

***CP* violation in the semileptonic B_{14} ($B \rightarrow D\pi l\nu$) decays: A model independent analysis**

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(Received 20 November 1998; published 16 April 1999)

CP violation from physics beyond the standard model is investigated in B_{14} decays: $B \rightarrow D\pi l\bar{\nu}_l$. The semileptonic B -meson decay to a D meson with an emission of a single pion is analyzed with heavy quark effective theory and chiral perturbation theory. In the decay process, we include various excited states as intermediate states decaying to the final hadrons, $D + \pi$. *CP* violation is implemented in a model independent way, in which we extend the leptonic current by including complex couplings of the scalar sector and those of the vector sector in extensions of the standard model. With these complex couplings, we calculate the *CP*-odd rate asymmetry and the optimal asymmetry. We find that the optimal asymmetry is sizable and can be detected at 1σ level with about 10^6 – 10^7 B -meson pairs, for some reference values of new physics effects. [S0556-2821(99)02809-X]

PACS number(s): 13.20.He, 11.30.Er, 12.39.Fe, 12.39.Hg

I. INTRODUCTION

Semileptonic 4-body decays of mesons, such as K_{14} , D_{14} , and B_{14} with the emission of a pion have been studied in detail by many authors [1–4]. B_{14} decays will be fully investigated soon at the forthcoming B -meson factories. The decays of K_{14} , D_{14} , and B_{14} have distinct characteristics in many theoretical aspects: K_{14} decays can be studied with chiral perturbation theory for full decay phase space [1]. However, in D_{14} decays the chiral perturbation theory can be applied only to a very small region of the final state phase space [2] since light mesons in final state have relatively large energies. The pions emitted in B_{14} decays have such a wide momentum range that one may have difficulties to analyze the decays over whole phase space. However, if we restrict our attention to the soft pion limit, we could investigate B_{14} decays, including the final state D meson, by the combined method of heavy quark expansion and chiral perturbation expansion [3,4]. The significance of the $B \rightarrow D\pi l\nu$ decay mode is seen from the observation that the elastic modes of $B \rightarrow De\nu$ and $B \rightarrow D^*e\nu$ account for less than 70% of the total semileptonic branching fraction. Goity and Roberts [4] have studied $B \rightarrow D\pi l\nu$ decays by including various intermediate states which are decaying to $D + \pi$, and found that the effects of higher excited intermediate states are substantial compared to the lowest state of D^* , as the invariant mass of $D + \pi$ grows away from the ground D^* resonance region.

In this work, we consider the possibility of probing direct *CP* violation in the decay of $B^\pm \rightarrow D\pi l^\pm \nu$ in a model independent way, in which we extend leptonic current by including complex couplings of the scalar sector and those of the vector sector in extensions of the standard model (SM). In our separate paper [5], we also consider the phenomenon within specific models such as multi-Higgs-doublet model and scalar leptoquark models. In order to observe direct *CP* violation effects, there should exist interferences not only through weak *CP*-violating phases but also different *CP*-conserving strong phases. In B_{14} decays, *CP*-violating phases can be generated through interference between W -exchange diagrams and scalar-exchange diagrams with complex couplings. The *CP*-conserving phases may come from the absorptive parts of the intermediate resonances in B_{14} decays.

CP violation can be also investigated through T -odd momentum or spin correlations in semileptonic heavy meson decays to three or more final state particles in some extended models [6,7]. Especially, the authors of Ref. [7] analyzed the possibility of probing *CP*-violation by extracting T -odd angular correlations in the lowest resonance decay of $B \rightarrow D^*(\rightarrow D\pi)l\nu$ and found that the effects can be detected in some cases. Here we include higher excited states, such as P -wave, D -wave states and the first radially excited S -wave state, as intermediate states in $B \rightarrow D\pi l\nu$ decay, since these intermediate states could contribute substantially to the decay B_{14} [4]. As can be seen later, including these higher excited states would significantly amplify the *CP* violation effects.

In previous studies of B_{14} decays to only light leptons (e and μ) have been considered, because the width is very small in the case of heavy leptonic decay, $B_{\tau 4}$. However, the

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CP violation effects implemented by interference between vector boson and scalar boson exchanges in leptonic current may in general be proportional to the lepton's mass. Therefore, we here consider the case including a tau lepton as well. We note the larger lepton mass implies the smaller available region of hadronic invariant mass in the final state phase space, which actually means the relatively larger region of phase space to which chiral perturbation theory is applicable in $B_{\tau 4}$ decays, compared to light leptonic decay cases.

In Sec. II, we briefly review the heavy quark effective theory and chiral perturbation theory to calculate the amplitude of $B \rightarrow D\pi l\nu$ decay. The detailed formalism dealing with B_{l4} decay is given in Sec. III and Appendices. And the observable asymmetries are shown in Sec. IV. Section V contains our results, discussions, and conclusions.

II. HEAVY QUARK EFFECTIVE THEORY AND CHIRAL PERTURBATION THEORY

The formalism of B_{l4} decay in the context of heavy quark effective theory (HQET) and chiral perturbation theory (ChPT) has been studied by many authors [3,4]. We follow the procedure described in that literature, and make it suitable for our analysis by including various intermediate states and retaining lepton masses. We briefly review these theories and describe explicitly our procedure of obtaining the amplitude of the decay.

The interactions of the octet of pseudo Goldstone (PG) bosons with hadrons containing a single heavy quark are constrained by two independent symmetries: spontaneously broken chiral $SU(3)_L \times SU(3)_R$ symmetry and heavy quark spin-flavor $SU(2N_h)$ symmetry [8], where N_h is the number of heavy quark flavors. Within the frame work of HQET [9], the spin J of a meson consists of the spin of a heavy quark (Q), the spin of a light quark (q), and relative angular momentum l :

$$J = S_Q + S_l + l. \quad (2.1)$$

We denote the meson state as J^P with its parity. If we define $j = S_l + l$ which corresponds to the spin of the light component of the meson, we have a multiplet for each j :

$$J = j \pm \frac{1}{2}. \quad (2.2)$$

Then, for a meson M , HQET predicts the following multiplets up to $l=2$:

$$\begin{aligned} l=0: & (0^-, 1^-) \quad (M, M^*), \\ l=1: & (0^+, 1^+) \quad (M_0, M_1^{(0)}) \\ & (1^+, 2^+) \quad (M_1^{(1)}, M_2), \\ l=2: & (1^-, 2^-) \quad (M_1, M_2^{(0)}) \\ & (2^-, 3^-) \quad (M_2^{(1)}, M_3), \end{aligned} \quad (2.3)$$

where we denote the corresponding meson states by the notation in the last column. Furthermore, there could be radially excited states. For example, the first radially excited states are

$$l=0(n=2): (0^-, 1^-) \quad (M', M'^*). \quad (2.4)$$

Among these resonances (denoted by \tilde{M}), $\tilde{D} \rightarrow D\pi$ or $B \rightarrow \tilde{B}\pi$ decay is possible only for $J^P = 0^+, 1^-, 2^+$, and 3^- resonances because of parity conservation. However, if we use chiral expansion, the decay amplitude of 2^+ state, M_2 , is proportional to $(p_\pi)^2$, and that of 3^- state is proportional to $(p_\pi)^3$ [4], so their contributions will be suppressed in the soft pion limit. Therefore, in the leading order in p_π , the resonances contributing to B_{l4} decays are

$$\tilde{M} = M^*, M_0, M_1, \text{ and } M'^*, \quad (2.5)$$

where M stands for B or D meson.

The weak decay matrix elements of B to D are given in the heavy quark limit [4] as

$$\begin{aligned} \langle D^*(v', \epsilon) | H_\mu | B(v) \rangle &= \xi(\omega) [-i\epsilon_{\mu\nu\alpha\beta} v'^\alpha v^\beta + g_{\mu\nu}(\omega+1) \\ &\quad - v'_\mu v_\nu] \epsilon^{*\nu}, \\ \langle D_0(v') | H_\mu | B(v) \rangle &= -\rho_1(\omega) (v-v')_\mu, \\ \langle D_1(v', \epsilon) | H_\mu | B(v) \rangle &= \frac{1}{\sqrt{6}} \rho_2(\omega) [i\epsilon_{\mu\nu\alpha\beta} v'^\alpha v^\beta (\omega-1) \\ &\quad + g_{\mu\nu}(\omega^2-1) - v_\nu \{ (2+\omega)v'_\mu \\ &\quad - 3v_\mu \}] \epsilon^{*\nu}, \\ \langle D'^*(v', \epsilon) | H_\mu | B(v) \rangle &= \xi^{(1)}(\omega) [-i\epsilon_{\mu\nu\alpha\beta} v'^\alpha v^\beta \\ &\quad + g_{\mu\nu}(\omega+1) - v'_\mu v_\nu] \epsilon^{*\nu}, \end{aligned} \quad (2.6)$$

where $H_\mu = \bar{c} \gamma_\mu (1 - \gamma_5) b = V_\mu - A_\mu$ and $\omega \equiv v \cdot v'$. The effective currents where a B -meson resonance decays into a ground-state D meson are easily obtained by simply taking the Hermitian conjugate of the currents given above, followed by interchanging the symbols $B \leftrightarrow D$ and $v \leftrightarrow v'$.

For the form factors $\xi(\omega)$, $\xi^{(1)}(\omega)$, $\rho_1(\omega)$, and $\rho_2(\omega)$ in Eq. (2.6), we adopt the forms of Ref. [4], which were derived from the quark model. The explicit forms are

$$\begin{aligned} \xi(\omega) &= \exp \left[\frac{\bar{\Lambda}^2}{4\beta^2} (\omega^2 - 1) \right], \\ \xi^{(1)}(\omega) &= -\sqrt{\frac{2}{3}} \left[\frac{\bar{\Lambda}^2}{4\beta^2} (\omega^2 - 1) \right] \exp \left[\frac{\bar{\Lambda}^2}{4\beta^2} (\omega^2 - 1) \right], \\ \rho_1(\omega) &= \frac{1}{\sqrt{2}} \frac{\bar{\Lambda}}{\beta} \left(\frac{2\beta\beta'_1}{\beta^2 + \beta'^2_1} \right)^{5/2} \exp \left[\frac{\bar{\Lambda}^2}{2(\beta^2 + \beta'^2_1)} \right. \\ &\quad \left. \times (\omega^2 - 1) \right], \end{aligned}$$

$$\rho_2(\omega) = \frac{1}{2\sqrt{2}} \left(\frac{\bar{\Lambda}}{\beta} \right)^2 \left(\frac{2\beta\beta'_2}{\beta^2 + \beta'^2_2} \right)^{7/2} \exp \left[\frac{\bar{\Lambda}^2}{2(\beta^2 + \beta'^2_2)} \right] \times (\omega^2 - 1), \quad (2.7)$$

where $\bar{\Lambda}$ is defined by writing the mass of the ground state as $M_{(0^-,1^-)} = m_Q + \bar{\Lambda}$ with $m_c = 1628$ MeV and $m_b = 4977$ MeV [10], and numerical values are $\beta = 0.29$, $\beta'_1 = 0.28$, and $\beta'_2 = 0.26$.

In order to deal with the interactions of PG bosons with hadrons containing a single heavy quark, one may construct an effective Lagrangian with the two symmetries above and perform a simultaneous expansion in the momenta of PG bosons and the inverse masses of the heavy quarks. Such a Lagrangian has been described in Ref. [11] for heavy hadrons with the light degrees of freedom in the ground state. For construction of such effective Lagrangians, it is convenient and now common to introduce superfields associated with each multiplet [12]. Using the superfield formalism, one may represent the spin-flavor symmetry explicitly, and according to a velocity superselection rule [13], one can have one such superfield assigned with each four velocity v_μ at the leading order in the inverse heavy mass expansion.

The superfield for the ground-state heavy meson multiplet $(0^-, 1^-)$ with velocity v_μ is

$$\mathcal{H}_- = \frac{1 + \not{v}}{2} (-\gamma_5 P + \gamma^\mu V_\mu^*), \quad (2.8)$$

where P and V_μ^* are the fields associated with the pseudo-scalar and vector partners, respectively. This multiplet transforms under spin-symmetry operations as

$$\mathcal{H}_- \rightarrow \exp(-i\vec{\epsilon} \cdot \vec{S}_v) \mathcal{H}_-, \quad \text{with} \quad S_v^j = i\epsilon^{jkl} [\not{e}_k, \not{e}_l] \frac{(1 + \not{v})}{2}, \quad (2.9)$$

where e_k^μ , $k = 1, 2, 3$, are spacelike vectors orthogonal to the four-velocity v_μ . Similarly, one can associate superfields with excited states [14]. The multiplets of our interest, $(0^+, 1^+)$, $(1^-, 2^-)$, and $(0^-, 1^-)'$ are described by the superfields:

$$\begin{aligned} \mathcal{H}_+ &= \frac{1 + \not{v}}{2} (-H_{0^+} + \gamma_\mu \gamma_5 H_{1^+}^\mu), \\ \mathcal{H}_-^\mu &= \frac{1 + \not{v}}{2} \left(\sqrt{\frac{3}{2}} H_{1^-}^\nu \left[g_\nu^\mu - \frac{1}{3} \gamma_\nu (\gamma^\mu + v^\mu) \right] \right. \\ &\quad \left. + \gamma_5 \gamma_\nu H_{2^-}^{\mu\nu} \right), \\ \mathcal{H}'_- &= \frac{1 + \not{v}}{2} (-\gamma_5 H'_{0^-} + \gamma_\mu H'_{1^-}{}^\mu), \end{aligned} \quad (2.10)$$

respectively. All the tensors are traceless, symmetric, and transverse to the four-velocity. These excited superfields

transform under the spin-symmetry operations in the same way as the ground state superfield does.

The strong interactions of heavy mesons with PG bosons are described by the so-called ‘‘heavy-light’’ chiral Lagrangian, which is written in terms of the usual exponentiated matrix of PG bosons,

$$\Sigma = \exp \left(\frac{2i\mathcal{M}}{\sqrt{2}f_\pi} \right), \quad (2.11)$$

where \mathcal{M} is a 3×3 matrix for the octet of PG bosons,

$$\mathcal{M} = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\sqrt{\frac{2}{3}}\eta \end{pmatrix}, \quad (2.12)$$

and

$$f_\pi = 93 \text{ MeV} \quad (2.13)$$

is the pion decay constant. The Lagrangian for the PG bosons is

$$\mathcal{L}_\mathcal{M} = \frac{f_\pi^2}{4} \text{tr} \partial_\mu \Sigma^\dagger \partial^\mu \Sigma, \quad (2.14)$$

which contains all $SU(3)_L \times SU(3)_R$ invariant interactions up to two derivatives among the PG bosons. One can easily see the invariance of the Lagrangian $\mathcal{L}_\mathcal{M}$ under $SU(3)_L \times SU(3)_R$ chiral transformation, since in this representation the unitary matrix transforms as

$$\Sigma \rightarrow \Sigma' = L \Sigma R^\dagger, \quad (2.15)$$

where L and R are global transformations in $SU(3)_L$ and $SU(3)_R$, respectively. Since the chiral symmetry $SU(3)_L \times SU(3)_R$ is spontaneously broken down to $SU(3)_V$, we can deal with interactions of such PG bosons to the usual hadrons by introducing a new matrix [15]

$$\xi = \Sigma^{1/2}, \quad (2.16)$$

which transforms under the $SU(3)_L \times SU(3)_R$ as

$$\xi \rightarrow \xi' = L \xi U^\dagger = U \xi R^\dagger, \quad (2.17)$$

where U is a unitary matrix depending on L and R and non-linear functions of PG fields. From ξ we can construct a vector field V_μ and an axial vector field A_μ as follows:

$$V_\mu = \frac{1}{2} (\xi^\dagger \partial_\mu \xi + \xi \partial_\mu \xi^\dagger) \approx \frac{1}{4f_\pi^2} [\mathcal{M}, \partial_\mu \mathcal{M}], \quad (2.18)$$

$$A_\mu = \frac{i}{2} (\xi^\dagger \partial_\mu \xi - \xi \partial_\mu \xi^\dagger) \approx \frac{-1}{\sqrt{2}f_\pi} \partial_\mu \mathcal{M}. \quad (2.19)$$

The vector field transforms like a gauge field under chiral transformation

$$V_\mu \rightarrow V'_\mu = UV_\mu U^\dagger + U \partial_\mu U^\dagger, \quad (2.20)$$

and the axial vector field is an SU(3) octet which transforms as

$$A_\mu \rightarrow A'_\mu = UA_\mu U^\dagger. \quad (2.21)$$

The heavy quark spin-symmetry multiplet, \mathcal{H} , transforms under a chiral rotation as

$$\mathcal{H} \rightarrow U\mathcal{H}. \quad (2.22)$$

Note that all multiplets we are considering are isotriplets, and effectively reduce to isodoublets since we do not include the strange quark in our consideration.

In order to construct chirally invariant Lagrangian, we make use of a gauge-covariant derivative involving V_μ :

$$\nabla_\mu = \partial_\mu + V_\mu. \quad (2.23)$$

Using this covariant derivative and the axial vector field A_μ , one can construct an effective Lagrangian, which possesses spin-flavor and chiral symmetry explicitly. In Ref. [4], the strong interaction effective Lagrangian is given to the lowest chiral order, i.e., to $O(p_\pi)$ in the heavy meson sector and to $O(p_\pi^2)$ in the pion sector. Among many possible terms in the Lagrangian, the part of the interaction between heavy mesons and pions is

$$\begin{aligned} \mathcal{L}_{int} = & g \operatorname{tr}(\bar{\mathcal{H}}_+ A^\mu \mathcal{H}_- \gamma_\mu \gamma_5) + \alpha_1 \operatorname{tr}(\bar{\mathcal{H}}_+ A^\mu \mathcal{H}_- \gamma_\mu \gamma_5) \\ & + \alpha_2 \operatorname{tr}(\bar{\mathcal{H}}_- A^\mu \mathcal{H}_- \gamma_\mu \gamma_5) + \alpha_3 \operatorname{tr}(\bar{\mathcal{H}}_- A^\mu \mathcal{H}_- \gamma_\mu \gamma_5), \end{aligned} \quad (2.24)$$

where $\bar{\mathcal{H}} = \gamma^0 \mathcal{H}^\dagger \gamma^0$, and g and α_i 's are the low energy coupling constants. From the above Lagrangian, one can obtain vertices for soft neutral pion emission [4]:

$$\begin{aligned} (DD^{*\mu}) \pi^0: & \frac{g}{f_\pi} p_\pi^\mu, \\ (DD^{0+}) \pi^0: & \frac{\alpha_1}{f_\pi} p_\pi \cdot v, \\ (DD_1^{\mu-}) \pi^0: & \sqrt{\frac{2}{3}} \frac{\alpha_2}{f_\pi} p_\pi^\mu, \\ (DD'^{*\mu}) \pi^0: & \frac{\alpha_3}{f_\pi} p_\pi^\mu. \end{aligned} \quad (2.25)$$

Using these vertex factors and the heavy meson transition matrix elements in Eq. (2.6), one can calculate B_{l4} decay amplitudes.

III. B_{l4} DECAY RATES WITH SCALAR COUPLINGS

We consider B_{l4} decays of $B^- \rightarrow D^+ \pi^- l \bar{\nu}$ and $B^- \rightarrow D^0 \pi^0 l \bar{\nu}$. We here show theoretical expressions for $B^- \rightarrow D^0 \pi^0 l \bar{\nu}$. For $B^- \rightarrow D^+ \pi^- l \bar{\nu}$, we can easily derive them

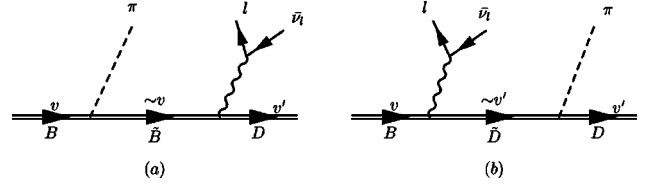


FIG. 1. Feynman diagrams for B_{l4} decays.

from those for $B^- \rightarrow D^0 \pi^0 l \bar{\nu}$ at the amplitude level by using isospin relation. The amplitude has the general form:

$$T = \kappa[(1 + \chi)j_\mu \Omega_V^\mu + \eta j_s \Omega_s], \quad \kappa = V_{cb} \frac{G_F}{\sqrt{2}} \sqrt{m_B m_D}, \quad (3.1)$$

where j_μ is the $(V-A)$ charged leptonic current and j_s is a Yukawa-type scalar current in the leptonic sector. Here the parameters χ and η , which parametrize contributions from physics beyond the SM, are in general complex. Note that the SM values are $\chi = \eta = 0$. We retain charged lepton mass since we also consider decays of $B_{\tau 4}$. The vector interaction part of the hadronic amplitude, Ω_V^μ , receives contributions from two types of diagrams, illustrated in Figs. 1(a) and 1(b), respectively:

$$\begin{aligned} \Omega_V^\mu = & \langle D \pi | \bar{c} \gamma^\mu (1 - \gamma_5) b | B \rangle = \sum_{\tilde{B}_i} \langle D | \bar{c} \gamma^\mu (1 - \gamma_5) b | \tilde{B}_i \rangle \\ & \times \langle \tilde{B}_i \pi | B \rangle + \sum_{\tilde{D}_i} \langle D \pi | \tilde{D}_i \rangle \langle \tilde{D}_i | \bar{c} \gamma^\mu (1 - \gamma_5) b | B \rangle, \end{aligned} \quad (3.2)$$

and Ω_s is the corresponding scalar current matrix element:

$$\Omega_s = \langle D \pi | \bar{c} (1 - \gamma_5) b | B \rangle, \quad (3.3)$$

where \tilde{D}_i and \tilde{B}_i stand for intermediate excited states of our interest. We can obtain Yukawa interaction form factors by multiplying the $(V-A)$ currents with momentum transfer q^μ :

$$\begin{aligned} q^\mu = & p_B - p_D = m_B v^\mu - m_D v'^\mu, \quad (3.4) \\ \langle D^*(v', \epsilon) | q^\mu A_\mu | B(v) \rangle = & -\xi(\omega)(m_B + m_D)(v \cdot \epsilon^*), \\ \langle D_0(v') | q^\mu A_\mu | B(v) \rangle = & \rho_1(\omega)(m_B + m_D)(1 - \omega), \\ \langle D_1(v', \epsilon) | q^\mu A_\mu | B(v) \rangle = & -\frac{2}{\sqrt{6}} \rho_2(\omega)(m_B + m_D)(1 - \omega) \\ & \times (v \cdot \epsilon^*), \\ \langle D'^*(v', \epsilon) | q^\mu A_\mu | B(v) \rangle = & -\xi^{(1)}(\omega)(m_B + m_D)(v \cdot \epsilon^*); \quad (3.5) \\ \langle \text{all states} | q^\mu V_\mu | B(v) \rangle = & 0. \quad (3.6) \end{aligned}$$

And using

$$q^\mu V_\mu = (m_B - m_D) \bar{c} b, \quad q^\mu A_\mu = -(m_B + m_D) \bar{c} \gamma_5 b, \quad (3.7)$$

in the heavy quark limit, one can relate any Yukawa-type interaction with the vector (axial) current ones. Consequently, we get the following relation:

$$q_\mu \Omega_V^\mu = (m_B + m_D) \Omega_s. \quad (3.8)$$

We define the dimensionless parameter ζ , which determines the size of the scalar contributions relative to the vector ones:

$$\zeta = \frac{\eta}{1 + \chi}. \quad (3.9)$$

Note that the SM corresponds to the case with $\chi = \eta = \zeta = 0$. Then the amplitude can be written as

$$T = \kappa(1 + \chi) j_\mu \Omega^\mu, \quad \Omega^\mu = \Omega_V^\mu + \zeta \Omega_s \frac{L^\mu}{m_l}, \quad (3.10)$$

where we used the Dirac equation for leptonic current, $L^\mu j_\mu = m_l j_s$ with $L^\mu = p_l^\mu + p_\nu^\mu$.

Then, the relevant Feynman diagrams read

$$\begin{aligned} \Omega_\mu = & \frac{g}{f_\pi} \xi(\omega) p_\nu \left\{ \Theta^{\nu\rho}(v) \Delta(B^*) \left[i v^\alpha v'^\beta \epsilon_{\mu\alpha\beta\rho} + g_{\mu\rho}(1 + \omega) - v_\mu v'_\rho - \zeta v'_\rho \frac{L_\mu}{m_l} \right] + \Theta^{\nu\rho}(v') \Delta(D^*) \right. \\ & \times \left[i v^\alpha v'^\beta \epsilon_{\mu\alpha\beta\rho} + g_{\mu\rho}(1 + \omega) - v_\rho v'_\mu + \zeta v_\rho \frac{L_\mu}{m_l} \right] \left. + \frac{\alpha_1}{f_\pi} \rho_1(\omega) \left[(v - v')_\mu + \zeta(1 - \omega) \frac{L_\mu}{m_l} \right] \left[-\Delta(B_0) p \cdot v + \Delta(D_0) p \cdot v' \right] \right. \\ & + \frac{\alpha_2}{3f_\pi} \rho_2(\omega) p_\rho \left\{ \Theta^{\nu\rho}(v) \Delta(B_1) \left[i \epsilon_{\mu\nu\alpha\beta} v^\alpha v'^\beta (\omega - 1) + g_{\mu\nu} (\omega^2 - 1) - v'_\nu (v_\mu (2 + \omega) - 3v'_\mu) + 2\zeta(1 - \omega) v'_\nu \frac{L_\mu}{m_l} \right] \right. \\ & + \Theta^{\nu\rho}(v') \Delta(D_1) \left[i \epsilon_{\mu\nu\alpha\beta} v^\alpha v'^\beta (\omega - 1) + g_{\mu\nu} (\omega^2 - 1) - v_\nu (v'_\mu (2 + \omega) - 3v_\mu) + 2\zeta(1 - \omega) v_\nu \frac{L_\mu}{m_l} \right] \left. \right\} \\ & + \frac{\alpha_3}{f_\pi} \xi^{(1)}(\omega) p_\nu \left\{ \Theta^{\nu\rho}(v) \Delta(B'^*) \left[i v^\alpha v'^\beta \epsilon_{\mu\alpha\beta\rho} + g_{\mu\rho}(1 + \omega) - v_\mu v'_\rho - \zeta v'_\rho \frac{L_\mu}{m_l} \right] + \Theta^{\nu\rho}(v') \Delta(D'^*) \right. \\ & \times \left. \left[i v^\alpha v'^\beta \epsilon_{\mu\alpha\beta\rho} + g_{\mu\rho}(1 + \omega) - v_\rho v'_\mu + \zeta v_\rho \frac{L_\mu}{m_l} \right] \right\}, \end{aligned} \quad (3.11)$$

where $\Theta^{\mu\nu}(v) = g^{\mu\nu} - v^\mu v^\nu$, and for numerical values of the coupling constants we use the predictions of Ref. [4]:

$$g = 0.5, \quad \alpha_1 = -1.43, \quad \alpha_2 = -0.14, \quad \alpha_3 = 0.69. \quad (3.12)$$

Here the functions representing the propagator of the intermediate resonance, $\Delta(\tilde{M})$, are defined as follows:

$$\begin{aligned} \Delta(\tilde{B}) &= \frac{M_{\tilde{B}}}{(p_B - p)^2 - M_{\tilde{B}}^2 + i M_{\tilde{B}} \Gamma_{\tilde{B}}} = \frac{1}{-2(v \cdot p + \delta m_{\tilde{B}}) + i \Gamma_{\tilde{B}}}, \\ \Delta(\tilde{D}) &= \frac{M_{\tilde{D}}}{(p_D + p)^2 - M_{\tilde{D}}^2 + i M_{\tilde{D}} \Gamma_{\tilde{D}}} = \frac{1}{2(v' \cdot p - \delta m_{\tilde{D}}) + i \Gamma_{\tilde{D}}}, \end{aligned} \quad (3.13)$$

where we have incorporated the finite width of the resonances, $\Gamma_{\tilde{D}}$ and $\Gamma_{\tilde{B}}$. This inclusion of resonance widths corresponds to considering the overall strong interaction contribution, and plays an important role in our investigation. Those widths serve as CP -conserving phases needed for direct CP violations. Here the mass differences between the resonances and the ground-state mesons, $\delta m_{\tilde{M}}$, are defined as $\delta m_{\tilde{M}} = m_{\tilde{M}} - m_M$, and $\Gamma_{\tilde{M}}$ is the total width of the resonance. For the specific values of the above quantities, we adopt the predictions in Ref. [4]:

$$\delta m_{M_0} = 0.39 \text{ GeV}, \quad \Gamma_{M_0} = 1040 \text{ MeV},$$

$$\delta m_{M_1} = 0.71 \text{ GeV}, \quad \Gamma_{M_1} = 405 \text{ MeV},$$

$$\delta m_{M'^*} = 0.56 \text{ GeV}, \quad \Gamma_{M'^*} = 191 \text{ MeV}, \quad (3.14)$$

where M stands for B or D . Note that we used the same numerical values for \tilde{B} and \tilde{D} , for simplicity, following Ref. [4]. For B_{14} decays, values for \tilde{D} are much more important, and are also more precisely known from experiments compared to values of \tilde{B} . In order to do detailed calculations, first, we now define the kinematic variables

$$P = p_D + p_\pi, \quad \text{with } p_D = m_D v', \quad p_\pi = p,$$

$$Q = p_D - p,$$

$$L = p_l + p_{\bar{\nu}},$$

$$N = p_l - p_{\bar{\nu}}. \quad (3.15)$$

In terms of these variables, the most general form of Ω_μ can be written as

$$\Omega_\mu = \frac{i}{2} H \epsilon_{\mu\nu\rho\sigma} L^\nu Q^\rho P^\sigma + F P_\mu + G Q_\mu + R L_\mu, \quad (3.16)$$

where H , F , G , and R are the form factors depending on three invariants ω , $p \cdot v$, and $p \cdot v'$. These form factors are easily

obtained from the expressions in Eq. (3.11), and their explicit forms are given in Appendix A. We get the following invariants:

$$\begin{aligned}
P^2 &= s_M, \\
Q^2 &= 2(m_D^2 + m_\pi^2) - s_M, \\
L^2 &= s_L, \\
N^2 &= -s_L + 2m_l^2, \\
P \cdot Q &= m_D^2 - m_\pi^2, \\
P \cdot L &= \frac{1}{2}(m_B^2 - s_L - s_M), \\
L \cdot N &= m_l^2. \tag{3.17}
\end{aligned}$$

It is well known that there are five independent kinematic variables for these processes when the spins of the initial and final states are zero or not observed. From the momenta of the B meson, D meson, the pion, the lepton, and its neutrino p_B , p_D , p , p_l , and $p_{\bar{\nu}}$, for the five independent variables

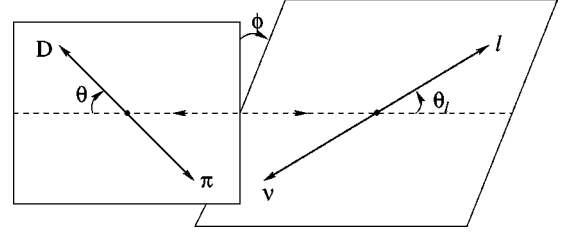


FIG. 2. Decay planes for the kinematic variables.

we choose $s_M = (p_D + p)^2$, $s_L = (p_l + p_{\bar{\nu}})^2$, θ (i.e., the angle between the D momentum in the $D\pi$ rest frame and the moving direction of the $D\pi$ system in the B -meson's rest frame), θ_l (i.e., the angle between the lepton momentum in the $l\bar{\nu}$ rest frame and the moving direction of the $l\bar{\nu}$ system in the B -meson's rest frame) and ϕ [i.e., the angle between the two decay planes defined by the pairs $(\mathbf{p}, \mathbf{p}_D)$ and $(\mathbf{p}_l, \mathbf{p}_{\bar{\nu}})$ in the rest frame of the B meson]. This is the set of variables initially introduced by Cabibbo and Maksymowicz [16] in the analysis of K_{l4} decays. All the angles above are explicitly defined in Fig. 2. Then, the remaining invariants are

$$\begin{aligned}
P \cdot N &= \frac{1}{2s_L} [m_l^2(m_B^2 - s_M - s_L) + \cos \theta_l (s_L - m_l^2) \lambda^{1/2}(m_B^2, s_M, s_L)], \\
Q \cdot L &= \frac{1}{2s_M} [(m_D^2 - m_\pi^2)(m_B^2 - s_M - s_L) + \cos \theta \lambda^{1/2}(s_M, m_D^2, m_\pi^2) \lambda^{1/2}(m_B^2, s_M, s_L)], \\
Q \cdot N &= \frac{m_l^2}{2s_M s_L} [(m_D^2 - m_\pi^2)(m_B^2 - s_M - s_L) + \cos \theta \lambda^{1/2}(m_B^2, s_M, s_L) \lambda^{1/2}(s_M, m_D^2, m_\pi^2)] \\
&\quad + \frac{m_D^2 - m_\pi^2}{2s_M s_L} \cos \theta_l \lambda^{1/2}(m_B^2, s_M, s_L) \lambda^{1/2}(s_L, m_l^2, 0) + \frac{(m_B^2 - s_M - s_L)}{2s_M s_L} \cos \theta_l \cos \theta \lambda^{1/2}(s_M, m_D^2, m_\pi^2) \lambda^{1/2}(s_L, m_l^2, 0) \\
&\quad - \frac{1}{\sqrt{s_M s_L}} \cos \phi \sin \theta_l \sin \theta \lambda^{1/2}(s_M, m_D^2, m_\pi^2) \lambda^{1/2}(s_L, m_l^2, 0), \\
\epsilon_{\mu\nu\rho\sigma} P^\mu Q^\nu L^\rho N^\sigma &= -\frac{1}{2\sqrt{s_L s_M}} \lambda^{1/2}(m_B^2, s_M, s_L) \lambda^{1/2}(s_M, m_D^2, m_\pi^2) \lambda^{1/2}(s_L, m_l^2, 0) \sin \phi \sin \theta_l \sin \theta, \tag{3.18}
\end{aligned}$$

where $\lambda(x, y, z) = x^2 + y^2 + z^2 - 2xy - 2yz - 2xz$. In particular, the amplitudes are functions of three invariants, $\omega \equiv v \cdot v'$, $p \cdot v$, and $p \cdot v'$, which are given by

$$\begin{aligned}
v \cdot v' &= \frac{1}{4m_B m_D s_M} [(s_M + m_D^2 - m_\pi^2)(m_B^2 + s_M - s_L) + \lambda^{1/2}(m_B^2, s_M, s_L) \lambda^{1/2}(s_M, m_D^2, m_\pi^2) \cos \theta], \\
p \cdot v &= \frac{1}{4m_B s_M} [(s_M + m_\pi^2 - m_D^2)(m_B^2 + s_M - s_L) - \lambda^{1/2}(m_B^2, s_M, s_L) \lambda^{1/2}(s_M, m_D^2, m_\pi^2) \cos \theta], \\
p \cdot v' &= \frac{1}{2m_D} (s_M - m_D^2 - m_\pi^2). \tag{3.19}
\end{aligned}$$

Finally, the decay rate is proportional to

$$|T|^2 = \tilde{\kappa}^2 L^{\mu\nu} \Omega_\mu \Omega_\nu^*, \tag{3.20}$$

where $\tilde{\kappa} = \kappa|1 + \chi|$, and the leptonic tensor $L_{\mu\nu}$ is given by

$$L_{\mu\nu} = 4(L_\mu L_\nu - N_\mu N_\nu - (s_L - m_l^2)g_{\mu\nu} - i\epsilon_{\mu\nu\rho\sigma}L^\rho N^\sigma), \quad (3.21)$$

retaining lepton mass m_l . The explicit result is

$$\begin{aligned} |T|^2/\tilde{\kappa}^2 = & 4|F|^2[(P \cdot L)^2 - (s_L - m_l^2)s_M - (P \cdot N)^2] + 4|G|^2[(Q \cdot L)^2 - Q^2(s_L - m_l^2) - (Q \cdot N)^2] + |H|^2[(s_L - m_l^2) \\ & \times (2P \cdot LP \cdot QQ \cdot L - (P \cdot L)^2 Q^2 - (P \cdot Q)^2 s_L - (Q \cdot L)^2 s_M + Q^2 s_M s_L) - (\epsilon_{\mu\nu\rho\sigma} L^\mu N^\nu P^\rho Q^\sigma)^2] + 8\text{Re}(FG^*) \\ & \times [P \cdot LQ \cdot L - P \cdot NQ \cdot N - (s_L - m_l^2)P \cdot Q] + 4\text{Re}(FH^*)[s_L P \cdot QP \cdot N + (P \cdot L)^2 Q \cdot N - P \cdot LQ \cdot LP \cdot N - s_M s_L Q \cdot N \\ & - m_l^2(P \cdot QP \cdot L - s_M Q \cdot L)] + 4\text{Re}(GH^*)[-P \cdot N(Q \cdot L)^2 + P \cdot LQ \cdot LQ \cdot N + Q^2 s_L P \cdot N - s_L P \cdot QQ \cdot N - m_l^2 \\ & \times (Q^2 P \cdot L - P \cdot QQ \cdot L)] + 4m_l^2[|R|^2(s_L - m_l^2) + 2\text{Re}(F^*R)(P \cdot L - P \cdot N) + 2\text{Re}(G^*R)(Q \cdot L - Q \cdot N) \\ & - \text{Im}(RH^*)\epsilon_{\mu\nu\rho\sigma}P^\mu Q^\nu L^\rho N^\sigma] + 4\text{Im}(2FG^* + F^*HP \cdot N + G^*HQ \cdot N)\epsilon_{\mu\nu\rho\sigma}P^\mu Q^\nu L^\rho N^\sigma. \end{aligned} \quad (3.22)$$

The differential partial width of interest can be expressed as

$$d\Gamma_{B_{14}} = \frac{N_\pi}{2m_B} J(s_M, s_L) |T|^2 d\Phi_4, \quad (3.23)$$

where the 4-body phase space $d\Phi_4$ is

$$d\Phi_4 \equiv ds_M \cdot ds_L \cdot d\cos\theta \cdot d\cos\theta_l \cdot d\phi, \quad (3.24)$$

and

$$N_\pi = \begin{cases} 2 & \text{for charged pions,} \\ 1 & \text{for neutral pions,} \end{cases} \quad (3.25)$$

and the Jacobian $J(x, y)$ is

$$\begin{aligned} J(x, y) = & \frac{1}{2^{14}\pi^6 x y m_B^2} \lambda^{1/2}(m_B^2, x, y) \\ & \times \lambda^{1/2}(x, m_D^2, m_\pi^2) \lambda^{1/2}(y, m_l^2, 0). \end{aligned} \quad (3.26)$$

Kinematically allowed regions of the variables are

$$\begin{aligned} (m_D + m_\pi)^2 & < s_M < (m_B - m_l)^2, \\ m_l^2 & < s_L < (m_B - \sqrt{s_M})^2, \\ -1 & < \cos\theta, \cos\theta_l < 1, \\ 0 & < \phi < 2\pi. \end{aligned} \quad (3.27)$$

Since the initial B^- system is not CP self-conjugate, any genuine CP -odd observable can be constructed only by considering both the B_{14}^- decay and its charge-conjugated B_{14}^+ decay, and by identifying the CP relations of their kinematic distributions. Before constructing possible CP -odd asymmetries explicitly, we calculate the transition probability for the charge-conjugated process $B^+ \rightarrow \bar{D}^0 \pi^0 l^+ \nu_l$ or $B^+ \rightarrow D^- \pi^+ l^+ \nu_l$. Following previous explicit expressions, the transition probability $|\bar{T}|^2$ for the B^+ decay in the same reference frame as in the B^- decay is given by simple modification of the transition probability $|T|^2$ of the B^- decay:

$$|\bar{T}|^2 = |T|^2 \begin{cases} \text{(i) change signs in front of the terms proportional to imaginary part} \\ \text{(ii) } \zeta \rightarrow \zeta^*. \end{cases} \quad (3.28)$$

It is easy to see that if the parameter ζ is real, the transition probability (3.22) for the B^- decay and Eq. (3.28) for the B^+ decay satisfy CP relation:

$$|T|^2(\theta, \theta_l, \phi) = |\bar{T}|^2(\theta, \theta_l, -\phi). \quad (3.29)$$

Note that all the imaginary parts are being multiplied by the quantity $\epsilon_{\mu\nu\rho\sigma}P^\mu Q^\nu L^\rho N^\sigma$ which is proportional to $\sin\phi$, as can be seen in Eq. (3.18). And then, $d\Gamma/d\Phi_4$ can be decomposed into a CP -even part \mathcal{S} and a CP -odd part \mathcal{D} :

$$\frac{d\Gamma}{d\Phi_4} = \frac{1}{2}(\mathcal{S} + \mathcal{D}). \quad (3.30)$$

The CP -even part \mathcal{S} and the CP -odd part \mathcal{D} can be easily identified by making use of the CP relation (3.29) between the B^- and B^+ decay probabilities and they are expressed as

$$\mathcal{S} = \frac{d(\Gamma + \bar{\Gamma})}{d\Phi_4}, \quad \text{and} \quad \mathcal{D} = \frac{d(\Gamma - \bar{\Gamma})}{d\Phi_4}, \quad (3.31)$$

where we have used the same kinematic variables $\{s_M, s_L, \theta, \theta_l\}$ for the $d\bar{\Gamma}/d\Phi_4$ except for the replacement of ϕ by $-\phi$, as shown in Eq. (3.29). Here Γ and $\bar{\Gamma}$ are the decay rates for B^- and B^+ , respectively. The CP -even \mathcal{S} term and the CP -odd \mathcal{D} term can be obtained from B^\mp decay probabilities and their explicit form is listed in Appendix B. Note that the CP -odd term is proportional to the imaginary

part of the parameter ζ in Eq. (3.9) and lepton mass m_l [note also that \mathcal{D} in Eq. (B2) looks proportional to m_l^2 , but the quantity A_4 has $1/m_l$ factor in it]. Therefore, there exists no CP violation in B_{14} decays within the SM since the SM corresponds to the case with $\zeta=0$.

IV. ASYMMETRIES

An easily constructed CP -odd asymmetry is the rate asymmetry

$$A \equiv \frac{\Gamma - \bar{\Gamma}}{\Gamma + \bar{\Gamma}}, \quad (4.1)$$

which has been used as a probe of CP violation in Higgs and top quark sectors [17]. Here Γ and $\bar{\Gamma}$ are the decay rates for B^- and B^+ , respectively. The statistical significance of the asymmetry can then be computed as

$$N_{SD} = \frac{N_- - N_+}{\sqrt{N_- + N_+}} = \frac{N_- - N_+}{\sqrt{N \cdot Br}}, \quad (4.2)$$

where N_{SD} is the number of standard deviations, N_{\pm} is the number of events predicted in B_{14} decay for B^{\pm} meson, N is the number of B mesons produced, and Br is the branching fraction of the relevant B decay mode. For a realistic detection efficiency ϵ , we have only to rescale the number of events by this parameter, $N_- + N_+ \rightarrow \epsilon(N_- + N_+)$. Taking $N_{SD}=1$, we obtain the number N_B of the B mesons needed to observe CP violation at $1\text{-}\sigma$ level:

$$N_B = \frac{1}{Br \cdot A^2}. \quad (4.3)$$

Next, we consider the so-called optimal observable. An appropriate real weight function $w(s_M, s_L; \theta, \theta_l, \phi)$ is usually employed to separate the CP -odd \mathcal{D} contribution and to enhance its analysis power for the CP -odd parameter $\text{Im}(\zeta)$ through the CP -odd quantity:

$$\langle w\mathcal{D} \rangle \equiv \int [w\mathcal{D}] d\Phi_4, \quad (4.4)$$

of which the analysis power is determined by the parameter

$$\varepsilon = \frac{\langle w\mathcal{D} \rangle}{\sqrt{\langle \mathcal{S} \rangle \langle w^2 \mathcal{S} \rangle}}. \quad (4.5)$$

For the analysis power ε , the number N_B of the B mesons needed to observe CP violation at $1\text{-}\sigma$ level is

$$N_B = \frac{1}{Br \cdot \varepsilon^2}. \quad (4.6)$$

Certainly, it is desirable to find the optimal weight function with the largest analysis power. It is known [18] that when the CP -odd contribution to the total rate is relatively small, the optimal weight function is approximately given as

$$w_{\text{opt}}(s_M, s_L; \theta, \theta_l, \phi) = \frac{\mathcal{D}}{\bar{\mathcal{S}}} \varepsilon_{\text{opt}} = \sqrt{\frac{\langle \mathcal{D}^2 / \mathcal{S} \rangle}{\langle \mathcal{S} \rangle}}. \quad (4.7)$$

We adopt this optimal weight function in the following numerical analyses.

V. NUMERICAL RESULTS AND CONCLUSIONS

Let us first consider the decay to heavy lepton, τ , since CP -odd asymmetry is proportional to m_l , and heavy lepton may be more susceptible to effects of new physics. The contributions of each multiplet to the $B_{\tau 4}$ decays are shown in Figs. 3(a) and 3(b), for $B^- \rightarrow D^+ \pi^- \tau \bar{\nu}_\tau$ and $B^- \rightarrow D^0 \pi^0 \tau \bar{\nu}_\tau$, respectively. We found that the contributions from $(0^+, 1^+)$ and $(0^-, 1^-)$ multiplets are comparable to the lowest one $(0^-, 1^-)$, and the contribution of the D-wave $(1^-, 2^-)$ multiplet is very small. However, the latter may give non-negligible contribution through the interference with the other dominant parts, so we retain it.

We restrict ourselves to the soft-pion limit by considering only the region $s_M \leq 6.5 \text{ GeV}^2$, which is about one half of the maximum value. This restriction corresponds to the pion momentum less than about 0.6 GeV. The decay rates we obtain (including all of the resonances we have discussed) are $1.79 \times 10^{-16} \text{ GeV}$ for $B^- \rightarrow D^+ \pi^- \tau \bar{\nu}_\tau$ and $5.56 \times 10^{-15} \text{ GeV}$ for $B^- \rightarrow D^0 \pi^0 \tau \bar{\nu}_\tau$, where we use $\Gamma_{D^{*0}} = 60 \text{ keV}$ for the total width of the D^{*0} [4]. These correspond to branching fractions of 0.045% and 1.37%, respectively, which were obtained by using the recently published lifetime of the charged B meson ($\tau_{B^\pm} = 1.65 \times 10^{-12} \text{ s}$) [19]. We notice that, although $B^- \rightarrow D^+ \pi^- \tau \bar{\nu}_\tau$ decay amplitude is larger than that of $B^- \rightarrow D^0 \pi^0 \tau \bar{\nu}_\tau$ by factor $\sqrt{2}$ from isospin relation, for the actual decay rate the latter mode has much larger decay rate. This is so because the most dominant resonance D^{*0} cannot decay into $D^+ \pi^-$ on its mass shell [19].

In Table I, we show the results of $B_{\tau 4}$ decays for the two CP -violating asymmetries: the rate asymmetry A and the optimal asymmetry ε_{opt} . We estimated the number of B meson pairs, N_B , needed for detection at 1σ level for CP -violating values $|\text{Im}(\zeta)| = 0.5$ and 2.0. These values are chosen from our rough estimates of current experimental bounds in the multi-Higgs-doublet model and scalar leptoquark models. Thorough investigation on specific models can be found in Ref. [5]. As we can expect, the optimal observable gives much better result than the simple rate asymmetry. We also found that although the decay rate of neutral pion mode is larger than that of the charged pion mode, the latter case gives better detection results because of large CP -violating effects in charged pion decay mode. Since one can expect about 10^8 orders of B -meson pairs produced yearly in the asymmetric B factories, one could probe the CP -violation effect by using the optimal asymmetry observable.

We also estimated CP -violation effects in the B_{14} decays with light leptons. The results for $B_{\mu 4}$ decays are shown in Table II. We found that relative sizes of CP -violating asymmetry, which is proportional to the lepton mass, are smaller

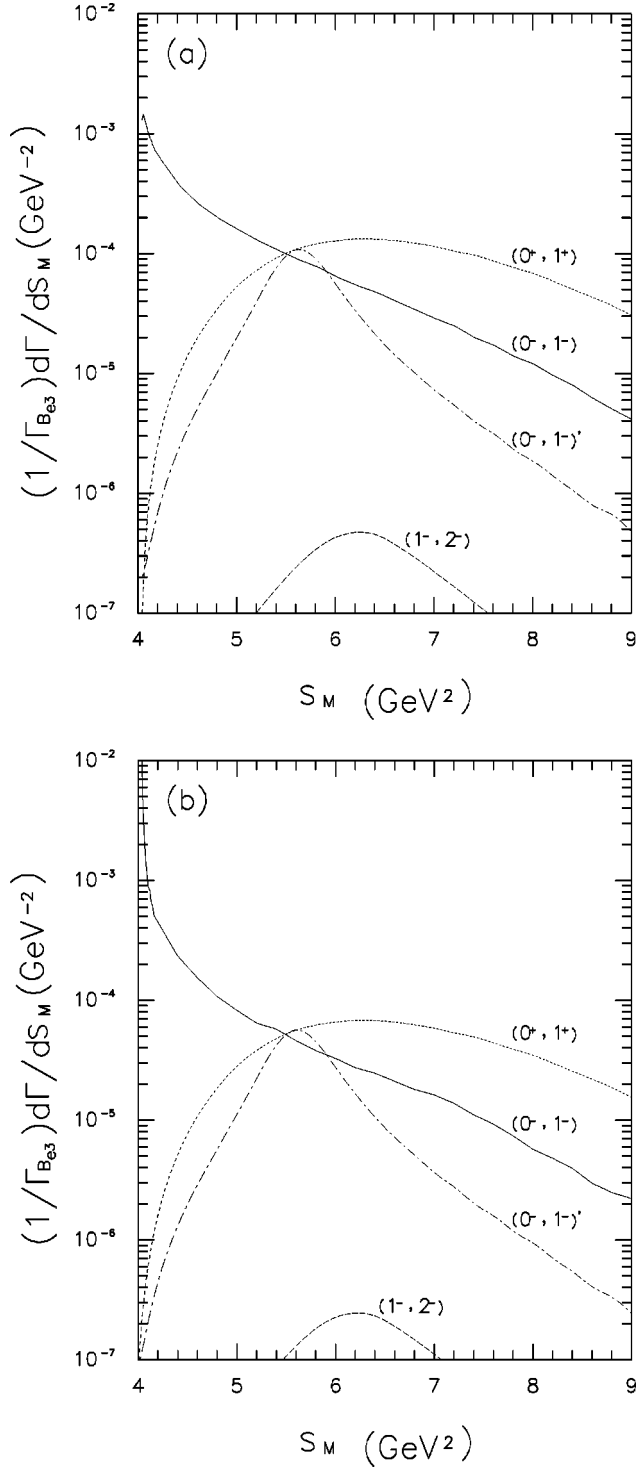


FIG. 3. $[1/\Gamma(B \rightarrow D e \nu)](d\Gamma_{B_{\tau 4}}/ds_M)$ as a function of the invariant mass of $D\pi$, s_M : (a) for $B^- \rightarrow D^+ \pi^- \tau \nu$ and (b) for $B^- \rightarrow D^0 \pi^0 \tau \nu$. The curves correspond to the individual resonance doublets, as indicated in the figures.

in $B_{\mu 4}$ decays than in $B_{\tau 4}$ decays. However, due to the larger branching fractions, at the same value of $\text{Im}(\zeta)$ one needs in the $B_{\mu 4}$ case the number of B -meson pairs of the same order of magnitude as in the $B_{\tau 4}$ case to probe the CP -violating effect. For $B_{e 4}$ decays, however, electron mass is so small that we find that $\sim 10^{10}$ B -meson pairs are needed for the

TABLE I. The CP -violating rate asymmetry A and the optimal asymmetry ε_{opt} , determined in the soft pion limit, and the number of charged B meson pairs, N_B , needed for detection at 1σ level, at reference values (a) $\text{Im}(\zeta)=0.5$ and (b) $\text{Im}(\zeta)=2.0$, for the $B_{\tau 4}$ decays.

(a) Modes	$B^- \rightarrow D^+ \pi^- \tau \bar{\nu}_\tau$		$B^- \rightarrow D^0 \pi^0 \tau \bar{\nu}_\tau$	
Asymmetry	Size(%)	N_B	Size(%)	N_B
A	0.08	3.31×10^9	0.002	3.22×10^{11}
ε_{opt}	1.13	1.77×10^7	0.15	3.33×10^7
(b) Modes	$B^- \rightarrow D^+ \pi^- \tau \bar{\nu}_\tau$		$B^- \rightarrow D^0 \pi^0 \tau \bar{\nu}_\tau$	
Asymmetry	Size(%)	N_B	Size(%)	N_B
A	0.33	2.07×10^8	0.006	2.01×10^{10}
ε_{opt}	4.52	1.11×10^6	0.59	2.08×10^6

same input $\text{Im}(\zeta)$ values, even when using the optimal observable. Thus one may conclude that both $B_{\tau 4}$ and $B_{\mu 4}$ decays could serve as equally good probes of CP -violation effects. However, in some models such as multi-Higgs-doublet models, the scalar coupling, i.e., effectively ζ itself, is proportional to the lepton mass, and therefore the CP violation in the $B_{\mu 4}$ decays is highly suppressed compared with the $B_{\tau 4}$ decays. Therefore, $B_{\tau 4}$ decays could serve as a much better probe of CP -violating effects in such models.

As mentioned earlier, the pure contribution of the D -wave $(1^-, 2^-)$ multiplet is very small (cf. Fig. 3), but we included that multiplet since its contribution could be rather sizable through interference with other dominant ones. However, we compared the result obtained by dropping off the $(1^-, 2^-)$ multiplet with the previous results including all the multiplets, and found that the interference effect is also very small. So one could safely neglect the $(1^-, 2^-)$ multiplet for B_{l4} decays.

We here included various higher excited states other than just the ground state D^* vector mesons as intermediate states in the decay processes. In order to estimate effects of these higher excited states on CP violation effects, we show in Table III the results with only the ground state included as an intermediate state. It can be easily seen, by comparing with the values in Table I(b), that those higher excited states have highly amplified the effect of CP violation.

In conclusion, we investigated CP violation from physics beyond the standard model through semileptonic B_{l4} decays: $B \rightarrow D \pi l \bar{\nu}_l$. In the decay process, we included various excited states as intermediate states decaying to the final hadrons. The CP violation is implemented in a model independent way, in which we extend the leptonic current by including complex couplings of the scalar sector and those of the vector sector in extensions of the standard model. We calculated the CP -odd rate asymmetry and the optimal asymmetry, and found that the optimal asymmetry is sizable and can be detected at 1σ level with about 10^6 - 10^7 B -meson pairs, for some reference values of new physics effect. Since $\sim 10^8$ B -meson pairs are expected to be produced yearly at the forthcoming asymmetric B facto-

TABLE II. The CP -violating rate asymmetry A and the optimal asymmetry ε_{opt} , determined in the soft pion limit, and the number of charged B -meson pairs, N_B , needed for detection at 1σ level, at reference values (a) $\text{Im}(\zeta)=0.5$ and (b) $\text{Im}(\zeta)=2.0$, for the $B_{\mu 4}$ decays.

(a) Modes		$B^- \rightarrow D^+ \pi^- \mu \bar{\nu}_\mu$		$B^- \rightarrow D^0 \pi^0 \mu \bar{\nu}_\mu$	
Asymmetry	Size(%)	N_B	Size(%)	N_B	
A	0.014	7.27×10^9	0.0005	3.57×10^{11}	
ε_{opt}	0.24	2.35×10^7	0.045	4.53×10^7	
(b) Modes		$B^- \rightarrow D^+ \pi^- \mu \bar{\nu}_\mu$		$B^- \rightarrow D^0 \pi^0 \mu \bar{\nu}_\mu$	
Asymmetry	Size(%)	N_B	Size(%)	N_B	
A	0.054	4.54×10^8	0.002	1.83×10^{10}	
ε_{opt}	0.95	1.47×10^6	0.2	2.83×10^6	

ries, one could investigate CP -violation effects by using our formalism.

ACKNOWLEDGMENTS

We thank H.Y. Cheng and G. Cvetič for a careful reading of the manuscript and their valuable comments. C.S.K. wishes to thank the Korea Institute for Advanced Study for warm hospitality. The work of C.S.K. was supported in part by Non-Directed-Research-Fund, KRF, in part by the CTP, Seoul National University, in part by the BSRI Program, Ministry of Education, Project No. 98-015-D00061, in part by the KOSEF-DFG large collaboration project, Project No. 96-0702-01-01-2. J.L. acknowledges the financial support of Sughak program of Korean Research Foundation. The work

TABLE III. The CP -violating rate asymmetry A and the optimal asymmetry ε_{opt} , determined in the soft pion limit, and the number of charged B -meson pairs, N_B , needed for detection at 1σ level, at the reference value $\text{Im}(\zeta)=2.0$, for the $B_{\tau 4}$ decays, including only the ground state $(0^-, 1^-)$ multiplet as an intermediate state.

Modes		$B^- \rightarrow D^+ \pi^- \tau \bar{\nu}_\tau$		$B^- \rightarrow D^0 \pi^0 \tau \bar{\nu}_\tau$	
Asymmetry	Size(%)	N_B	Size(%)	N_B	
A	0.0004	2.20×10^{14}	0.0002	1.75×10^{13}	
ε_{opt}	0.007	6.06×10^{11}	0.01	5.39×10^9	

of W.N. was supported by the BSRI Program, Ministry of Education, Project No. 98-015-D00061.

APPENDIX A: FORM FACTORS

We give the form factors needed in Eq. (3.16). If we write

$$\Omega_\mu = i h m_B m_D \epsilon_{\mu\nu\rho\sigma} v^\nu v'^\rho p^\sigma + A_1 p_\mu + A_2 m_B v_\mu + A_3 m_D v'_\mu + \zeta A_4 L_\mu, \quad (\text{A1})$$

then, the form factors in Eq. (3.16) are expressed as

$$\begin{aligned} H &= h, \\ F &= A_2 + \frac{1}{2}(A_1 + A_3), \\ G &= \frac{1}{2}(A_3 - A_1), \\ R &= A_2 + \zeta A_4, \end{aligned} \quad (\text{A2})$$

where

$$h = \frac{g \xi(\omega)}{f_\pi m_B m_D} (\Delta(B^*) + \Delta(D^*)) + \frac{\alpha_2 \rho_2(\omega)}{3 f_\pi m_B m_D} (\omega - 1) (\Delta(B_1) + \Delta(D_1)) + \frac{\alpha_3 \xi^{(1)}(\omega)}{f_\pi m_B m_D} (\Delta(B'^*) + \Delta(D'^*)), \quad (\text{A3})$$

$$A_1 = \frac{g \xi(\omega)}{f_\pi} (1 + \omega) (\Delta(B^*) + \Delta(D^*)) + \frac{\alpha_2 \rho_2(\omega)}{3 f_\pi} (\omega^2 - 1) (\Delta(B_1) + \Delta(D_1)) + \frac{\alpha_3 \xi^{(1)}(\omega)}{f_\pi} (1 + \omega) (\Delta(B'^*) + \Delta(D'^*)), \quad (\text{A4})$$

$$\begin{aligned} A_2 &= -\frac{g \xi(\omega)}{f_\pi m_B} p \cdot (v + v') \Delta(B^*) - \frac{\alpha_1 \rho_1(\omega)}{f_\pi m_B} (p \cdot v \Delta(B_0) - p \cdot v' \Delta(D_0)) - \frac{\alpha_2 \rho_2(\omega)}{f_\pi m_B} \\ &\quad \times \left\{ \Delta(B_1) \left[\frac{1}{3} (\omega p \cdot v' - p \cdot v) + \frac{2}{3} (p \cdot v' - \omega p \cdot v) \right] - \Delta(D_1) (p \cdot v - \omega p \cdot v') \right\} - \frac{\alpha_3 \xi^{(1)}(\omega)}{f_\pi m_B} p \cdot (v + v') \Delta(B^*), \end{aligned} \quad (\text{A5})$$

$$\begin{aligned} A_3 &= -\frac{g \xi(\omega)}{f_\pi m_D} p \cdot (v + v') \Delta(D^*) - \frac{\alpha_1 \rho_1(\omega)}{f_\pi m_D} (p \cdot v' \Delta(D_0) - p \cdot v \Delta(B_0)) - \frac{\alpha_2 \rho_2(\omega)}{f_\pi m_D} \\ &\quad \times \left\{ \Delta(D_1) \left[\frac{1}{3} (\omega p \cdot v - p \cdot v') + \frac{2}{3} (p \cdot v - \omega p \cdot v') \right] - \Delta(B_1) (p \cdot v' - \omega p \cdot v) \right\} - \frac{\alpha_3 \xi^{(1)}(\omega)}{f_\pi m_D} p \cdot (v + v') \Delta(D^*), \end{aligned} \quad (\text{A6})$$

$$\begin{aligned}
A_4 = & \frac{g\xi(\omega)}{f_\pi m_l} [-\Delta(B^*)(p \cdot v' - \omega p \cdot v) + \Delta(D^*)(p \cdot v - \omega p \cdot v')] + \frac{\alpha_1 \rho_1(\omega)}{f_\pi m_l} (1 - \omega) [-p \cdot v \Delta(B_0) + p \cdot v' \Delta(D_0)] \\
& + \frac{2\alpha_2 \rho_2(\omega)}{3f_\pi m_l} (1 - \omega) [-\Delta(B_1)(p \cdot v' - \omega p \cdot v) + \Delta(D_1)(p \cdot v - \omega p \cdot v')] + \frac{\alpha_3 \xi^{(1)}(\omega)}{f_\pi m_l} \\
& \times [-\Delta(B'^*)(p \cdot v' - \omega p \cdot v) + \Delta(D'^*)(p \cdot v - \omega p \cdot v')]. \tag{A7}
\end{aligned}$$

APPENDIX B: CP-EVEN AND CP-ODD QUANTITIES

The CP-even quantity \mathcal{S} is

$$\begin{aligned}
\mathcal{S} = & 2C(s_M, s_L) \cdot (R\text{-independent terms of } |T|^2) + 8C(s_M, s_L) m_l^2 [2(s_L - m_l^2) \text{Re}(\zeta) \text{Re}(A_2 A_4^*) + 2(P \cdot L - P \cdot N) \{ \text{Re}(F^* A_2) \\
& + \text{Re}(\zeta) \text{Re}(F^* A_4) \} + 2(Q \cdot L - Q \cdot N) \{ \text{Re}(G^* A_2) + \text{Re}(\zeta) \text{Re}(G^* A_4) \} - \epsilon_{\mu\nu\rho\sigma} P^\mu Q^\nu L^\rho N^\sigma \{ \text{Re}(H^* A_2) \\
& + \text{Re}(\zeta) \text{Re}(H^* A_4) \}], \tag{B1}
\end{aligned}$$

and the CP-odd quantity \mathcal{D} is

$$\begin{aligned}
\mathcal{D} = & 8C(s_M, s_L) m_l^2 \text{Im}(\zeta) [2(s_L - m_l^2) \text{Im}(A_2 A_4^*) - 2(P \cdot L - P \cdot N) \text{Im}(F^* A_4) - 2(Q \cdot L - Q \cdot N) \text{Im}(G^* A_4) \\
& + \epsilon_{\mu\nu\rho\sigma} P^\mu Q^\nu L^\rho N^\sigma \text{Im}(H^* A_4)], \tag{B2}
\end{aligned}$$

where the overall function $C(s_M, s_L)$ is given by

$$C(s_M, s_L) = \kappa^2 |1 + \chi|^2 \frac{N_\pi}{2m_B} J(s_M, s_L). \tag{B3}$$

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