Radiative decays of heavy hadrons from light-cone QCD sum rules in the leading order of HQET

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The radiative decays of heavy baryons and the lowest three doublets of heavy mesons are studied with the light cone QCD sum rules in the leading order of heavy quark effective theory. [S0556-2821(99)05311-4]

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I. INTRODUCTION

Heavy quark effective theory (HQET) [1] provides a framework to study the heavy hadron spectra and the transition amplitude with a systematic expansion in terms of $1/m_Q$. Using the light cone photon wave function (PWF), radiative decay processes such as $B\to l\nu\gamma$, $B\to\rho\gamma$ have been studied [2–6] with QCD sum rules (QSR) [7]. Recently a similar approach was employed to analyze the couplings of pions with heavy hadrons [8]. In this work we will study the radiative decays of heavy baryons and the lowest three doublets of heavy mesons using the light cone QSR (LCQSR) [9] in the leading order of HQET. With LCQSR the continuum and excited states contribution is subtracted more cleanly, which is in contrast with the analysis of meson radiative decays using the external field method in QSR [10]. The LCQSRs for the radiative decays of heavy baryons are presented in Sec. II. Section III discusses radiative decays of lowest three doublets of heavy mesons. The following section is a discussion of the parameters and the photon wave functions. The last section is the numerical analysis and a short summary.

II. RADIATIVE DECAYS OF HEAVY BARYONS

We first introduce the interpolating currents for the heavy baryons:

$$\eta_\Lambda(x)=\epsilon_{abc}[u^{aT}(x)C\gamma_5 d^b(x)]h_v^c(x), \tag{1}$$

$$\eta_{\Sigma^+}(x)=\epsilon_{abc}[u^{aT}(x)C\gamma_\mu d^b(x)]\gamma_t^\mu\gamma_5 h_v^c(x), \tag{2}$$

$$\eta^\mu_{\Sigma^{++*}}(x)=\epsilon_{abc}[u^{aT}(x)C\gamma_\nu u^b(x)]\times\left(-g_t^{\mu\nu}+\frac{1}{3}\gamma_t^\mu\gamma_t^\nu\right)h_v^c(x), \tag{3}$$

where a, b, c is the color index, $u(x)$, $d(x)$, $h_v(x)$ is the up, down and heavy quark fields, T denotes the transpose, C is the charge conjugate matrix, $g_t^{\mu\nu}=g^{\mu\nu}-v^\mu v^\nu$, $\gamma_t^\mu=\gamma_\mu-\hat{v}v^\mu$, and v^μ is the velocity of the heavy hadron.

The overlap amplititudes of the interpolating currents with the heavy baryons is defined as

$$\langle 0|\eta_\Lambda|\Lambda\rangle=f_\Lambda u_\Lambda, \tag{4}$$

$$\langle 0|\eta_\Sigma|\Sigma\rangle=f_\Sigma u_\Sigma, \tag{5}$$

$$\langle 0|\eta^\mu_{\Sigma*}|\Sigma*\rangle=\frac{f_{\Sigma*}}{\sqrt{3}}u^\mu_{\Sigma*}, \tag{6}$$

where $u^\mu_{\Sigma*}$ is the Rarita-Schwinger spinor in HQET. In the leading order of HQET, $f_\Sigma=f_{\Sigma*}$ [11].

The coupling constants η_i are defined through the following amplitudes:

$$M(\Sigma_c\to\Lambda_c\gamma)=ie\,\eta_1\overline{u}_{\Lambda_c}\sigma^{\mu\nu}q_\mu e_\nu u_{\Sigma_c}, \tag{7}$$

$$M(\Sigma_c^*\to\Lambda_c\gamma)=ie\,\eta_2\epsilon_{\mu\nu\alpha\beta}\overline{u}_{\Lambda_c}\gamma^\nu q^\alpha e^\beta u^\mu_{\Sigma_c^*}, \tag{8}$$

$$M(\Sigma_c^*\to\Sigma_c\gamma)=ie\,\eta_3\epsilon_{\mu\nu\alpha\beta}\overline{u}_{\Sigma_c}\gamma^\nu q^\alpha e^\beta u^\mu_{\Sigma_c^*}, \tag{9}$$

where e_μ and q_μ are the photon polarization vector and momentum, respectively, and e is the charge unit.

In order to derive the sum rules for the coupling constants we consider the correlator

$$\int d^4x e^{-ik\cdot x}\langle\gamma(q)|T(\eta_\Sigma(0)\overline{\eta}_\Lambda(x))|0\rangle =e\frac{1+\hat{v}}{2}\gamma_t^\mu\gamma_5\epsilon_{\mu\alpha\beta\sigma}e^\alpha q^\beta v^\sigma G_{\Sigma,\Lambda}(\omega,\omega'), \tag{10}$$

$$\int d^4x e^{-ik\cdot x}\langle\gamma(q)|T(\eta^\mu_{\Sigma*}(0)\overline{\eta}_\Lambda(x))|0\rangle =e(e_\alpha q_\nu-e_\nu q_\alpha)\frac{1+\hat{v}}{2}\left(-g_t^{\mu\nu}+\frac{1}{3}\gamma_t^\mu\gamma_t^\nu\right)\times\epsilon_{\nu\alpha\beta\sigma}e^\alpha q^\beta v^\sigma G_{\Sigma*,\Lambda}(\omega,\omega'), \tag{11}$$

$$\int d^4x e^{-ik\cdot x}\langle\gamma(q)|T(\eta^\mu_{\Sigma*}(0)\overline{\eta}_\Sigma(x))|0\rangle =e\frac{1+\hat{v}}{2}\gamma_t^\alpha\gamma_5\left(-g_t^{\mu\nu}+\frac{1}{3}\gamma_t^\mu\gamma_t^\nu\right)\times(e_\alpha q_\nu-e_\nu q_\alpha)G_{\Sigma*,\Sigma}(\omega,\omega'), \tag{12}$$

where $k'=k-q$, $q^t_\mu=q_\mu-(q\cdot v)v_\mu$, $\omega=2v\cdot k$, $\omega'=2v\cdot k'$ and $q^2=0$.

Let us first consider the functions $G_{\Sigma,\Lambda}(\omega,\omega')$ etc. in Eqs. (10)–(12). As functions of two variables, they have the following pole terms from double dispersion relation:

$$\frac{-4i\eta_1 f_\Sigma f_\Lambda}{(2\bar\Lambda_\Sigma-\omega')(2\bar\Lambda_\Lambda-\omega)}+\frac{c}{2\bar\Lambda_\Sigma-\omega'}+\frac{c'}{2\bar\Lambda_\Lambda-\omega}, \quad (13)$$

$$\frac{-4i\eta_2}{\sqrt{3}}\frac{f_{\Sigma*}f_\Lambda}{(2\bar\Lambda_{\Sigma*}-\omega')(2\bar\Lambda_\Lambda-\omega)}+\frac{c}{2\bar\Lambda_{\Sigma*}-\omega'}+\frac{c'}{2\bar\Lambda_\Lambda-\omega}, \quad (14)$$

$$\frac{4\eta_3}{\sqrt{3}}\frac{f_{\Sigma*}f_\Sigma}{(2\bar\Lambda_{\Sigma*}-\omega')(2\bar\Lambda_\Sigma-\omega)}+\frac{c}{2\bar\Lambda_{\Sigma*}-\omega'}+\frac{c'}{2\bar\Lambda_\Sigma-\omega}, \quad (15)$$

where $f_{\Sigma*}$, etc. are constants defined in Eqs. (4)–(6), $\bar\Lambda_{\Sigma*}=m_{\Sigma*}-m_Q$.

Keeping the two particle component of the photon wave function, the expression for $G_{\Sigma*,\Lambda}(\omega,\omega')$ with the tensor structure reads

$$-2\int_0^\infty dt\int dx e^{-ikx}\delta(-x-vt)$$
$$\times\mathbf{Tr}\{[C\langle\gamma(q)|u(0)\bar u(x)|0\rangle^T C\gamma_\mu iS(-x)\gamma_5]$$
$$+[CiS^T(-x)C\gamma_\mu]\langle\gamma(q)|u(0)\bar u(x)|0\rangle\gamma_5]\}, \quad (16)$$

where $iS(-x)$ is the full light quark propagator with both perturbative term and contribution from vacuum fields:

$$iS(x)=\langle 0|T[q(x),\bar q(0)]|0\rangle$$
$$=i\frac{\hat x}{2\pi^2x^4}-\frac{\langle\bar qq\rangle}{12}-\frac{x^2}{192}\langle\bar qg_s\sigma\cdot Gq\rangle$$
$$-ig_s\frac{1}{16\pi^2}\int_0^1 du\left\{\frac{\hat x}{x^2}\sigma\cdot G(ux)\right.$$
$$\left.-4iu\frac{x_\mu}{x^2}G^{\mu\nu}(ux)\gamma_\nu\right\}+\cdots. \quad (17)$$

The light cone two-particle photon wave functions are [2]

$$\langle\gamma(q)|\bar q(x)\sigma_{\mu\nu}q(0)|0\rangle$$
$$=ie_qe\langle\bar qq\rangle\int_0^1 du e^{iuqx}\{(e_\mu q_\nu-e_\nu q_\mu)$$
$$\times[\chi\phi(u)+x^2g_1(u)]+[(qx)(e_\mu x_\nu-e_\nu x_\mu)$$
$$+(ex)(x_\mu q_\nu-x_\nu q_\mu)-x^2(e_\mu q_\nu-e_\nu q_\mu)]g_2(u)\}, \quad (18)$$

$$\langle\gamma(q)|\bar q(x)\gamma_\mu\gamma_5 q(0)|0\rangle$$
$$=\frac{f}{4}e_qe\epsilon_{\mu\nu\rho\sigma}e^\nu q^\rho x^\sigma\int_0^1 du e^{iuqx}\psi(u). \quad (19)$$

Because of the choice of the gauge $x^\mu A_\mu(x)=0$, the path-ordered gauge factor $P\exp(ig_s\int_0^1 dux^\mu A_\mu(ux))$ has been omitted. The $\phi(u),\psi(u)$ is associated with the leading twist two photon wave function, while $g_1(u)$ and $g_2(u)$ are twist-4 PWFs. All these PWFs are normalized to unity, $\int_0^1 duf(u)=1$.

Expressing Eq. (16) with the photon wave functions, we arrive at

$$G_{\Sigma*,\Lambda}(\omega,\omega')=-(e_u-e_d)$$
$$\times\int_0^\infty dt\int_0^1 du e^{i(1-u)(\omega t/2)}e^{iu(\omega' t/2)}$$
$$\times\left\{\langle\bar qq\rangle\left[\frac{1}{\pi^2t^3}\chi\phi(u)+\frac{1}{\pi^2 t}\right.\right.$$
$$\left.\times(g_1(u)-g_2(u))\right]+\frac{f}{24}\psi(u)t$$
$$\left.\times\left(\langle\bar qq\rangle+\frac{t^2}{16}\langle\bar qg_s\sigma\cdot Gq\rangle\right)\right\}+\cdots. \quad (20)$$

Similarly we have

$$G_{\Sigma,\Lambda}(\omega,\omega')=G_{\Sigma*,\Lambda}(\omega,\omega'), \quad (21)$$

$$G_{\Sigma*,\Sigma}(\omega,\omega')=(e_u+e_d)$$
$$\times\int_0^\infty dt\int_0^1 du e^{i(1-u)(\omega t/2)}e^{iu(\omega' t/2)}$$
$$\times\left\{\frac{f}{4\pi^2t^2}\psi(u)+\frac{\langle\bar qq\rangle}{6}\right.$$
$$\times\left(\langle\bar qq\rangle+\frac{t^2}{16}\langle\bar qg_s\sigma\cdot Gq\rangle\right)$$
$$\left.\times[\chi\phi(u)+t^2(g_1(u)-g_2(u))]\right\}+\cdots, \quad (22)$$

where $\langle\bar qq\rangle=-(225\text{ MeV})^3$, $\langle\bar qg_s\sigma\cdot Gq\rangle=m_0^2\langle\bar qq\rangle$, and $m_0^2=0.8\text{ GeV}^2$. For large Euclidean values of ω and ω' this integral is dominated by the region of small t, therefore it can be approximated by the first a few terms.

After Wick rotations and making double Borel transformation with the variables ω and ω' the single-pole terms in Eqs. (13)–(15) are eliminated. Subtracting the continuum contribution which is modeled by the dispersion integral in the region $\omega,\omega'\geqslant\omega_c$, we arrive at

$$\eta_1 f_\Sigma f_\Lambda = -\frac{1}{64\pi^4}(e_u-e_d)ae^{(\Lambda_\Sigma+\Lambda_\Lambda)/T} \times\left\{\chi\phi(u_0)T^4 f_3\left(\frac{\omega_c}{T}\right)-4[g_1(u_0)-g_2(u_0)] \times T^2 f_1\left(\frac{\omega_c}{T}\right)+\frac{2\pi^2}{3}f\psi(u_0)\left(1-\frac{m_0^2}{4T^2}\right)\right\}, \quad (23)$$

$$\eta_2 f_{\Sigma*} f_\Lambda = -\frac{\sqrt{3}}{64\pi^4}(e_u-e_d)ae^{(\Lambda_{\Sigma*}+\Lambda_\Lambda)/T} \times\left\{\chi\phi(u_0)T^4 f_3\left(\frac{\omega_c}{T}\right)-4[g_1(u_0)-g_2(u_0)] \times T^2 f_1\left(\frac{\omega_c}{T}\right)+\frac{2\pi^2}{3}f\psi(u_0)\left(1-\frac{m_0^2}{4T^2}\right)\right\}, \quad (24)$$

$$\eta_3 f_{\Sigma*} f_\Sigma = \frac{\sqrt{3}}{32\pi^2}(e_u+e_d)e^{(\Lambda_{\Sigma*}+\Lambda_\Sigma)/T} \times\left\{f\psi(u_0)T^3 f_2\left(\frac{\omega_c}{T}\right)-\frac{a^2}{6\pi^2}\left(1-\frac{m_0^2}{4T^2}\right) \times\left[\chi\phi(u_0)Tf_0\left(\frac{\omega_c}{T}\right)-\frac{4}{T}(g_1(u_0)-g_2(u_0))\right]\right\}, \quad (25)$$

where $f_n(x)=1-e^{-x}\sum_{k=0}^{n}x^k/k!$ is the factor used to subtract the continuum, ω_c is the continuum threshold. $u_0=T_1/(T_1+T_2)$, $T\equiv T_1T_2/(T_1+T_2)$, T_1,T_2 are the Borel parameters $a=-(2\pi)^2\langle\bar{q}q\rangle$. We have used the Borel transformation formula: $\hat{\mathcal{B}}_\omega^T e^{\alpha\omega}=\delta(\alpha-1/T)$.

Due to the heavy quark symmetry, $\Lambda_\Sigma=\Lambda_{\Sigma*}$ and $f_\Sigma=f_{\Sigma*}$ in the limit $m_Q\to\infty$. So from Eqs. (23) and (24) we have $\eta_2=\sqrt{3}\eta_1$. For the decays $\Sigma_c^{*0}\to\Sigma_c^0\gamma$ and $\Sigma_c^{*++}\to\Sigma_c^{++}\gamma$, we need make replacement $(e_u+e_d)\to 2e_d, 2e_u$ in Eq. (25).

III. RADIATIVE DECAYS OF HEAVY MESONS

We shall confine ourselves to the lowest lying three doublets and consider all possible radiative decay processes among them in the leading order of $1/m_Q$ expansion. Denote the doublet $(1^+,2^+)$ with $j_l=3/2$ by (B_1,B_2^*), the doublet $(0^+,1^+)$ with $j_l=1/2$ by (B_0',B_1') and the doublet $(0^-,1^-)$ by (B,B^*).

The interpolating currents are given in [12] as

$$J^{\dagger\alpha}_{1,+,3/2}=\sqrt{\frac{3}{4}}\bar{h}_v\gamma^5(-i)\left(\mathcal{D}_t^\alpha-\frac{1}{3}\gamma_t^\alpha\mathcal{D}_t\right)q, \quad (26)$$

$$J^{\dagger\alpha_1,\alpha_2}_{2,+,3/2}=\sqrt{\frac{1}{2}}\bar{h}_v\frac{(-i)}{2}\left(\gamma_t^{\alpha_1}\mathcal{D}_t^{\alpha_2}+\gamma_t^{\alpha_2}\mathcal{D}_t^{\alpha_1}-\frac{2}{3}g_t^{\alpha_1\alpha_2}\mathcal{D}_t\right)q, \quad (27)$$

$$J^{\dagger\alpha}_{1,-,1/2}=\sqrt{\frac{1}{2}}\bar{h}_v\gamma_t^\alpha q, \quad J^{\dagger\alpha}_{0,-,1/2}=\sqrt{\frac{1}{2}}\bar{h}_v\gamma_5 q, \quad (28)$$

$$J^{\dagger}_{0,+,1/2}=\frac{1}{\sqrt{2}}\bar{h}_v q, \quad J^{\dagger\alpha}_{1,+,1/2}=\frac{1}{\sqrt{2}}\bar{h}_v\gamma^5\gamma_t^\alpha q. \quad (29)$$

$(1^+,2^+)\to(0^-,1^-)+\gamma$

The decay amplitudes are

$$M(B_1\to B^*\gamma)=e_q e e^{*\mu}v^\sigma\epsilon_\beta^*\eta_\alpha \times\Big\{[\epsilon_{\mu\nu\beta\sigma}q_t^\alpha+(\alpha\leftrightarrow\beta)]q_t^\nu g_D^1(B_1,B^*) +\left[\epsilon_{\mu\nu\beta\sigma}\left(q_t^\alpha q_t^\nu-\frac{1}{3}q_t^2 g_t^{\alpha\nu}\right)-(\alpha\leftrightarrow\beta)\right] \times g_D^2(B_1,B^*)+\epsilon_{\mu\alpha\beta\sigma}g_S(B_1,B^*)\Big\}, \quad (30)$$

where the tensor structure associated with $g_D^1(B_1,B^*)$ and $g_D^2(B_1,B^*)$ is symmetric and antisymmetric under the exchange of $(\alpha\leftrightarrow\beta)$, respectively:

$$M(B_1\to B\gamma)=e_q e e_\beta^*\eta_\alpha\left\{\left(q_t^\alpha q_t^\beta-\frac{1}{3}q_t^2 g_t^{\alpha\beta}\right) \times g_D(B_1,B)+g_t^{\alpha\beta}g_S(B_1,B)\right\}, \quad (31)$$

$$M(B_2^*\to B\gamma)=e_q e e_\beta^*\eta_{\alpha_1\alpha_2}q_\nu v_\sigma[\epsilon^{\beta\nu\alpha_1\sigma}q_t^{\alpha_2} +(\alpha_1\leftrightarrow\alpha_2)]g_D(B_2^*,B), \quad (32)$$

$$M(B_2^*\to B^*\gamma)=e_q e\,\eta_{\alpha_1\alpha_2}\eta_\beta^* \times\left\{e_t^\beta\left(q_t^{\alpha_1}q_t^{\alpha_2}-\frac{1}{3}q_t^2 g_t^{\alpha_1\alpha_2}\right) \times g_D^1(B_2^*,B^*)+\left[e_t^{\alpha_1}\left(q_t^\beta q_t^{\alpha_2}-\frac{1}{3}q_t^2 g_t^{\beta\alpha_2}\right) +(\alpha_1\leftrightarrow\alpha_2)\right]g_D^2(B_2^*,B^*) +[e_t^{\alpha_1}g_t^{\alpha_2\beta}+(\alpha_1\leftrightarrow\alpha_2)]g_S(B_2^*,B^*)\right\},$$

where $\eta_{\mu\nu}$, η_μ and ϵ_μ are polarization tensors for states 2^+, 1^+, and 1^-, respectively, and $e_q e$ is the light quark electric charge.

Due to heavy quark symmetry, there exist only two independent coupling constants for the D-wave and S-wave decay respectively. Let $g_d\equiv g_D(B_2^*,B)$ and $g_s\equiv -g_S(B_2^*,B^*)$. Then we have

$$\frac{\sqrt{6}}{2}g_S(B_1,B^*)=\frac{\sqrt{6}}{4}g_S(B_1,B)=g_s, \quad (33)$$

$$\frac{\sqrt{6}}{3}g_D^1(B_1,B^*)=\sqrt{6}g_D^2(B_1,B^*)$$
$$=\frac{\sqrt{6}}{2}g_D(B_1,B)$$
$$=-\frac{1}{2}g_D^1(B_2^*,B^*)=g_D^2(B_2^*,B^*)$$
$$=g_d. \tag{34}$$

The above relation is confirmed by our detailed calculation.

For deriving the sum rules for the coupling constants we consider the correlator

$$\int d^4x e^{-ik\cdot x}\langle \gamma(q)|T(J_{0,-,1/2}(0)J^{\dagger\alpha}_{1,+,3/2}(x))|0\rangle$$
$$=e_q e\left\{e^*_\beta\left(q_t^\alpha q_t^\beta-\frac{1}{3}q_t^2 g_t^{\alpha\beta}\right)G^D_{B_1B}(\omega,\omega')\right.$$
$$\left.+e_t^{*\alpha}G^S_{B_1B}(\omega,\omega')\right\}. \tag{35}$$

The functions $G^{D,S}_{B_1B}(\omega,\omega')$ in Eq. (35) have the following double dispersion relation:

$$\frac{f_{-,1/2}f_{+,3/2}g_{D,S}(B_1B)}{(2\bar{\Lambda}_{-,1/2}-\omega')(2\bar{\Lambda}_{+,3/2}-\omega)}+\frac{c}{2\bar{\Lambda}_{-,1/2}-\omega'}$$
$$+\frac{c'}{2\bar{\Lambda}_{+,3/2}-\omega}, \tag{36}$$

where $\bar{\Lambda}_{P,j_l}=m_{P,j_l}-m_Q$ and f_{P,j_l} are constants defined as

$$\langle 0|J^{\alpha_1\cdots\alpha_j}_{j,P,j_l}(0)|j',P',j_l'\rangle=f_{Pj_l}\delta_{jj'}\delta_{PP'}\delta_{j_lj_l'}\eta^{\alpha_1\cdots\alpha_j}. \tag{37}$$

Applying the same procedure as in Sec. II we obtain

$$G^D_{B_1B}(\omega,\omega')=-\frac{\sqrt{6}}{24}\int_0^\infty dt\int_0^1 du e^{i(1-u)(\omega t/2)}$$
$$\times e^{iu(\omega' t/2)}u\left\{-\frac{it}{4}f\psi(u)+\langle\bar{q}q\rangle\right.$$
$$\left.\times[\chi\phi(u)+t^2(g_1(u)-g_2(u))]\right\}+\cdots, \tag{38}$$

$$G^S_{B_1B}(\omega,\omega')=-\frac{2}{3}q_t^2 G^D_{B_1B}(\omega,\omega')$$
$$=\frac{1}{6}(\omega-\omega')^2 G^D_{B_1B}(\omega,\omega'). \tag{39}$$

Finally we have

$$g_d f_{-,1/2}f_{+,3/2}=\frac{1}{8}e^{(\Lambda_{-,1/2}+\Lambda_{+,3/2})/T}$$
$$\times\left\{fu_0\psi(u_0)+\frac{a}{2\pi^2}u_0\right.$$
$$\times\left[\chi\phi(u_0)Tf_0\left(\frac{\omega_c}{T}\right)\right.$$
$$\left.\left.-\frac{4}{T}(g_1(u_0)-g_2(u_0))\right]\right\}, \tag{40}$$

$$g_s f_{-,1/2}f_{+,3/2}=-\frac{1}{96}e^{(\Lambda_{-,1/2}+\Lambda_{+,3/2})/T}$$
$$\times\left\{f\frac{d^2(u\psi(u))}{du^2}T^2f_1\left(\frac{\omega_c}{T}\right)\right.$$
$$+\frac{a}{2\pi^2}\left[\chi\frac{d^2(u\phi(u))}{du^2}T^3f_2\left(\frac{\omega_c}{T}\right)\right.$$
$$-4\frac{d^2([ug_1(u)-ug_2(u)])}{du^2}$$
$$\left.\left.\times Tf_0\left(\frac{\omega_c}{T}\right)\right]\right\}\Bigg|_{u=u_0}. \tag{41}$$

Here we have used integration by parts to absorb the factor $(q\cdot v)^2$, which leads to the second derivatives in Eq. (41). In this way we arrive at the simple form after double Borel transformation.

$(0^+,1^+)\to(0^-,1^-)+\gamma$

There exists only one independent coupling constant, corresponding to S-wave decay. The decay amplitudes are

$$M(B_1'\to B^*\gamma)=e_q e\epsilon^{\mu\sigma\alpha\beta}e_\mu v_\sigma\eta'_\alpha\epsilon^*_\beta g_S(B_1',B^*), \tag{42}$$

where η'_α is the polarization vector of B_1'.

$$M(B_1'\to B\gamma)=e_q e e^\alpha\eta'_\alpha g_S(B_1',B), \tag{43}$$

$$M(B_0'\to B^*\gamma)=e_q e e_t^\beta\epsilon_\beta g_S(B_0',B^*). \tag{44}$$

The process $B_0'\to B\gamma$ is forbidden due to parity and angular momentum conservation. Due to heavy quark symmetry, we have

$$g_S(B_1',B^*)=g_S(B_1',B)=-g_S(B_0',B^*)\equiv g_1. \tag{45}$$

We consider the correlator

$$\int d^4x e^{-ik\cdot x}\langle\gamma(q)|T(J_{0,+,1/2}(0)J^{\dagger\beta}_{1,-,1/2}(x))|0\rangle$$
$$=e_q e e^\beta G_{B_0'B^*}(\omega,\omega'), \tag{46}$$

where

$$G_{B_0'B^*}(\omega,\omega')=-\frac{1}{4}\langle\bar{q}q\rangle(q\cdot v)\times\int_0^\infty dt\int_0^1 du\, e^{i(1-u)(\omega t/2)}e^{iu(\omega' t/2)}\times\{\chi\phi(u)+t^2 g_1(u)\}+\cdots. \tag{47}$$

$$g_1 f_{-,1/2}f_{+,1/2}=\frac{a}{16\pi^2}e^{(\Lambda_{-,1/2}+\Lambda_{+,1/2})/T}\times\left\{\chi\frac{d\phi(u)}{du}T^2 f_1\left(\frac{\omega_c}{T}\right)-4\frac{dg_1(u)}{du}\right\}\Bigg|_{u=u_0}. \tag{48}$$

$(1^+,2^+)\to(0^+,1^+)+\gamma$

There exists only one independent coupling constant, corresponding to P-wave decay. The decay amplitudes are

$$M(B_1\to B_1'\gamma)=e_q e\,\eta_\alpha\eta'^*_\beta e^*_\mu\left\{\left[\left(q_t^\alpha g_t^{\beta\mu}-\frac{1}{3}q_t^\mu g_t^{\alpha\beta}\right)+(\alpha\leftrightarrow\beta)\right]g_P^1(B_1,B_1')+(q_t^\alpha g_t^{\beta\mu}-q_t^\beta g_t^{\alpha\mu})g_P^2(B_1,B_1')\right\}, \tag{49}$$

$$M(B_1\to B_0'\gamma)=e_q e\,\epsilon_{\mu\sigma\nu\alpha}\eta^\alpha e^{*\mu}v^\sigma q^\nu g_P(B_1,B_0'), \tag{50}$$

$$M(B_2^*\to B_0'\gamma)=e_q e e^{*\mu}\eta_{\alpha_1\alpha_2}\left[\left(q_t^{\alpha_1}g_t^{\alpha_2\mu}-\frac{1}{3}q_t^\mu g_t^{\alpha_1\alpha_2}\right)+(\alpha_1\leftrightarrow\alpha_2)\right]g_P(B_2^*,B_0'), \tag{51}$$

$$M(B_2^*\to B_1'\gamma)=e_q e\,\epsilon_{\mu\sigma\rho\beta}\eta'^\beta e^{*\mu}\eta_{\alpha_1\alpha_2}\times\left[\left(q_t^{\alpha_1}g_t^{\alpha_2\rho}-\frac{1}{3}q_t^\rho g_t^{\alpha_1\alpha_2}\right)+(\alpha_1\leftrightarrow\alpha_2)\right]\times g_P(B_2^*,B_1'). \tag{52}$$

Due to heavy quark symmetry we have

$$\frac{\sqrt{6}}{3}g_P^1(B_1,B_1')=\sqrt{6}g_P^2(B_1,B_1')=\sqrt{6}g_P(B_1,B_0')=g_P(B_2^*,B_0')=g_P(B_2^*,B_1')\equiv g_2. \tag{53}$$

We consider the correlator

$$\int d^4x\, e^{-ik\cdot x}\langle\pi(q)|T(J^\alpha_{0,+,1/2}(0)J^{\dagger\alpha_1\alpha_2}_{2,+,3/2}(x))|0\rangle=e_q e e^\mu\left[\left(q_t^{\alpha_1}g_t^{\alpha_2\mu}-\frac{1}{3}q_t^\mu g_t^{\alpha_1\alpha_2}\right)+(\alpha_1\leftrightarrow\alpha_2)\right]G_{B_2^*B_0'}(\omega,\omega'), \tag{54}$$

where

$$G_{B_2^*B_0'}(\omega,\omega')=-\frac{1}{8}\langle\bar{q}q\rangle(q\cdot v)\times\int_0^\infty dt\int_0^1 du\, e^{i(1-u)(\omega t/2)}\times e^{iu(\omega' t/2)}u\{\chi\phi(u)+t^2 g_1(u)\}+\cdots, \tag{55}$$

$$g_2 f_{+,1/2}f_{+,3/2}=\frac{a}{32\pi^2}e^{(\Lambda_{+,1/2}+\Lambda_{+,3/2})/T}\times\left\{\chi\frac{d(u\phi(u))}{du}T^2 f_1\left(\frac{\omega_c}{T}\right)-4\frac{d(ug_1(u))}{du}\right\}\Bigg|_{u=u_0}. \tag{56}$$

$B_1'\to B_0'\gamma$

$$M(B_1'\to B_0'\gamma)=e_q e\,\epsilon^{\alpha\mu\nu\sigma}\eta'_\alpha e^*_\mu q_\nu v_\sigma g_3. \tag{57}$$

In order to derive g_3, we consider the correlator

$$\int d^4x\, e^{-ik\cdot x}\langle\gamma(q)|T(J_{0,+,1/2}(0)J^{\dagger\alpha}_{1,+,1/2}(x))|0\rangle=e_q e\,\epsilon^{\alpha\mu\nu\sigma}e_\mu q_\nu v_\sigma G_{B_1'B_0'}(\omega,\omega'), \tag{58}$$

where

$$G_{B_1'B_0'}(\omega,\omega')=\frac{i}{4}\int_0^\infty dt\int_0^1 du\, e^{i(1-u)(\omega t/2)}e^{iu(\omega' t/2)}\times\left\{\frac{it}{4}f\psi(u)+\langle\bar{q}q\rangle[\chi\phi(u)+t^2(g_1(u)-g_2(u))]\right\}+\cdots, \tag{59}$$

$$g_3 f^2_{+,1/2}=-\frac{1}{4}e^{2\Lambda_{+,1/2}/T}\left\{-f\psi(u_0)+\frac{a}{2\pi^2}\left[\chi\phi(u_0)Tf_0\left(\frac{\omega_c}{T}\right)-\frac{4}{T}(g_1(u_0)-g_2(u_0))\right]\right\}. \tag{60}$$

$B^*\to B\gamma$

$$M(B^*\to B\gamma)=e_q e\epsilon^{\alpha\mu\nu\sigma}\epsilon_\alpha e^*_\mu q_\nu v_\sigma g_4. \tag{61}$$

In order to derive g_4, we consider the correlator

$$\int d^4x e^{-ik\cdot x}\langle\gamma(q)|T(J_{0,-,1/2}(0)J^{\dagger\alpha}_{1,-,1/2}(x))|0\rangle$$
$$=e_q e\epsilon^{\alpha\mu\nu\sigma}e_\mu q_\nu v_\sigma G_{B*B}(\omega,\omega'), \tag{62}$$

where

$$G_{B*B}(\omega,\omega')=\frac{i}{4}\int_0^\infty dt\int_0^1 du e^{i(1-u)(\omega t/2)} \times e^{iu(\omega' t/2)}\left\{-\frac{it}{4}f\psi(u)+\langle\bar{q}q\rangle \times[\chi\phi(u)+t^2(g_1(u)-g_2(u))]\right\}+\cdots. \tag{63}$$

Note this coupling was calculated in [3] using LCQSR. But there the contribution from the photon wave function $\psi(u)$ has not been taken into account:

$$g_4 f^2_{-,1/2}=-\frac{1}{4}e^{2\Lambda_{-,1/2}/T}\left\{f\psi(u_0)+\frac{a}{2\pi^2}\times\left[\chi\phi(u_0)Tf_0\left(\frac{\omega_c}{T}\right)-\frac{4}{T}(g_1(u_0)-g_2(u_0))\right]\right\}. \tag{64}$$

$B_2^*\to B_1\gamma$

$$M(B_2^*\to B_1\gamma)=e_q e\epsilon^{\alpha\mu\nu\sigma}e_\mu q_\nu v_\sigma\eta^*_\beta\eta_{\alpha_1\alpha_2}\times\left(g_t^{\alpha_1\rho}q_t^{\alpha_2}+g_t^{\alpha_2\rho}q_t^{\alpha_1}-\frac{2}{3}g_t^{\alpha_1\alpha_2}q_t^\rho\right)\times(2q_t^\beta g_t^{\rho\alpha}+g_t^{\alpha\beta}q_t^\rho)g_5. \tag{65}$$

In order to derive g_5, we consider the correlator

$$\int d^4x e^{-ik\cdot x}\langle\gamma(q)|T(J^\beta_{1,+,3/2}(0)J^{\dagger\alpha_1\alpha_2}_{2,+,3/2}(x))|0\rangle$$
$$=e_q e\epsilon^{\alpha\mu\nu\sigma}e_\mu q_\nu v_\sigma\times\left(g_t^{\alpha_1\rho}q_t^{\alpha_2}+g_t^{\alpha_2\rho}q_t^{\alpha_1}-\frac{2}{3}g_t^{\alpha_1\alpha_2}q_t^\rho\right)\times(2q_t^\beta g_t^{\rho\alpha}+g_t^{\alpha\beta}q_t^\rho)G_{B_2^*B_1}(\omega,\omega'), \tag{66}$$

where

$$G_{B_2^*B_1}(\omega,\omega')=-i\frac{\sqrt{6}}{16}\int_0^\infty dt\int_0^1 du e^{i(1-u)(\omega t/2)}\times e^{iu(\omega' t/2)}u(1-u)\left\{-\frac{it}{4}f\psi(u)+\langle\bar{q}q\rangle\times[\chi\phi(u)+t^2(g_1(u)-g_2(u))]\right\}+\cdots, \tag{67}$$

$$g_5 f^2_{+,3/2}=\frac{\sqrt{6}}{16}e^{2\Lambda_{+,3/2}/T}u_0(1-u_0)\times\left\{f\psi(u_0)+\frac{a}{2\pi^2}\left[\chi\phi(u_0)Tf_0\left(\frac{\omega_c}{T}\right)-\frac{4}{T}(g_1(u_0)-g_2(u_0))\right]\right\}. \tag{68}$$

IV. DETERMINATION OF THE PARAMETERS

The leading photon wave functions receive only small corrections from the higher conformal spins [9] so they do not deviate much from the asymptotic form. We shall use [4]

$$\phi(u)=6u\bar{u}, \tag{69}$$

$$\psi(u)=1, \tag{70}$$

$$g_1(u)=-\frac{1}{8}\bar{u}(3-u), \tag{71}$$

$$g_2(u)=-\frac{1}{4}\bar{u}^2, \tag{72}$$

with $\bar{u}=1-u$, $f=0.028$ GeV^2, and $\chi=-4.4$ GeV^2 [13] at the scale $\mu=1$ GeV. Using this value of χ, the octet, decuplet and heavy baryon magnetic moments have been calculated to a good accuracy [14–16].

We need the mass parameters $\bar{\Lambda}$'s and the coupling constants f's of the corresponding interpolating currents in the leading order of α_s as input. The results are [11,8]

$$\bar{\Lambda}_\Lambda=0.8\ \text{GeV},\quad f_\Lambda=(0.018\pm0.002)\ \text{GeV}^3,$$
$$\bar{\Lambda}_\Sigma=1.0\ \text{GeV},\quad f_\Sigma=(0.04\pm0.004)\ \text{GeV}^3. \tag{73}$$

$$\bar{\Lambda}_{+,3/2}=0.82\ \text{GeV},\quad f_{+,3/2}=0.19\pm0.03\ \text{GeV}^{5/2},$$
$$\bar{\Lambda}_{+,1/2}=1.1\ \text{GeV},\quad f_{+,1/2}=0.40\pm0.06\ \text{GeV}^{3/2},$$
$$\bar{\Lambda}_{-,1/2}=0.5\ \text{GeV},\quad f_{-,1/2}=0.25\ \text{GeV}^{3/2}. \tag{74}$$

We choose to work at the symmetric point $T_1=T_2=2T$, i.e., $u_0=\frac{1}{2}$. Such a choice is very reasonable for the symmetric sum rules (25), (60), (64), and (68) since Σ_c^* and Σ_c, and the three meson doublets are degenerate in the leading order

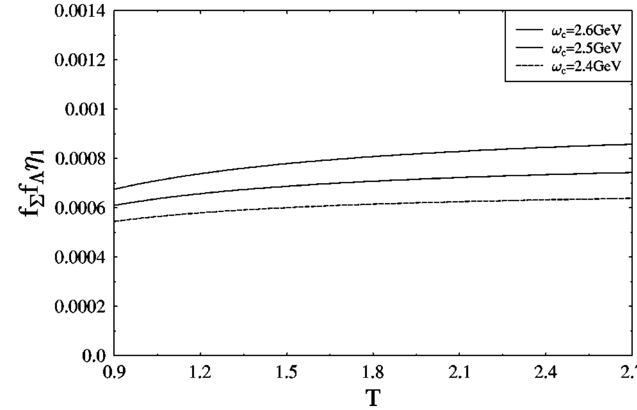

FIG. 1. Dependence of $f_\Sigma f_\Lambda \eta_1$ on the Borel parameter T for different values of the continuum threshold ω_c. From top to bottom the curves correspond to $\omega_c=2.6$, 2.5, and 2.4 GeV.

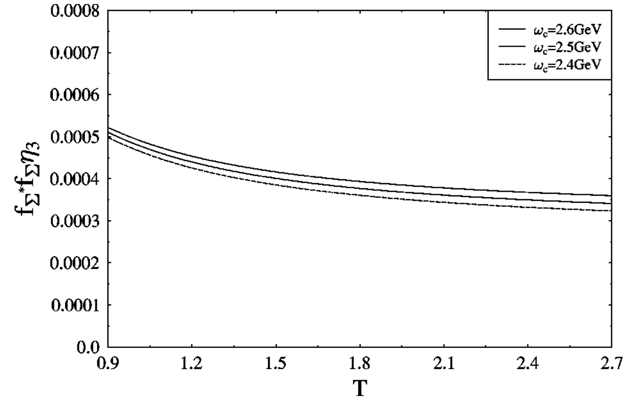

FIG. 2. Dependence of $f_{\Sigma *} f_\Sigma \eta_3$ on T with $\omega_c=2.6$, 2.5, and 2.4 GeV.

of HQET. Moreover, the mass difference between Σ_c^* and Λ_c is only about 0.2 GeV. The $(0^+,1^+)$ doublet lies only slightly below $(1^+,2^+)$ doublet. Due to the large values of T_1,T_2 used below, the choice of $T_1=T_2$ is also reasonable for sum rules (23) and (56).

Note the choice $T_1=T_2$ is not unique for the asymmetric sum rules (40), (41), and (48) since the initial and final mesons have different masses. But the choice $T_1=T_2$ will enable the clean subtraction of the continuum contribution, which is cruicial for the numerical analysis of the sum rules. In our case the sum rules are stable with reasonable variations of the Borel parameter T_1 and T_2. Such a choice does not alter significantly the numerical results. Based on these considerations we adopt $u_0=\frac{1}{2}$ for for the sum rules (40), (41), and (48), too.

V. NUMERICAL RESULTS AND DISCUSSION

A. Numerical analysis of the baryon sum rules

We now turn to the numerical evaluation of the sum rules for the coupling constants. Since the spectral density of the sum rules (23)–(25) $\rho(s)$ is either proptional to s^2 or s^3, the continuum has to be subtracted carefully. We use the value of the continuum threshold ω_c determined from the corresponding mass sum rule at the leading order of α_s and $1/m_Q$ [11].

The lower limit of T is determined by the requirement that the terms of higher twists in the operator expansion is reasonably smaller than the leading twist, say $\leqslant 1/3$ of the latter. This leads to $T>1.3$ GeV for the sum rules (23)–(25). In fact the twist-four terms contribute only a few percent to the sum rules. The upper limit of T is constrained by the requirement that the continuum contribution is less than 50%. This corresponds to $T<2.2$ GeV.

The variation of $\eta_{1,3}$ with the Borel parameter T and ω_c is presented in Fig. 1 and Fig. 2. The curves correspond to $\omega_c=2.4,2.5,2.6$ GeV from bottom to top, respectively. Stability develops for the sum rules (23) and (25) in the region 1.3 GeV $<T<2.2$ GeV, we get

$$\eta_1 f_\Sigma f_\Lambda=(7.0\pm 0.9)\times 10^{-4}\ \ \mathrm{GeV}^5, \tag{75}$$

$$\eta_3 f_{\Sigma *} f_\Sigma=(3.9\pm 0.5)\times 10^{-4}\ \ \mathrm{GeV}^5, \tag{76}$$

where the errors refer to the variations with T and ω_c in this region. And the central value corresponds to $T=1.6$ GeV and $\omega_c=2.5$ GeV.

Combining Eq. (73) we arrive at

$$\eta_1=(1.0\pm 0.2)\ \ \mathrm{GeV}^{-1}, \tag{77}$$

$$\eta_3=(0.24\pm 0.05)\ \ \mathrm{GeV}^{-1}. \tag{78}$$

B. Numerical analysis of the meson sum rules

We now turn to the numerical evaluation of the sum rules for the coupling constants. The lower limit of T is determined by the requirement that the terms of higher twists in the operator expansion is less than one third of the whole sum rule. This leads to $T>1.0$ GeV for the sum rules (40), (41), (48), (56), (60), (64), and (68). In fact the twist-four terms contribute only a few percent to the sum rules for such T values. The upper limit of T is constrained by the requirement that the continuum contribution is less than 30%. This corresponds to $T<2.5$ GeV. With the values of photon wave functions at $u_0=\frac{1}{2}$ we obtain the left hand side of these sum rules as functions of T. The continuum threshold is $\omega_c=3.0\pm 0.2$ GeV except $\omega_c=2.4\pm 0.2$ GeV for the sum rule (64). Stability develops for the sum rules in the region 1.0 GeV $<T<2.5$ GeV. The results are shown in Figs. 3–9. Numerically we have

$$g_d f_{-,1/2} f_{+,3/2}=-(3.0\pm 0.2)\times 10^{-2}\ \ \mathrm{GeV}^2, \tag{79}$$

$$g_s f_{-,1/2} f_{+,3/2}=-(1.9\pm 0.2)\times 10^{-2}\ \ \mathrm{GeV}^4, \tag{80}$$

$$g_1 f_{-,1/2} f_{+,1/2}=-(1.5\pm 0.5)\times 10^{-2}\ \ \mathrm{GeV}^3, \tag{81}$$

$$g_2 f_{+,1/2} f_{+,3/2}=-(5.5\pm 0.4)\times 10^{-2}\ \ \mathrm{GeV}^3, \tag{82}$$

$$g_3 f_{+,1/2}^2=(0.28\pm 0.04)\ \ \mathrm{GeV}^2, \tag{83}$$

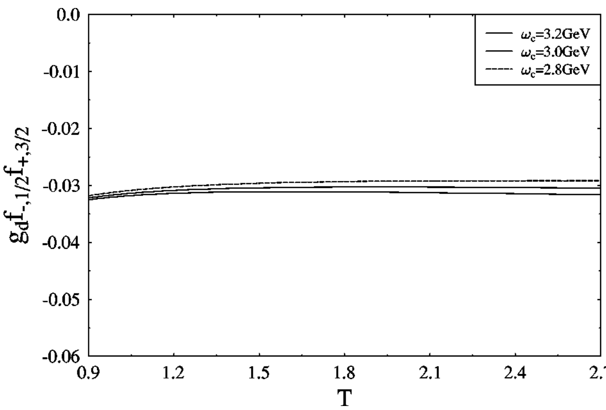

FIG. 3. Dependence of $g_d f_{-,1/2} f_{+,3/2}$ on T with $\omega_c = 3.2$, 3.0, and 2.8 GeV.

$$g_4 f^2_{-,1/2} = (8.9 \pm 0.5) \times 10^{-2} \ \ \text{GeV}^2, \tag{84}$$

$$g_5 f^2_{+,3/2} = -(2.3 \pm 0.3) \times 10^{-2} \ \ \text{GeV}^2, \tag{85}$$

where the errors refer to the variations with T in this region and the uncertainty in ω_c. And the central value corresponds to $T = 1.5$ GeV and $\omega_c = 3.0 \pm 0.2$ GeV except that we use $\omega_c = 2.4 \pm 0.2$ GeV for the sum rule (64).

With the central values of f's in Eq. (74) we get the absolute value of the coupling constants:

$$g_d = -(0.63 \pm 0.10) \ \ \text{GeV}^{-2}, \tag{86}$$

$$g_s = -(0.40 \pm 0.05), \tag{87}$$

$$g_1 = -(0.20 \pm 0.06), \tag{88}$$

$$g_2 = -(0.72 \pm 0.07) \ \ \text{GeV}^{-1}, \tag{89}$$

$$g_3 = (1.8 \pm 0.3) \ \ \text{GeV}^{-1}, \tag{90}$$

$$g_4 = (1.4 \pm 0.2) \ \ \text{GeV}^{-1}, \tag{91}$$

$$g_5 = -(0.64 \pm 0.08) \ \ \text{GeV}^{-3}. \tag{92}$$

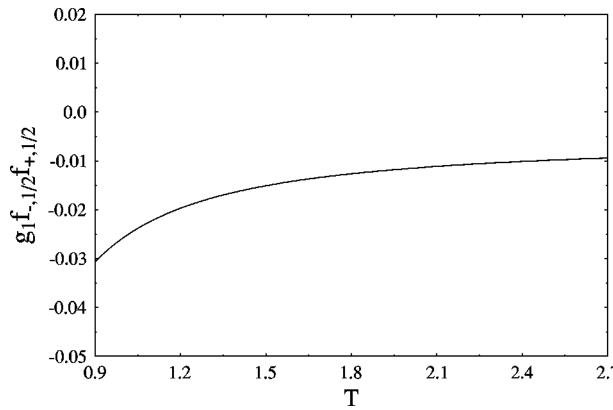

FIG. 5. Dependence of $g_1 f_{-,1/2} f_{+,1/2}$ on T.

Note we have only considered the uncertainty due to the variations of the Borel parameter and the continuum threshold in the above expressions. There are other sources of uncertainty. The input parameters χ and f are associated with the photon distribution amplitude. Especially the value of χ has been estimated with QCD sum rules [13] and with the octet baryon magnetic moments as inputs using the external field method [15]. Both approaches yield consistent results $\chi \approx -4.4$ GeV. With this value the octet, decuplet and heavy baryon magnetic moments derived using the external field method are in good agreement with the experimental data. So we expect its accuracy is better than 30%. The value of f has been estimated with the vector meson dominance model also with an accuracy of 30% [2].

The light cone sum rules for the coupling constants g_i and the mass sum rules for the heavy hadrons in HQET both receive large perturbative QCD corrections. But their ratio does not depend on radiative corrections strongly because of large cancellation [17]. In the present case, the uncertainty of the coupling constants g_i due to radiative corrections is expected to around 10% while the couplings f_Λ, etc. are affected significantly.

Another possible source of error is the truncation of the light cone expansion at twist four. We take the sum rules for η_1 for example. At $T = 1.5$ GeV, the twist-four term involved with g_1, g_2 is only -2.5% of the leading twist term

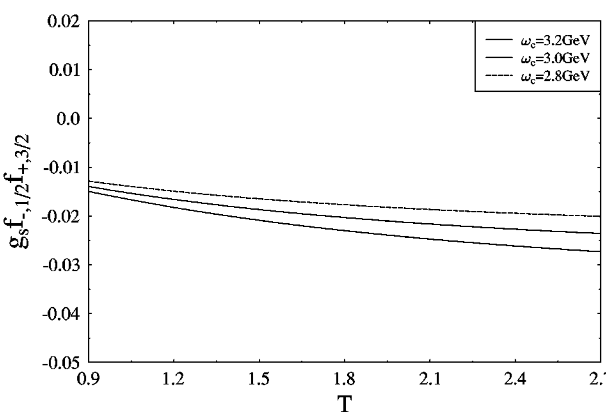

FIG. 4. Dependence of $g_s f_{-,1/2} f_{+,3/2}$ on T with $\omega_c = 3.2$, 3.0, and 2.8 GeV.

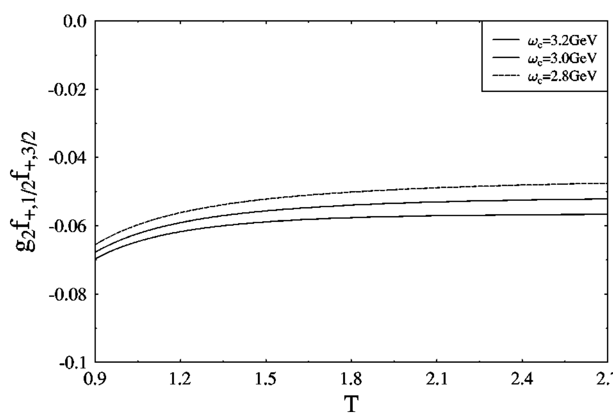

FIG. 6. Dependence of $g_2 f_{+,1/2} f_{+,3/2}$ on T with $\omega_c = 3.2$, 3.0, and 2.8 GeV.

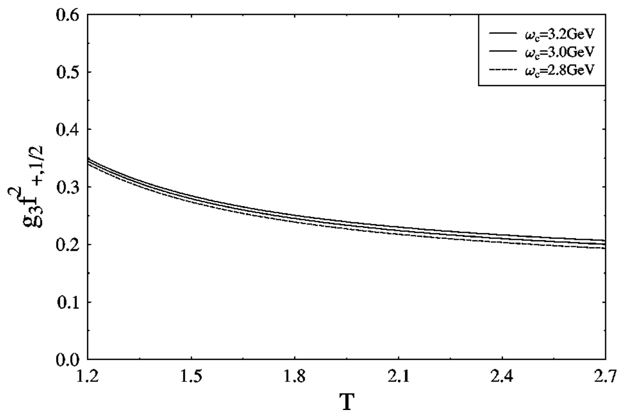

FIG. 7. Dependence of $g_3 f^2_{+,1/2}$ on T with $\omega_c = 3.2$, 3.0, and 2.8 GeV.

after making double Borel transformation to Eq. (20). Even after the subtraction of the continuum and excited states contribution the twist-four term is only -15% of the twist-two one in Eq. (23). So the light cone expansion converges quickly. We expect the contribution of higher twist distribution amplitudes to be small.

We have calculated the coupling constant in the leading order of HQET. The $1/m_Q$ correction is sizable for the charm system. But for the bottom system the $1/m_Q$ correction is typically around 5–10 % for the pionic coupling constants [8]. We expect the $1/m_Q$ correction to the electromagnetic coupling constants is of the same order. The inherent uncertainty of the method of QCD sum rules is not included, which is typically about 10%.

C. Decay widths of heavy hadrons

With these coupling constants we can calculate the decay widths of heavy hadrons.

The decay width formulas in the leading order of HQET are

$$\Gamma(\Sigma_b \to \Lambda_b \gamma) = 4\,\eta_1^2 \alpha |\vec{q}|^3,$$

$$\Gamma(\Sigma_b^* \to \Lambda_b \gamma) = \eta_2^2 \alpha |\vec{q}|^3 \frac{3m_i^2 + m_f^2}{3m_i^2},$$

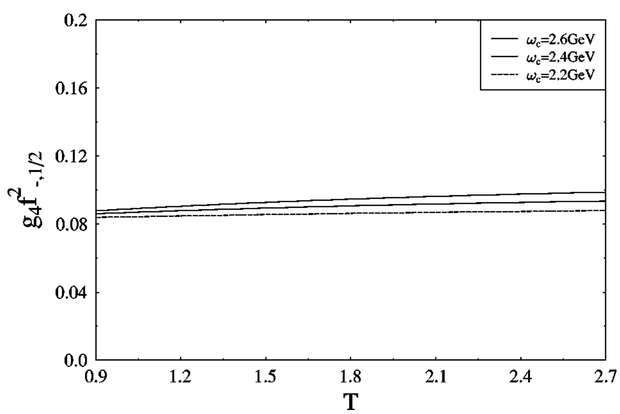

FIG. 8. Dependence of $g_4 f^2_{-,1/2}$ on T with $\omega_c = 2.6$, 2.4, and 2.2 GeV.

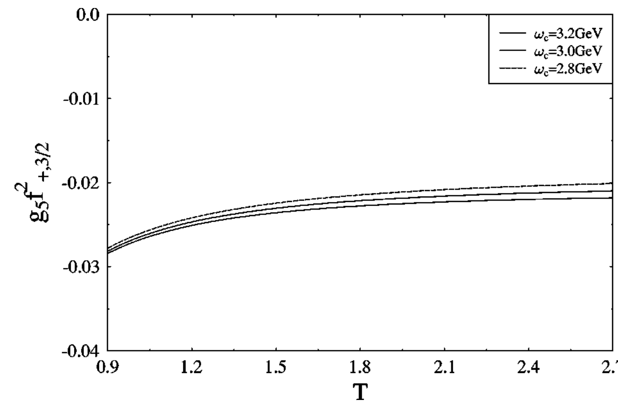

FIG. 9. Dependence of $g_5 f^2_{+,3/2}$ on T with $\omega_c = 3.2$, 3.0, and 2.8 GeV.

$$\Gamma(\Sigma_b^* \to \Sigma_b \gamma) = \eta_3^2 \alpha |\vec{q}|^3 \frac{3m_i^2 + m_f^2}{3m_i^2},$$

$$\Gamma(B_1 \to B^* \gamma) = \frac{2}{3} e_q^2 \alpha \left(\frac{14}{9} g_d^2 \left|\vec{q}\right|^5 + g_s^2 \left|\vec{q}\right| \right),$$

$$\Gamma(B_1 \to B \gamma) = \frac{4\alpha}{3} e_q^2 \alpha \left(\frac{1}{18} g_d^2 |\vec{q}|^5 + g_s^2 |\vec{q}| \right),$$

$$\Gamma(B_2^* \to B \gamma) = \frac{2}{5} e_q^2 \alpha g_d^2 |\vec{q}|^5,$$

$$\Gamma(B_2^* \to B^* \gamma) = e_q^2 \alpha \left(\frac{64}{45} g_d^2 |\vec{q}|^5 + 4 g_s^2 |\vec{q}| \right),$$

$$\Gamma(B_1' \to B^* \gamma) = e_q^2 \alpha g_1^2 |\vec{q}|,$$

$$\Gamma(B_1' \to B \gamma) = \frac{1}{2} e_q^2 \alpha g_1^2 |\vec{q}|,$$

$$\Gamma(B_0' \to B^* \gamma) = \frac{3}{2} e_q^2 \alpha g_1^2 |\vec{q}|,$$

$$\Gamma(B^* \to B \gamma) = \frac{1}{3} e_q^2 \alpha g_4^2 |\vec{q}|^3, \tag{93}$$

where $|\vec{q}| = (m_i^2 - m_f^2)/2m_i$, m_i, m_f is the parent and decay heavy hadron mass.

We apply the leading order formulas obtained above to the excited states of bottomed hadrons using the central values of the coupling constants in the previous section:

$$\Gamma(\Sigma_b \to \Lambda_b \gamma) = 131 \times \left(\frac{|\vec{q}|}{165\ \text{MeV}} \right)^3\ \text{keV},$$

$$\Gamma(\Sigma_b^{*0} \to \Lambda_b \gamma) = 313 \times \left(\frac{|\vec{q}|}{224\ \text{MeV}} \right)^3\ \text{keV},$$

$$\Gamma(\Sigma_b^{*+}\to\Sigma_b^+\gamma)=2.2\times\left(\frac{|\vec{q}|}{63.4\ \ \text{MeV}}\right)^3\ \ \text{keV},$$

$$\Gamma(\Sigma_b^{*0}\to\Sigma_b^0\gamma)=0.14\times\left(\frac{|\vec{q}|}{63.4\ \ \text{MeV}}\right)^3\ \ \text{keV},$$

$$\Gamma(\Sigma_b^{*-}\to\Sigma_b^-\gamma)=0.56\times\left(\frac{|\vec{q}|}{63.4\ \ \text{MeV}}\right)^3\ \ \text{keV},$$

$$\Gamma(B^{*0}\to B^0\gamma)=1.4\times\left(\frac{|\vec{q}|}{137\ \ \text{MeV}}\right)\ \ \text{keV},$$

$$\Gamma(B^{*+}\to B^+\gamma)=5.5\times\left(\frac{|\vec{q}|}{137\ \ \text{MeV}}\right)\ \ \text{keV},$$

$$\Gamma(B_1^0\to B^0\gamma)=84.4\times\left(\frac{|\vec{q}|}{490\ \ \text{MeV}}\right)\ \ \text{keV},$$

$$\Gamma(B_1^+\to B^+\gamma)=338\times\left(\frac{|\vec{q}|}{490\ \ \text{MeV}}\right)\ \ \text{keV},$$

$$\Gamma(B_1^0\to B^{*0}\gamma)=42.2\times\left(\frac{|\vec{q}|}{377\ \ \text{MeV}}\right)\ \ \text{keV},$$

$$\Gamma(B_1^+\to B^{*+}\gamma)=169\times\left(\frac{|\vec{q}|}{377\ \ \text{MeV}}\right)\ \ \text{keV},$$

$$\Gamma(B_2^{*0}\to B^0\gamma)=5.8\times\left(\frac{|\vec{q}|}{537\ \ \text{MeV}}\right)\ \ \text{keV},$$

$$\Gamma(B_2^{*+}\to B^+\gamma)=23\times\left(\frac{|\vec{q}|}{537\ \ \text{MeV}}\right)\ \ \text{keV},$$

$$\Gamma(B_2^{*0}\to B^{*0}\gamma)=211\times\left(\frac{|\vec{q}|}{408\ \ \text{MeV}}\right)\ \ \text{keV},$$

$$\Gamma(B_2^{*+}\to B^{*+}\gamma)=844\times\left(\frac{|\vec{q}|}{408\ \ \text{MeV}}\right)\ \ \text{keV}. \tag{94}$$

The uncertainty of the decay width is typically about 30%.

We do not present numerical results for the radiative decay widths for the charmed hadrons since $1/m_Q$ corrections are sizable for the charm system while such corrections are only a few percent of the leading order term for the bottom system [8].

In summary we have calculated the coupling constants of photons with the heavy baryons and the lowest three heavy meson doublets using the light cone QCD sum rules with the photon wave functions in the leading order of HQET. We hope these calculations will be tested in the future experiments.

ACKNOWLEDGMENTS

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