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Helicity and partial wave amplitude analysis of $D \to K^* \rho$ decay

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We have carried out an analysis of helicity and partial-wave amplitudes for the process $D \to K^* \rho$ in the factorization approximation using several models for the form factors. All the models, with the exception of one, generate partial-wave amplitudes with the hierarchy $|S|>|P|>|D|$. The one exception gives $|S|>|D|>|P|$. Even though in most models the D-wave amplitude is an order of magnitude smaller than the S-wave amplitude, its effect on the longitudinal polarization could be as large as 30%. Because of a misidentification of the partial-wave amplitudes in terms of the Lorentz structures in the relevant literature, we cast doubt on the veracity of the listed data, particularly the partial-wave branching ratios. [S0556-2821(99)03711-X]

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I. INTRODUCTION

The weak hadronic decay of the charm D meson to two resonant vector particles is difficult to analyze experimentally, as well as to understand theoretically. At the theoretical level much of the effort of the past was devoted to understanding mainly the rate $\Gamma(D \to V_1 V_2)$ (V stands for vector meson). Studies based on the factorization model were carried out by Bauer *et al.* [1] and Kamal *et al.* [2]; approaches based on flavor $SU(3)$ symmetry and broken $SU(3)$ symmetry were pursued also by Kamal *et al.* [2] and by Hinchliffe and Kaeding [3]; Bedaque *et al.* [4] made a pole-dominance model calculation.

One peculiarity of a pseudoscalar meson, P, decaying into two vector mesons is that the final-state particles are produced in different correlated polarization states. The hadronic matrix element, $A = \langle V_1 V_2 | H_{\text{weak}} | P \rangle$, involves three invariant amplitudes which can be expressed in terms of three different, but equivalent, bases; the helicity basis $|++\rangle, |--\rangle, |00\rangle$, the partial-wave basis (or the LS-basis) $|S\rangle, |P\rangle, |D\rangle$ and the transversity basis $|0\rangle, |\parallel\rangle, |\perp\rangle$. The interrelations between the amplitudes in these bases are presented in the next section. The data [5] for $D \to K^* \rho$ decay are quoted either in terms of the helicity branching ratios or the partial-wave branching ratios. Hence our study of the process $D \to K^* \rho$ is carried out in these two bases. We have undertaken a theoretical analysis for the particular decay, $D \to K^* \rho$, assuming factorization (see Sec. II) and using a variety of models for the form factors. Such a study has not been undertaken in the past.

The experimental analysis of $D \to K^* \rho$ (measurement of the branching ratio, partial-wave branching ratios, polarization etc.) is done by considering the resonant substructure of the four-body decays $D \to K\pi\pi\pi$ [6,7]. There are several two-body decay modes (for example, $D \to K^* \rho$ and $D \to K a_1$) which contribute to the final states in $D^0 \to \overline{K^0} \pi^- \pi^+ \pi^0$, $D^+ \to K^- \pi^+ \pi^+ \pi^0$, $D^0 \to K^- \pi^+ \pi^+ \pi^-$. Following the standard practice, Refs. [6,7] took the signal terms of the probability density to be a coherent sum of complex amplitudes for each decay chain leading to the four-body decays of D. Hence, the different contributing amplitudes can interfere among themselves. In general, the interference terms do not integrate to zero (see [8] for more details about the three-body decay $D \to K\pi\pi$). Consequently, the sum of the fractions f_i does not add up to unity: $\Sigma f_i \neq 1$ (see Refs. [6,8,9]). The branching fractions into two-body channels are then determined by maximizing the likelihood function. The branching fraction into any particular two-body channel, such as $D \to K^* \rho$, can be analyzed in terms of the helicity amplitdes (H_{++}, H_{--}, H_{00}), or the partial-wave amplitudes (S,P,D), or the transversity amplitudes $(A_0, A_\parallel, A_\perp)$. As a result of the completeness of each one of these bases, the decay rate $\Gamma(D \to K^* \rho)$ is expressed as an incoherent sum of the helicity, or the partial-wave, or the transversity amplitudes [10–12]. This imposes some constraints on the helicity and the partial-wave branching fractions B as they should add up to the total branching fraction for the mode $D \to K^* \rho$ as follows: $B_{++} + B_{--} + B_{00} = B_0 + B_\parallel + B_\perp = B_S + B_P + B_D = B_{K^* \rho}$. A similar situation occurs in $\bar{n}p$ annihilation to 3 (and 5) pions [13] where S and P waves are treated incoherently. An obvious problem with the $D^0 \to K^{*0} \rho^0$ data [5] is that this constraint is violated: the sum of the branching fractions into S and D states exceeds the total branching fraction. The fact that this sum also exceeds the transverse branching fraction is, by itself, not a problem due to the interference between the S and D waves. However, the problem with the data [5] is that the transverse branching fraction saturates the total branching fraction. There is, therefore, an internal inconsistency in the data: all the data listings cannot be correct. The Particle Data Group listing of $D \to K^* \rho$ data has remained unchanged since 1992.

We believe that the source of the inconsistency in the data [5–7] has to do with the identification of the partial-wave amplitudes, S, P, and D, with the Lorentz structures in the decay amplitude (see Table II and, especially, Eqs. (32)–(34) of [6]). The decay amplitude A for the process $P \to V_1 V_2$ is expressed in terms of three independent Lorentz structures and their coefficients, represented in the notation of [14,15] by a,b,c, and in the notation of [16] by the form factors $A_1(q^2)$, $A_2(q^2)$, and $V(q^2)$. We discuss this point in detail in the next section, but suffice it to say here that in [6] the P-wave amplitude is identified with c of [14,15] (or V of [16]), which is correct; however, they identify the S-wave amplitude with a of [14,15] (or A_1 of [16]) and D-wave amplitude with b of [14,15] (or A_2 of [16]), which is incorrect. We discuss this point in some detail in Secs. III and IV.

Part of the problem could also be that the transverse amplitudes H_{++}, H_{--} and the longitudinal amplitude H_{00} were fitted independently in [6]. Their argument for doing so was the large measured polarization of K^* in semileptonic decay of the D meson [17]. However, later measurements [18] of the polarization of K^* being much smaller vitiate this procedure.

II. METHOD

The decay $D \to K^* \rho$ is Cabibbo-favored and is induced by the effective weak Hamiltonian which can be reduced to the following color-favored (CF) and color-suppressed (CS) forms [19]:

$$H_{\mathrm{CF}} = \frac{G_F}{\sqrt{2}} V_{cs} V_{ud}^* [a_1 (\bar{u}d)(\bar{s}c) + c_2 O_8], \tag{1}$$

$$H_{\mathrm{CS}} = \frac{G_F}{\sqrt{2}} V_{cs} V_{ud}^* [a_2 (\bar{u}c)(\bar{s}d) + c_1 \tilde{O}_8], \tag{2}$$

where $V_{qq'}$ are the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements. The brackets $(\bar{u}d)$ represent $(V-A)$ color-singlet Dirac bilinears. O_8 and $\tilde{O}_8$ are products of color octet currents: $O_8 = \frac{1}{2}\Sigma_{a=1}^{8} (\bar{u}\lambda^a d)(\bar{s}\lambda^a c)$ and $\tilde{O}_8 = \frac{1}{2}\Sigma_{a=1}^{8} (\bar{u}\lambda^a c)(\bar{s}\lambda^a d)$. λ^a are the Gell-Mann matrices. a_1 and a_2 are the Wilson coefficients for which we use the values $a_1 = 1.26 \pm 0.04$ and $a_2 = -0.51 \pm 0.05$ [19]. In general a_1 and a_2 are related to the coefficients c_1 and c_2 [20] by

$$a_1 = c_1 + \frac{c_2}{N}, \quad a_2 = c_2 + \frac{c_1}{N}, \tag{3}$$

where N is an effective number of colors, and $c_1 = 1.26$, $c_2 = -0.51$ [20]. Using a value of N different from 3 is a way to parametrize nonfactorization effects. Our parametrization amounts to setting $N \to \infty$. This particular decay, $D \to K^* \rho$, has also been studied by Kamal *et al.* [19] and by Cheng [21] from the viewpoint of explicit (rather than implicit as here) nonfactorization.

In the factorization approximation one neglects the contribution from O_8 and $\tilde{O}_8$, and the matrix element of the first term is written as a product of two current matrix elements. Since we are effectively working with $N \neq 3$, one could argue that the nonfactorization arising from O_8 and $\tilde{O}_8$ is being included. We consider the following three decays: (i) $D^0 \to K^{-*} \rho^+$, a color-favored decay which gets contribution from external W exchange, known as a class I process; (ii) $D^0 \to \bar{K}^{0*} \rho^0$, a color-suppressed process which gets contribution from internal W exchange, known as a class II process; and (iii) $D^+ \to \bar{K}^{0*} \rho^+$, a class III process which gets contribution from external as well as internal W-exchange mechanisms. The decay amplitudes are given by

$$A(D^0 \to K^{-*} \rho^+) = \frac{G_F}{\sqrt{2}} V_{cs} V_{ud}^* a_1 \langle \rho^+ | \bar{u}d | 0 \rangle \langle K^{-*} | \bar{s}c | D^0 \rangle. \tag{4}$$

$$A(D^0 \to \bar{K}^{0*} \rho^0) = \frac{G_F}{\sqrt{2}} V_{cs} V_{ud}^* \frac{a_2}{\sqrt{2}} \langle \bar{K}^{0*} | \bar{s}d | 0 \rangle \langle \rho^0 | \bar{u}c | D^0 \rangle. \tag{5}$$

$$\begin{aligned} A(D^+ \to \bar{K}^{0*} \rho^+) &= \frac{G_F}{\sqrt{2}} V_{cs} V_{ud}^* \{ a_1 \langle \rho | \bar{u}d | 0 \rangle \langle \bar{K}^{*0} | \bar{s}c | D^+ \rangle \\ &\quad + a_2 \langle \bar{K}^{0*} | \bar{s}d | 0 \rangle \langle \rho^+ | \bar{u}c | D^+ \rangle \} \\ &= A(D^0 \to K^{-*} \rho^+) + \sqrt{2} A(D^0 \to \bar{K}^{0*} \rho^0). \end{aligned} \tag{6}$$

The extra $\sqrt{2}$ in Eq. (5) comes from the flavor part of the wave function of ρ^0. Each of the current matrix elements can be expressed in terms of meson decay constants and invariant form factors. We use the following definitions:

$$\langle V | \bar{u}d | 0 \rangle = m_V f_V \varepsilon_\mu^* \tag{7}$$

$$\begin{aligned} \langle V | \bar{s}c | D \rangle &= \frac{2}{m_D + m_V} \epsilon_{\mu\nu\rho\sigma} \varepsilon_V^{*\nu} P_D^\rho P_V^\sigma V(q^2) \\ &\quad + i \Bigg\{ \varepsilon_{V\mu}^* (m_D + m_V) A_1(q^2) - \frac{\varepsilon_V^* \cdot q}{m_D + m_V} \\ &\quad \times (P_V + P_D)_\mu A_2(q^2) - \frac{\varepsilon_V^* \cdot q}{q^2} 2 m_V q_\mu A_3(q^2) \\ &\quad + \frac{\varepsilon_V^* \cdot q}{q^2} 2 m_V q_\mu A_0(q^2) \Bigg\}, \end{aligned} \tag{8}$$

where $q = P_D - P_V$ is the momentum transfer, f_V is the decay constant of the vector meson V, ε_V is its polarization, $A_i(q^2), (i = 1,2,3)$ and $V(q^2)$ are invariant form factors defined in [16]. In terms of the helicity amplitudes the decay rate is given by

$$\Gamma(D \to V_1 V_2) = \frac{p}{8 \pi m_D^2} \{ |H_{00}|^2 + |H_{++}|^2 + |H_{--}|^2 \}, \tag{9}$$

where p is the center-of-mass momentum in the final state. H_{00}, H_{++} and H_{--} are the longitudinal and the two transverse helicity amplitudes, respectively, and for the decay $D^0 \to K^{-*} \rho^+$ they are given by

$$\begin{aligned} H_{00}(D^0 \to K^{-*} \rho^+) &= -i \frac{G_F}{\sqrt{2}} V_{cs} V_{ud}^* m_\rho f_\rho (m_D + m_{K^{-*}}) \\ &\quad \times a_1 \{ \alpha A_1^{DK^*}(m_\rho^2) - \beta A_2^{DK^*}(m_\rho^2) \} \end{aligned} \tag{10}$$

$$H_{\pm\pm}(D^0 \to K^{*-}\rho^+) = i\frac{G_F}{\sqrt{2}}V_{cs}V_{ud}^* m_\rho f_\rho (m_D + m_{K^{*-}}) \times a_1\{A_1^{DK^*}(m_\rho^2) \mp \gamma V(m_\rho^2)^{DK^*}\}, \tag{11}$$

where α, β, and γ are function of r and t and given by

$$\alpha = \frac{1-r^2-t^2}{2rt}, \quad \beta = \frac{k^2}{2rt(1+r)^2} \quad \text{and} \quad \gamma = \frac{k}{(1+r)^2} \tag{12}$$

with r, t, and k defined as follows:

$$r = \frac{m_{K^*}}{m_D}, \quad t = \frac{m_\rho}{m_D}, \quad k^2 = (1+r^4+t^4-2r^2-2t^2-2r^2t^2). \tag{13}$$

For $D \to K^*\rho$ the values of the parameters α, β, and γ are

$$\alpha = 1.52, \quad \beta = 0.24, \quad \gamma = 0.24. \tag{14}$$

Equivalently one can work with the partial wave amplitudes which are related to the helicity amplitudes by [11,22]

$$H_{00} = -\frac{1}{\sqrt{3}}S + \sqrt{\frac{2}{3}}D, \quad H_{\pm\pm} = \frac{1}{\sqrt{3}}S \pm \frac{1}{\sqrt{2}}P + \frac{1}{\sqrt{6}}D. \tag{15}$$

The partial waves are in general complex and can be expressed in terms of their phases as follows:

$$S = |S|\exp(i\delta_S), \quad P = |P|\exp(i\delta_P), \quad D = |D|\exp(i\delta_D). \tag{16}$$

For completeness, we introduce here the transversity basis, A_0, $A_\parallel$, and $A_\perp$, through

$$A_0 = H_{00} = -\sqrt{\frac{1}{3}}S + \sqrt{\frac{2}{3}}D$$

$$A_\parallel = \sqrt{\frac{1}{2}}(H_{++}+H_{--}) = \sqrt{\frac{2}{3}}S + \sqrt{\frac{1}{3}}D$$

$$A_\perp = \sqrt{\frac{1}{2}}(H_{++}-H_{--}) = P. \tag{17}$$

The longitudinal polarization is defined by the ratio of the longitudinal decay rate to the total decay rate

$$P_L = \frac{\Gamma_{00}}{\Gamma} = \frac{|H_{00}|^2}{|H_{++}|^2+|H_{--}|^2+|H_{00}|^2}. \tag{18}$$

Using Eqs. (10), (11), and (15) to solve for S, P, and D in term of form factors, we obtain (we drop a common factor of $i(G_F/\sqrt{2})V_{cs}V_{ud}^* m_\rho f_\rho (m_D + m_{K^{-*}})a_1$):

$$S = \frac{1}{\sqrt{3}}\{(2+\alpha)A_1(q^2) - \beta A_2(q^2)\}, \quad P = -\sqrt{2}\gamma V(q^2)$$

and

$$D = \sqrt{\frac{2}{3}}\{(1-\alpha)A_1(q^2) + \beta A_2(q^2)\}. \tag{19}$$

These real amplitudes are assumed to be the magnitudes of the partial wave amplitudes. The phases are then fed in by hand. The decay rate given by an incoherent sum, $\Gamma \propto (|H_{++}|^2+|H_{--}|^2+|H_{00}|^2) = (|S|^2+|P|^2+|D|^2) = (|A_0|^2 + |A_\parallel|^2 + |A_\perp|^2)$, is independent of the partial-wave phases. However, the polarization does depend on the phase difference, $\delta_{SD} = \delta_S - \delta_D$, arising from the interference between S and D waves,

$$P_L = \frac{1}{3}\frac{|S|^2+2|D|^2-2\sqrt{2}|S||D|\cos\delta_{SD}}{|S|^2+|P|^2+|D|^2}. \tag{20}$$

The knowledge of the different form factors is required to proceed further with the numerical estimate of the decay rate, Γ, and the longitudinal polarization P_L. Since it is not yet possible to obtain the q^2 dependence of these form factors from experimental data, and a rigorous theoretical calculation is still lacking, we have relied on several theoretical models for the form factors in our analysis. They are the following (i) Bauer, Stech, and Wirbel (BSWI) [16], where an infinite momentum frame is used to calculate the form factors at $q^2=0$, and a monopole form (pole masses are as in [16]) for q^2 dependence is assumed to extrapolate all the form factors to the desired value of q^2. (ii) BSWII [20] is a modification of BSWI, where while $F_0(q^2)$ and $A_1(q^2)$ are the same as in BSWI, a dipole q^2 dependence is assumed for $A_2(q^2)$ and $V(q^2)$. (iii) Altomari and Wolfenstein (AW) model [23], where the form factors are evaluated in the limit of zero recoil, and a monopole form is used to extrapolate to the desired value of q^2. (iv) Casalbuoni, Deandrea, Di Bartolomeo, Feruglio, Gatto, and Nardulli (CDDGFN) model [24], where the form factors are evaluated at $q^2=0$ in an effective Lagrangian satisfying heavy quark spin-flavor symmetry in which light vector particles are introduced as gauge particles in a broken chiral symmetry. A monopole form is used for the q^2 dependence. The experimental inputs for this model are from the semileptonic decay $D \to K^* l \nu$, and we have used the recent experimental values [25] of the form factors $A_1^{DK^*}(0) = 0.55 \pm 0.03$, $A_2^{DK^*}(0) = 0.40 \pm 0.08$, and $V^{DK^*}(0) = 1.0 \pm 0.2$, and $f_D = 194^{+14}_{-10} \pm 10$ MeV [26] in calculating the weak coupling constants of the model at $q^2=0$ [24], which are subsequently used in evaluating the required form factors. (v) Isgur, Scora, Grinstein, and Wise (ISGW) model [27], where a nonrelativistic quark model is used to calculate the form factors at zero recoil and an exponential

TABLE I. Model and experimental predictions for the form factors: ${A_{1,2}}^{DK^*(\rho)}(m_\rho^2(m_{K^*}^2)), V^{DK^*(\rho)}(m_\rho^2(m_{K^*}^2))$ and the ratios $x = A_2(0)/A_1(0), y = V(0)/A_1(0)$ for the process $D \to K^* \rho$.

	BSI	BSII	AW	CDD	ISG	BFO	Exp.F [29]
$A_1^{DK^*}(m_\rho^2)$	0.969	0.969	0.887	0.606	0.909	0.578	0.606
$A_2^{DK^*}(m_\rho^2)$	1.264	1.392	0.707	0.441	0.929	3.747	0.441
$V^{DK^*}(m_\rho^2)$	1.414	1.630	1.602	1.153	1.25	0.773	1.153
$x = \frac{A_2(0)}{A_1(0)}$	1.30	1.30	0.80	0.73	1.02	6.5	0.73 [31]
$y = \frac{V(0)}{A_1(0)}$	1.39	1.39	1.73	1.82	1.38	1.16	1.87 [31]
$A_1^{D\rho}(m_{K^*}^2)$	0.898	0.898	0.835	0.732	0.766	0.605	0.637
$A_2^{D\rho}(m_{K^*}^2)$	1.070	1.240	0.846	0.487	0.958	3.574	0.464
$V^{D\rho}(m_{K^*}^2)$	1.529	1.908	1.343	1.326	1.41	0.713	1.248

q^2 dependence, based on a potential-model calculation of the meson wave function, is used to extrapolate them to the desired q^2. (vi) Bajc, Fajfer, and Oakes (BFO) model [28], where the form factors $A_1(q^2)$ and $A_2(q^2)$ are assumed to be flat, and a monopole behavior is assumed for $V(q^2)$. Finally (vii), a parametrization that uses experimental values (Exp.F) [25] of the form factors at $q^2 = 0$ and extrapolates them using monopole forms.

III. RESULTS

A. Parameters

For the numerical calculations we use the following values for the CKM matrix elements and meson decay constants:

$$V_{cs} = 0.974, \qquad V_{ud} = 0.975,$$

$$f_\rho = 0.212 \text{ GeV}, \quad f_{K^*} = 0.221 \text{ GeV}. \tag{21}$$

In Table I we present the predicted values of the form factors in the different models as well as their experimental values [29]. One observes that while the model predictions for the form factors $A_1(q^2)$ and $V(q^2)$ are in the range $(0.6-1)$ and $(0.8-1.6)$, respectively, the model-dependence of $A_2(q^2)$ shows a spread over a larger range: $(0.4-3.7)$. A striking feature of the BFO model [28] is the large value of the form factor A_2, which is incompatible with its experimental determination.

B. $D^0 \to K^{*-} \rho^+$

We calculate the experimental value of polarization from the listing of Ref. [5]:

TABLE II. Decay rates for $D^{+,0} \to \overline{K}^0{}^* \rho^{+,0}$. The values of Γ must be multiplied by 10^{11}s^{-1}. The parameter $z = \cos\delta_{SD}$. The experimental values of P_L are listed only if measurements of longitudinal or transverse branching ratios are available [5].

		BSI	BSII	AW	CDD	ISG	BFO	Exp.F [29]	Expt.
D^0	Γ	4.99	4.96	4.63	2.20	4.56	1.03	2.20	$1.47 \pm .58$
$\downarrow$	P_L	$0.319 - 0.084z$	$0.313 - 0.071z$	$0.316 - 0.122z$	$0.315 - 0.127z$	$0.324 - 0.108z$	$0.418 - 0.417z$	$0.315 - 0.127z$	0.475 ± 0.271
K^{-*}	$\frac{\lvert S\rvert}{\lvert P\rvert}$	4.3	3.7	3.6	3.5	4.7	2.9	3.5	
ρ^+	$\frac{\lvert S\rvert}{\lvert D\rvert}$	10.6	12.3	7.0	6.7	8.3	1.4	6.7	
D^+	Γ	1.56	1.54	1.50	0.409	1.69	0.268	0.559	0.20 ± 0.12
$\downarrow$	P_L	$0.326 - 0.086z$	$0.325 - 0.079z$	$0.319 - 0.141z$	$0.318 - 0.128z$	$0.333 - 0.129z$	$0.416 - 0.416z$	$0.321 - 0.134z$	
$\overline{K}^0{}^*$	$\frac{\lvert S\rvert}{\lvert P\rvert}$	5.5	5.3	3.6	3.7	6.9	2.8	4.0	>2 [32]
ρ^+	$\frac{\lvert S\rvert}{\lvert D\rvert}$	10.6	11.5	6.1	6.7	7.0	1.4	6.5	1.3 ± 0.8 [32]
	Γ	0.481	0.488	0.426	0.353	0.351	0.124	0.267	0.354 ± 0.080
D^0 $\downarrow$	P_L	$0.309 - 0.080z$	$0.294 - 0.060z$	$0.314 - 0.097z$	$0.313 - 0.125z$	$0.379 - 0.074z$	$0.420 - 0.419z$	$0.307 - 0.119z$	$0.0^{+0.4}_{-0}$
$\overline{K}^0{}^*$	$\frac{\lvert S\rvert}{\lvert P\rvert}$	3.4	2.7	3.6	3.3	3.1	3.0	3.0	>2.8 [32]
ρ^0	$\frac{\lvert S\rvert}{\lvert D\rvert}$	10.7	13.7	8.9	6.8	11.5	1.4	7.1	1.21 ± 0.23 [32]

$$P_L = \frac{\Gamma(D^0 \to \rho^+ \overline{K}^{*-}_{\text{longitudinal}})}{\Gamma(D^0 \to \rho^+ \overline{K}^{*-})} = \frac{2.9 \pm 1.2}{6.1 \pm 2.4} = 0.475 \pm 0.271. \tag{22}$$

In Table II we have summarized the results for the decay rates Γ, logitudinal polarization P_L, and partial-wave ratios $|S|/|P|$ and $|S|/|D|$ in different models.

We note from Table II that the models CDDGFN, BFO, and the scheme that uses experimentally measured form factors, predict a decay rate within a standard deviation of the central measured value. All other models overestimate the rate by several standard deviations. As for the longitudinal polarization, given the freedom of the unknown $\cos\delta_{SD}$, all models are able to fit the data. In particular, all models except BFO are able to predict the polarization correctly for $\delta_{SD}=0$; in the BFO model for $\delta_{SD}=0$, $D^0 \to K^{*-}\rho^+$ becomes totally transversely polarized. This circumstance arises from the fact that BFO model predicts a large D-wave contribution, $|S|/|D| \approx \sqrt{2}$. It then becomes evident from Eq. (15) that H_{00} vanishes. All models except BFO also display the partial-wave-amplitude hierarchy: $|S|>|P|>|D|$; the BFO model on the other hand predicts $|S|>|D|>|P|$, which we believe is less likely. The reasoning goes as folows: For decays close to threshold, one anticipates the Lth partial-wave amplitude to behave like $(p/\Lambda)^L$, where p is the center-of-mass momentum and Λ a mass scale. For $p \sim 0.4$ GeV and $\Lambda \sim m_D$, one expects the hierarchy $|S|>|P|>|D|$.

C. $D^+ \to \overline{K}^{0*}\rho^+$

In contrast to the decay mode $D^0 \to \overline{K}^{-*}\rho^+$, here the data listing [5] is at best confused. First, since the longitudinal and/or transverse branching ratios are not listed, it is not possible to calculate the longitudinal polarization. Second, though in Refs. [6,9] the identification of the transversity amplitudes (A_T, A_L, and $A_{l=1}$ in the notation of Ref. [6]) in terms of the partial-wave amplitudes is correct (see Eqs. (20)–(26) of Ref. [6]), their identification of the partial-wave amplitudes S and D in terms of the Lorentz structure of the decay amplitude is incorrect. In Table II, and more succinctly in Eqs. (32) and (34) of Ref. [6], S-wave amplitude is identified with the Lorentz structure that goes with the form factor A_1, and D-wave amplitude with that of A_2. In fact, the correct identification of the S- and D-wave amplitudes given in Eq. (19) shows that they are both linear superpositions of A_1 and A_2.

With the caveat that the identification of the partial waves in Refs. [6,9] is incorrect (note also that the listing of Ref. [5] uses these references only), we take the S-, P-, and D-wave branching ratios at their face value and calculate the ''experimental'' ratios $|S|/|P|$ and $|S|/|D|$.

In Table II, we have shown the calculated decay rate, the longitudinal polarization and the ratios of the partial-wave amplitudes in different models and compared them with the data. The BFO [28] model is the only one that reproduces the total rate correctly. This model also generates a large D-wave amplitude, with the partial-wave hierarchy $|S|>|D|>|P|$. This feature of the BFO model is due to the exceptionally large value of the form factor A_2, which is in contradiction with the experimental determination of the form factor as shown in Table I.

D. $D^0 \to \overline{K}^{0*}\rho^0$

Reference [5] lists the branching ratio, and the transverse branching ratio. This enables us to calculate the longitudinal polarization from

$$P_L = 1 - P_T = 1 - \frac{B(D^0 \to \overline{K}^{0*}\rho^0{}_{\text{transverse}})}{B(D^0 \to \overline{K}^{0*}\rho^0)} = 0.0 \pm^{0.4}_{0.0}. \tag{23}$$

Reference [5] also lists the S- and D-wave branching ratios. However, our criticism of these numbers in the previous subsection applies also to $D^0 \to \overline{K}^{0*}\rho^0$ decay. With this caution, we have taken their [5] numbers at face value and calculated the experimental and theoretical ratios of the partial wave amplitudes and listed them in Table II.

We note from Table II that the rate in the BFO model is too low by three standard deviations; the rates predicted in BSWI and BSWII models are 1.5 standard deviations too high, while all other models fit the rate within one standard deviation. As for the longitudinal polarization, all models predict a value consistent with the data. All models also satisfy the $|S|/|P|$ bound, but only the BFO model fits the $|S|/|D|$ ratio. This is because the BFO model generates a large D-wave amplitude.

A final comment: The inconsistency of the data is evident in the listing [5] of the total branching ratio and the individual partial-wave branching ratios. We know that the total branching ratio is an incoherent sum of the individual branching ratios in S, P, and D waves. Yet, in the Particle Data Group listing [5], the sum of S- and D-wave branching ratios exceeds the total. This by itself should cast doubt on the veracity of the data.

IV. S-WAVE AND $A_1(q^2)$ DOMINANCE

Since S-wave and D-wave amplitudes are linear superpositions of the form factors A_1 and A_2, see Eq. (19), the concept of S-wave dominance is different from that of A_1 dominance. All the models we have discussed, with the exception of the BFO model [28], predict that S-wave amplitude is the dominant partial-wave amplitude. Further, since Ref. [6] identifies $S \sim A_1$ and $D \sim A_2$, we need to look at what is meant by S-wave dominance and contrast it with A_1 dominance.

Consider first the concept of S-wave dominance. We see from Eqs. (9), (15) that in this approximation, $\Gamma \propto |S|^2$, and $|H_{00}| = |H_{++}| = |H_{--}| = |S/\sqrt{3}|$. In practice, most of the models predict the S-wave amplitude to be roughly an order of magnitude larger than the D-wave amplitude. Consequently, D wave would contribute only 1% to the rate relative to the S wave. However, it could influence the longitudinal polarization considerably through its interference with the S wave. Depending on the value of δ_{SD} the interference term could amount to a 30% correction to P_L (see also Ref. [30]). However, regardless of the exact size of the D-wave amplitude, S-wave dominance would lead to $P_L \to \frac{1}{3}$, for $\delta_{SD} = \pi/2$.

Consider now the concept of A_1 dominance. From Eqs. (10) and (11), we see that $H_{00} \propto \alpha A_1$ and $H_{++} = H_{--} \propto A_1$. With $\alpha = 1.52$, the longitudinal helicity amplitude is the largest, and the longitudinal polarization becomes

$$P_L = \frac{\alpha^2}{2+\alpha^2} = 0.54, \tag{24}$$

in contrast to a value 1/3 (with an error from $S-D$ interference) for S-wave dominance. Further, from Eq. (19), we note that in A_1 dominance,

$$S \propto \frac{2+\alpha}{\sqrt{3}} A_1(q^2), \quad D \propto \sqrt{\frac{2}{3}}(1-\alpha)A_1(q^2), \tag{25}$$

which makes the S-wave amplitude five times larger than the D-wave amplitude—not quite what one would term ''S-wave dominance.''

V. CONCLUSION

We have carried out an analysis of the process $D \to K^* \rho$ in terms of the helicity, and partial-wave amplitudes. We used several models for the form factors, as well as their experimental values, when available, from semileptonic decay. A general feature of our calculation is that all the models, with the exception of the BFO model [28], are consistent with the expected threshold behavior $|S| > |P| > |D|$; the BFO model, on the other hand, gives $|S| > |D| > |P|$. Even though in most models the D-wave amplitude is almost an order of magnitude smaller than the S-wave amplitude, it could effect the polarization prediction significantly through $S-D$ interference.

As we see from Table II, models BSWI, BSWII, AW, and ISGW grossly overestimate the rate for $D^0 \to K^{*-} \rho^+$, while models CDD, BFO, and the model that uses experimental form factor input, more or less agree with the measured rate. For this decay mode, we trust the measurement of the longitudinal branching ratio as the identification of the transversity amplitudes in Ref. [6] is correct. Due to the large error in P_L, and the uncertainty arising from the $S-D$ interference, all models are consistent with the polarization measurement.

For the mode $D^+ \to \overline{K}^{*0} \rho^+$, all the models, with the exception of the BFO model [28], grossly overestimate the rate. Before one gets the impression that the BFO model does well, we would like to point out that its prediction for the form factor A_2 is in sharp disagreement with the measurements from the semileptonic decays. There are no direct measurements of the longitudinal (or transverse) polarization for this mode. The predicted values of the polarization in every model are almost the same as for the mode $D^0 \to K^{*-} \rho^+$.

For the mode $D^0 \to \overline{K}^{*0} \rho^0$, BSWI and BSWII models predict a rate within 1.5 standard deviations. The remaining models, with the exception of the BFO model, predict a rate in agreement with data within one standard deviation. The BFO model underestimates the rate by three standard deviations. The transverse rate has been measured [5], from which we have calculated the longitudinal polarization. The measured value of P_L has large errors, but it is consistent with the longitudinal polarization in $D^0 \to K^{*-} \rho^+$. Given the freedom of the $S-D$ interference, all models are consistent with the measured polarization. The predicted longitudinal polarization is almost decay-mode independent.

A final comment: Because of the misidentification of the S and D waves with the Lorentz structures in [6,9], we do not trust the partial-wave branching ratios listed in [5]. For this reason the listings of $|S|/|P|$ and $|S|/|D|$ ratios in the last column of Table II have to be read with this caveat.

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