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CP violation in the semileptonic B_{l4} $(B\to D\pi l\nu)$ decays: Multi-Higgs-doublet model and scalar-leptoquark models

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CP violation from physics beyond the standard model is investigated in B_{l4} decays: $B\to D\pi l\overline{\nu_l}$. In the decay process, we include various excited states as intermediate states decaying to the final hadrons, $D+\pi$. We consider the semileptonic decay to a tau lepton family as well. CP violation is implemented through complex scalar-fermion couplings in the multi-Higgs-doublet model and scalar-leptoquark models beyond the standard model. With these complex couplings, we calculate the CP-odd rate asymmetry and the optimal asymmetry. We find that for $B_{\tau 4}$ decays the optimal asymmetry is sizable and can be detected at 1σ level with about 10^6–10^7 B-meson pairs, for maximally allowed values of CP-odd parameters in those extended models. [S0556-2821(99)02909-4]

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I. INTRODUCTION

The pions emitted in B_{l4} decays $(B\to D\pi l\overline{\nu_l})$ have such a wide momentum range that one may have difficulty analyzing the decays over whole phase space. However, if we restrict our attentions to the soft pion limit, we could investigate B_{l4} decays including the final state D meson by the combined method of heavy quark expansion and chiral perturbation expansion [1,2]. So far, the significance of the $B\to D\pi l\nu$ decay mode has been seen from the observation that the elastic modes $B\to De\nu$ and $B\to D^*e\nu$ account for less than 70% of the total semileptonic branching fraction. Goity and Roberts [2] have studied $B\to D\pi l\nu$ decays including various intermediate states which are decaying to $D+\pi$, and found that the effects of higher excited intermediate states are substantial compared to the lowest state of D^*, as the invariant mass of $D+\pi$ grows away from the ground D^* resonance regions.

In Ref. [3], we considered the possibility of probing direct CP violation in the decay of $B\to D\pi l\nu$ in a model independent way, in which we extended leptonic current by including complex couplings of the scalar sector and those of the vector sector in extensions of the standard model (SM). In the present paper, we would like to consider specific models such as mult-Higgs-doublet model and scalar-leptoquark models. In order to observe direct CP violation effects, there should exist interferences not only through weak CP-violating phases but also with different CP-conserving strong phases. In B_{l4} decays, CP-violating phases can be generated through interference between W-exchange diagrams and Scalar-exchange diagrams with complex couplings. The CP-conserving phases may come from the absorptive parts of the intermediate resonances in B_{l4} decays.

We include higher excited states, such as P-wave, D-wave states and the first radially excited S-wave state, as intermediate states in $B\to D\pi l\nu$ decay, since these intermediate states could contribute substantially to the decay B_{l4} [2]. And we have found in our previous work [3] that CP-violation effects are highly amplified by including those higher excited states.

The interactions of the octet of pseudogoldstone bosons with hadrons containing a single heavy quark are constrained by two independent symmetries: spontaneously broken chiral $SU(3)_L\times SU(3)_R$ symmetry and heavy quark spin-flavor $SU(2N_h)$ symmetry [4], where N_h is the number of heavy quark flavors. Within the frame work of heavy quark effective theory (HQET) [5], the spin J of a meson consists of the spin of a heavy quark (Q), the spin of a light quark (q) and relative angular momentum l:

$$J=S_Q+S_l+l. \tag{1.1}$$

We denote the meson state as J^P with its parity. If we define $j=S_l+l$ which corresponds to the spin of the light component of the meson, we have a multiplet for each j:

$$J=j\pm\frac{1}{2}. \tag{1.2}$$

Then, for a meson M, HQET predicts the following multiplets up to $l=2$:

$$l=0:(0^-,1^-)\quad (M,M^*),$$

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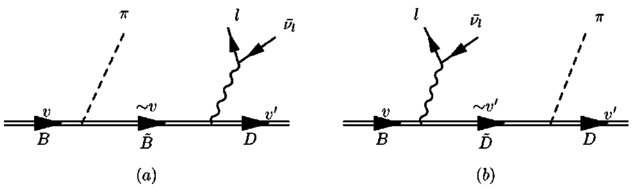

FIG. 1. Feynman diagrams for B_{l4} decays.

$$\begin{aligned} l=1&:(0^+,1^+) \quad (M_0,M_1^{(0)}),\\ &(1^+,2^+) \quad (M_1^{(1)},M_2),\\ l=2&:(1^-,2^-) \quad (M_1,M_2^{(0)}),\\ &(2^-,3^-) \quad (M_2^{(1)},M_3), \end{aligned} \tag{1.3}$$

where we denote the corresponding meson states by the notation in the last column. Furthermore, there could be radially excited states. For example, the first radially excited states are

$$l=0(n=2):(0^-,1^-) \quad (M',M'^*). \tag{1.4}$$

Among these resonances (denoted by $\tilde{M}$), $\tilde{D}\to D\pi$ or $B\to\tilde{B}\pi$ decay is possible only for $J^P=0^+$, 1^-, 2^+, and 3^- resonances because of parity conservation. However, if we use chiral expansion, the decay amplitude of 2^+ state, M_2, is proportional to $(p_\pi)^2$, and that of 3^- state is proportional to $(p_\pi)^3$ [2], so their contributions will be suppressed in the soft pion limit. Therefore, in the leading order in p_π, the resonances contributing to B_{l4} decays are

$$\tilde{M}=M^*,M_0,M_1 \text{ and } M'^*, \tag{1.5}$$

where M stands for B or D meson. We include all the possible multipets above as intermediate states in B_{l4} decays.

In Sec. II, we briefly review on our formalism dealing with B_{l4} decays. The multi-Higgs-doublet model and scalar-leptoquark models are introduced as new sources of CP violation in Sec. III, and the observable asymmetries are considered in Sec. IV. Section V contains our numerical results and conclusions.

II. DECAY RATES

We consider B_{l4} decays of $B^-\to D^+\pi^-l\overline{\nu}$ and $B^-\to D^0\pi^0l\overline{\nu}$. The amplitude has the general form:

$$T=\kappa[(1+\chi)j_\mu\Omega_V^\mu+\eta j_s\Omega_s], \quad \kappa=V_{cb}\frac{G_F}{\sqrt{2}}\sqrt{m_Bm_D}, \tag{2.1}$$

where j_μ is the $(V-A)$ charged leptonic current and j_s is a Yukawa type scalar current in leptonic sector. Here the parameters χ and η, which parametrize contributions from physics beyond the SM are in general complex. Note that the SM values are $\chi=\eta=0$. We retain charged lepton mass since we also consider decays of $B_{\tau 4}$. The vector interaction part of the hadronic amplitude, Ω_V^μ, receives contributions from two types of diagrams, illustrated in Figs. 1(a) and 1(b), respectively:

$$\begin{aligned}\Omega_V^\mu=\langle D\pi|\overline{c}\gamma^\mu(1-\gamma_5)b|B\rangle=&\sum_{\tilde{B}_i}\langle D|\overline{c}\gamma^\mu(1-\gamma_5)b|\tilde{B}_i\rangle\\ &\times\langle\tilde{B}_i\pi|B\rangle+\sum_{\tilde{D}_i}\langle D\pi|\tilde{D}_i\rangle\langle\tilde{D}_i|\overline{c}\gamma^\mu(1-\gamma_5)b|B\rangle,\end{aligned} \tag{2.2}$$

and Ω_s is the corresponding scalar current matrix element:

$$\Omega_s=\langle D\pi|\overline{c}(1-\gamma_5)b|B\rangle, \tag{2.3}$$

where $\tilde{D}_i$ and $\tilde{B}_i$ stand for intermediate excited states of our interest. One can obtain Yukawa interaction form factors by multiplying the $V-A$ currents with momentum transfer $q^\mu=p_B-p_D=m_Bv^\mu-m_Dv'^\mu$. Consequently, we get the following relation:

$$q_\mu\Omega_V^\mu=(m_B+m_D)\Omega_s. \tag{2.4}$$

We define the dimensionless parameter ζ, which determines the relative size of the scalar contributions to the vector ones:

$$\zeta=\frac{\eta}{1+\chi}. \tag{2.5}$$

Then the amplitude can be written as

$$T=\kappa(1+\chi)j_\mu\Omega^\mu, \quad \Omega^\mu=\Omega_V^\mu+\zeta\Omega_s\frac{L^\mu}{m_l}, \tag{2.6}$$

where we used the Dirac equation for leptonic current, $L^\mu j_\mu=m_lj_s$, with $L^\mu=p_l^\mu+p_\nu^\mu$. All the explicit expressions of the form factors and the amplitudes can be found in Ref. [3].

Then, the differential partial width of interest can be expressed as

$$d\Gamma_{B_{l4}}=\frac{N_\pi}{2m_B}J(s_M,s_L)|T|^2d\Phi_4, \tag{2.7}$$

where the 4 body phase space $d\Phi_4$ is

$$d\Phi_4\equiv ds_M\cdot ds_L\cdot d\cos\theta\cdot d\cos\theta_l\cdot d\phi, \tag{2.8}$$

and

$$N_\pi=\begin{cases}2 & \text{for charged pions,}\\ 1 & \text{for neutral pions,}\end{cases} \tag{2.9}$$

and the Jacobian $J(x,y)$ is

$$\begin{aligned}J(x,y)=&\frac{1}{2^{14}\pi^6xym_B^2}\lambda^{1/2}(m_B^2,x,y)\\ &\times\lambda^{1/2}(x,m_D^2,m_\pi^2)\lambda^{1/2}(y,m_l^2,0).\end{aligned} \tag{2.10}$$

Here, from the momenta of the B meson, D meson, the pion, the lepton, and its neutrino p_B, p_D, p, p_l, and $p_{\overline{\nu}}$, for five independent kinematic variables we choose $s_M=(p_D+p)^2$,

$s_L=(p_l+p_{\bar{\nu}})^2$, θ (i.e., the angle between the D momentum in the $D\pi$ rest frame and the moving direction of the $D\pi$ system in the B-meson's rest frame), θ_l (i.e., the angle between the lepton momentum in the $l\bar{\nu}$ rest frame and the moving direction of the $l\bar{\nu}$ system in the B-meson's rest frame) and ϕ [i.e., the angle between the two decay planes defined by the pairs $(\mathbf{p},\mathbf{p}_D)$ and $(\mathbf{p}_l,\mathbf{p}_{\bar{\nu}})$ in the rest frame of the B meson].

Since the initial B^- system is not CP self-conjugate, any genuine CP-odd observable can be constructed only by considering both the B^-_{l4} decay and its charge-conjugated B^+_{l4} decay ($B^+\to\overline{D^0}\pi^0 l^+\nu_l$ or $B^+\to D^-\pi^+ l^+\nu_l$), and by identifying the CP relations of their kinematic distributions. The transition probability $|\overline{T}|^2$ for the B^+ decay in the same reference frame as in the B^- decay is given by simple modification of the transition probability $|T|^2$ of the B^- decay [3]:

$$|\overline{T}|^2=|T|^2\begin{cases}\text{(i) change signs in front of the terms proportional to imaginary part,}\\ \text{(ii)}\quad \zeta\to\zeta^*.\end{cases} \tag{2.11}$$

We note that all the imaginary parts are being multiplied by the quantity proportional to $\sin\phi$. If the parameter ζ is real, the transition probability for the B^- decay and that of the B^+ decay satisfy the following CP relation:

$$|T|^2(\theta,\theta_l,\phi)=|\overline{T}|^2(\theta,\theta_l,-\phi). \tag{2.12}$$

Then with CP violating complex parameter ζ, $d\Gamma/d\Phi_4$ can be decomposed into a CP-even part $\mathcal{S}$ and a CP-odd part $\mathcal{D}$:

$$\frac{d\Gamma}{d\Phi_4}=\frac{1}{2}(\mathcal{S}+\mathcal{D}). \tag{2.13}$$

The CP-even part $\mathcal{S}$ and the CP-odd part $\mathcal{D}$ can be easily identified by making use of the CP relation (2.12) between the B^- and B^+ decay probabilities and they are expressed as

$$\mathcal{S}=\frac{d(\Gamma+\overline{\Gamma})}{d\Phi_4},\quad \mathcal{D}=\frac{d(\Gamma-\overline{\Gamma})}{d\Phi_4}, \tag{2.14}$$

where we have used the same kinematic variables $\{s_M,s_L,\theta,\theta_l\}$ for the $d\overline{\Gamma}/d\Phi_4$ except for the replacement of ϕ by $-\phi$, as shown in Eq. (2.12). Here Γ and $\overline{\Gamma}$ are the decay rates for B^- and B^+, respectively. The CP-even $\mathcal{S}$ term and the CP-odd $\mathcal{D}$ can be obtained from $B^{\mp}$ decay probabilities and their explicit form is also listed in Ref. [3]. We notice that the CP-odd term is proportional to the imaginary part of the parameter ζ in Eq. (2.5) and lepton mass m_l. Therefore, there exists no CP violation in B_{l4} decays within the SM since the SM corresponds to the case with $\zeta=0$.

III. MODELS

As possible new sources of CP violation detectable in the B_{l4} decays, we consider new scalar-fermion interactions which preserve the symmetries of the SM. Then it can be proven that only four types of scalar-exchange models [6] contribute to the $B\to D\pi l\bar{\nu}_l$. One of them is the mult-Higgs-doublet (MHD) model [7] and the other three models are scalar-leptoquark (SLQ) models [8,9]. The authors of Ref. [10] investigated CP violations in τ decay processes within these extended models. We follow their description on the models and make it to be appropriate for our analysis.

A. Multi-Higgs doublet model

We first consider MHD model with n Higgs doublets. The Yukawa interaction of the MHD model is

$$\mathcal{L}_{MHD}=\overline{Q}_{L_i}F^D_{ij}\Phi_d D_{R_j}+\overline{Q}_{L_i}F^U_{ij}\tilde{\Phi}_u U_{R_j}+\overline{L}_{L_i}F^E_{ij}\Phi_e E_{R_j}+\text{H.c.} \tag{3.1}$$

Here Q_{L_i} denotes left-handed quark doublets, and L_{L_i} denotes left-handed lepton doublets, $D_{R_i}(U_{R_i})$ and E_{R_i} are for right-handed down (up) quark singlets and right-handed charged lepton singlets, respectively. The index i is a generation index ($i=1,2,3$), $\Phi_j(j=1$ to $n)$ are n Higgs doublets and $\tilde{\Phi}_j=i\sigma_2\Phi_j^*$. And among the n doublets we denote by indices d, u, and e the Higgs doublets that couple to down-type quarks, up-type quarks, and charged leptons, respectively. F^U and F^D are general 3×3 Yukawa matrices of which one matrix can be taken to be real and diagonal. Since neutrinos are massless, F^E can be chosen real and diagonal. The MHD model has $2(n-1)$ charged and $(2n-1)$ neutral physical scalars. The Yukawa interactions of the $2(n-1)$ physical charged scalar with fermions in the mass eigenstates read as

$$\mathcal{L}_{MHD}=\sqrt{2\sqrt{2}G_F}\sum_{i=2}^{n}[X_i(\overline{U}_L V\mathcal{M}_D D_R)+Y_i(\overline{U}_R\mathcal{M}_U V D_L)+Z_i(\overline{N}_L\mathcal{M}_E E_R)]H_i^+ +\text{H.c.} \tag{3.2}$$

Here $\mathcal{M}_D$, $\mathcal{M}_U$, and $\mathcal{M}_E$ denote diagonal mass matrices of down-type quarks, up-type quarks, and charged leptons, respectively. H_i^+ are positively charged Higgs particles, N_L are for left-handed neutrino fields, and V for the Cabibbo-Kobayashi-Maskawa (CKM) matrix. X_i, Y_i, and Z_i are complex coupling constants which arise from the mixing matrix for charged scalars.

Within the framework of the MHD model, CP violation in charged scalar exchange can arise for more than two Higgs doublets [11,12]. There are two mechanisms which give rise to CP violation in the scalar sector. In one mechanism [13,11], CP symmetry is maintained at the Lagrangian level but broken through complex vacuum expectation values. However, this possibility has been shown to have some phenomenological difficulties [14,7]. In the other mechanism CP is broken by complex Yukawa couplings and possibly by complex vacuum expectation values so that CP violation can arise both from charged scalar exchange and from $W^{\pm}$ exchange. CP violation in both mechanisms is commonly manifest in phases that appear in the combinations XY^*, XZ^*, and YZ^*.

One crucial condition for CP violation in the MHD model is that not all the charged scalars should be degenerate. Then, without loss of generality and for simplicity, we can assume that all but the lightest of the charged scalars effectively decouple from fermions. The couplings of the lightest charged scalar to fermions are described by a simple Lagrangian

$$\mathcal{L}_{MHD}=\sqrt{2\sqrt{2}G_F}[X(\overline{U}_L V\mathcal{M}_D D_R)+Y(\overline{U}_R\mathcal{M}_U VD_L)+Z(\overline{N}_L\mathcal{M}_E E_R)]H^+ +\text{H.c.} \tag{3.3}$$

This Lagrangian gives the effective Lagrangian contributing to the decay $B\to D\pi l\overline{\nu_l}$,

$$\mathcal{L}_{eff}=2\sqrt{2}G_F V_{cb}\frac{m_l}{M_H^2}[m_b XZ^*(\overline{c}_L b_R)+m_c YZ^*(\overline{c}_R b_L)]\times(\overline{l}_R\nu_L), \tag{3.4}$$

at energies considerably low compared to the mass of the charged Higgs boson. Then, one can show that the contribution from the MHD model to the B_{l4} decay rate is represented by the parameters

$$\chi_{MHD}=0, \quad \eta_{MHD}=\frac{m_l m_b}{M_H^2}\left\{XZ^*-\left(\frac{m_c}{m_b}\right)YZ^*\right\}, \tag{3.5}$$

and CP violation in the MHD model is determined by the parameter

$$\text{Im}(\zeta_{MHD})=\frac{m_l m_b}{M_H^2}\left\{\text{Im}(XZ^*)-\left(\frac{m_c}{m_b}\right)\text{Im}(YZ^*)\right\}. \tag{3.6}$$

The constraint on the CP-violation parameter (3.6) depends upon the values chosen for the c and b quark masses. In the present work, we use for the heavy c and b quark masses [15],

$$m_c=1628 \text{ MeV}, \quad m_b=4977 \text{ MeV}. \tag{3.7}$$

Clearly, sizable CP-violating effects require that Im(XZ^*) and Im(YZ^*) are large and M_H is small. In the MHD model the strongest constraint [7] on Im(XZ^*) comes from the measurement of the branching ratio $\mathcal{B}(b\to X\tau\nu_\tau)$ which actually gives a constraint on $|XZ|$. For $M_H<440$ GeV, the bound on Im(XZ^*) is given by

$$\text{Im}(XZ^*)<|XZ|<0.23M_H^2 \text{ GeV}^{-2}. \tag{3.8}$$

On the other hand, the bound on Im(YZ^*) is mainly given by $K^+\to\pi^+\nu\overline{\nu}$. The present bound [7] is

$$\text{Im}(YZ^*)<|YZ|<110. \tag{3.9}$$

Combining the above bounds, we obtain the following bounds on Im(ζ_{MHD}) as

$$|\text{Im}(\zeta_{MHD})|<1.88 \text{ for } \tau \text{ family},$$
$$|\text{Im}(\zeta_{MHD})|<0.11 \text{ for } \mu \text{ family}. \tag{3.10}$$

B. Scalar leptoquark models

There are three types of scalar leptoquark (SLQ) models [6,8] which can contribute to the $B\to D\pi l\nu$ at the tree level. The quantum numbers of the three leptoquarks under the gauge group $SU(3)_C\times SU(2)_L\times U(1)_Y$ are

$$\Phi_1=\left(3,2,\frac{7}{6}\right) \text{ (model I)},$$
$$\Phi_2=\left(3,1,-\frac{1}{3}\right) \text{ (model II)},$$
$$\Phi_3=\left(3,3,-\frac{1}{3}\right) \text{ (model III)}, \tag{3.11}$$

respectively. The hypercharge Y is defined to be $Q=I_3+Y$. The Yukawa couplings of the leptoquarks to fermions are given by

$$\mathcal{L}_{SLQ}^{I}=[-x_{ij}\overline{Q}_{L_i}i\sigma_2 E_{R_j}+x'_{ij}\overline{U}_{R_i}L_{L_j}]\Phi_1+\text{H.c.},$$
$$\mathcal{L}_{SLQ}^{II}=[y_{ij}\overline{Q}_{L_i}i\sigma_2 L^c_{L_j}+y'_{ij}\overline{U}_{R_i}E^c_{R_j}]\Phi_2+\text{H.c.},$$
$$\mathcal{L}_{SLQ}^{III}=z_{ij}[\overline{Q}_{L_i}i\sigma_2\vec{\sigma}L^c_{L_j}]\cdot\vec{\Phi}_3+\text{H.c.} \tag{3.12}$$

Here the coupling constants $x_{ij}^{(\prime)}$, $y_{ij}^{(\prime)}$, and z_{ij} are complex when CP violation arises from the Yukawa interactions. $\overline{Q}_{L_i}=(\overline{u}_i,\overline{d}_i)_L$ and $L_{L_i}=(\overline{\nu}_i,\overline{e}_i)_L$. The superscript c denotes charge conjugation, i.e., $\psi^c_{R,L}=i\gamma^0\gamma^2\overline{\psi}^T_{R,L}$ for a spinor field ψ. $\vec{\sigma}=(\sigma_1,\sigma_2,\sigma_3)$ and $\sigma_i(i=1,2,3)$ are the Pauli matrices. In terms of the charge component of the leptoquarks, the Lagrangian relevant to the $B\to D\pi l\overline{\nu_l}$ decay is given by

$$\mathcal{L}_{SLQ}^{I}=[x_{3j}\overline{b}_L l_R+x'_{2j}\overline{c}_R\nu_{lL}]\phi_1^{(2/3)}+\text{H.c.},$$
$$\mathcal{L}_{SLQ}^{II}=[-y_{3j}(\overline{t}_L l^c_L-\overline{b}_L\nu^c_{lL})+y'_{2j}\overline{c}_R l^c_R]\phi_2^{(-1/3)}+\text{H.c.},$$
$$\mathcal{L}_{SLQ}^{III}=-[z_{2j}(\overline{c}_L l^c_L+\overline{s}_L\nu^c_{lL})+z_{3j}(\overline{t}_L l^c_L+\overline{b}_L\nu^c_{lL})]\phi_3^{(-1/3)}+\text{H.c.}, \tag{3.13}$$

where $j=2,3$ for $l=\mu,\tau$, respectively. After Fierz rearrangement we obtain the effective SLQ Lagrangians which can give contribution to B_{l4} decay:

$$\mathcal{L}_{eff}^{I}=-\frac{x_{3j}x'^*_{2j}}{2M_{\phi_1}^2}\left[(\overline{b}_L c_R)(\overline{\nu}_{lL}l_R)+\frac{1}{4}(\overline{b}_L\sigma^{\mu\nu}c_R)(\overline{\nu}_{lL}\sigma_{\mu\nu}l_R)\right]+\text{H.c.},$$

$$\mathcal{L}_{eff}^{II} = -\frac{y_{3j}y_{2j}'^{*}}{2M_{\phi_2}^2}\left[(\overline{b_L}c_R)(\overline{l_R^c}\nu_{lL}^c) + \frac{1}{4}(\overline{b_L}\sigma^{\mu\nu}c_R)\right.$$
$$\left.\times(\overline{l_R^c}\sigma_{\mu\nu}\nu_{lL}^c)\right] + \frac{y_{3j}y_{2j}^*}{2M_{\phi_2}^2}(\overline{b_L}\gamma_\mu c_L)(\overline{l_L^c}\gamma^\mu\nu_{lL}^c) + \text{H.c.},$$

$$\mathcal{L}_{eff}^{III} = -\frac{z_{3j}z_{2j}^*}{2M_{\phi_3}^2}(\overline{b_L}\gamma_\mu c_L)(\overline{l_L^c}\gamma^\mu\nu_{lL}^c) + \text{H.c.} \tag{3.14}$$

The tensor parts in model I and model II may contribute to the $B\to D\pi l\overline{\nu_l}$ decay through the spin-2 resonances such as D_2^*. In the present work, however, spin-2 resonances do not enter, so we will neglect these tensor contributions, which take the same form and have the same couplings as the scalar resonance contributions.

Then the size of new contributions from these three SLQ models is parametrized as

$$\chi_{SLQ}^I = 0, \quad \eta_{SLQ}^I = -\frac{x_{3j}^* x_{2j}'}{4\sqrt{2}G_F V_{cb} M_{\phi_1}^2},$$

$$\chi_{SLQ}^{II} = -\frac{y_{3j}y_{2j}^*}{4\sqrt{2}G_F V_{cb} M_{\phi_2}^2},$$

$$\eta_{SLQ}^{II} = -\frac{y_{3j}^* y_{2j}'}{4\sqrt{2}G_F V_{cb} M_{\phi_2}^2},$$

$$\chi_{SLQ}^{III} = \frac{z_{3j}z_{2j}^*}{4\sqrt{2}G_F V_{cb} M_{\phi_3}^2}, \quad \eta_{SLQ}^{III} = 0. \tag{3.15}$$

Note that model I acquires the contribution only from the scalar-type interaction. On the other hand, model III does only from the vector-type interactions, so the model III does not contribute to CP violation in the B_{l4} decay. The terms with $y_{3j}y_{2j}^*$ in model II and $z_{3j}z_{2j}^*$ in model III modify the size of the vector contributions. Taking the approximation that these new contributions are negligible compared to the SM contributions, we find that the size of the SLQ models CP-violation effects is dictated by the CP-odd parameters

$$\text{Im}(\zeta_{\text{SLQ}}^I) \approx -\frac{\text{Im}[x_{3j}x_{2j}'^*]}{4\sqrt{2}G_F V_{cb} M_{\phi_1}^2},$$

$$\text{Im}(\zeta_{\text{SLQ}}^{II}) \approx -\frac{\text{Im}[y_{3j}y_{2j}'^*]}{4\sqrt{2}G_F V_{cb} M_{\phi_2}^2},$$

$$\text{Im}(\zeta_{\text{SLQ}}^{III}) \approx 0. \tag{3.16}$$

This approximation is justified because the contributions from new physics are expected to be very small compared to those from the SM.

Although there are at present no direct constraints on the SLQ models CP-odd parameters in Eq. (3.16), a rough constraint to the parameters can be provided on the assumption [16] that $|x_{2j}'| \sim |x_{2j}|$ and $|y_{2j}'| \sim |y_{2j}|$, that is to say, the leptoquark couplings to quarks and leptons belonging to the same generation are of a similar size; then the experimental upper bound from $B\to l\overline{l}X$ decay for model I, and $B\to l\nu X$ for model II yields [16]

$$|\text{Im}(\zeta_{\text{SLQ}}^I)| < 0.38, \quad |\text{Im}(\zeta_{\text{SLQ}}^{II})| < 1.52 \quad \text{for } \tau \text{ family},$$
$$|\text{Im}(\zeta_{\text{SLQ}}^I)| < 0.03, \quad |\text{Im}(\zeta_{\text{SLQ}}^{II})| < 1.52 \quad \text{for } \mu \text{ family}. \tag{3.17}$$

Based on the constraints (3.17) to the CP-odd parameters, we quantitatively estimate the number of the semileptonic B_{l4} decays to detect CP violation for the maximally-allowed values of the CP-odd parameters.

IV. ASYMMETRIES

An easily constructed CP-odd asymmetry is the rate asymmetry

$$A \equiv \frac{\Gamma - \overline{\Gamma}}{\Gamma + \overline{\Gamma}}, \tag{4.1}$$

where Γ and $\overline{\Gamma}$ are the decay rates for B^- and B^+, respectively. The statistical significance of the asymmetry can then be computed as

$$N_{SD} = \frac{N_- - N_+}{\sqrt{N_- + N_+}} = \frac{N_- - N_+}{\sqrt{N\cdot Br}}, \tag{4.2}$$

where N_{SD} is the number of standard deviations, $N_\pm$ is the number of events predicted in B_{l4} decay for $B^\pm$ meson, N is the number of B meson produced, and Br is the branching fraction of the relevant B decay mode. Taking $N_{SD}=1$, we obtain the number N_B of the B mesons needed to observe CP violation at 1-σ level:

$$N_B = \frac{1}{Br\cdot A^2}. \tag{4.3}$$

Next, we consider the so-called optimal observable. An appropriate real weight function $w(s_M, s_L; \theta, \theta_l, \phi)$ is usually employed to separate the CP-odd $\mathcal{D}$ contribution and to enhance its analysis power for the CP-odd parameter Im(ζ) through the CP-odd quantity:

$$\langle w\mathcal{D}\rangle \equiv \int [w\mathcal{D}]d\Phi_4, \tag{4.4}$$

of which the analysis power is determined by the parameter

$$\varepsilon = \frac{\langle w\mathcal{D}\rangle}{\sqrt{\langle\mathcal{S}\rangle\langle w^2\mathcal{S}\rangle}}. \tag{4.5}$$

For the analysis power ε, the number N_B of the B mesons needed to observe CP violation at 1-σ level is

$$N_B = \frac{1}{Br\cdot\varepsilon^2}. \tag{4.6}$$

TABLE I. The CP-violating rate asymmetry A and the optimal asymmetry $\varepsilon_{\rm opt}$, determined in the soft pion limit, and the number of charged B meson pairs, N_B, needed for detection at 1σ level, at reference values $\text{Im}(\zeta_{MHD})=1.88$, $\text{Im}(\zeta^I_{SLQ})=0.38$, and $\text{Im}(\zeta^{II}_{SLQ})=1.52$, for the $B_{\tau 4}$ decays.

(a) $B^-\to D^+\pi^-\tau\overline{\nu_\tau}$ mode						
Model	MHD		SLQ I		SLQ II	
Asymmetry	Size(%)	N_B	Size(%)	N_B	Size(%)	N_B
A	0.31	2.34×10^8	0.06	5.74×10^9	0.25	3.59×10^8
$\varepsilon_{\rm opt}$	4.25	1.25×10^6	0.86	3.06×10^7	3.44	1.91×10^6
(b) $B^-\to D^0\pi^0\tau\overline{\nu_\tau}$ mode						
Model	MHD		SLQ I		SLQ II	
Asymmetry	Size(%)	N_B	Size(%)	N_B	Size(%)	N_B
A	0.006	2.28×10^{10}	0.001	5.58×10^{11}	0.005	3.49×10^{10}
$\varepsilon_{\rm opt}$	0.6	2.36×10^6	0.1	5.77×10^7	0.45	3.61×10^6

Certainly, it is desirable to find the optimal weight function with the largest analysis power. It is known [17] that when the CP-odd contribution to the total rate is relatively small, the optimal weight function is approximately given by

$$w_{\rm opt}(s_M,s_L;\theta,\theta_l,\phi)=\frac{\mathcal{D}}{\mathcal{S}}\Rightarrow\varepsilon_{\rm opt}=\sqrt{\frac{\langle\mathcal{D}^2/\mathcal{S}\rangle}{\langle\mathcal{S}\rangle}}. \quad (4.7)$$

We adopt this optimal weight function in the following numerical analyses.

V. NUMERICAL RESULTS AND CONCLUSIONS

We first consider decay to heavy lepton, τ, since CP-odd asymmetry is proportional to m_l, and heavy lepton may be more susceptible to effects of new physics. We restrict ourselves to the soft-pion limit by considering only the region $s_M\leqslant 6.5$ GeV^2 which is about one half of the maximum value. This restriction corresponds to the pion momentum less than about 0.6 GeV.

In Table I, we show the results of $B_{\tau 4}$ decays for the two CP-violating asymmetries; the rate asymmetry A and the optimal asymmetry $\varepsilon_{\rm opt}$. We estimated the number of B meson pairs, N_B, needed for detection at 1σ level for maximally-allowed values of CP-odd parameters in Eqs. (3.10) and (3.17). As we can expect, the optimal observable gives much better result than the simple rate asymmetry. We also found that although the decay rate of neutral pion mode is larger than that of the charged pion mode because the most dominant resonance D^{0*} cannot decay into $D^+\pi^-$ in its mass-shell, the latter case gives better detection results because of large CP-violating effects in charged pion decay mode. Since one expect about 10^8 orders of B meson pairs produced yearly in the asymmetric B factories, one could probe the CP-violation effects down to the current bounds of the multi-Higgs doublet (MHD) model and SLQ models by using the optimal asymmetry observable.

We also estimated CP-violation effects in the B_{l4} decays with light leptons. The results for $B_{\mu 4}$ decays are shown in Table II. We found that $B_{\mu 4}$ decay modes give worse results than $B_{\tau 4}$ cases. In Ref. [3], we found that both $B_{\tau 4}$ and $B_{\mu 4}$ decay modes could be equally good probes for the same values of CP-odd parameter $\text{Im}(\zeta)$. At the same time, we discussed that in some extensions of the SM the CP-odd parameter itself would be proportional to lepton mass. And therefore, $B_{\tau 4}$ decay modes would serve as definitely much better probes of CP violation than light lepton cases in those models. This is the case in the MHD model where $\text{Im}(\zeta_{MHD})=1.88(0.11)$ for $\tau(\mu)$ family, directly resulted

TABLE II. The CP-violating rate asymmetry A and the optimal asymmetry $\varepsilon_{\rm opt}$, determined in the soft pion limit, and the number of charged B meson pairs, N_B, needed for detection at 1σ level at reference values $\text{Im}(\zeta_{MHD})=0.11$, $\text{Im}(\zeta^I_{SLQ})=0.03$, and $\text{Im}(\zeta^{II}_{SLQ})=1.52$, for the $B_{\mu 4}$ decays.

(a) $B^-\to D^+\pi^-\mu\overline{\nu_\mu}$ mode						
Model	MHD		SLQ I		SLQ II	
Asymmetry	Size(%)	N_B	Size(%)	N_B	Size(%)	N_B
A	0.003	1.5×10^{11}	0.0008	2.02×10^{12}	0.04	7.87×10^8
$\varepsilon_{\rm opt}$	0.05	4.87×10^8	0.014	6.54×10^9	0.72	2.55×10^6
(b) $B^-\to D^0\pi^0\mu\overline{\nu_\mu}$ mode						
Model	MHD		SLQ I		SLQ II	
Asymmetry	Size(%)	N_B	Size(%)	N_B	Size(%)	N_B
A	0.0001	6.07×10^{12}	0.00004	8.15×10^{13}	0.002	3.18×10^{10}
$\varepsilon_{\rm opt}$	0.01	9.37×10^8	0.003	1.26×10^{10}	0.15	4.91×10^6

from its dependence on lepton mass. But there is no such dependence in SLQ models. Although the numerical values of CP-odd parameters in SLQ model I, $\mathrm{Im}(\zeta^I_{SLQ}) = 0.38$ (0.03) for the $\tau(\mu)$ family, look like they contain some lepton mass dependence, actually different values are from current experimental bounds. For instance, the constraints on the CP-odd parameter in SLQ model I come from $B \to \tau\bar{\tau}X$ for τ family and $B \to \mu\bar{\mu}X$ for μ family, respectively, and so the smaller CP-odd value for μ family implies current experimental constraint on muon mode is more strict. For the SLQ model II, however, current bounds are roughly the same for τ and μ families. Therefore, if one uses optimal asymmetry observable, $B_{\tau 4}$ decay modes would provide more stringent constraints to all the extended models we have considered, and one could also use $B_{\mu 4}$ decays to constrain at least SLQ model II.

In conclusion, we investigated CP violation from physics beyond the standard model through semileptonic B_{l4} decays: $B \to D\pi l\bar{\nu}_l$. We considered as new sources of CP violation multi-Higgs-doublet model and scalar-leptoquark models. In the decay process, we included various excited states as intermediate states decaying to the final hadrons. The CP violation is implemented through interference between W-exchange diagrams and scalar-exchange diagrams with complex couplings in the extended models above. We calculated the CP-odd rate asymmetry and the optimal asymmetry for $B_{\tau 4}$ and $B_{\mu 4}$ decay modes, and found that the optimal asymmetry for $B_{\tau 4}$ decays is sizable and can be detected at 1σ level with about 10^6–10^7 orders of B-meson pairs, for maximally-allowed values of CP-odd parameters in the models of our interest. Since $\sim 10^8$ B-meson pairs are expected to be produced yearly at the forthcoming B factories, one could investigate CP-violation effects by using our formalism. In other words, one could give more stringent constraints to all the models we have considered using $B_{\tau 4}$ decay modes.

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