

# Pion and photon light-cone wave functions from the instanton vacuum

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The leading-twist wave functions of the pion and the photon at a low normalization point are calculated in the effective low-energy theory derived from the instanton vacuum. The pion wave function is found to be close to the asymptotic one, consistent with the recent CLEO measurements. The photon wave function is nonzero at the end points. This different behavior is a consequence of the momentum dependence of the dynamical quark mass suggested by the instanton vacuum. We comment on the relation of meson wave functions and off-forward parton distributions in this model. [S0556-2821(99)09309-1]

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## I. INTRODUCTION

Hadron light-cone wave functions<sup>1</sup> parametrize the non-perturbative information entering in the amplitudes for exclusive hard scattering processes in QCD [1–3]. The pion wave function appears in the description of the pion electromagnetic form factor and pion-meson transition form factors (for a review see Ref. [4]), and in the amplitudes for exclusive pion production in photon-photon [5–9] and photon-nucleon processes [10]. The photon wave function enters, for instance, in the amplitudes for radiative hyperon decays [11], or in power-suppressed contributions to  $\gamma^* \gamma$  processes such as  $\gamma^* \gamma \rightarrow \pi^0$  [8] or deeply virtual Compton scattering [12,13].

A calculation of the meson and photon wave functions from first principles requires a theory of nonperturbative effects giving rise to a hadron structure. Meson wave functions have extensively been studied using QCD sum rules. The original suggestion by Chernyak and Zhitnitsky of a “double-humped” wave function of the pion at a low scale, far from the asymptotic form, was based on an extraction of the first few moments from a standard QCD sum rule approach [4], which has been criticized and revised in Refs. [14,15]. Additional arguments in favor of a form of the pion wave functions close to the asymptotic one came from the analysis of the transition form factor  $\gamma \gamma^* \rightarrow \pi^0$  [8]. The recent measurements of this form factor by the CLEO Collabo-

ration are consistent with a near-asymptotic form of the wave function [16].

About the leading-twist wave function of the photon little is known at present from experiment. Its asymptotic form is fixed by the approximate conformal invariance of QCD for large virtualities (see, e.g., Ref. [4]). Balitsky *et al.* [11] attempted to estimate corrections to the asymptotic form using QCD sum rules and local duality relations but found that in this approach higher moments could not be estimated with sufficient accuracy. The photon wave function has also been discussed in the context of a constituent quark model for photon-meson transition form factors by Anisovich *et al.* [17].

In this paper we study the pion and photon wave functions in the instanton vacuum. The picture of the QCD vacuum as a dilute medium of instantons explains the dynamical breaking of chiral symmetry, which is the nonperturbative phenomenon most important for hadron structure at low energies [18]. Quarks interact with the fermionic zero modes of the individual instantons in the medium, which leads to the formation of a chiral condensate. One derives from the instanton vacuum an effective theory of quarks with a dynamical mass which drops to zero at Euclidean momenta of the order of the inverse average instanton size,  $\bar{\rho}^{-1} \simeq 600$  MeV [19].

The pion and photon wave functions can be extracted from correlation functions of light-ray operators with the mesonic viz. electromagnetic current, which can be computed in the effective low-energy theory. The normalization point of the wave functions obtained in this approach is of the order of  $\bar{\rho}^{-1} \simeq 600$  MeV. The pion wave function has been computed in Ref. [20]; it was found to be close to the asymptotic one. Our intention here is twofold. First, we wish to expand the investigation of the pion wave function, discuss its scale dependence and compare with the recent CLEO measurements [16]. Second, we compute also the photon wave function. In particular, we shall be interested in comparing the photon and the pion wave functions. As will be seen below,

<sup>1</sup>In this article we use the term “light-cone wave function” to denote functions describing matrix elements of QCD light-ray operators between hadron (or photon) states and the vacuum, depending only on longitudinal momentum fractions. These objects are frequently also referred to as “distribution amplitudes” in the literature. Note that some authors use the term “wave function” for more elementary functions depending on longitudinal as well as on transverse momenta, from which the functions considered here would be obtained by integration over transverse momenta.

the two exhibit different behavior at the end points,  $u \rightarrow 0$  and 1. This is a consequence of the momentum dependence of the dynamical quark mass implied by the instanton vacuum.

Additional motivation for studying the photon and pion wave function comes from their importance for deeply virtual Compton scattering and hard meson production [10,12,13]. The factorization of the Compton amplitude in the deeply virtual domain involves the off-forward parton distributions (OFPD's) of the nucleon. Recently, Radyushkin has argued, on the basis of a ‘‘meson exchange’’ contribution to the so-called double distributions related to the OFPD's [21], that the OFPD's may be discontinuous at  $x = \pm \xi/2$  ( $\xi$  is the longitudinal component of the momentum transfer to the nucleon) if the meson wave function were nonzero at the end points. Such discontinuities had indeed been observed in a calculation of the isosinglet OFPD's of the nucleon in the effective low-energy theory derived from the instanton vacuum, where the nucleon is described as a chiral soliton [22]. It was seen there that near  $x = \pm \xi/2$  the behavior of the OFPD is governed by the momentum dependence of the dynamical quark mass, which turns the would-be discontinuity into a sharp but continuous cross-over. Here we shall see that the same physical mechanism—the momentum dependence of the dynamical quark mass obtained from the instanton vacuum—is responsible for the end point behavior of the meson wave function, showing that this approach provides a consistent realization of Radyushkin's general arguments.

## II. PION AND PHOTON WAVE FUNCTION

The basic objects arising in the factorization of hard scattering amplitudes involving mesons or photons in the initial or final state are matrix elements of certain gauge-invariant nonlocal operators between the meson (photon) states and the vacuum. Their classification in structures of different twist and their respective role in the asymptotic limit has been discussed, e.g., in Ref. [4]. The twist-2 wave function of the pion is defined through the matrix element

$$\begin{aligned} \langle 0 | \bar{d}(z) \gamma_\mu \gamma_5 [z, -z] u(-z) | \pi^+(P) \rangle \\ = i \sqrt{2} F_\pi P_\mu \int_0^1 du e^{i(2u-1)P \cdot z} \phi_\pi(u). \end{aligned} \quad (1)$$

Here  $z$  is a lightlike four-vector ( $z^2=0$ ), and

$$[z, -z] \equiv \mathcal{P} \exp \left[ \int_{-1}^1 dt z^\mu A_\mu(tz) \right] \quad (2)$$

denotes the path-ordered exponential of the gluon field, required by strong gauge invariance;<sup>2</sup> the path here is defined to be along the lightlike direction  $z$ . Furthermore,  $P$  is the

<sup>2</sup>Here and in the following we have omitted the corresponding phase factors connected with electromagnetic gauge invariance; they are not important in the discussion here.

pion four-momentum; we shall consider the chiral limit  $P^2=0$ . Finally, in Eq. (1)  $F_\pi$  denotes the usual weak pion decay constant

$$\langle 0 | \bar{d}(0) \gamma_\mu \gamma_5 u(0) | \pi^+(P) \rangle = i \sqrt{2} F_\pi P_\mu \quad (3)$$

( $F_\pi=93$  MeV), and the wave function is normalized according to

$$\int_0^1 du \phi_\pi(u) = 1. \quad (4)$$

The matrix element in Eq. (1) involves a chiral-even operator with Dirac matrix  $\gamma_\mu \gamma_5$ ; the corresponding matrix element with a chiral-odd operator ( $\gamma_5$ ) is of higher twist.

The twist decomposition of the matrix elements for the photon parallels that for the rho meson (see, e.g., Ref. [23]), the only difference—except isospin—being that the real photon has only two transverse polarization states. The twist-2 wave function of the photon is defined by the matrix element [11]

$$\begin{aligned} \langle 0 | \bar{u}(z) \sigma_{\mu\nu} [z, -z] u(-z) | \gamma(P, \lambda) \rangle \\ = i \frac{2}{3} f_{\gamma\perp} (e_{\perp\mu}^{(\lambda)} P_\nu - e_{\perp\nu}^{(\lambda)} P_\mu) \\ \times \int_0^1 du e^{i(2u-1)P \cdot z} \phi_{\gamma\perp}(u) + \text{higher twists}. \end{aligned} \quad (5)$$

Here the photon state is characterized by the four-momentum,  $P$  ( $P^2=0$ ), and polarization vector  $e_{\perp}^{(\lambda)}$ , where  $e_{\perp}^{(\lambda)}$  is transverse with respect to  $z$  and  $P$  ( $z \cdot P \neq 0$ ).

We have not written explicitly in Eq. (5) the terms with tensor structures corresponding to contributions of twist 3 and 4. The matrix element of the chiral-odd operator with  $\sigma_{\mu\nu}$  represents the only possible twist-2 matrix element for the on-shell photon; a twist-2 structure with a chiral-even operator ( $\gamma_\mu$ ) is possible only for a virtual photon or rho meson with longitudinal polarization. In Eq. (5), the normalization constant  $f_{\gamma\perp}$  is defined through the matrix element of the corresponding local operator ( $z=0$ )

$$\langle 0 | \bar{u}(0) \sigma_{\mu\nu} u(0) | \gamma(P, \lambda) \rangle = i \frac{2}{3} f_{\gamma\perp} (e_{\perp\mu}^{(\lambda)} P_\nu - e_{\perp\nu}^{(\lambda)} P_\mu), \quad (6)$$

so that the photon wave function is normalized analogous to Eq. (4).

## III. EFFECTIVE LOW-ENERGY THEORY FROM THE INSTANTON VACUUM

We now compute the matrix elements Eqs. (1) and (5) in the effective low-energy theory which has been derived from the instanton vacuum in the large  $N_c$  limit. This effective theory describes the interaction of a quark field  $\psi$  with a pion field  $\pi$  by an effective action

$$S_{\text{eff}} = \int d^4x \bar{\psi}(x) [i \gamma^\mu \partial_\mu - M U^{\gamma_5}(x)] \psi(x), \quad (7)$$

where

$$U^{\gamma_5}(x) = \exp[i \gamma_5 \tau^a \pi^a(x)]. \quad (8)$$

Note that the interaction is chirally invariant. Here,  $M$  denotes the dynamical quark mass, which appears due to the spontaneous breaking of chiral symmetry. On general grounds, this effective theory is valid up to some ultraviolet cutoff. In the instanton vacuum this cutoff is implemented in the form of a specific momentum dependence of the dynamical quark mass, which leads to the following form of the quark-pion coupling:

$$\begin{aligned} & \int d^4x \bar{\psi}(x) M U^{\gamma_5}(x) \psi(x) \\ & \rightarrow M \int \frac{d^4k}{(2\pi)^4} \int \frac{d^4l}{(2\pi)^4} \bar{\psi}(k) F(k) \\ & \quad \times U^{\gamma_5}(k-l) F(l) \psi(l). \end{aligned} \quad (9)$$

Here  $F(k)$  is a form factor,  $F(0) = 1$ , related to the Fourier transform of the instanton zero mode, which drops to zero for spacelike momenta larger than the inverse average instanton size<sup>3</sup>

$$F(k) \rightarrow 0 \quad \text{for} \quad -k^2 \gg \bar{\rho}^{-2}. \quad (10)$$

We note that, when working with the effective theory, Eq. (7), it is assumed that the dynamical quark mass is parametrically small compared to the ultraviolet cutoff; their ratio is proportional to the packing fraction of instantons in the vacuum

$$(M\bar{\rho})^2 \propto \left(\frac{\bar{\rho}}{\bar{R}}\right)^4. \quad (11)$$

In order to compute the photon wave function we need to couple an electromagnetic field to the quark fields of the effective Lagrangian, Eq. (7). The interaction of the electromagnetic field with the quarks is dominated by the pointlike interaction which derives from the kinetic term in the effective Lagrangian. The nonpointlike coupling arising from the momentum-dependent mass term, which would involve derivatives of the form factor  $F(k)$ , is parametrically suppressed in  $M\bar{\rho}$  relative to the pointlike one. Thus, in leading order in  $M\bar{\rho}$  we shall work with the pointlike electromagnetic current ( $\hat{Q}$  is the quark charge matrix)

$$J_\mu^{\text{e.m.}}(x) = \bar{\psi}(x) \gamma_\mu \hat{Q} \psi(x). \quad (12)$$

One should note the different role of the form factors in the coupling of the pion and the electromagnetic field to the quarks. While in the pion-quark coupling, Eq. (9), the form factors act *multiplicatively*, suppressing the coupling for large quark virtualities, in the case of the photon the form factors inherent in Eq. (7) make only an *additive* (and parametrically small) contribution to the pointlike coupling. This has important consequences for the behavior of the pion and photon wave functions, in particular at the end points  $u \rightarrow 0$  and 1.

#### IV. COMPUTATION OF WAVE FUNCTIONS IN THE EFFECTIVE LOW-ENERGY THEORY

To compute the photon wave function, Eq. (5), we proceed in analogy to the calculation of the pion wave function in Ref. [20]. The matrix element between the one-photon state and the vacuum is extracted from the correlation function of the light-ray operator with the electromagnetic current

$$i \int d^4x e^{-iP \cdot x} \langle 0 | T \{ J_\rho^{\text{e.m.}}(x), \bar{u}(z) \sigma_{\mu\nu} [z, -z] u(-z) \} | 0 \rangle. \quad (13)$$

When computing this correlation function in the instanton vacuum we can drop the path-ordered exponential in the twist-2 operator, since its contribution is parametrically of order  $(\bar{\rho}/\bar{R})^4 \propto (M\bar{\rho})^2$  (see Refs. [24,25] for a discussion). The correlator can then be evaluated in the effective low-energy theory given by Eq. (7). In the large- $N_c$  limit, Eq. (13) is given by a simple quark loop, with the quark propagator subject to the momentum-dependent dynamical quark mass, cf. Eq. (7). Projecting on the twist-2 structure and contracting with the photon polarization vector, we obtain, for the matrix element on the right-hand side (RHS) of Eq. (5),

$$\begin{aligned} & z^\nu \langle 0 | \bar{u}(z) \sigma_{\mu\nu} u(-z) | \gamma(P, \lambda) \rangle \\ & = -\frac{8}{3} N_c M e_{\perp\mu}^{(\lambda)} \int \frac{d^4k}{(2\pi)^4} e^{-i(P-2k) \cdot z} D(k) D(k-P) \\ & \quad \times [z \cdot (P-k) F^2(k) + z \cdot k F^2(k-P)], \end{aligned} \quad (14)$$

where we have set

$$D(k) = \frac{1}{k^2 - M^2 F^4(k) + i0}. \quad (15)$$

Note that we take into account here the form factors (i.e., the momentum dependence of the dynamical quark mass) in the denominators of the quark propagators.

To evaluate the integral in Eq. (14) it is convenient to introduce lightlike vector components. Let  $n$  and  $\tilde{n}$  be dimensionless lightlike vectors parallel to  $z$  and  $P$ , with

$$n \cdot \tilde{n} = 2. \quad (16)$$

Then we can decompose

<sup>3</sup>Here and in the following, all momenta refer to the Minkowskian metric.

$$\begin{aligned}
 k_\mu &= \frac{k^+}{2} n_\mu + \frac{k^-}{2} \tilde{n}_\mu + k_\mu^\perp, \\
 k^+ &\equiv n \cdot k, \\
 k^- &\equiv \tilde{n} \cdot k.
 \end{aligned} \tag{17}$$

Expressing the integral in Eq. (14) in terms of the lightlike vector components, and inserting Eq. (14) in Eq. (5), we can read off the expression for the photon wave function

$$\begin{aligned}
 \phi_{\gamma\perp}(u) &= (-i) \frac{2N_c M P^+}{f_{\gamma\perp}} \int \frac{d^2 k^\perp}{(2\pi)^2} \int \frac{dk^-}{2\pi} \int \frac{dk^+}{2\pi} \\
 &\times \delta(k^+ - uP^+) D(k) D(k-P) \\
 &\times [(1-u)F^2(k) + uF^2(k-P)].
 \end{aligned} \tag{18}$$

The normalization constant can be obtained by computing the matrix element with the local operator, Eq. (6). After taking the limit  $P^2 \rightarrow 0$  one finds

$$f_{\gamma\perp} = (-i) 4N_c M \int \frac{d^4 k}{(2\pi)^4} D^2(k) \left[ F^2(k) - \frac{k^2}{2} \frac{dF^2(k)}{d(k^2)} \right]. \tag{19}$$

Integrating Eq. (18) over  $u$  and using the fact that the integral does not depend on the light-cone vector  $n$ , one easily verifies that Eq. (18) obeys the normalization condition, Eq. (4).

Equation (18) should be compared to the result for the pion wave function, Eq. (1), which was derived in Ref. [20]:

$$\begin{aligned}
 \phi_\pi(u) &= (-i) \frac{2N_c M^2 P^+}{F_\pi^2} \int \frac{d^2 k^\perp}{(2\pi)^2} \int \frac{dk^-}{2\pi} \int \frac{dk^+}{2\pi} \\
 &\times \delta(k^+ - uP^+) D(k) D(k-P) F(k) F(k-P) \\
 &\times [(1-u)F^2(k) + uF^2(k-P)].
 \end{aligned} \tag{20}$$

The expression for the normalization constant, the weak pion decay constant  $F_\pi$  has been obtained in Ref. [18]. Note the additional form factors  $F(k)F(k-P)$  in the integrand in Eq. (20), as compared to the photon wave function, Eq. (18). These are the form factors originating from the pion-quark coupling, Eq. (9).

The evaluation of the integrals defining the photon and pion wave functions, Eqs. (18) and (20), proceeds as follows. First the integral over  $k^+$  is taken, using up the delta function. Then the integral over  $k^-$  is performed by contour integration. A special property of the lightlike coordinates is that the denominators are linear in  $k^-$ . As shown in Ref. [20], the condition that the poles lie on different sides of the real axis ensures that Eqs. (18) and (20) are nonzero only for  $0 < u < 1$ . In the last step the integral over transverse momenta is computed, taking into account the form factors.

## V. END POINT BEHAVIOR

In the expressions for the photon and pion wave functions in the effective theory, Eqs. (18) and (20), the integral over

transverse momenta contains a logarithmic divergence which is regularized by the form factors. Let us analyze the integrands in order to see, for a given value of  $u$ , which regions of  $k^\perp$  make the main contribution to the integrals. For this we consider the virtualities of the quark propagators in the loop integrals in the vicinity of the two poles in  $k^-$ . Taking into account that  $k^+ = uP^+$  one easily sees that at the pole in  $k^-$  corresponding to  $k^2 = M^2$  the virtuality of the quark with momentum  $k-P$  is

$$(k-P)^2 = -\frac{|k^\perp|^2 + M^2}{u}, \tag{21}$$

while at the pole in  $k^-$  corresponding to  $(k-P)^2 = M^2$  the virtuality of the other quark is

$$k^2 = -\frac{|k^\perp|^2 + M^2}{1-u}. \tag{22}$$

We see that the kinematical boundaries  $u \rightarrow 0$  and  $u \rightarrow 1$  correspond to the situation that one of the quarks has a large spacelike momentum. In these limits the form factors in the integrand play a crucial role, since they suppress the contributions of large spacelike momenta. More precisely, for values of  $u$  parametrically of the order

$$u \sim (M\bar{\rho})^2 \quad \text{or} \quad 1-u \sim (M\bar{\rho})^2 \tag{23}$$

one of the quarks has a virtuality of the order  $\bar{\rho}^{-2}$  and the integral is cut by the form factors already at transverse momenta of order  $|k_\perp| \sim M$ . For values of  $u$  not close to the boundaries the integral over transverse momenta extends up to  $\bar{\rho}^{-1}$ , leading to the usual logarithmic dependence of the integral on the ultraviolet cutoff  $\bar{\rho}^{-1}$ .

In light of the above it is clear that the photon and the pion wave functions, Eqs. (18) and (20), behave differently at the end points. In the case of the pion, due to the form factors  $F(k)F(k-P)$  introduced by the coupling of the pion field to the quarks, Eq. (9), the contribution from large virtualities are suppressed, leading to the vanishing of the wave function at the end points  $u \rightarrow 0$  and 1. In the case of the photon, on the other hand, the multiplicative factors are absent, and the integral is not suppressed for  $u \rightarrow 0$  and 1. The remaining form factors in Eq. (18), which originate from the momentum dependence of the dynamical mass in the quark propagators, do not suffice to make the integral go to zero at  $u \rightarrow 0$  and 1. Hence there is no reason for the photon wave function to go to zero at the boundaries.

## VI. NUMERICAL ESTIMATES

To perform a numerical estimate of the pion and photon wave functions we need to put in the specific form of the form factor  $F(k)$ . This function has been derived for Euclidean (i.e., spacelike) momenta as the Fourier transform of the instanton zero mode [18]. One possibility would be to compute moments of the wave functions, which can be expressed as integrals over Euclidean momenta. However, we would like to compute the wave function directly, since, for in-

stance, very high moments would be needed in order to restore the end point behavior of the wave function. Thus we prefer to carry out the integrals Eqs. (18) and (20) over Minkowskian momenta. In principle one could continue the exact Fourier transform of the instanton zero mode to Minkowskian momenta; the function exhibits a cut at positive Minkowskian  $k^2 > 0$ . Since the numerical evaluation of the integrals with this function is rather tedious, we shall instead use a simple pole form

$$F(k) \rightarrow \frac{\Lambda^2}{\Lambda^2 - k^2 - i0} \quad (24)$$

( $k^2$  is the Minkowskian momentum). With  $\Lambda^2 \sim 2.0\bar{\rho}^{-2}$  this form gives a good overall approximation to the Fourier transform of the zero mode in the Euclidean domain ( $k^2 < 0$ ). With the form factors approximated by Eq. (24), the integrals Eqs. (18) and (20) can be evaluated by contour integration over  $k^-$ . We emphasize that the prescription for dealing with the poles of the form factors, cf. Eq. (24), follows unambiguously from the requirement that the moments of the wave function computed with Eq. (24) coincide with the corresponding Euclidean integrals. Since we have seen above that the end point behavior of the wave function is governed by the form factor at large spacelike momenta, cf. Eqs. (21) and (22), where the pole form Eq. (24) is a good approximation to the exact Fourier transform, we are confident that the use of Eq. (24) is at least qualitatively correct.

Dorokhov has discussed the pion electromagnetic form factor in connection with the instanton vacuum using a dispersion relation approach [26]. He quotes an expression for the pion wave function which involves the zero mode form factor corresponding to the instanton in regular gauge, which is not consistent with the superposition of instantons (sum ansatz) implied in the derivation of the instanton medium [18]. Furthermore, in the language of our approach, his result apparently amounts to neglecting the contributions from the singularities of the form factors  $F(k)$  in the Minkowskian loop integral, and thus seems to have no clear relation to a Euclidean calculation of moments of the wave function.

For the numerical estimates we use the standard parameters of the instanton vacuum ( $M = 350$  MeV,  $\bar{\rho} = 600$  MeV). The results for the pion and photon wave function are shown in Figs. 1 and 2. As can be seen from Fig. 1, the pion wave function obtained from Eq. (20) vanishes at the end points, in agreement with the general argument presented above. The wave function at the low normalization point is only slightly flatter than the asymptotic one<sup>4</sup> [1–3]

$$\phi_\pi^{\text{asympt}}(u) = 6u(1-u), \quad (25)$$

<sup>4</sup>In the calculation of the pion wave function in Ref. [20] the form factors inside the square bracket in Eq. (20) and in the denominators of the quark propagators were put to unity, since they are not essential for cutting off the integral. Here we take into account also those factors in Eq. (20). Our numerical results are nevertheless very close to those of Ref. [20].

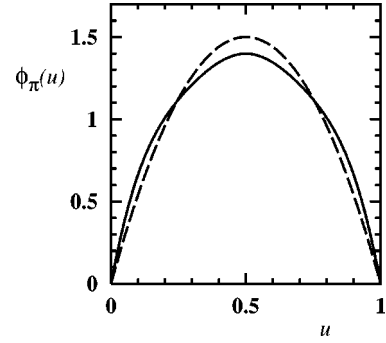


FIG. 1. The pion wave function,  $\phi_\pi(u)$ . Solid line: Wave function calculated in the low-energy effective theory, cf. Eq. (20). Dashed line: Asymptotic wave function  $\phi_\pi^{\text{asympt}}(u) = 6u(1-u)$ .

and far from the form suggested by Chernyak and Zhitnitsky [4]  $\phi_\pi^{\text{CZ}}(u) = 30u(1-u)(2u-1)^2$ .

In order to determine the scale dependence of the wave function one needs to expand it in eigenfunctions of the evolution equations. To one loop accuracy, both for the pion [2] and the photon [27] wave functions, Eqs. (1) and (5), the eigenfunctions are Gegenbauer polynomials of index  $3/2$ . The expansion coefficients are given by the moments

$$B_n = N_n^{-1} \int_0^1 du C_n^{3/2}(2u-1) \phi_\pi(u),$$

$$N_n = \int_0^1 du [C_n^{3/2}(2u-1)]^2 \phi_\pi^{\text{asympt}}(u). \quad (26)$$

Computing these coefficients for the pion wave function obtained from the effective low-energy theory, Eq. (20), we find the expansion

$$\phi_\pi(u) = \phi_\pi^{\text{asympt}}(u) [1 + 0.062 C_2^{3/2}(2u-1) + 0.01 C_4^{3/2}(2u-1) + \dots]. \quad (27)$$

Our value for the second moment  $B_2 = 0.062$  is considerably smaller than that of Chernyak and Zhitnitsky  $B_2 = 0.66$ . One notes that the coefficient of the fourth-order polynomial is

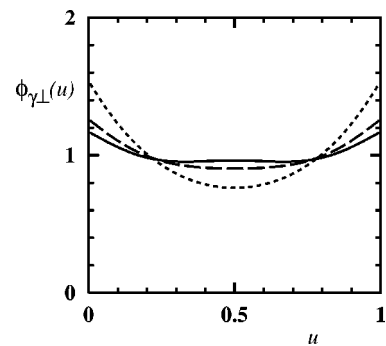


FIG. 2. The photon wave function  $\phi_{\gamma_\perp}(u)$  calculated in the low-energy effective theory, cf. Eq. (18). Solid line: real photon ( $P^2 = 0$ ). Dashed line: the corresponding function for a spacelike virtual photon with  $P^2 = -(250 \text{ MeV})^2$ . Dotted line:  $P^2 = -(500 \text{ MeV})^2$ .

TABLE I. The leading-order scale dependence of the integral  $I(\mu^2)$ , Eq. (28), computed with the pion wave function obtained from the effective low-energy theory (see Fig. 1).

$\mu^2/\text{GeV}^2$	0.6	1	4	8	10	100
$I(\mu^2)$	3.21	3.16	3.12	3.11	3.08	3.06

already very small numerically. We remark that Eq. (27) (with higher terms neglected) provides also a reasonable numerical representation of the computed wave function.

The recent CLEO measurements [16] of the transition form factor  $\gamma\gamma^*\rightarrow\pi^0$  provide a unique opportunity to extract information about the pion wave function. In the standard hard scattering approach with the tree-level coefficient function the  $1/Q^2$ -asymptotic behavior of the transition form factor is governed by the ‘‘inverse’’ moment of the pion wave function [1,6]

$$I(\mu^2) = \int_0^1 du u^{-1} \phi_\pi(u, \mu^2), \quad (28)$$

where  $\mu \simeq Q$ . With the asymptotic wave function, Eq. (25), one obtains a value of  $I^{\text{asympt}} = 3$ , while the Chernyak-Zhitnitsky wave function at the initial normalization point gives  $I^{\text{CZ}} = 5$  [4]. With the wave function computed in the effective low-energy theory, Eq. (20), we find a value of  $I = 3.21$ , which should be associated with a normalization point of the order of  $\mu \simeq \bar{\rho}^{-1} = 600$  MeV. In Table I we give the values of the integral Eq. (28) obtained by leading-order evolution ( $\Lambda_{\text{QCD}} = 250$  MeV,  $N_f = 3$ ) of our wave function, cf. Eq. (27), at a few values of experimentally relevant scales. (For details concerning the evolution see Ref. [4].) We note that the values are close to those obtained in a QCD sum rule approach with nonlocal condensates [28].

It is known that the inclusion of  $\alpha_s$  corrections to the coefficient function decreases the coefficient of the  $1/Q^2$ -asymptotic behavior of the transition form factor by about 15–20 %, see Refs. [7,9]. Including these corrections our value for  $I$  is consistent with the CLEO results, while that of Chernyak and Zhitnitsky seems to be ruled out [16]. Note also that our value is comparable with the one extracted from a QCD sum rule for the form factor  $\gamma\gamma^*\rightarrow\pi^0$  [8].

Finally, it is interesting to note that at  $u = 1/2$ , where the quark and antiquark in the pion carry equal momentum fraction, we obtain a value of the wave function of

$$\phi_\pi(1/2) = 1.4, \quad (29)$$

which is in good agreement with the bound obtained by Braun and Filyanov from QCD sum rules in exclusive kinematics  $\phi_\pi(1/2) = 1.2 \pm 0.3$  [15].

The photon wave function calculated in the effective low-energy theory, cf. Eq. (18), is shown in Fig. 2. It does not go to zero at the boundaries. Thus, the numerical results support the above general conclusions of different behavior of the photon and pion wave functions. We do not write a representation analogous to Eq. (27) for the photon wave function. Such an expansion would be meaningless—since the func-

tion does not vanish at the end points, the moments do not decrease rapidly, and a very large number of terms would be needed to represent the function even for values of  $u$  not close to the boundaries. In Ref. [11] an attempt was made to estimate the deviation of the photon wave function from its asymptotic form using standard QCD sum rules and local duality relations. While it was observed that higher moments could not be reliably estimated in that approach due to rapidly increasing mass scales in the coefficient functions, the authors of Ref. [11] expressed the belief that they are small, and that the photon wave function at a normalization point of  $\mu \sim 1$  GeV is close to the asymptotic one. Our results do not support this conjecture.

For the normalization constant of the photon wave function, Eq. (19), we obtain a value of

$$f_{\gamma\perp} = 0.036 N_c M \simeq 38 \text{ MeV} \quad (30)$$

at the low normalization point. Due to the nonconservation of the tensor current  $f_{\gamma\perp}$  is actually scale dependent [27]. This quantity is directly related to the so-called magnetic susceptibility of the quark condensate,  $\chi_q$ , introduced in Refs. [29,30], namely,  $f_{\gamma\perp} = \langle \bar{u}u \rangle \chi_u$ . One should compare our result with the value obtained in Refs. [31,32] from a QCD sum rule approach,  $f_{\gamma\perp} = 68$  MeV at  $\mu = 1$  GeV [using a value of  $\langle \bar{u}u \rangle = -(250 \text{ MeV})^3$  at  $\mu = 1$  GeV]. Assuming a normalization point of  $\mu \simeq \bar{\rho}^{-1} = 600$  MeV for  $f_{\gamma\perp}$  calculated in the effective theory the value given in Eq. (30) should be reduced by a few percent at  $\mu = 1$  GeV.

## VII. OFF-SHELL BEHAVIOR OF THE PHOTON WAVE FUNCTION

The photon wave function, Eq. (5), is defined as the matrix element of a twist-2 operator between a physical photon state ( $P^2 = 0$ ) and the vacuum. It is interesting to consider the corresponding correlation function of the light-cone operators with the electromagnetic current also at spacelike momentum transfers ( $P^2 < 0$ ). We define

$$\begin{aligned} & \int d^4x e^{-iP \cdot x} \langle 0 | T \{ J_\rho^{e.m.}(x), \bar{u}(z) \sigma_{\mu\nu} [z, -z] u(-z) \} | 0 \rangle \\ &= i \frac{2}{3} f_{\gamma\perp}(P^2) (g_{\rho\mu} P_\nu - g_{\rho\nu} P_\mu) \\ & \times \int_0^1 du e^{i(2u-1)p \cdot z} \phi_{\gamma\perp}(P^2, u) + \dots, \end{aligned} \quad (31)$$

where  $p$  is a lightlike vector defined in such a way that it coincides with  $P$  in the limit  $P^2 \rightarrow 0$ ,

$$\begin{aligned} p_\mu &= P_\mu - \frac{P^2}{2(z \cdot P)} z_\mu, \\ p^2 &= 0, \end{aligned} \quad (32)$$

and we have not written out terms with other tensor structures which correspond to higher twists. The function  $\phi_{\gamma\perp}(P^2, u)$ , which we define to be normalized according to

Eq. (4) also for  $P^2 < 0$ , reduces to the photon wave function,  $\phi_{\gamma\perp}(u)$  in the limit  $P^2 \rightarrow 0$ . In the effective low-energy theory it is given by the expression Eq. (18) with  $P^2 < 0$ . The numerical results for momenta  $P^2 = -(250 \text{ MeV})^2$  and  $P^2 = -(500 \text{ MeV})^2$  are shown in Fig. 2. One sees that the wave function becomes larger at the boundaries for increasing spacelike photon momentum. The result for the normalization constant is for momenta  $0 < -P^2 < 1 \text{ GeV}^2$  well approximated by the form

$$f_{\gamma\perp}(P^2) \simeq f_{\gamma\perp} \left( 1 - \frac{P^2}{2.2\bar{\rho}^{-2}} \right)^{-1}, \quad (33)$$

where  $f_{\gamma\perp}$  is given by Eq. (30).

### VIII. MESON WAVE FUNCTIONS AND OFF-FORWARD PARTON DISTRIBUTIONS

The description of deeply virtual Compton scattering or hard meson production requires the so-called off-forward parton distributions (OFPD's) of the nucleon [12,13]. A novel feature of these compared to the usual parton distribution functions is the dependence on the longitudinal component of the momentum transfer,  $\xi$  (see Ref. [13] for definitions). A convenient language to understand general aspects of the  $\xi$  dependence of the OFPD's, according to Radyushkin [21], are the so-called double distributions. In particular, he discusses a ‘‘meson exchange’’ type contribution to the double distributions which contributes to the OFPD's in the kinematical region  $-\xi/2 < x < \xi/2$ . This argument relates the behavior of the meson wave function at the boundaries,  $u = 0, 1$ , to that of the OFPD at  $x = \pm \xi/2$ . In particular, if the meson wave function at  $u = 0, 1$  were nonzero, the OFPD would be discontinuous at  $x = \pm \xi/2$ , which would spoil the factorization of the amplitude.

Strong variations of the OFPD of the nucleon near  $x = \pm \xi/2$  have been observed in a calculation in the effective low-energy theory in the large- $N_c$  limit, where the nucleon is described as a chiral soliton [22]. It was seen there that near  $x = \pm \xi/2$  the behavior of the OFPD is governed by the momentum dependence of the dynamical quark mass, which turns a would-be discontinuity into a sharp but continuous crossover. This is consistent with the observation made in the above calculation of wave functions, namely that it is the momentum dependence of the dynamical quark mass that determines also the end point behavior of the meson wave function. One important difference between the wave functions and the OFPD's in this approach is due to the role of the formal parameter  $N_c$  (number of colors): While in the case of the meson wave function the parametric range of those values of  $u$  close to the boundaries essentially affected by the momentum-dependent dynamical quark mass is given by Eq. (23), in the case of the OFPD the crossover region in  $x$  near  $\pm \xi/2$  where the momentum-dependent mass is essential is parametrically of the order [22]

$$|x| - \xi/2 \sim \frac{(M\bar{\rho})^2 M}{M_N} \sim \frac{(M\bar{\rho})^2}{N_c}. \quad (34)$$

The ‘‘crossover’’ region of the OFPD is parametrically smaller than the ‘‘boundary’’ region of the wave function. Nevertheless, the physical mechanism—the suppression of large quark virtualities due to the momentum-dependent quark mass—is the same in both cases. Thus the effective theory derived from the instanton vacuum, with the ensuing fully field-theoretic description of the nucleon as a chiral soliton, provide a consistent realization of the general relations noted in Ref. [21].

### IX. CONCLUSIONS AND OUTLOOK

In this paper we have computed the photon and pion wave function in the effective low-energy theory derived from the instanton vacuum. We have exhibited the reason for the vanishing of the pion wave function at the end points—the suppression of large quark virtualities by the momentum-dependent dynamical quark mass—and seen that the corresponding mechanism is absent in the photon.<sup>5</sup>

As to the numerical reliability of the calculated wave functions, we would like to take a very modest point of view. There is an intrinsic uncertainty in the parameters of the effective low-energy theory, related to the approximations made in the instanton model of the QCD vacuum, which is based on the smallness of the packing fraction  $\bar{\rho}/\bar{R} \simeq 1/3$ . Nevertheless, our qualitative conclusions concerning the different behavior of the photon and pion wave functions stand up, since they follow from the general structure of the dynamical quark mass and the quark-pion coupling in the effective low-energy theory, which is unambiguous at least to leading order in  $\bar{\rho}/\bar{R}$ .

Our result for the pion wave function at the low normalization point is close to the asymptotic form and consistent with the CLEO measurements. The fact that we obtain a shape substantially different from the Chernyak-Zhitnitsky one is due to a significantly smaller value of the second moment, and, more importantly, the taking into account of *all* moments of the wave function (which is to say, the avoidance of working with explicit moments) by our approach. In this sense our results support conclusions reached previously in Refs. [14,15].

We have pointed out that the physical mechanism determining the end point behavior of the meson wave function and the behavior of the off-forward parton distribution at the transition points  $x = \pm \xi/2$  are the same—the momentum dependence of the dynamical quark mass. The fact that the low-energy effective theory allows to calculate both quantities in a consistent framework makes it a particularly valu-

<sup>5</sup>Qualitative arguments in favor of an important role of the momentum-dependent quark mass in hadron wave functions have been presented in Ref. [34].

able tool for investigating the unknown off-forward distributions.

The effective chiral theory, Eq. (7), allows us to compute also the light-cone wave functions of many-pion states. Recently the two-pion wave function has been studied in this approach, which is needed to describe exclusive pion production in processes such as  $\gamma^* \gamma \rightarrow \pi\pi$  or  $\gamma^* p \rightarrow p + 2\pi, 3\pi$  etc. [33].

The approach outlined in this paper can be extended to study also the higher-twist components of the meson and photon wave functions. In this case, however, one has to take into account also explicit contributions from the path-ordered exponentials of the gauge field, Eq. (2). This can be done using the method of effective gluon operators in the instanton vacuum developed in Ref. [35].

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- [1] G. P. Lepage and S. J. Brodsky, Phys. Lett. **87B**, 359 (1979); Phys. Rev. Lett. **43**, 545 (1979); **43**, 1625(E) (1979); Phys. Rev. D **22**, 2157 (1980).
  - [2] A. V. Efremov and A. V. Radyushkin, Phys. Lett. **94B**, 245 (1980).
  - [3] V. L. Chernyak and A. R. Zhitnitsky, JETP Lett. **25**, 510 (1977); Yad. Fiz. **31**, 1053 (1980).
  - [4] V. L. Chernyak and A. R. Zhitnitsky, Phys. Rep. **112**, 173 (1984).
  - [5] S. Ong, Phys. Rev. D **52**, 3111 (1995).
  - [6] R. Jakob, P. Kroll, and M. Raulfs, J. Phys. G **22**, 45 (1996).
  - [7] P. Kroll and M. Raulfs, Phys. Lett. B **387**, 848 (1996).
  - [8] A. V. Radyushkin and R. Ruskov, Phys. Lett. B **374**, 173 (1996); Nucl. Phys. **B481**, 625 (1996).
  - [9] I. V. Musatov and A. V. Radyushkin, Phys. Rev. D **56**, 2713 (1997).
  - [10] J. C. Collins, L. Frankfurt, and M. Strikman, Phys. Rev. D **56**, 2982 (1997); L. Mankiewicz, G. Piller, and T. Weigl, Eur. Phys. J. C **5**, 119 (1998).
  - [11] I. I. Balitsky, V. M. Braun, and A. V. Kolesnichenko, Nucl. Phys. **B312**, 509 (1989).
  - [12] A. V. Radyushkin, Phys. Lett. B **380**, 417 (1996); **385**, 333 (1996).
  - [13] X. Ji, Phys. Rev. Lett. **78**, 610 (1997); Phys. Rev. D **55**, 7114 (1997).
  - [14] S. V. Mikhailov and A. V. Radyushkin, JETP Lett. **43**, 712 (1986); Sov. J. Nucl. Phys. **49**, 494 (1989); Phys. Rev. D **45**, 1754 (1992).
  - [15] V. M. Braun and I. E. Filyanov, Z. Phys. C **44**, 157 (1989).
  - [16] CLEO Collaboration, J. Gronberg *et al.*, Phys. Rev. D **57**, 33 (1998).
  - [17] V. V. Anisovich, D. I. Melikhov, and V. A. Nikonov, Phys. Rev. D **55**, 2918 (1997).
  - [18] D. Diakonov and V. Petrov, Nucl. Phys. **B272**, 457 (1986); in *Hadron Matter Under Extreme Conditions* (Naukova Dumka, Kiev, 1986) (in Russian), p. 192.
  - [19] For a review, see D. I. Diakonov, talk given at the International School of Physics, ‘‘Enrico Fermi,’’ Course 80, Selected Topics in Nonperturbative QCD, Varenna, Italy, 1995, hep-ph/9602375.
  - [20] V. Yu. Petrov and P. V. Pobylitsa, hep-ph/9712203.
  - [21] A. V. Radyushkin, Phys. Rev. D **59**, 014030 (1999).
  - [22] V. Yu. Petrov, P. V. Pobylitsa, M. V. Polyakov, I. Börnig, K. Goeke, and C. Weiss, Phys. Rev. D **57**, 4325 (1998).
  - [23] P. Ball, V. M. Braun, Y. Koike, and K. Tanaka, Nucl. Phys. **B529**, 323 (1998).
  - [24] D. I. Diakonov, V. Yu. Petrov, P. V. Pobylitsa, M. V. Polyakov, and C. Weiss, Nucl. Phys. **B480**, 341 (1996).
  - [25] M. V. Polyakov and C. Weiss, in *Proceedings of the 37th Cracow School of Theoretical Physics: Dynamics of Strong Interactions*, Zakopane, Poland, 1997 [Acta Phys. Pol. B **28**, 2751 (1997)].
  - [26] A. E. Dorokhov, Nuovo Cimento A **109**, 391 (1996).
  - [27] M. A. Shifman and M. I. Vysotsky, Nucl. Phys. **B186**, 475 (1981).
  - [28] A. P. Bakulev and S. V. Mikhailov, Phys. Lett. B **436**, 351 (1998).
  - [29] I. I. Balitsky and A. V. Yung, Phys. Lett. **129B**, 328 (1983).
  - [30] B. L. Ioffe and A. V. Smilga, Nucl. Phys. **B232**, 109 (1984).
  - [31] I. I. Balitsky, A. V. Kolesnichenko, and A. V. Yung, Yad. Fiz. **41**, 282 (1985).
  - [32] V. M. Belyaev and Ya. I. Kogan, Yad. Fiz. **40**, 1035 (1984).
  - [33] M. V. Polyakov and C. Weiss, Phys. Rev. D **59**, 091502 (1999).
  - [34] A. Zhitnitsky, Talk given at 10th Summer School and Symposium on Nuclear Physics: QCD, Light cone Physics and Hadron Phenomenology (NuSS 97), Seoul, Korea, 1997, hep-ph/9801228.
  - [35] D. I. Diakonov, M. V. Polyakov, and C. Weiss, Nucl. Phys. **B461**, 539 (1996); J. Balla, M. V. Polyakov, and C. Weiss, *ibid.* **B510**, 327 (1997).