an $\Upsilon(4S)$ decay. When a tag was found, they searched for an accompanying electron with minimum momentum 0.6 GeV. The main sources of these electrons are (a) the secondary lepton from the same B, (b) the primary lepton from the other B, and (c) the secondary lepton from the other B. The lepton from (c) has the same charge as the tag lepton while leptons from (a) and (b) have opposite charge to the tag lepton. And leptons from (a) and (b) have different kinematic signatures so that their contributions are easy to separate. In the $\Upsilon(4S)$ decay, the B and the $\bar{B}$ are produced nearly at rest. Hence there is little correlation between the directions of a tag lepton and an accompanying electron if they are from different B mesons. If they are from the same B, there is a tendency for the tagged lepton and the electron to be back-to-back. They analyzed the data with double lepton tagging and separated the primary leptons from secondary leptons without model dependence.

In this paper by using the double lepton tagging data, we made a minimum χ^2 analysis to determine the value of the parameter λ_1. There is one difficulty in χ^2 fitting for the data, as is well known. Since nonperturbative correction up to $1/m_b^2$ cannot predict the correct shape of lepton distribution near the end point, we have to exclude the high energy data points of the distribution. Choosing E_{QCD}, the maximum lepton energy that one can trust the shape of $1/m_b$ expansion, is very important in this fitting. Following Ref. [5], we choose $E_{QCD}=2.0\ \mathrm{GeV}$. The double tagged data has larger statistical error than the single tagged one, but we can use low energy lepton data model-independently. Therefore, this work can complement the work of Ref. [3].

II. THEORETICAL DETAILS

Following the heavy quark effective theory (HQET) [6], the mass of a pseudoscalar or a vector meson M containing a heavy quark Q can be expanded as

$$m_M=m_Q+\bar{\Lambda}-\frac{\lambda_1+d_M\lambda_2}{2m_Q}+\cdots, \tag{1}$$

where $d_M=3,\ -1$ for pseudoscalar and vector mesons, respectively, and

*Email address: kkjeong@theory.yonsei.ac.kr
†Email address: kim@cskim.yonsei.ac.kr

0556-2821/99/59(11)/114019(6)/$15.00

$$\lambda_1 = \frac{1}{2m_M}\langle M(v)|\,\overline{h_v}(iD)^2 h_v|M(v)\rangle, \tag{2}$$

$$\lambda_2 = \frac{1}{2d_M m_M}\langle M(v)|\,\overline{h_v}\frac{g}{2}\sigma_{\mu\nu}G^{\mu\nu}h_v|M(v)\rangle, \tag{3}$$

where h_v is the heavy quark field in the HQET with velocity v. λ_1 parametrizes the mass shift due to the kinetic energy of heavy quark inside the meson, and λ_2 is related to the effect of chromomagnetic interaction between heavy quark and light degrees of freedom. In the case of a B meson, we can estimate the value of λ_2 quite accurately from the mass difference between B and B^* mesons:

$$m_{B^*} = m_b + \bar{\Lambda} - \frac{\lambda_1 - \lambda_2}{2m_b}, \tag{4}$$

$$m_B = m_b + \bar{\Lambda} - \frac{\lambda_1 + 3\lambda_2}{2m_b}, \tag{5}$$

and approximately

$$\frac{1}{4}(m_{B^*}^2 - m_B^2) = \lambda_2 + \mathcal{O}\left(\frac{1}{m_b}\right) \approx 0.12\ \ \mathrm{GeV}^2. \tag{6}$$

Within HQET the lepton spectrum of the semileptonic decays of a b-flavored hadron ($H_b \to X_q l\nu$) is calculated in Refs. [5,7], and the result is

$$\begin{aligned}\frac{d\Gamma}{dx} = {} & \Gamma_0\theta(1-x-\epsilon^2)2x^2\Bigg[(3-2x) - 3\epsilon^2 - \frac{3\epsilon^4}{(1-x)^2} \\ & + \frac{(3-x)\epsilon^6}{(1-x)^3} + G_b\Bigg\{\frac{6+5x}{3} - \frac{(6-4x)\epsilon^2}{(1-x)^2} + \frac{(3x-6)\epsilon^4}{(1-x)^3} \\ & + \frac{5(6-4x+x^2)\epsilon^6}{3(1-x)^4}\Bigg\} + K_b\Bigg\{-\frac{5x}{3} + \frac{(2x^2-5x)\epsilon^4}{(1-x)^4} \\ & + \frac{2(x^3-5x^2+10x)\epsilon^6}{3(1-x)^5}\Bigg\}\Bigg],\end{aligned} \tag{7}$$

where

$$\Gamma_0 = \frac{G_F^2 m_b^5}{192\pi^3}|V_{qb}|^2, \quad \epsilon = \frac{m_q}{m_b}, \quad x = \frac{2E_l}{m_b}, \tag{8}$$

$$K_b = -\lambda_1/m_b^2, \quad G_b = 3\lambda_2/m_b^2, \tag{9}$$

with m_q denoting the mass of the quark $q=u,\ c$ in the final state, and V_{qb} is the CKM matrix element [8]. The terms in the second line and third line of Eq. (7) correspond to nonperturbative corrections (NP) to leading order Born approximation of the first line.

Perturbative corrections of the electron spectrum from b decay were calculated in various references [9,10]. The analytic form of order α_s correction is given [9] as

$$\left(\frac{d\Gamma}{dx}\right)_{\alpha_s} = -\frac{2\alpha_s}{3\pi}\Gamma_0\int_0^{y_m} dy\,\frac{12}{(1-\xi y)^2+\gamma^2}F_1(x,y,\epsilon^2), \tag{10}$$

with

$$F_1(x,y,\epsilon^2) = H_1(x,y) + H_2(x,y,\epsilon^2) - H_2(x,y,z_m), \tag{11}$$

where

$$\begin{aligned}H_1(x,y) = {} & 2(x-y)(x_M - x + y)H_B(x,y) \\ & + \frac{\overline{Y_p}}{2\overline{p_3}}\{x(-4+5x) + y(4-6x-5x^2) \\ & + y^2(1+10x) - 5y^3 + \epsilon^2[1-2x+5x^2 \\ & + y(5-16x) + 11y^2] + \epsilon^4(-2+6x-7y) + \epsilon^6\} \\ & + \ln\epsilon[x(-1+2x) + y(1-4x) + 2y^2 \\ & + \epsilon^2(1+x-y) - \epsilon^4],\end{aligned} \tag{12}$$

and

$$\begin{aligned}H_2(x,y,z) = {} & \frac{f_1 + zf_2}{8(1-y)[p_3(z)]^2} + \frac{Y_p(z)(f_3+zf_4)}{8[p_3(z)]^3} \\ & + \frac{Y_p(z)(f_5+zf_6)}{4p_3(z)} + \frac{1}{4}\ln(z)f_7 + \frac{\epsilon^2 f_8}{2(1-y)z} \\ & + [\mathrm{Li}_2(w_+(z)) + \mathrm{Li}_2(w_-(z))]f_9 - yz \\ & + 4yp_3(z)Y_p(z),\end{aligned} \tag{13}$$

with

$$H_B(x,y)=1-\ln(1-x)-\ln(1-y/x)-2(\overline{p_0}/\overline{p_3}\overline{Y}_p-1)\ln[(1-x)(1-y/x)-\epsilon^2]+\frac{\overline{p_0}}{\overline{p_3}}\Bigg[\mathrm{Li}_2\left(1-\frac{\overline{p}_-\overline{w}_-}{\overline{p}_+\overline{w}_+}\right)-\mathrm{Li}_2\left(1-\frac{\overline{w}_-}{\overline{w}_+}\right)$$
$$-\mathrm{Li}_2\left(1-\frac{\overline{p}_-}{\overline{p}_+}\right)+\mathrm{Li}_2\left(1-\frac{1-x}{\overline{p}_+}\right)+\mathrm{Li}_2\left(1-\frac{x-y}{x\overline{p}_+}\right)-\mathrm{Li}_2\left(1-\frac{1-x}{\overline{p}_-}\right)-\mathrm{Li}_2\left(1-\frac{x-y}{x\overline{p}_-}\right)+2\overline{Y}_p(\overline{Y}_w+2\ln\epsilon)$$
$$+\ln\epsilon\ln\frac{(\overline{w}_+-x)(\overline{w}_+-y/x)}{(x-\overline{w}_-)(y/x-\overline{w}_-)}\Bigg], \tag{14}$$

and

$$f_1=(1-y)^3[5x^2+y(5-2x)]-4\epsilon^2(1-y)\times[x^2+y(1-5x+2x^2)+y^2(2-x)]+\epsilon^4[-x^2+y(-1+6x-3x^2)+y^2(-3+2x)], \tag{15}$$

$$f_2=-(1-y)[5x^2+y(5+18x+3x^2)+y^2(3-2x)]+4\epsilon^2(1-y)[x^2+y(1-x)]+\epsilon^4[x^2+y(1-2x)], \tag{16}$$

$$f_3=(1-y)^2[-5x^2+y(-5-8x+x^2)+y^2]+2\epsilon^2(1-y)[2x^2+y(2-6x+x^2)+y^2]+\epsilon^4[x^2+y(1-4x+x^2)+y^2], \tag{17}$$

$$f_4=5x^2+y(5+28x+12x^2)+y^2(12+4x-x^2)-y^3+\epsilon^2[-4x^2+y(-4-4x+2x^2)+2y^2]-\epsilon^4(x^2+y), \tag{18}$$

$$f_5=-5+10x+y(5+24x+8x^2)+y^2(5-18x)+3y^2+\epsilon^2[4-10x-4x^2+y(-8+18x)-4y^2]+\epsilon^4(1-4x+y), \tag{19}$$

$$f_6=5+10x-4x^2+y(14+10x)-3y^2-2\epsilon^2(2+3x-2y)-\epsilon^4, \tag{20}$$

$$f_7=-5+4x-4x^2+6yx-y^2+\epsilon^2[4(1+x)-10y]+\epsilon^4, \tag{21}$$

$$f_8=x(1-x)+y(-1+x+x^2)-2y^2x+y^3+\epsilon^2(1-x)(x-y), \tag{22}$$

$$f_9=x+y(1-2x)+y^2+\epsilon^2(x-y). \tag{23}$$

All the parameters and the kinematic variables in the above expressions are listed in the Appendix.

After using all the above formulas, the electron distribution in semileptonic decay of the B meson can be written as

$$\frac{d\Gamma_{theory}}{dE_l}=\left(\frac{d\Gamma}{dE_l}\right)_{Born}+\left(\frac{d\Gamma}{dE_l}\right)_{NP}+\left(\frac{d\Gamma}{dE_l}\right)_{\alpha_s}, \tag{24}$$

where $(d\Gamma/dE_l)_{Born}$ is the leading order Born approximation, $(d\Gamma/dE_l)_{NP}$ is the nonperturbative correction using the HQET, and $(d\Gamma/dE_l)_{\alpha_s}$ is the perturbative α_s correction. We define the CKM-matrix independent decay rate

$$\gamma_q=\frac{\Gamma_{theory}(B\to X_q l\nu)}{|V_{qb}|^2}, \tag{25}$$

and then, the semileptonic decay rate Γ_{SL} can be written as

$$\Gamma_{SL}=\gamma_c|V_{cb}|^2+\gamma_u|V_{ub}|^2. \tag{26}$$

Since $|V_{ub}|^2\ll|V_{cb}|^2$, we can neglect $b\to u$ decay. Integrating over E_l of Eq. (24), we obtain [11]

$$\Gamma_{SL}=\gamma_c|V_{cb}|^2=\Gamma_0\left[z_0\left\{1-\frac{2\alpha_s(m_b)}{3\pi}g(\epsilon)\right\}+\frac{1}{2}z_0(G_b-K_b)-2z_1G_b\right], \tag{27}$$

where z_0 and z_1 are defined as

$$z_0=1-8\epsilon^2+8\epsilon^6-\epsilon^8-24\epsilon^4\ln\epsilon, \tag{28}$$

$$z_1=(1-\epsilon^2)^4, \tag{29}$$

and $g(\epsilon)$ is a complicated function of ϵ, which can be approximated [12] to

$$g(\epsilon)=\left(\pi^2-\frac{31}{4}\right)(1-\epsilon)^2+\frac{3}{2}. \tag{30}$$

To obtain the mass ratio $\epsilon=m_c/m_b$, we use the relation

$$m_b-m_c=(m_B-m_D)-\frac{1}{2}(\lambda_1+3\lambda_2)\left(\frac{1}{m_c}-\frac{1}{m_b}\right) \tag{31}$$
$$\simeq(m_B-m_D)\pm(\sim 1\%)\approx m_B-m_D$$
$$=3.41\ \text{GeV}.$$

We note that if we use instead the other relations, e.g.,

$$m_b - m_c = (m_{B*} - m_{D*}) - \frac{1}{2}(\lambda_1 - \lambda_2)\left(\frac{1}{m_c} - \frac{1}{m_b}\right)$$
$$\approx m_{B*} - m_{D*} = 3.32 \text{ GeV}, \tag{32}$$

or

$$m_b - m_c = (\overline{m}_B - \overline{m}_D) - \frac{\lambda_1}{2}\left(\frac{1}{m_c} - \frac{1}{m_b}\right) \approx \overline{m}_B - \overline{m}_D$$
$$= 3.35 \text{ GeV}, \tag{33}$$

where

$$\overline{m}_B = \frac{1}{4}(m_B + 3m_{B*})$$

and

$$\overline{m}_D = \frac{1}{4}(m_D + 3m_{D*}),$$

the values of the correction would become as large as $\sim +(2\text{–}4\,\%)$ depending on λ_1.

For $b \to c l \nu$, Figs. 1(a) and 1(b) illustrate the dependencies of various corrections on m_b and λ_1. All figures in Fig. 1 are with $|V_{cb}| = 0.04$. The value of m_b determines mainly the overall size of decay width, while other parameters determine the shape of the distribution. As can be seen from Fig. 1(a), we find that the dependence of Γ_{SL} on m_b is rather weak on the contrary to the naive estimation of $\Gamma_{SL} \propto m_b^5$, and is very sensitive to the quark mass difference $(m_b - m_c)$ [11]. Note also that the shapes of Born approximation and perturbative correction are almost insensitive to the value of m_b, while nonperturbative correction is quite sensitive to both m_c/m_b and λ_1.

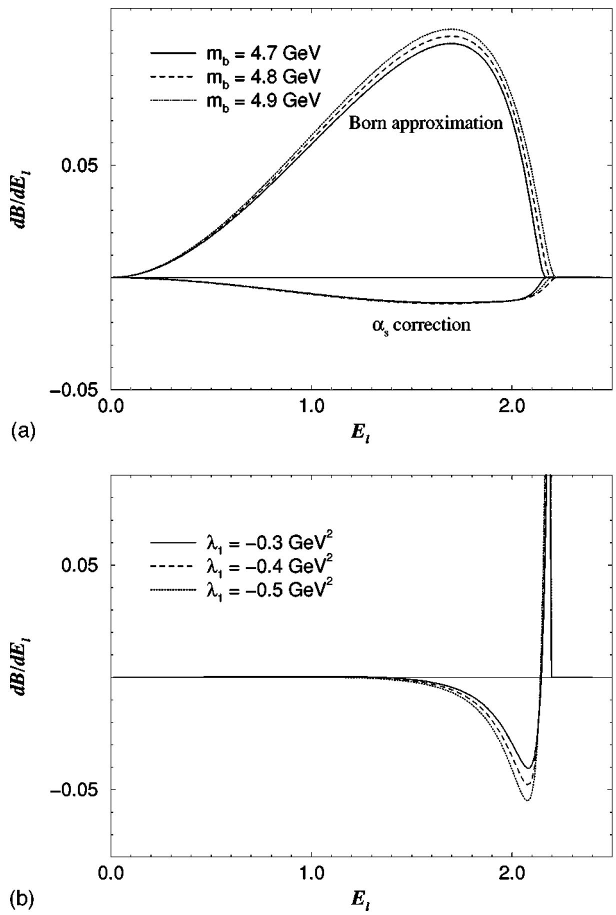

FIG. 1. Contributions of each term in lepton spectra of $b \to c l \nu$ decay. In all figures, $|V_{cb}| = 0.04$. (a) Born approximation and perturbative α_s correction with $m_b = 4.7$ GeV (solid line), $m_b = 4.8$ GeV (dashed line), $m_b = 4.9$ GeV (dotted line), and $\alpha_s = 0.22$. (b) Nonperturbative correction with $\lambda_1 = 0.3$ GeV^2 (solid line), $\lambda_1 = 0.4$ GeV^2 (dashed line), and $\lambda_1 = 0.5$ GeV^2 (dotted line). λ_2 and m_b are fixed with the values $\lambda_2 = 0.12$ GeV^2 and $m_b = 4.8$ GeV.

III. RESULTS AND DISCUSSIONS

To compare CLEO data with theoretical calculation, we use the minimum χ^2 method with

$$\chi^2 = \sum_{E_i < E_{QCD}} \frac{[\mathcal{B}(E_i) - F^{\text{theory}}(E_i;\epsilon,\lambda_1)]^2}{\sigma(E_i)^2}, \tag{34}$$

where $\mathcal{B}(E_i)$ and $\sigma(E_i)$ are experimental data of the differential branching ratio and error at lepton energy E_i, and $F^{\text{theory}}(E_i;\epsilon,\lambda_1)$ is the theoretical prediction at E_i as a function of parameters $\epsilon \equiv m_c/m_b$ and λ_1. We normalized the decay distribution to have a branching ratio of 10.49% as in Ref. [2], $\mathcal{B}(B \to X l \nu) = (10.49 \pm 0.17 \pm 0.43)\%$, i.e.,

$$F^{\text{theory}}(E_i;\epsilon \equiv m_c/m_b, \lambda_1) = \frac{0.1049}{\Gamma_{SL}} \left[\frac{d\Gamma_{theory}}{dE_l}\right]_{E_l = E_i}. \tag{35}$$

We note that, because of exact cancellation between Γ_{SL} and Γ_{theory}, F^{theory} is independent of Γ_0, and therefore independent of $|V_{cb}|$ and m_b^5. F^{theory} is only indirectly dependent on m_b through the definition of x in Eq. (8). Following Ref. [11], we use the b quark mass, $m_b = 4.8 \pm 0.1$ GeV, which has been derived from the QCD sum rule analysis of the Υ system [13]. For α_s, we use $\alpha_s = 0.22$, as in Ref. [3].

We comment here on the determination of E_{QCD}, which is crucial for this analysis. As we can see in Figs. 1(b) and 2, nonperturbative corrections are significant only in the large electron energy region (i.e., $E_l > 1.5$ GeV). Therefore, we have to include as many data points up to E_{QCD}, in which theory can give the correct shape of the lepton energy distribution. Otherwise, we cannot fully see the effect of the nonperturbative correction which determines the value of λ_1. However, if we include the data points over $E_l > E_{QCD}$, the result will be meaningless because the shape of the lepton energy spectrum is not reliable above the E_{QCD} region. The numerical value of E_{QCD} can be estimated from the value of m_b [5]: For $b \to u$ decay with $m_u = 0$, $E_{QCD} \approx 0.9 m_b/2 \sim 2.15$ GeV. For $b \to c$ decay, a smaller smearing range is required near the end point, and

$$E_{QCD} \approx 0.9(m_b^2 - m_c^2)/2m_b \sim 2.0 \text{ GeV}. \tag{36}$$

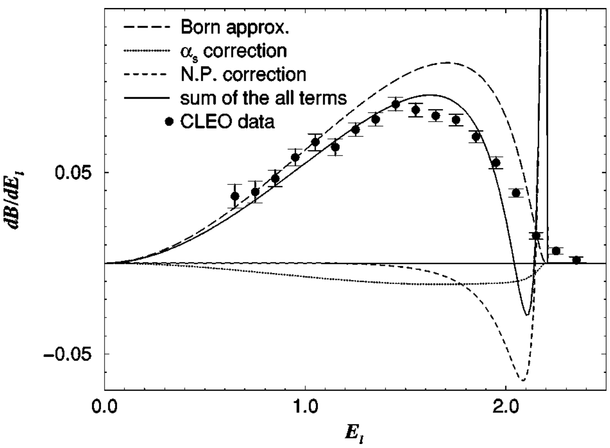

FIG. 2. Best fit result for $m_b = 4.8$ GeV with Born approximation (long dashed line), nonperturbative correction (short dashed line), perturbative correction (dotted line), and sum of all (solid line). Dots with error bars represent CLEO data. Parameter values are $\lambda_1 = -0.58$ GeV^2, $\lambda_2 = 0.12$ GeV^2, and $\alpha_s = 0.22$.

Since we are dealing only with lepton energies less than 2.15 GeV, we neglect $b \to u$ decay and set $0.6 < E_i < 2.0$ GeV.

We tabulated the results in Table I. Since we fixed $(m_b - m_c)$, changing the value of m_b means changing mass ratio $\epsilon \equiv m_c/m_b$ together, and this mass ratio affects the results. The values of $\bar{\Lambda}$ are determined from the mass relation Eq. (1). All values of λ_1 in Table I are much larger than the value in Ref. [3] which is $\lambda_1 = -0.19 \pm 0.10$ GeV^2, or the values in [14,15] which are ~ -0.1 GeV^2, but consistent with [16–18] which are in the range -0.4 to -0.7 GeV^2.

The values of λ_1 show significant dependencies on the input value of m_b, but still each value is consistent within 1σ error range. As explained before, this large sensitivity comes from mass ratio m_c/m_b. Indeed, for $m_b = 4.8$ GeV, if we change the value of $(m_b - m_c)$ to 3.35 GeV, i.e., $m_c^2/m_b^2 = 0.091$, then $\lambda_1 = -0.69 \pm 0.22$ GeV^2. Changing the value of m_b with fixed $m_c^2/m_b^2 = 0.084$, we obtain $\lambda_1 = -0.55 \pm 0.18$ GeV^2 for $m_b = 4.7$ GeV and $\lambda_1 = -0.52 \pm 0.29$ GeV^2 for $m_b = 4.9$ GeV, which are very similar to the case with $m_b = 4.8$ GeV, as shown in Table 1.

Once we know the parameter values m_b, λ_1, we can extract $|V_{cb}|$ from the relation

$$|V_{cb}|^2 = \frac{\mathcal{B}(B \to X l \nu)}{\tau_B \gamma_c}. \tag{37}$$

TABLE I. Results of the fitting with $m_b = 4.7$–4.9 GeV and the fixed $m_b - m_c = 3.41$ GeV.

m_b	$\epsilon^2 \equiv m_c^2/m_b^2$	λ_1	$\bar{\Lambda}$
4.7 GeV	0.075	$-(0.45 \pm 0.19)$ GeV^2	0.57 ± 0.018 GeV
4.8 GeV	0.084	$-(0.58 \pm 0.23)$ GeV^2	0.46 ± 0.022 GeV
4.9 GeV	0.092	$-(0.70 \pm 0.27)$ GeV^2	0.34 ± 0.026 GeV

For τ_B, we averaged $\tau_{B^\pm}$ and τ_{B^0} from Particle Data Book [19]

$$\tau_{B^\pm} = (1.62 \pm 0.06) \times 10^{-12} \text{ sec},$$

$$\tau_{B^0} = (1.56 \pm 0.06) \times 10^{-12} \text{ sec}.$$

This gives the value

$$|V_{cb}| = 0.041 \pm 0.002, \tag{38}$$

where the error includes the errors from the semileptonic branching ratio of CLEO data, B meson lifetime, uncertainties from λ_1 and b quark mass. This result is consistent with the CLEO result with the ISGW model which is $|V_{cb}| = 0.040 \pm 0.001 \pm 0.002$ [2], and with the recent Particle Data Book result $|V_{cb}| = 0.0395 \pm 0.0017$ [19].

Figure 2 shows the best fit result of the differential branching ratio compared with CLEO data as a function of charged lepton energy, with $m_b = 4.8$ GeV and $\lambda_1 = -0.58$ GeV^2. It shows the relative size and shape of the various corrections for $m_b = 4.8$ GeV. The nonperturbative term is about $\sim -4.5\%$ and the perturbative term is $\sim -12\%$ from leading approximation. From these facts, it is clear that the nonperturbative correction determines the shape and does not have much effect on the total decay rate, while the perturbative term has little effect on the shape but its contribution on the total decay rate is quite large. Fitting with data points between 1.0 GeV$< E_i <$2.0 GeV, we obtain $\lambda_1 = -0.57 \pm 0.19$ GeV^2 and $\bar{\Lambda} \simeq 0.46$ for $m_b = 4.8$ GeV, which are almost the same with the results from 0.6 GeV$< E_i <$2.0 GeV. Finally we note the dependence on α_s is very weak. Changing α_s to 0.35 we get $\lambda_1 \sim -0.54$ GeV^2 and $\bar{\Lambda} \sim 0.46$ GeV for $m_b = 4.8$ GeV, which are almost the same as the values in Table I.

We finally note that recently the CLEO Collaboration measured [20] the first and the second moments of the hadronic mass-squared distribution in the inclusive decay $B \to X_c l \nu$ and also made a preliminary determination of the first and the second moments of the lepton energy distribution from the spectrum in Ref. [2]. Using those four moments, they obtained the values of $\bar{\Lambda}$ and λ_1, but there appeared to be inconsistencies in the results which suggest either experimental error or problems in the HQET. However, if we consider only the moments of the lepton energy distribution, the preliminary CLEO analysis [20] gives $\lambda_1 \sim -0.75 \pm 0.20$ GeV^2, which is in rather good agreement with our results.

ACKNOWLEDGMENTS

We thank A. Falk for a careful reading of the manuscript and his valuable comments. C.S.K. wishes to thank the Korea Institute for Advanced Study for warm hospitality. The work of K.K.J. was supported in part by Non-Directed-Research-Fund, KRF, in part by the CTP, Seoul National University, in part by the BSRI Program, Ministry of Education, Project No. 98-015-D00061, and in part by the KOSEF-DFG large collaboration project, Project No. 96-

0702-01-01-2. C.S.K. wishes to acknowledge the financial support of Sughak Program from Korean Research Foundation.

APPENDIX: KINEMATIC VARIABLES

In Ref. [9], the kinematic variables are defined as follows: b,q,G,l,ν are the four-momenta of the b-quark, lighter quark, gluon, lepton, neutrino; $P=q+G$, $W=l+\nu$ are the four-momentum of the quark-gluon system and the virtual W; λ_G stands for the scaled gluon mass $(\lambda_G\equiv m_G/m_b\ll\epsilon)$.

The scaled masses and lepton energies

$$\epsilon\equiv\frac{m_q}{m_b}=\left(\frac{q^2}{b^2}\right)^{1/2},\quad x\equiv\frac{2E_l}{m_b},\quad y\equiv\frac{W^2}{b^2},$$

$$z\equiv\frac{P^2}{b^2},\quad \xi=\frac{m_b^2}{M_W^2},\quad \gamma\equiv\frac{\Gamma_W}{M_W} \tag{A1}$$

vary in the region

$$0\leqslant x\leqslant x_M\equiv 1-\epsilon^2, \tag{A2}$$

$$0\leqslant y\leqslant y_m\equiv x(x_M-x)/(1-x), \tag{A3}$$

$$(\epsilon+\lambda_G)^2\leqslant z\leqslant z_m\equiv(1-x)(1-y/x). \tag{A4}$$

Frequently used kinematic variables which characterize the quark-gluon system are

$$p_0(z)\equiv\tfrac{1}{2}(1-y+z),$$

$$p_3(z)\equiv\tfrac{1}{2}[1+y^2+z^2-2(y+z+yz)]^{1/2},$$

$$p_\pm(z)\equiv p_0(z)\pm p_3(z),$$

$$Y_p(z)\equiv\frac{1}{2}\ln\frac{p_+(z)}{p_-(z)}=\ln\frac{p_+(z)}{\sqrt{z}}, \tag{A5}$$

and similarly for the virtual W

$$w_0(z)\equiv\tfrac{1}{2}(1+y-z),$$

$$w_3(z)\equiv\tfrac{1}{2}[1+y^2+z^2-2(y+z+yz)]^{1/2},$$

$$w_\pm(z)\equiv w_0(z)\pm w_3(z),$$

$$Y_w(z)\equiv\frac{1}{2}\ln\frac{w_+(z)}{w_-(z)}=\ln\frac{w_+(z)}{\sqrt{z}}. \tag{A6}$$

For $G=0$, which implies $z=\epsilon^2$, the abbreviations

$$\overline{p_0}\equiv p_0(\epsilon^2),\quad \overline{p_3}\equiv p_3(\epsilon^2),\quad \text{etc.},$$

$$\overline{w_0}\equiv w_0(\epsilon^2),\quad \overline{w_3}\equiv w_3(\epsilon^2),\quad \text{etc.}, \tag{A7}$$

will be useful. Polylogarithms are defined as real functions, and in particular

$$\mathrm{Li}_2(x)=-\int_0^x\frac{dt}{t}\ln|1-t|. \tag{A8}$$

[1] CLEO Collaboration, J. Bartelt *et al.*, CLEO/CONF 93-19, 1993.

[2] CLEO Collaboration, B. Barish *et al.*, Phys. Rev. Lett. **76**, 1570 (1996).

[3] M. Gremm, A. Kapustin, Z. Ligeti, and M.B. Wise, Phys. Rev. Lett. **77**, 20 (1996).

[4] R. Wang, Ph.D. thesis, University of Minnesota, 1994.

[5] I.I. Bigi, M. Shifman, N.G. Uraltsev, and A. Vainshtein, Phys. Rev. Lett. **71**, 496 (1993).

[6] E. Eichten and B. Hill, Phys. Lett. B **232**, 113 (1989); H. Georgi, *ibid.* **240**, 447 (1990); for a review see M. Neubert, Phys. Rep. **245**, 259 (1994).

[7] B. Blok, L. Koyrakh, M. Shifman, and A.I. Vainshtein, Phys. Rev. D **49**, 3356 (1994); A.V. Manohar and M.B. Wise, *ibid.* **49**, 1310 (1994).

[8] N. Cabibbo, Phys. Rev. Lett. **10**, 531 (1963); M. Kobayashi and M. Maskawa, Prog. Theor. Phys. **49**, 652 (1973).

[9] M. Jezabek and J.H. Kühn, Nucl. Phys. **B320**, 20 (1989); A. Czarnecki and M. Jezabek, *ibid.* **B427**, 3 (1994).

[10] A. Ali and E. Pietarinen, Nucl. Phys. **B154**, 519 (1979); G. Corbò, Phys. Lett. **116B**, 298 (1982); Nucl. Phys. **B212**, 99 (1983).

[11] M. Shifman, N.G. Uraltsev, and A. Vainshtein, Phys. Rev. D **51**, 2217 (1995); **52**, 3149(E) (1995).

[12] C.S. Kim and A.D. Martin, Phys. Lett. B **225**, 186 (1989).

[13] M. Voloshin and Y. Zaitsev, Usp. Fiz. Nauk **152**, 361 (1987) [Sov. Phys. Usp. **30**, 553 (1987)].

[14] A. Falk, M. Luke, and M.J. Savage, Phys. Rev. D **53**, 6316 (1996).

[15] M. Neubert, Phys. Lett. B **389**, 727 (1996).

[16] P. Ball and V.M. Braun, Phys. Rev. D **49**, 2472 (1994).

[17] D.S. Hwang, C.S. Kim, and W. Namgung, Z. Phys. C **69**, 107 (1995); Phys. Rev. D **53**, 4951 (1996); **54**, 5620 (1996); Phys. Lett. B **406**, 117 (1997).

[18] F. De Fazio, Mod. Phys. Lett. A **11**, 2693 (1996).

[19] Particle Data Group, C. Caso *et al.*, Eur. Phys. J. C **3**, 1 (1998).

[20] CLEO Collaboration, J. Bartelt *et al.*, CLEO/CONF 98-21, 1998.