Polarized and unpolarized baryon production: SU(3) revisited

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The production of the octet baryons in ep collisions is analyzed based on an SU(3) model for quark fragmentation into octet baryons. A good fit to currently available data on Λ production in ep scattering is obtained. The model is extended to include polarization effects and all polarized fragmentation functions $\Delta D_q^h(z,Q^2)$ describing the polarized fragmentation into members of the baryon octet are expressed in terms of three SU(3) symmetric functions $\Delta\alpha(z,Q^2)$, $\Delta\beta(z,Q^2)$, and $\Delta\gamma(z,Q^2)$ and an SU(3) breaking parameter λ. From suitable comparisons with Λ polarization data in e^+e^- annihilations at the Z pole, $\Delta\alpha(z,Q^2)$ has been extracted. The fragmentation functions are then evolved using leading order polarized evolution equations and the polarization of Λ in e^+e^- and ep reactions has been predicted. [S0556-2821(99)08509-4]

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Polarization phenomena in particle interactions play a fundamental role in determining the spin structure of hadrons. Such studies have posed very interesting problems, the most intriguing being the proton spin puzzle, which led to the so-called ''spin crisis.'' Measurements of the first moment of the spin-dependent structure function of the proton $\int_0^1 g_1^p(x)dx$ (=0.126) by the European Muon Collaboration (EMC) [1] indicated that very little of the proton spin resided on the constituent partons. This contradicts the naive parton model assumption that all of the spin of the proton is carried by the constituent partons, hence yielding the famous Ellis-Jaffe sum rule—$g_1^p=0.19$.

The other area of ignorance lies in the spin-transfer mechanism during the process of fragmentation of a quark into a hadron. Large transverse polarization of hyperons in unpolarized $p\bar{p}$ interactions was reported by Fermilab [2]. To leading order, if the initial state particles are unpolarized, QCD predicts a zero polarization for final state particles.

Recently, several studies have been reported on Λ polarization. A review of polarized Λ production can be found in Refs. [3,4]. Most of the investigations have reported polarization of Λ because the self-analyzing property of the decay of Λ into p and π makes it easy to reconstruct its spin. It would be interesting to investigate the polarization of other baryons such as the proton and Ξ, in addition to Λ. Such an investigation would provide a firmer foundation to the proposed theoretical model. The present work is an effort in this direction.

In the present work, we try to build a model to explain the transfer of spin from the quark to the baryon it fragments into. We use an SU(3) model to explain the polarized fragmentation. This is an extension of an earlier study [5] done for unpolarized fragmentation functions using SU(3) symmetry of the baryon octet. The fragmenting quark is considered to be an SU(3) triplet q_i which fragments into a baryon octet B_i^j and X, so that the probability for the interaction $q_i\rightarrow B_i^j+X$ is $\alpha(z)[\beta(z),\gamma(z)]$ when X is a triplet [antisixplet, fifteenplet]. The model yields good agreement with all unpolarized data for baryon production in e^+e^- experiments. More importantly, it shows that there is indeed an SU(3) symmetry among the members of the baryon octet, up to a symmetry breaking factor λ [which arises because the more massive s-quark breaks the SU(3) symmetry]. Encouraged by the success of this very simple model, we extend it to include longitudinally polarized fragmentation functions. We again rely on the SU(3) symmetry of the baryons. Since data on baryon production (polarized and unpolarized) do not distinguish between baryon and anti-baryon production, we calculate only the sum of these. Throughout the text, the term baryon production implies baryon + antibaryon production.

The fragmenting quark q_i is longitudinally polarized and is a member of the quark triplet ($q_1=u$, $q_2=d$, $q_3=s$) and the hadron under study h_i^j is a member of the baryon octet, so that the process is

$$q\rightarrow h+X.$$

Analogous to the unpolarized functions $\alpha(z,Q)$, $\beta(z,Q)$, and $\gamma(z,Q)$ of Ref. [5], we introduce the polarized functions $\Delta\alpha(z,Q)$, $\Delta\beta(z,Q)$, and $\Delta\gamma(z,Q)$. All polarized fragmentation functions can be expressed in terms of these functions and the suppression factor λ. $\Delta\alpha(z,Q)$ is defined as

$$D_{q_i^+}^{B_i^{j+}+X}-D_{q_i^+}^{B_i^{j-}+X}, \tag{1}$$

where B_i^j is a member of the baryon octet, X is a triplet and D_q^{B+X} is the probability that the parton q fragments into the baryon B through the channel $q\rightarrow B+X$. $\Delta\beta(z,Q)$ and $\Delta\gamma(z,Q)$ are similarly defined with X being antisixplet and fifteenplet, respectively. The polarized quark fragmentation into B_i^j; $i,j=1,...,3$ in terms of $\Delta\alpha$, $\Delta\beta$, and $\Delta\gamma$ is given in Table I.

Since the (more) massive strange quark is known to break SU(3) symmetry, we introduce symmetry breaking effects as in Ref. [5]. The fragmentation function is suppressed by a factor λ whenever a strange quark belonging to the valence of the hadron is produced. All nonstrange fragmentation functions of strange hadrons are suppressed by λ. For example, ΔD_u^Λ is suppressed by a factor λ compared to ΔD_s^Λ. Gluons being flavor singlets couple to all baryons with the same strength. λ can be different for both the polarized and unpolarized case. It can even depend on energy of the frag-

TABLE I. Polarized quark fragmentation functions into members of the baryon octet in terms of the SU(3) functions $\Delta\alpha$, $\Delta\beta$, and $\Delta\gamma$.

	p		n
$u\to p$	: $\Delta\alpha+\Delta\beta+\frac{3}{4}\Delta\gamma$	$u\to n$	: $2\Delta\beta+\Delta\gamma$
$d\to p$	: $2\Delta\beta+\Delta\gamma$	$d\to n$	: $\Delta\alpha+\Delta\beta+\frac{3}{4}\Delta\gamma$
$s\to p$	: $2\Delta\gamma$	$s\to n$	: $2\Delta\gamma$
	Λ^0		Σ^0
$u\to\Lambda^0$	: $\frac{1}{6}\Delta\alpha+\frac{9}{6}\Delta\beta+\frac{9}{8}\Delta\gamma$	$u\to\Sigma^0$	: $\frac{1}{2}\Delta\alpha+\frac{1}{2}\Delta\beta+\frac{11}{8}\Delta\gamma$
$d\to\Lambda^0$	: $\frac{1}{6}\Delta\alpha+\frac{9}{6}\Delta\beta+\frac{9}{8}\Delta\gamma$	$d\to\Sigma^0$	: $\frac{1}{2}\Delta\alpha+\frac{1}{2}\Delta\beta+\frac{11}{8}\Delta\gamma$
$s\to\Lambda^0$	: $\frac{4}{6}\Delta\alpha+\frac{9}{6}\Delta\gamma$	$s\to\Sigma^0$	: $2\Delta\beta+\Delta\gamma$
	Σ^+		Σ^-
$u\to\Sigma^+$	: $\Delta\alpha+\Delta\beta+\frac{3}{4}\Delta\gamma$	$u\to\Sigma^-$	: $2\Delta\gamma$
$d\to\Sigma^+$	: $2\Delta\gamma$	$d\to\Sigma^-$	: $\Delta\alpha+\Delta\beta+\frac{3}{4}\Delta\gamma$
$s\to\Sigma^+$	: $2\Delta\beta+\Delta\gamma$	$s\to\Sigma^-$	: $2\Delta\beta+\Delta\gamma$
	Ξ^0		Ξ^-
$u\to\Xi^0$	: $2\Delta\beta+\Delta\gamma$	$u\to\Xi^-$	: $2\Delta\gamma$
$d\to\Xi^0$	: $2\Delta\gamma$	$d\to\Xi^-$	: $2\Delta\beta+\Delta\gamma$
$s\to\Xi^0$	: $\Delta\alpha+\Delta\beta+\frac{3}{4}\Delta\gamma$	$s\to\Xi^-$	: $\Delta\alpha+\Delta\beta+\frac{3}{4}\Delta\gamma$

menting quark. We follow the procedure adopted in Ref. [5] to extract these parameters. There is not enough data to enable the extraction of all these parameters and we have to rely on a few assumptions. For the present study, we assume λ to be the same as for the unpolarized case. The antiquark fragmentation functions are given by $\Delta\overline{\alpha}$, $\Delta\overline{\beta}$, and $\Delta\overline{\gamma}$ and are related to $\Delta\gamma$ as in Ref. [5]

$$\Delta\overline{\gamma}=\Delta\gamma,\ \Delta\overline{\alpha}=0.75\Delta\gamma,\ \Delta\overline{\beta}=0.5\Delta\gamma. \tag{2}$$

Thus, all polarized quark fragmentation functions for the baryon octet are given by three SU(3) symmetric functions $\Delta\alpha$, $\Delta\beta$, and $\Delta\gamma$ and a suppression factor λ, which are to be determined by comparison with data. $\Delta\gamma$ defines the polarized sea fragmentation just as γ describes unpolarized sea fragmentation [5]. We separate the quark fragmentation functions into sea and valence components as

$$\Delta\alpha=\Delta\alpha_V+\Delta\alpha_S,\ \Delta\beta=\Delta\beta_V+\Delta\beta_S,\ \Delta\gamma=\Delta\gamma_V+\Delta\gamma_S,$$

$$\Delta\overline{\alpha}=\Delta\alpha_S,\ \Delta\overline{\beta}=\Delta\beta_S,\ \Delta\overline{\gamma}=\Delta\gamma_S. \tag{3}$$

As mentioned earlier, there is not enough data on polarization of baryons to enable the extraction of all of these fragmentation functions. Only Λ polarization data in e^+e^- annihilation experiment at the Z pole exists [6]. We make a few simplifying assumptions to extract information regarding polarized fragmentation functions from the data.

As mentioned in Ref. [7], strange baryons produced in high energy reactions show strong polarization effects transverse to the scattering plane. However, there is very little polarization in the longitudinal direction. Therefore, we assume that quarks produced in the hadronization process are only very weakly polarized. For this reason, we assume all polarized sea fragmentation functions to be zero. Only valence fragmentation functions contribute to polarization effects. Thus, $\Delta\gamma$ is equal to zero and hence $\Delta\overline{\alpha}$, $\Delta\overline{\beta}$, and $\Delta\overline{\gamma}$ are all zero.

We use the unpolarized fragmentation functions given in Ref. [5] and use the same suppression factor λ (=0.07) for the polarized case. Consider the Λ baryon. Due to the smallness of λ, $D_s^\Lambda(z)\gg D_{u,d}^\Lambda(z)$. Even the polarized gluon fragmentation function is suppressed. Heavy quarks (c and b) also fragment to produce Λ. However, the branching ratio of $b\to\Lambda$ is very small $[BR(b\to\Lambda X)=(5.87\pm0.46\pm0.48)\%]$ [8]. Λ_c decays to yield Λ. Λ_c data at $\sqrt{s}=91.2$ GeV is between 15 to 100 times smaller than the Λ data in the overlapping z range of about 0.3–0.8 [9]. Due to this reason, the contribution of c and b fragmentation to Λ has been ignored in the analysis. Therefore, for this study, we shall assume that Λ production in e^+e^- annihilations is almost entirely due to s-quark fragmentation and in particular s-valence fragmentation. Thus, the polarization of Λ can be written as [7,10]

$$P^\Lambda(\sqrt{s},\cos\theta,z)$$
$$=\frac{(d\sigma(\sqrt{s})_{s^+}/d\cos\theta-d\sigma(\sqrt{s})_{s^-}/d\cos\theta)\Delta D_{s_V}^\Lambda(z)}{(d\,\sigma(\sqrt{s})_{s^+}/d\cos\theta+d\,\sigma(\sqrt{s})_{s^-}/d\cos\theta)D_{s_V}^\Lambda(z)}, \tag{4}$$

where $\sqrt{s}$ is the c.m. energy, z is $E_{\rm hadron}/E_{\rm beam}$, θ is the scattering angle, $\Delta D_q^h(z)$ is the polarized fragmentation function, $D_q^h(z)$ is the unpolarized fragmentation function, and a sum over quarks and antiquarks is implied.

Using the expression for polarized fragmentation functions given in Table I and unpolarized fragmentation functions given in Ref. [5], $\Delta D_{s_V}^\Lambda(z)=\frac{4}{6}\Delta\alpha_V(z)$ and $D_{s_V}^\Lambda=\frac{4}{6}\alpha_V(z)$. Our model predicts very small fragmentation functions for u and d quarks unlike those of Refs. [11,12], where it is shown that u and d carry significant amount of the spin of Λ. At the Z pole, for unpolarized e^+ and e^- beams, the polarization of Λ can be expressed as

$$P^\Lambda(\sqrt{s}=91.2\,\text{GeV},\cos\theta,z)$$
$$=\frac{[a(1+\cos^2\theta)+b\cos\theta]\Delta\alpha_V(z)}{[c(1+\cos^2\theta)+d\cos\theta]\alpha_V(z)}. \tag{5}$$

The expressions for a, b, c, and d can be found in Ref. [7].

On averaging over θ,

$$P^\Lambda(\sqrt{s}=91.2\,\text{GeV},z)=\frac{a\Delta\alpha_V(z)}{c\,\alpha_V(z)}. \tag{6}$$

We extract $\Delta\alpha_V(z)$ from the polarization data of Λ in e^+e^- annihilations at the Z pole [6]. The polarization of Λ is parametrized as

$$P^\Lambda(z)=-2.9(1-z)^{0.97}z^{2.05},$$

so that at 91.2 GeV, $\Delta\alpha_V(z)=(c/a)\,\alpha_V(z)P^\Lambda(z)$. $\Delta\alpha_V(z)$ and $\alpha_V(z)$ are shown in Fig. 1. $|\Delta\alpha_V(z)|\ll\alpha_V(z)$ in the small z region. To extract $\Delta\beta_V$, polarization data for at least

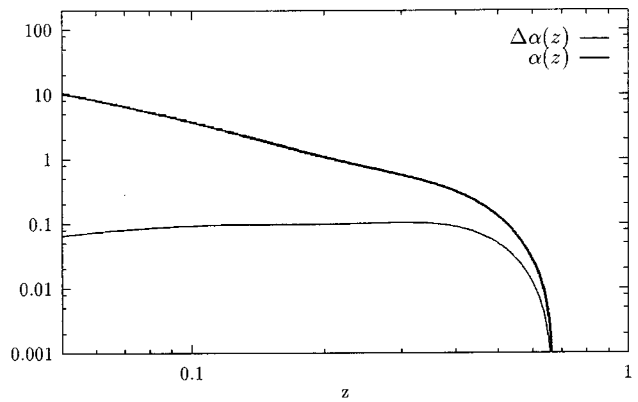

FIG. 1. The figure shows the unpolarized fragmentation function α_V and the polarized fragmentation function $\Delta\alpha_V$ at Q=91.2 GeV.

one more baryon is required. We would like to remark that if the polarization of one more baryon were known, then using this model, a leading order approximation of polarization of all the octet baryons can be obtained.

At $Q^2=2\ \mathrm{GeV}^2$, we parametrize $\Delta\alpha_V(z)$ as

$$6.9(1-z)^{5.51}z^{1.7}(1-3.96z+13.12z^2).$$

The input parametrization is obtained by tuning the parameters so that on evolution, we get back the fragmentation functions at 91.2 GeV. Using this polarized fragmentation function we calculate the polarization of Λ in e^+e^- annihilations at 161 GeV. The polarized fragmentation functions evolve with energy and a review of their evolution is given, for instance, in Ref. [13]. The polarization data [6] of Λ at the Z pole is shown in Fig. 2.

The cross section for the reaction $e^+e^-_{h_1}\to q_{h_2}\bar{q}$ is given by

$$\frac{d\sigma^{h_1}_{q_{h_2}}}{d\Omega}=\frac{1}{64\pi^2 s}\Sigma^{h_1}_{q_{h_2}}, \tag{7}$$

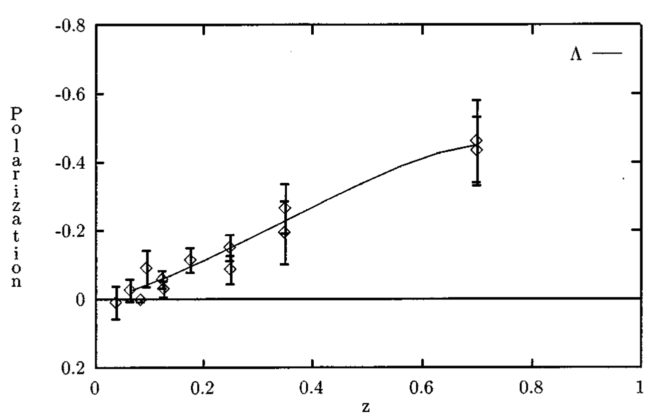

FIG. 2. The figure shows the polarization of Λ in e^+e^- annihilation at the Z pole. The data is the polarization data for Λ taken from Ref. [6]. The fit to the data is obtained by the Cernlib routine Minsq.

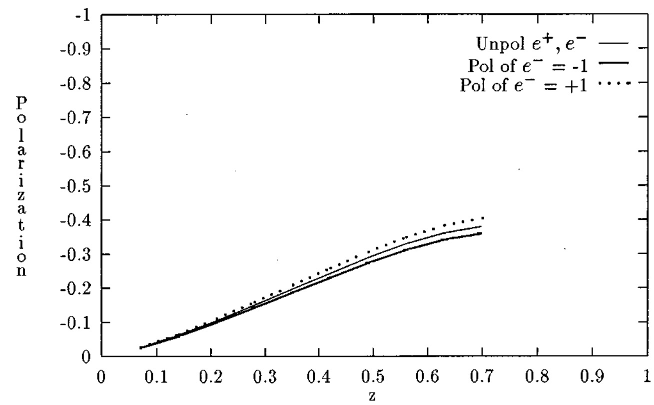

FIG. 3. The figure shows the polarization of Λ in e^+e^- annihilation at $\sqrt{s}$ = 161 GeV. The central graph corresponds to unpolarized e^+e^- scattering, the graph at the top corresponds to the case when the e^- beam has positive helicity and the bottom graph corresponds to the case when the e^- has negative helicity.

where h_1 is the helicity of the electron, h_2 is the helicity of the quark q, Ω is the solid angle, $\sqrt{s}$ is the c.m. energy, and $\Sigma^{h_1}_{qh_2}$ is given in Ref. [10]. We note that for $\sqrt{s}<91.2$ GeV (i.e., only γ exchange) the helicity-dependent term in the cross section turns out to be zero on averaging over the scattering angle.

For the case of unpolarized e^+e^- annihilation at the Z pole, the polarization of Λ is negative. To study polarization effects in e^+e^- process below the Z pole, it is necessary to study the angular distribution of the cross section.

At $\sqrt{s}=161$ GeV and for the unpolarized e^+e^- interaction, the polarization decreases in magnitude. This is due to the fact that the γ-exchange term is significant at this energy, but does not contribute to the polarized case. The resulting polarization is shown in Fig. 3.

The case where the e^- beam is polarized is also studied. The results are almost the same as those for the unpolarized case and are shown in Fig. 3.

We now turn our attention to baryon production in semi inclusive deep inelastic ep process. We study the production of unpolarized and longitudinally polarized baryons in ep scattering. To leading order the unpolarized cross section is expressed as

$$\frac{1}{\sigma_{\mathrm{tot}}}\frac{d\sigma^h}{dz}=\frac{\sum_q e_q^2 q(x,Q^2)D_q^h(z,Q^2)}{\sum_q e_q^2 q(x,Q^2)}, \tag{8}$$

where e_q is the charge of the quark q, $q(x,Q^2)$ is the distribution function for a quark of flavor q, $D_q^h(z,Q^2)$ is the fragmentation function for a quark q into a hadron h, and a sum over q and $\bar{q}$ is implied. x and z are the usual DIS variables. $Q^2=xys$, $z=P_HP_N/P_Nq$, y is the energy fraction of the incident lepton carried by the virtual photon, and x is the energy fraction of the incident nucleon carried by the interacting quark. P_H and P_N are the outgoing hadron and

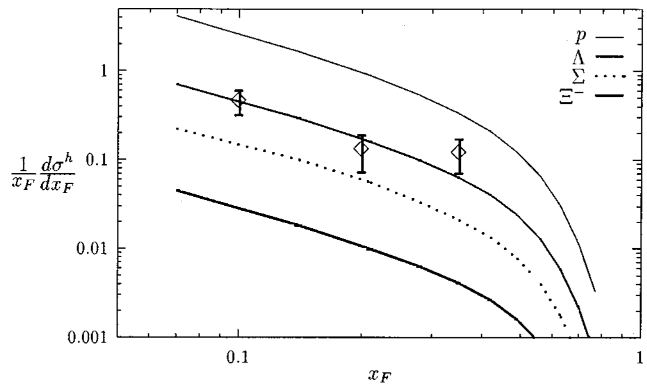

FIG. 4. The figure shows the unpolarized cross section of the baryons in ep scattering for $10<Q^2<70$ GeV^2. The data is the Λ production data at HERA taken from Ref. [14].

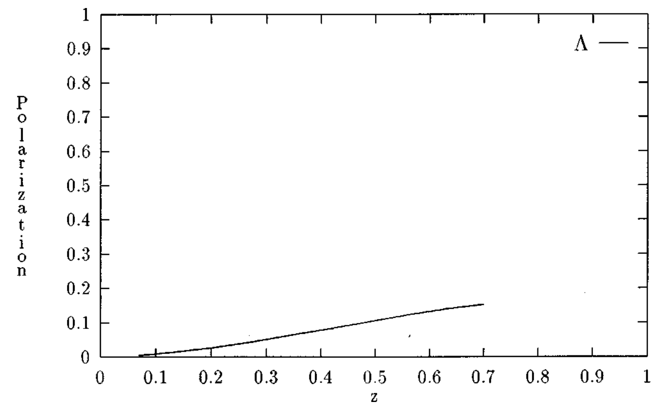

FIG. 5. The figure shows the polarization of Λ in ep processes for the case when the electron is polarized and the proton is unpolarized for $Q^2=18$ GeV^2 and $x=5.6\times10^{-4}$. The x and Q^2 values are from Ref. [15].

incoming nucleon momentum, s is the usual Mandelstam variable and $q^2=-Q^2$. Equation (8) is valid only in the current fragmentation region.

The study of unpolarized particle production in ep scattering can give a more concrete determination of the flavor dependence of fragmentation functions, which the e^+e^- data fails to provide. We calculate the cross section for p, Λ, Σ, and Ξ^-. The cross section for Λ is compared with the data [14] and the results show good agreement with data. The data shows the inclusive production of Λ, i.e., it also shows Λ's which are produced in nonstrange decays of higher resonances (Σ's and Ξ's). To compare our results with the data, we calculate the inclusive cross section for Λ:

$$\Lambda_{\mathrm{inc}}=\Lambda_{\mathrm{exc}}+1.0\Sigma^0+1.0\Xi^-+1.0\Xi^0.$$

Unfortunately, there is not enough data to enable us to make a clearer statement regarding production of baryons in ep scattering. The three points in the data are in the Q^2 range $10<Q^2<70$ GeV^2. In effect, each point could be at a different Q^2. However, evolution does not change the fragmentation functions significantly in this region. We note that the momentum fraction x used in the distribution functions does not make a significant change in the cross section. In the result, we have included the contribution due to decay of Σ^0 and Ξ into Λ and Σ^+ and Λ into p. Hence the fragmentation functions do not contain the contribution of these decays into Λ. The contribution of the higher resonances (Σ's and Ξ's) should also not be included in the fragmentation functions. Our model is very simple and cannot incorporate decuplet baryons. However, the data of Σ^* and Ξ is very small compared to Λ. So we do not expect a significant contribution from these decays. In the analysis, we have ignored this contribution. The various cross sections are shown in Fig. 4.

We now consider longitudinal polarization in ep processes. To leading order, the longitudinal polarization of a hadron h in polarized SIDIS is given by [3]

$$P^h(x,y,z)=\frac{\sum_q e_q^2[P_B D(y)q(x)+P_T\Delta q(x)]\Delta D_q^h(z)}{\sum_q e_q^2[q(x)+P_B D(y)P_T\Delta q(x)]D_q^h(z)}, \tag{9}$$

where P_B and P_T are the beam and target longitudinal polarizations, e_q is the charge of the quark q, $q(x)$ and $\Delta q(x)$ are the unpolarized and polarized quark distribution functions. $D(y)$ is referred to as the longitudinal depolarization factor

$$D(y)=\frac{1-(1-y)^2}{1+(1-y)^2},$$

y being the energy fraction of the incident lepton carried by the virtual photon.

We calculate the polarization of Λ in the ep experiment at DESY ep collider HERA energy. At present, the proton beam is unpolarized, although the electron-positron beam has a natural polarization. It would be interesting to study the polarization of the baryons for the case when the colliding particles are polarized. The x, y, and Q^2 values for the calculation are taken from Ref. [15].

The polarization of Λ for the case when the proton beam is unpolarized is shown in Fig. 5. The polarization is shown for $x=5.6\times10^{-4}$ and $Q^2=18$ GeV^2. The x, y, and Q^2 values are taken from Ref. [15]. We assume that e^- beam has a polarization of 0.7. The unpolarized quark distribution functions are from Ref. [16], and the polarized quark distribution functions are from Ref. [17]. When the electron-positron beam is unpolarized and the proton is polarized, the polarization of Λ is very small and consistent with zero.

Finally, we also calculate the polarization of Λ for the case where both the electron as well as the proton beam is polarized. The polarization is same as in the first case (i.e., proton unpolarized, e^-/ positron polarized) when we assume P_B and $P_T=0.7$. This is because at small x, the polarized quark distribution functions are very small [17] so that the target polarization term does not contribute significantly to the polarization of Λ. The results are similar to those obtained using the fragmentation functions of scenario 3 of Ref. [4].

In conclusion we have proposed a simple model for longitudinally polarized quark fragmentation into a longitudinally polarized octet baryon using the SU(3) symmetry of quarks and the baryons in the octet. All the polarized quark fragmentation functions have been described in terms of three SU(3) symmetric functions $\Delta\alpha_V(z,Q^2)$, $\Delta\beta_V(z,Q^2)$, and $\Delta\gamma_V(z,Q^2)$ and an SU(3) breaking parameter λ. The model is very simple and is able to express all the 48 fragmentation functions of three quarks and three antiquarks into eight baryons in terms of these SU(3) symmetric functions and the parameter λ, thus, vastly reducing the number of unknown parameters. We have chosen λ for the polarized case to be the same as that for the unpolarized case (= 0.07). In effect the suppression factor λ for the two cases can be different. However, in the absence of more data on polarization of baryons, we cannot make any guess about the value of λ and hence choose it to be the same as in the unpolarized case.

We have extracted the function $\Delta\alpha_V(z,Q^2)$ from the data on Λ polarization at the Z pole [6], and have used this to predict the polarization of Λ in polarized and unpolarized e^+e^- processes and polarized ep processes.

The polarization of Λ is found to be negative in e^+e^- processes at $\sqrt{s} = 91.2$ GeV and 161 GeV. The polarization of Λ is smaller at $\sqrt{s}=161$ GeV than at 91.2 GeV.

Three cases of polarized ep processes have been studied. We have analyzed the process for HERA kinematics for $x=5.6\times10^{-4}$ and $Q^2=18$ GeV^2. The results are in agreement with those predicted by scenario 3 of Ref. [4]. When more data on the polarization of various baryons is available, a more complete determination of the various parameters used in the study will be possible. We can then parametrize a complete set of fragmentation functions for the u and d quarks as well. However, because of the smallness of λ, we expect ΔD_u^Λ and ΔD_d^Λ to be negligible compared to the ΔD_s^Λ. This is in contradiction to the model proposed in Ref. [11], where the possibility that the polarized quark and antiquark content of Λ is reflected in the polarized fragmentation functions is examined. We do not delve into this correlation and instead use a different approach.

We have also predicted the unpolarized cross sections for the octet baryons in ep scattering experiments using the fragmentation functions of Ref. [5]. The cross section for Λ production in ep processes shows good agreement with data [14].

In the above analysis, we have considered polarization of baryons to be a result of direct fragmentation. We have not considered the contribution to polarization resulting from the decay of heavier resonances into these baryons as our model is very simple and there is not enough data to enable such an analysis. Experiments on other polarized processes will provide deeper insight into the spin transfer mechanism and into whether or not the SU(3) symmetry extends to polarized phenomena.

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