Nonfactorizable contributions in hadronic weak decays of charm mesons

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Two body decays of charm mesons are studied by describing their amplitude in terms of a sum of factorizable and nonfactorizable ones. The former is estimated by using a naive factorization while the latter is calculated by using a hard pseudoscalar-meson approximation. The hard pseudoscalar-meson amplitude is given by a sum of the so-called equal-time commutator term and surface term which contains all possible pole contributions of various mesons, not only the ordinary $\{q\bar{q}\}$ but also four-quark $\{qq\bar{q}\bar{q}\}$, hybrid $\{q\bar{q}g\}$ and glueballs. Naively factorized amplitudes for the spectator decays which lead to too big rates can interfere destructively with exotic meson pole amplitudes and the total amplitudes can reproduce their observed rates. The nonfactorizable contributions can supply sufficiently large contributions to the color suppressed decays which are strongly suppressed in the naive factorization. A possible solution to the long standing puzzle that the ratio of decay rates for $D^0\to K^+K^-$ to $D^0\to\pi^+\pi^-$ is around 2.5 is given by different contributions of exotic meson poles. [S0556-2821(98)06923-9]

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I. INTRODUCTION

Nonleptonic weak decays of charm mesons have been studied extensively by using the so-called factorization (or vacuum insertion) prescription [1,2]. However, recent semi-phenomenological analyses [3,4] in two-body decays of B mesons within the framework of the factorization suggest that the value of a_2 to reproduce the observed branching ratios for these decays [3,5] should be larger by about a factor 2 than the one with the leading order (LO) QCD corrections [1,2,6] where the color degree of freedom $N_c=3$ and that its sign should be opposite to the one in the large N_c limit although the phenomenological value of a_1 is very close to the one expected in the same approximation. [a_1 and a_2 are the coefficients of four quark operators in the effective weak Hamiltonian in the Bauer-Stech-Wirbel (BSW) scheme [1,2] which will be reviewed briefly in the next section.] The above fact implies that the large N_c argument fails, at least, in hadronic weak decays of B mesons. Since the large N_c argument is independent of flavors, it also does not work in nonleptonic weak decays of charm mesons. Therefore dominance of the factorized amplitude in charm decays looses its theoretical support since, in charm meson decays, the large N_c argument is the only one known theoretical background to support the factorization prescription [7]. In fact, a naive application of the factorization to charm decay amplitudes causes many problems, for example, too strong suppression for the color mismatched decays like $D^0\to\pi^0\bar{K}^0$ and $D^0\to\pi^0\pi^0$, too big rates for decays described by the so-called spectator diagrams such as the Cabibbo-angle favored $c\to s+(\bar{d}u)_1$ and Cabibbo-angle suppressed $c\to d+(\bar{d}u)_1$ and $c\to s+(\bar{s}u)_1$, too small ratio (less than unity) of the rates $\Gamma(D^0\to K^+K^-)$ to $\Gamma(D^0\to\pi^+\pi^-)$ although the observed value is around 2.5, etc., where $(\overline{q'}q)_1$ denotes a color singlet pair of $\overline{q'}$ and q. To get rid of these problems, the factorization has been implemented by multiplying the factorized amplitudes by phase factors arising from final state interactions. However, the final state interactions are realized by dynamical contributions of various hadron states.

Using a hard pseudo-scalar-meson approximation, the present author has studied dynamical contributions of various hadron states to charm meson decays and has given a hint to solve the above problems in charm meson decays [8,9]. However, in these analyses, the amplitudes did not include the factorizable contributions so that the results were not necessarily satisfactory, i.e., these analyses could not satisfactorily provide an overall fit to the observed rates of Cabibbo-angle favored and suppressed decays of charm mesons. On the other hand, a recent analysis in hadronic weak decays of B mesons [10] by assuming that their amplitude can be given by a sum of factorizable and nonfactorizable ones suggests that, only in some processes under a particular kinematical condition (a heavy quark goes to another heavy quark plus a pair of light quark and antiquark with sufficiently high energies like $\bar{B}\to D\pi$ and $D^*\pi$), the factorization works well while nonfactorizable contributions are important in the other decays, in particular, in color suppressed decays. It seems to imply that the naive factorization is not guaranteed by the large N_c arguments but it works well under some special kinematical condition [11]. In this article, we reanalyze two body decays of charm mesons describing their amplitude by a sum of factorizable and nonfactorizable ones. We will review briefly the naive factorization and list the factorized amplitudes for two body decays of charm mesons in the next section. Nonfactorizable amplitudes in a hard pseudoscalar-meson approximation will be presented in Sec. III. In Sec. IV, we will compare our result with experiments. A brief summary will be given in the final section.

II. FACTORIZED AMPLITUDES

Our starting point in this article is to describe the two body decay amplitude by a sum of factorizable and nonfactorizable ones [10]:

$$M_{\rm total}=M_{\rm fact}+M_{\rm non\text{-}f}. \tag{1}$$

The factorizable amplitude $M_{\rm fact}$ is evaluated by using the factorization in the BSW scheme [1,2] in which the relevant part of the effective weak Hamiltonian responsible for the charm decays is given by

$$H_w^{BSW}=\frac{G_F}{\sqrt{2}}\{a_1O_1^{(s'c)H}+a_2O_2^{(s'c)H}+(\text{penguin term})+\text{H.c.}\}. \tag{2}$$

It can be obtained by applying the Fierz reordering to the usual effective Hamiltonian,

$$H_w=\frac{G_F}{\sqrt{2}}\{c_1O_1^{(s'c)}+c_2O_2^{(s'c)}+(\text{penguin term})+\text{H.c.}\}, \tag{3}$$

where c_1 and c_2 are the Wilson coefficients of the normal ordered four quark operators,

$$O_1^{(s'c)}=:(\bar{u}d')_{V-A}(\bar{s'}c)_{V-A}:,$$
$$O_2^{(s'c)}=:(\bar{s'}d')_{V-A}(\bar{u}c)_{V-A}: \ . \tag{4}$$

Here d' and s' denote weak eigenstates of the down and strange quarks, respectively, and $(\bar{u}d)_{V-A}=\bar{u}\gamma_\mu(1-\gamma_5)d$, etc. We do not explicitly show the possible penguin term through which s-channel pole contributions of a scalar glueball can play a role in Cabibbo-angle suppressed decays, since its explicit expression will not be needed. The quark bilinears in $O_1^{(s'c)H}$ and $O_2^{(s'c)H}$ are treated as interpolating fields for the mesons and therefore should be no longer Fierz reordered. The coefficients a_1 and a_2 in Eq. (2) are given by

$$a_1=c_1+\frac{c_2}{N_c},\quad a_2=c_2+\frac{c_1}{N_c}. \tag{5}$$

The leading order (LO) QCD corrections lead to $a_1\simeq1.09$ and $a_2\simeq-0.09$ for $N_c=3$ [2].

When H_w^{BSW} is obtained by using the Fierz reordering, an extra term $\tilde{H}_w$ which is given by a sum of products of colored currents comes out,

$$H_w\to H_w^{BSW}+\tilde{H}_w, \tag{6}$$

where

$$\tilde{H}_w=\frac{G_F}{\sqrt{2}}\{c_2\tilde{O}_1^{(s'c)}+c_1\tilde{O}_2^{(s'c)}\} \tag{7}$$

with

$$\tilde{O}_1^{(s'c)}=2\sum_a :(\bar{u}t^ad')_{V-A}(\bar{s'}t^ac)_{V-A}:,$$
$$\tilde{O}_2^{(s'c)}=2\sum_a :(\bar{s'}t^ad')_{V-A}(\bar{u}t^ac)_{V-A}: \ . \tag{8}$$

Here t^a's are the generators of color $SU_c(N_c)$ symmetry group.

The factorization prescription in the BSW scheme leads to the following factorized amplitude, for example, for the $D^+(p)\to\bar{K}^0(p')\pi^+(q)$ decay,

$$\begin{aligned}M_{\rm fact}(D^+(p)\to\bar{K}^0(p')\pi^+(q))&\\ =\frac{G_F}{\sqrt{2}}U_{cs}U_{ud}\{a_1\langle\pi^+(q)|(\bar{u}d)_{V-A}|0\rangle&\\ \times\langle\bar{K}^0(p')|(\bar{s}c)_{V-A}|D^+(p)\rangle&\\ +a_2\langle\bar{K}^0(p')|(\bar{s}d)_{V-A}|0\rangle&\\ \times\langle\pi^+(q)|(\bar{u}c)_{V-A}|D^+(p)\rangle\},&\end{aligned} \tag{9}$$

where U_{ij} is the Cabibbo-Kobayashi-Maskawa (CKM) matrix element [12] which is taken to be real in this article since CP invariance is always assumed. Factorizable amplitudes for the other two-body decays also can be calculated in the same way.

To evaluate the factorized amplitudes, we use the following parametrization of matrix elements of currents,

$$\langle\pi(q)|A_\mu^\pi|0\rangle=-if_\pi q_\mu,\ \text{etc.}, \tag{10}$$

and

$$\langle\bar{K}(p')|V_\mu|D(p)\rangle=(p+p')_\mu f_+^{\bar{K}D}(q^2)+q_\mu f_-^{\bar{K}D}(q^2),\ \text{etc.}, \tag{11}$$

as usual, where $q=p-p'$. Using these expressions of current matrix elements, we obtain the factorized amplitudes for two-body decays of charm mesons listed in Table I, where terms proportional to f_- are neglected since their coefficients are small or since f_- is expected to be small for large values of its argument. It is seen that the factorized amplitudes for $D^0\to\pi^0\pi^0$ and $D^0\to\pi^0\bar{K}^0$ described by the color mismatched diagrams, $c\to(d\bar{d})_1+u$ and $c\to(s\bar{d})_1+u$, respectively, are much smaller (the color suppression) than those for the spectator decays (because of $|a_1|\gg|a_2|$). The factorized amplitude for $D_s^+\to K^+\bar{K}^0$ is described by a sum of the color mismatched diagram and the so-called annihilation diagram in the weak boson mass $m_W\to\infty$ limit. The former is again proportional to a_2. The latter is proportional to $f_-^{KK}(m_{D_s}^2)$ and neglected. However the observed rates for these decays are not very small. The vanishing factorized amplitude for $D^0\to K^0\bar{K}^0$ reflects a cancellation between two possible annihilation diagrams while the measured rate for this decay is a little smaller than the ordinary ones but not extremely suppressed. To get rid of these problems, the factorization has been implemented by multiplying the amplitudes by phase factors arising from final state interactions [2]. However, the final state interactions are realized by dynamical contributions of various hadron states. Therefore, we study explicitly the dynamical contributions of various hadrons as the nonfactorizable amplitudes in the next section.

TABLE I. Factorized amplitudes for two-body decays of charm mesons. The ellipses denote neglected contributions proportional to f_-.

Decay	$M_{\rm fact}$
$D^+\to\pi^+\overline{K}^0$	$iU_{cs}U_{ud}\frac{G_F}{\sqrt{2}}a_1f_\pi(m_D^2-m_K^2)f_+^{\overline{K}D}(m_\pi^2)\times\left\{1+\left(\frac{a_2}{a_1}\right)\left(\frac{f_K}{f_\pi}\right)\left(\frac{m_D^2-m_\pi^2}{m_D^2-m_K^2}\right)\left(\frac{f_+^{(\pi D)}(m_K^2)}{f_+^{(\overline{K}D)}(m_\pi^2)}\right)\right\}+\cdots$
$D^0\to\pi^+K^-$	$iU_{cs}U_{ud}\frac{G_F}{\sqrt{2}}a_1f_\pi(m_D^2-m_K^2)f_+^{\overline{K}D}(m_\pi^2)+\cdots$
$D^0\to\pi^0\overline{K}^0$	$iU_{cs}U_{ud}\frac{G_F}{\sqrt{2}}\sqrt{\frac{1}{2}}a_2f_K(m_D^2-m_\pi^2)f_+^{\pi D}(m_K^2)+\cdots$
$D_s^+\to K^+\overline{K}^0$	$iU_{cs}U_{ud}\frac{G_F}{\sqrt{2}}a_2f_K(m_{D_s}^2-m_K^2)f_+^{KD_s}(m_K^2)+\cdots$
$D^0\to\pi^+\pi^-$	$-iU_{cs}U_{us}\frac{G_F}{\sqrt{2}}a_1f_\pi(m_D^2-m_\pi^2)f_+^{\pi D}(m_\pi^2)+\cdots$
$D^0\to\pi^0\pi^0$	$0 \qquad +\cdots$
$D^+\to\pi^+\pi^0$	$iU_{cs}U_{us}\frac{G_F}{\sqrt{2}}\sqrt{\frac{1}{2}}(a_1+a_2)f_\pi(m_d^2-m_\pi^2)f_+^{\pi D}(m_\pi^2)+\cdots$
$D^0\to K^0\overline{K}^0$	0
$D^0\to K^+K^-$	$iU_{cs}U_{us}\frac{G_F}{2}a_1f_K(m_D^2-m_K^2)f_+^{\overline{K}D}(m_K^2)+\cdots$
$D^+\to K^+\overline{K}^0$	$iU_{cb}U_{ud}\frac{G_F}{\sqrt{2}}a_1f_K(m_D^2-m_K^2)f_+^{\overline{K}D}(m_K^2)+\cdots$
$D_s^+\to\pi^+K^0$	$-iU_{cs}U_{us}\frac{G_F}{\sqrt{2}}a_1f_\pi(m_{D_s}^2-m_K^2)f_+^{KD_s}(m_\pi^2)+\cdots$
$D_s^+\to\pi^0K^+$	$iU_{cs}U_{us}\frac{G_F}{\sqrt{2}}\sqrt{\frac{1}{2}}a_2f_\pi(m_{D_s}^2-m_K^2)f_+^{KD_s}(m_\pi^2)+\cdots$

III. NONFACTORIZABLE AMPLITUDES

Now we study nonfactorizable amplitudes for two-body decays of charm mesons, $P_1(p)\to P_2(p')+P_3(q)$. As mentioned in the previous section, we assume that they are dominated by dynamical contributions of various hadron states. Then they can be estimated by using a hard pseudoscalar-meson approximation in the infinite momentum frame (IMF, i.e., $\mathbf{p}\to\infty$). It is an innovation of the old soft pion technique [13]. The nonfactorizable amplitude, which is given by a matrix element of $\tilde{H}_w$ symmetrized with respect to exchange of two meson states in the final states, is written as

$$M_{\text{non-f}}(P_1\to P_2P_3)\simeq M_{\rm ETC}(P_1\to P_2P_3)+M_{\rm S}(P_1\to P_2P_3) \tag{12}$$

in this approximation [14,15]. The equal-time commutator term (M_{ETC}) and the surface term (M_S) are given by

$$M_{\rm ETC}(P_1\to P_2P_3)=\frac{i}{\sqrt{2}f_{P_3}}\langle P_2|[V_{\overline{P}_3},\tilde{H}_w]|P_1\rangle+(P_2\leftrightarrow P_3) \tag{13}$$

and

$$\begin{aligned}M_{\rm S}(P_1\to P_2P_3)&\\ =\frac{i}{\sqrt{2}f_{P_3}}&\left\{\sum_n\left(\frac{m_{P_2}^2-m_{P_1}^2}{m_n^2-m_{P_1}^2}\right)\langle P_2|A_{\overline{P}_3}|n\rangle\langle n|\tilde{H}_w|P_1\rangle\right.\\ &\left.+\sum_l\left(\frac{m_{P_2}^2-m_{P_1}^2}{m_l^2-m_{P_2}^2}\right)\langle P_2|\tilde{H}_w|l\rangle\langle l|A_{\overline{P}_3}|P_1\rangle\right\}\\ &+(P_2\leftrightarrow P_3),\end{aligned} \tag{14}$$

respectively, where $[V_\pi+A_\pi,\tilde{H}_w]=0$ has been used. (See Refs. [14] and [15] for notations.) $M_{\rm ETC}$ has the same form as the one in the old soft pion approximation but now has to be evaluated in the infinite momentum frame (IMF). The surface term has been given by a sum of all possible pole amplitudes, i.e., n and l run over all possible single meson states, not only ordinary $\{q\bar{q}\}$, but also hybrid $\{q\bar{q}g\}$, four-quark $\{qq\bar{q}\bar{q}\}$ and glue-balls. Since the value of wave function of orbitally excited $\{q\bar{q}\}_{L\neq 0}$ state at the origin is expected to vanish in the nonrelativistic quark model, or more generally, wave function overlappings between the ground-state $\{q\bar{q}\}_{L=0}$ and their excited states are expected to be small, however, we neglect contributions of these states. In the *u*-channel [the second line of the right-hand-side of Eq. (14)], excited meson contributions will be not very important because of $m_l^2 \gtrsim m_{P_1}^2 \gg m_{P_2}^2$ if l is an excited-state meson. In contrast, in the *s* channel, we need to treat carefully contributions of exotic (non-$\{q\bar{q}\}$) mesons to charm decays (if they exist) since they have been predicted around charm masses. The *s* channel of the color favored and mismatched spectator decays proceeds via four quark states after the weak interactions and therefore four-quark meson poles can contribute to these decays. However, in the annihilation decays in the weak boson mass $m_W\to\infty$ limit, their *s* channel is given by $\{q\bar{q}\}$ state just after the weak interactions. Therefore we expect that the ground-state $\{q\bar{q}\}_0$ and hybrid mesons can give important *s*-channel pole contributions to these decays. (However, we neglect contributions of scalar hybrids in this article since their masses have been expected to be considerably lower than the charm ones [16] and their contributions will be small.) The *s*-channel penguin can induce an *s*-channel pole contribution of glueball. In this way, the hard pseudoscalar-meson amplitude in Eq. (12) with Eqs. (13) and (14) as the nonfactorizable contribution is described in terms of *asymptotic matrix elements* (matrix elements taken between single hadron states with infinite momentum) of charges V_i and A_i, ($i=\pi$ and K), and the effective weak Hamiltonian H_w.

Asymptotic matrix elements of isospin and flavor $SU_f(3)$ charges, V_π and V_K, are parametrized as

$$\begin{aligned}\langle\pi^0|V_{\pi^+}|\pi^-\rangle &= \sqrt{2}\langle K^+|V_{\pi^+}|K^0\rangle\\ &= -\sqrt{2}\langle D^+|V_{\pi^+}|D^0\rangle\\ &= -2\langle K^+|V_{K^+}|\pi^0\rangle\\ &= -\sqrt{2}\langle D_s^+|V_{K^+}|D^0\rangle = \cdots = \sqrt{2}. \end{aligned}\tag{15}$$

The above parametrization can be obtained by applying *asymptotic* $SU_f(4)$ symmetry [17] or $SU_f(4)$ extension of the nonet symmetry in $SU_f(3)$ to the matrix elements or by using the quark counting. Matrix elements of axial counterpart, A_π and A_K, of the above V_π and V_K are parametrized as

$$\begin{aligned}\langle\rho^0|A_{\pi^+}|\pi^-\rangle &= \sqrt{2}\langle K^{*+}|A_{\pi^+}|K^0\rangle = -\sqrt{2}\langle D^{*+}|A_{\pi^+}|D^0\rangle\\ &= -2\langle K^{*+}|A_{K^+}|\pi^0\rangle = -\sqrt{2}\langle D_s^{*+}|A_{K^+}|D^0\rangle\\ &= \cdots = h \end{aligned}\tag{16}$$

in the same way as the above. The above parametrization reproduces well [15,18] the observed decay rates for $D^*\to D\pi$ and $D^*\to D\gamma$.

Next, we parametrize asymptotic matrix elements of $\tilde{H}_w$ using quark counting [8]. We expect that the factorization will not be a good approximation to estimate asymptotic matrix elements of H_w since one of the external states in these matrix elements contains only light quarks and weak interactions occur in a deep sea of soft gluons where color degree of freedom of quarks will be compensated by soft gluons. Therefore, we forget the color degree of freedom of quarks for the moment and count only their flavors (and hence connected quark-line diagrams). Now we review the procedure to parametrize the asymptotic matrix elements of H_w. To this, first, we rewrite the effective weak Hamiltonian in Eq. (7) as

$$\tilde{H}_w = \frac{G_F}{\sqrt{2}}\{c_-\tilde{O}_-^{(s'c)} + c_+\tilde{O}_+^{(s'c)} + (\text{penguin term}) + \text{H.c.}\},\tag{17}$$

where

$$\tilde{O}_\pm^{(s'c)} = \tilde{O}_1^{(s'c)} \pm \tilde{O}_2^{(s'c)}.\tag{18}$$

The four-quark operators $\tilde{O}_\pm^{(s'c)}$ belong to **64** and **20**″, respectively, of $SU_f(4)$ in its symmetry limit. The normal ordered four-quark operators $\tilde{O}_\pm^{(s'c)}$ can be expanded into a sum of products of (a) two creation operators in the left and two annihilation operators in the right, (b) three creation operators in the left and one annihilation operator in the right, (c) one creation operator in the left and three annihilation operators in the right, and (d) all (four) creation operators or annihilation operators of quarks and antiquarks. We associate (a)−(d) with quark-line diagrams describing different types of matrix elements of $\tilde{O}_\pm^{(s'c)}$. For (a), we utilize the two creation and annihilation operators to create and annihilate, respectively, the quarks and antiquarks belonging to the meson states $|\{q\bar{q}\}\rangle$ and $\langle\{q\bar{q}\}|$ in the asymptotic matrix elements of $\tilde{O}_\pm^{(s'c)}$. For (b) and (c), we need to add a spectator quark or antiquark to reach *physical* processes, $\langle\{qq\bar{q}\bar{q}\}|\tilde{O}_\pm^{(s'c)}|\{q\bar{q}\}$ and $\langle\{q\bar{q}\}|\tilde{O}_\pm^{(s'c)}|\{qq\bar{q}\bar{q}\}$, where $\{qq\bar{q}\bar{q}\}$ denotes four-quark mesons [19]. They can be classified into the following four types, $\{qq\bar{q}\bar{q}\} = [qq][\bar{q}\bar{q}] \oplus (qq)(\bar{q}\bar{q}) \oplus \{[qq](\bar{q}\bar{q}) \pm (qq)[\bar{q}\bar{q}]\}$. Here () and [] denote symmetry and anti-symmetry, respectively, under the exchange of flavors between them. Since only the first two can have $J^{P(C)}=0^{+(+)}$, we here consider contributions of them. These two types of four-quark mesons are again classified into two different types with two different combinations of color degree of freedom, *i.e.*, one consists of color singlet $\{q\bar{q}\}$ pairs and the other consists of color octet $\{q\bar{q}\}$ pairs. They can mix with each other. Their mass eigenstates are listed in Tables III and IV in Appendix A. As seen in these tables, the predicted masses of the lighter components with relevant quantum numbers are much smaller than the charm mass while some of the heavier ones (with * in the tables) can have masses close to the charm mass. We here take into account only contributions of the latter as an approximation. (For more precise arguments, we need all contributions of these exotic mesons.)

Counting all possible quark-line diagrams, we obtain sum rules which should be satisfied by asymptotic matrix elements of $\tilde{O}_{\pm}^{(s'c)}$. In this process, symmetry (or antisymmetry) property of wave functions of external meson states under exchange of their quark and antiquark plays an important role and therefore we have to be careful with the order of the quark(s) and antiquark(s) in $\tilde{O}_{\pm}^{(s'c)}$. Noting that the wave function of the ground-state $\{q\bar{q}\}_0$ meson is antisymmetric [20] under the exchange of its quark and antiquark, we obtain the following constraints on asymptotic matrix elements of $\tilde{O}_{\pm}^{(s'c)}$ [8]

$$\langle\{q\bar{q}\}_0|\tilde{O}_{+}^{(s'c)}|\{q\bar{q}\}_0\rangle=0, \tag{19}$$

$$\langle[qq][\overline{qq}]|\tilde{O}_{+}^{(s'c)}|\{q\bar{q}\}_0\rangle=\langle\{q\bar{q}\}_0|\tilde{O}_{+}^{(s'c)}|[qq][\overline{qq}]\rangle=0, \tag{20}$$

$$\langle(qq)(\overline{qq})|\tilde{O}_{-}^{(s'c)}|\{q\bar{q}\}_0\rangle=\langle\{q\bar{q}\}_0|\tilde{O}_{+}^{(s'c)}|(qq)(\overline{qq})\rangle=0, \tag{21}$$

and then, from them, we can obtain selection rules of asymptotic matrix elements of H_w. We summarize and parametrize them in Appendix B which have been given separately in Refs. [8] and [21]. Inserting the above parametrizations of asymptotic matrix elements of A_π, A_K and H_w into M_S in Eq. (14), we obtain pole amplitudes including contributions of the $\{q\bar{q}\}_0$, $[qq][\overline{qq}]$ and $(qq)(\overline{qq})$ mesons.

A scalar glueball can give an important contribution, as an s-channel pole, to Cabibbo-angle suppressed decays through the s-channel penguin diagram. It can mix with scalar isosinglet $\{q\bar{q}\}$ mesons. The glue-rich component of the mixture is described by S^*. We parametrize the ratio $\langle K|A_K|S^*\rangle$ to $\langle\pi|A_\pi|S^*\rangle$ by

$$Z=\frac{\langle K^+|A_{K^+}|S^*\rangle}{\langle\pi^+|A_\pi^+|S^*\rangle}, \tag{22}$$

and then the residue of S^* meson pole as

$$\langle K^+|A_{K^+}|S^*\rangle\langle S^*|\tilde{H}_w|D^0\rangle=-k_g\langle\pi^+|\tilde{H}_w|D^+\rangle. \tag{23}$$

Then the glueball contributions to the $D\to K\bar{K}$ and $D\to\pi\pi$ decays are given by

$$M_S^{(glue)}(D^0\to K^0\bar{K}^0)=\frac{i}{f_K}\langle\pi^+|\tilde{H}_w|D^+\rangle\left(\frac{m_D^2-m_K^2}{m_D^2-m_{S^*}^2}\right)2k_g \tag{24}$$

$$=M_S^{(glue)}(D^0\to K^+K^-), \tag{25}$$

$$M_S^{(glue)}(D^0\to\pi^0\pi^0)=-\frac{i}{f_\pi}\langle\pi^+|\tilde{H}_w|D^+\rangle\frac{1}{Z}\left(\frac{m_D^2-m_\pi^2}{m_D^2-m_{S^*}^2}\right)2k_g \tag{26}$$

$$=\sqrt{\frac{1}{2}}M_S^{(glue)}(D^0\to\pi^+\pi^-). \tag{27}$$

An amplitude for a dynamical hadronic process can be decomposed into (continuum contribution)+(Born term). In the present case, $M_{\rm S}$ is given by a sum of pole amplitudes and therefore $M_{\rm ETC}$ corresponds to the continuum contribution [22] which can develop a phase relative to the Born term. Therefore we here parametrize the equal-time commutator (ETC) terms using isospin eigen amplitudes and their phases; e.g., for the $M_{\rm ETC}(D\to\pi\bar{K})$'s,

$$M_{\rm ETC}(D^0\to\pi^+K^-)=\frac{1}{3}M_{\rm ETC}^{(3)}(D\to\pi\bar{K})e^{i\delta_3(\pi\bar{K})}+\frac{2}{3}M_{\rm ETC}^{(1)}(D\to\pi\bar{K})e^{i\delta_1(\pi\bar{K})}, \tag{28}$$

$$M_{\rm ETC}(D^0\to\pi^0\bar{K}^0)=-\frac{\sqrt{2}}{3}M_{\rm ETC}^{(3)}(D\to\pi\bar{K})e^{i\delta_3(\pi\bar{K})}+\frac{\sqrt{2}}{3}M_{\rm ETC}^{(1)}(D\to\pi\bar{K})e^{i\delta_1(\pi\bar{K})}, \tag{29}$$

$$M_{\rm ETC}(D^+\to\pi^+\bar{K}^0)=M_{\rm ETC}^{(3)}(D\to\pi\bar{K})e^{i\delta_3(\pi\bar{K})}, \tag{30}$$

where $M_{\rm ETC}^{(2I)}$'s are the isospin eigen amplitudes with isospin I and δ_{2I}'s are the corresponding phase shifts introduced [23]. $M_{\rm ETC}$'s for decays into $K\bar{K}$ and $\pi\pi$ final states can be parametrized in a similar way. Since $M_{ETC}(D^0\to K^0\bar{K}^0)=0$, we obtain $M_{\rm ETC}^{(0)}(D\to K\bar{K})=M_{\rm ETC}^{(2)}(D\to K\bar{K})$ and $\delta_0(K\bar{K})=\delta_2(K\bar{K})$, which lead to

$$M_{\rm ETC}(D^0\to K^+K^-)=M_{\rm ETC}(D^+\to K^+\bar{K}^0)=-\frac{i}{\sqrt{2}f_K}\langle\pi^+|\tilde{H}_w|D^+\rangle e^{i\delta_2(K\bar{K})}. \tag{31}$$

The parametrization of $M_{\rm ETC}(D\to\pi\pi)$ is taken to be compatible with that of the $D\to\pi\pi$ amplitudes in Ref. [24]. Because of the selection rule from Eq. (19), we obtain

$$M_{\rm ETC}^{(4)}(D\to\pi\pi)=0, \tag{32}$$

$$M_{\rm ETC}^{(0)}(D\to\pi\pi)=\frac{i}{\sqrt{2}f_\pi}\langle\pi^+|\tilde{H}_w|D^+\rangle e^{i\delta_0(\pi\pi)}, \tag{33}$$

$$M_{\rm ETC}(D_s^+\to K^+\bar{K}^0)=-\frac{i}{\sqrt{2}f_K}\langle\pi^+|\tilde{H}_w|D_s^+\rangle e^{i\delta_2(K\bar{K})}, \tag{34}$$

since the final state $K^+\overline{K^0}$ is of $I=1$. For the Cabibbo-angle favored $D\to\pi\overline{K}$ and suppressed $D_s^+\to(\pi K)^+$ decays, their ETC terms are given by

$$M_{\rm ETC}^{(1)}(D\to\pi\overline{K})=\frac{i}{\sqrt{2}f_\pi}\langle\pi^+|\tilde{H}_w|D_s^+\rangle\times\left[2-\frac{1}{2}\left(\frac{f_\pi}{f_K}\right)\right]e^{i\delta_1(\pi\overline{K})},\tag{35}$$

$$M_{\rm ETC}^{(3)}(D\to\pi\overline{K})=-\frac{i}{\sqrt{2}f_\pi}\langle\pi^+|\tilde{H}_w|D_s^+\rangle\times\left[1-\left(\frac{f_\pi}{f_K}\right)\right]e^{i\delta_3(\pi\overline{K})},\tag{36}$$

$$M_{\rm ETC}^{(1)}(D_s^+\to(\pi K)^+)=\frac{i}{\sqrt{2}f_\pi}\langle\pi^+|\tilde{H}_w|D^+\rangle\times\left[2\left(\frac{f_\pi}{f_K}\right)-1\right]e^{i\delta_1(\pi K)},\tag{37}$$

$$M_{\rm ETC}^{(3)}(D_s^+\to(\pi K)^+)=-\frac{i}{\sqrt{2}f_\pi}\langle\pi^+|\tilde{H}_w|D^+\rangle\times[0]e^{i\delta_3(\pi K)}.\tag{38}$$

In this way, the final state interactions are included in the nonfactorizable amplitudes in the present perspective. It should be noted that ETC terms for decays into exotic final states vanish [in the $SU_f(3)$ symmetry limit, i.e., $f_K=f_\pi$].

In the approximation in which pole contributions of the ground-state $\{q\bar{q}\}_0$, scalar $[qq][\overline{qq}]$ and $(qq)(\overline{qq})$ mesons and a glueball to M_S are taken into account, we obtain the non-factorizable amplitudes for the $D\to\pi\overline{K}$, $\pi\pi$ and $K\overline{K}$ and $D_s^+\to(K\overline{K})^+$ and $(\pi K)^+$ decays inserting the above parametrizations of asymptotic matrix elements of charges and the effective weak Hamiltonian and the ETC terms implemented by the phase factors into Eq. (12) with Eqs. (13) and (14). The result is listed in Appendix C.

IV. BRANCHING RATIOS

Now we compare our result with experiments. To this, we need to know values of parameters involved in our total amplitude (a sum of the factorized amplitude listed in Table I and the corresponding hard pseudoscalar-meson amplitude listed in Appendix C). We take the following values of the CKM matrix elements and the decay constants [5]; $U_{us}=-U_{cd}=0.21$, $U_{cs}=U_{ud}=0.98$ and $f_\pi=132$ MeV, $f_K=159$ MeV. To calculate decay branching ratios, we use the central values of the observed lifetimes of charm mesons [5]; $\tau(D^+)=(1.057\pm0.015)\times10^{-12}$ s, $\tau(D^0)=(0.415\pm0.004)\times10^{-12}$ s and $\tau(D_s^+)=(0.467\pm0.017)\times10^{-12}$ s.

The factorized amplitudes listed in Table I contain the form factors, $f_+(q^2)$'s. We put $f_+(m_\pi^2)\simeq f_+(0)$ since m_π^2 is very small. The estimated values of $f_+(0)$'s are summarized in Ref. [5] as

TABLE II. Branching ratios (%) for two-body decays of charm mesons where $a_1=1.09$ and $a_2=-0.09$ have been used. Values of parameters introduced are tentatively taken as follows: the matrix element $\langle\pi^+|\tilde{H}_w|D_s^+\rangle\simeq0.0695\times10^{-5}$ (GeV)^2, the phases $\delta_0(\pi\pi)=\delta_1(\pi\overline{K})=\delta_1(\pi K)=\delta_3(\pi\overline{K})=\delta_3(\pi K)=85°$, and the parameters providing the residues of various meson poles $k_0=0.71$, $k_a^*=-0.20$, $k_s^*=-0.02$, $k_g=0.085$. $B_{\rm fact}$, $B_{\text{non-f}}$ and $B_{\rm total}$ include only the factorized amplitude, only the nonfactorizable one [involving the $\{q\bar{q}\}_0$, $[qq][\overline{qq}]$ and $(qq)(\overline{qq})$ meson poles] and the sum of them, respectively. The data values are taken from Ref. [28].

Decay	$B_{\rm fact}$	$B_{\text{non-}f}$	B_{total}	$B_{\rm expt}$
$D^+\to\pi^+\overline{K^0}$	9.66	2.15	2.71	2.74 ± 0.29
$D^0\to\pi^+K^-$	4.66	6.75	3.91	3.83 ± 0.12
$D^0\to\pi^0\overline{K^0}$	0.03	1.94	2.26	2.11 ± 0.21
$D_s^+\to K^+\overline{K^0}$	0.06	3.82	2.93	3.6 ± 1.1
$D^0\to\pi^+\pi^-$	0.29	0.68	0.14	0.152 ± 0.011
$D^0\to\pi^0\pi^0$	0.00	0.05	0.05	0.084 ± 0.022
$D^+\to\pi^+\pi^0$	0.31	1.11	0.25	0.25 ± 0.07
$D^0\to K^+K^-$	0.32	0.19	0.41	0.43 ± 0.03
$D^0\to K^0\overline{K^0}$	0.00	0.12	0.12	0.13 ± 0.04
$D^+\to K^+\overline{K^0}$	0.81	1.29	0.98	0.72 ± 0.12
$D_s^+\to\pi^+K^0$	0.32	0.14	0.22	<0.70
$D_s^+\to\pi^0K^+$	0.00	0.02	0.02	<0.70

$$f_+^{\overline{K}D}(0)_{expt}=0.75\pm0.02\pm0.02,\tag{39}$$

$$\left[\frac{f_+^{\pi D}(0)}{f_+^{\overline{K}D}(0)}\right]_{expt}=1.0^{+0.3}_{-0.2}\pm0.4\quad[\text{Mark III}],\tag{40}$$

$$=1.3\pm0.2\pm0.1\quad[\text{CLEO}].\tag{41}$$

The above result is compatible with the $SU_f(3)$ symmetry and therefore we assume that

$$f_+^{\overline{K}D}(m_\pi^2)\simeq f_+^{\pi D}(m_\pi^2)\simeq f_+^{KD_s}(m_\pi^2)\simeq f_+^{\pi D_s}(m_\pi^2)\simeq f_+^{\overline{K}D}(0).\tag{42}$$

In this way, we can obtain the factorized amplitudes which provide the branching ratios $B_{\rm fact}$'s in Table II. It is seen that the branching ratios for the so-called spectator decays, $D^+\to\pi^+\pi^0$ and $D^+\to\pi^+\overline{K^0}$, are too big and that the color suppressed decays, $D^0\to\pi^0\pi^0$ and $D^+\to\pi^0\overline{K^0}$, are too strongly suppressed as mentioned before.

Now we evaluate the nonfactorizable amplitudes listed in Appendix C. The asymptotic matrix elements of A_π and A_K which have been parametrized in Eq. (16) are estimated to be

$|h|\simeq 1.0$ [14,15] by using partially conserved axial-vector current (PCAC) and the observed rate [5], $\Gamma(\rho\to\pi\pi)_{expt}\simeq 150$ MeV.

We here assign $f_J(1710)$ to the glue-rich scalar S^* and take $\Gamma_{f_J}=175\pm 9$ MeV [5] as the width Γ_{S^*} of S^*. Then $|\langle K|A_K|S^*\rangle|$ can be estimated from the observed width of f_J and the ratio of its partial widths

$$\left[\frac{\Gamma(f_J\to\pi\pi)}{\Gamma(f_J\to K\bar{K})}\right]_{expt}=0.39\pm 0.14 \tag{43}$$

as $|\langle K^+|A_{K^+}|S^*\rangle|\lesssim 0.15$ by using PCAC (partial conservation of axial vector current). They are really small as expected. It implies that overlapping between wave functions of the ground-state-meson and the glueball (or glue-rich meson) is very small. Therefore size of matrix element of H_w taken between S^* and K will be much smaller than that between π and K, and hence the size of k_g will be much smaller than unity. Z in Eq. (22) also can be estimated to be $Z\sim 2.0$ [26] from the above value of the observed ratio of decay rates, Eq. (43), where we have taken the same sign between $\langle K^+|A_{K^+}|S^*\rangle$ and $\langle\pi^+|A_\pi^+|S^*\rangle$ as expected in the $SU_f(3)$ symmetry.

Although existence of four-quark mesons has never been confirmed, indications of their existence are increasing [27]. We here take the predicted values of $[qq][\bar{q}\bar{q}]$ meson masses listed in Table III(a) in Appendix A. However, the predicted $(qq)(\bar{q}\bar{q})$ meson masses listed in Table IV(a) satisfy $m_{E^*_{\pi\pi}}<m_D<m_{E^*_{\pi K}}$. In this case, it is hard that the $(qq)(\bar{q}\bar{q})$ meson pole amplitudes for the spectator decays, $D^+\to\pi^+\bar{K}^0$ and $D^+\to\pi^+\pi^0$, interfere destructively with their factorized amplitudes, simultaneously. (The naively factorized amplitudes have lead to too big rates for the spectator decays as discussed before.) Since the predicted values of their masses still would have ambiguities, however, we here shift up the mass values of $(qq)(\bar{q}\bar{q})$ mesons by 100 MeV from the predicted ones to obtain $m_D<m_{E^*_{\pi\pi}}<m_{E^*_{\pi K}}$ which leads to a destructive interference between the factorized amplitudes and the four-quark meson pole amplitudes for the $D^+\to\pi^+\bar{K}^0$ and $D^+\to\pi^+\pi^0$. Masses of four-quark mesons containing a charm quark listed in Tables III(b) and IV(b) are estimated *crudely* by using the quark counting since our result is not very sensitive to them. Although widths of four-quark mesons are also still not known, the $[qq][\bar{q}\bar{q}]$ mesons with * are expected to be narrower than the corresponding $(qq)(\bar{q}\bar{q})$ mesons [19].

For the phases δ_{2I}'s arising from contributions of nonresonant multihadron intermediate states with isospin I, they will be restricted in the region $|\delta_{2I}|<90°$. (Resonant contributions have already been extracted as pole amplitudes in M_S.) We here treat them as adjustable parameters which are restricted in the region $|\delta_{2I}|<90°$ as mentioned above and satisfy the relations

$$\delta_0(\pi\pi)=\delta_0(K\bar{K})=\delta_2(K\bar{K})=\delta_1(\pi\bar{K})=\delta_1(\pi K)\equiv\delta. \tag{44}$$

The second equality has been obtained previously and the others are expected in the $SU_f(3)$ symmetry limit. However the exotic phase shift $\delta_3(\pi\bar{K})$ is treated as a parameter independent of nonexotic $\delta_1(\pi\bar{K})$.

The remaining parameters are k_a^*, k_s^* and k_g which provide the residues of poles of $[qq][\bar{q}\bar{q}]$, $(qq)(\bar{q}\bar{q})$ and glueball, respectively. Since overlappings between wave functions of these exotic mesons and the ground-state ones are expected to be very small, values of these parameters will be much smaller than unity, *i.e.*, $|k_g|,|k_a^*|,|k_s^*|\ll 1$. It was predicted [19] that the couplings of the four-quark $[qq][\bar{q}\bar{q}]$ and $(qq)(\bar{q}\bar{q})$ mesons with * to the ground-state $O^{-(+)}$ mesons would be small because of the structure of their wave functions with respect to spin and color degree of freedom and that the $(qq)(\bar{q}\bar{q})$ mesons with * would couple to the ground-state $O^{-(+)}$ mesons much more weakly than the $[qq][\bar{q}\bar{q}]$ mesons. The above statement implies that $|k_s^*|\ll|k_a^*|$.

For values of the asymptotic ground-state-meson matrix elements of $\tilde{H}_w$, $\langle\pi^+|\tilde{H}_w|D_s^+\rangle$, etc., we have no information. Therefore we here treat the above matrix element in addition to the parameters mentioned above as adjustable parameters and look for overall fits to the observed branching ratios for two body decays, where the latter parameters are not perfectly free but restricted as discussed above, *i.e.*, $|k_g|\ll 1$, $|k_s^*|\ll|k_a^*|\ll 1$ and $|\delta|$, $|\delta_{exotic}|<90°$, where δ is given in Eq. (44) and δ_{exotic} is the strong phase in the exotic πK (or $\bar{K}$) channel, $\delta_{exotic}=\delta_3(\pi K)=\delta_3(\pi\bar{K})$.

We can reproduce remarkably well the observed branching ratios for Cabibbo-angle favored and suppressed two body decays of charm mesons, simultaneously, by taking reasonable values of parameters involved, i.e., very small values of parameters providing the residues of poles of glueball, $[qq][\bar{q}\bar{q}]$ and $(qq)(\bar{q}\bar{q})$ mesons, $|k_g|\sim 0.1$, $|k_a^*|\sim 0.1$ and $|k_s^*|\sim 0.01$, respectively, the predicted mass values of $[qq][\bar{q}\bar{q}]$ mesons in Ref. [19] but $(qq)(\bar{q}\bar{q})$ meson masses larger by 100 MeV than the predicted ones, and their widths, relatively narrower $\Gamma_{[qq][\bar{q}\bar{q}]}\sim 0.2$ GeV and rather broader $\Gamma_{(qq)(\bar{q}\bar{q})}\sim 0.4$ GeV. These are compatible with the discussions in Ref. [19]. For the phases of the ETC term relative to the surface term in the narrow width limit, which are expected to arise from contributions of multihadron intermediate states, rather large values ($\gtrsim 70°$) of these phases in Eq. (44) are favored while our result is not very sensitive to the value of the exotic phases $\delta_3(\pi K)$ and $\delta_3(\pi\bar{K})$ since contributions of the ETC terms into exotic final states are small [vanishing in the $SU_f(3)$ symmetry limit, i.e., $f_K=f_\pi$]. For the asymptotic ground-state-meson matrix element of H_w, Cabibbo-angle favored and suppressed ones separately satisfy the charm counterparts of the (asymptotic) $\Delta I=\frac{1}{2}$ rule in the strangeness changing hadronic weak interactions of K mesons, and are related to each other by using (asymptotic) $SU_f(3)$ symmetry as discussed in Appendix B. Therefore it is sufficient to treat one of them, for example, $\langle\pi^+|\tilde{H}_w|D_s^+\rangle$, as an adjustable parameter.

As an example, a typical result is shown in Table II in which we have taken the following values of unknown parameters; the asymptotic matrix element of $\tilde{H}_w$, $\langle \pi^+|\tilde{H}_w|D_s^+\rangle \simeq 0.0695\times 10^{-5}$ (GeV)^2 which is not very far from the factorized matrix element,

$$\langle \pi^+|H_w^{BSW}|D_s^+\rangle_{\rm fact} = \frac{G_F}{\sqrt{2}} U_{ud}U_{cs}\left\{\frac{m_\pi^2+m_{D_s}^2}{2}\right\} f_\pi f_{D_s} a_1 \simeq 0.048\times 10^{-5}\ \ ({\rm GeV})^2, \tag{45}$$

where $a_1=1.09$ with the leading order QCD corrections [2] and a recent lattice result on the decay constant, $f_{D_s}\simeq 216$ MeV [25] have been taken, the strong phases, $\delta_0(\pi\pi) = \delta_1(\pi\overline{K}) = \delta_1(\pi K) = \delta_3(\pi\overline{K}) = \delta_3(\pi K) = 85°$, and the residues of meson poles, $k_0=0.71$, $k_a^*=-0.20$, $k_s^*=-0.02$, $k_g=0.085$. It is seen, from Table II, that the naively factorized amplitudes for the so-called spectator decays which lead to too big rates for these decays interfere destructively with the exotic meson pole amplitudes as expected. The nonfactorizable amplitudes can supply significant contributions to the so-called color suppressed $D^0\to\pi^0\overline{K}^0$ and $D^0\to\pi^0\pi^0$ which are strongly suppressed in the naive factorization. To solve the well-known puzzle that the observed ratio of rates for $D^0\to K^+K^-$ to $D^0\to\pi^+\pi^-$ is around 2.5, a peculiar $SU_f(3)$ symmetry breaking may have to be introduced [29]. In the present case, such a symmetry breaking can be realized dominantly by different contributions of $[qq][\overline{qq}]$ meson poles, i.e., $m_D^2-m_{\hat{\sigma}*}^2 \gg m_D^2 - m_{\hat{\sigma}s*}^2$. It is known that the $D^0\to K^0\overline{K}^0$ decay is described by two annihilation diagrams (in the $m_W\to\infty$ limit) which cancel each other. Therefore both the factorized and the nonfactorizable amplitudes for this decay vanish. However the *s*-channel penguin can induce a pole contribution of scalar glueball or glue-rich scalar meson which leads to a reasonable size of rate for this decay in consistency with the $D^0\to\pi\pi$ and K^+K^- decays.

V. SUMMARY

Two body decays of charm mesons have been studied by describing their amplitude in terms of a sum of factorized and nonfactorizable ones. The former has been estimated by using the naive factorization in the BSW scheme while the latter has been calculated by using a hard pseudoscalar-meson approximation. It has been given by a sum of the equal-time commutator term and the surface term which contains pole contributions of various meson states, not only the ground-state $\{q\bar{q}\}_0$ mesons but also a glueball and exotic $\{qq\overline{qq}\}$ mesons with $J^{P(C)}=0^{+(+)}$.

In this way, a possible solution to the long standing problems in charm meson decays has been given, at least, qualitatively. Factorizable contributions which lead to too big branching ratios $B_{\rm fact}$'s for the spectator decays like $D^+\to\pi^+\pi^0$ and $\pi^+\overline{K}^0$ can interfere destructively with nonfactorizable ones and a sum of these two contributions can reproduce their observed values of branching ratios. The naive factorization also leads to too strong color suppression. However, nonfactorizable amplitudes can supply sufficient contributions to the color suppressed decays. The observed branching ratios for the mixed decays which have both contributions from the spectator and the color mismatched diagrams can be reproduced by interferences between the factorizable and the nonfactorizable contributions.

Two body decays of charm mesons into final states including η or η', in particular, decays into $\pi\eta$ and $\pi\eta'$ are interesting. However nonfactorizable contributions to these decays are complicated [30] because of the η-η' mixing and therefore these decays should be investigated separately.

For quasi-two-body $D\to VP$ decays, the mixing between isosinglet mesons in the final states is rather simple, i.e., the ω-ϕ mixing is known to be approximately ideal. However, in these decays, all the four types of four-quark $\{qq\overline{qq}\} = [qq][\overline{qq}]\oplus(qq)(\overline{qq})\oplus\{[qq](\overline{qq})\pm(qq)[\overline{qq}]\}$ mesons can contribute except for annihilation decays. Fortunately, the decays, $D^0\to\overline{K}^0\phi$, $D_s^+\to\pi^+\rho^0$ and $\pi^+\omega$, are described approximately by annihilation diagrams and four-quark meson contributions can be neglected. Here the first one has been observed with a substantial rate but the last two are suppressed. Since the factorized amplitudes for these decays are always suppressed [1], the observed rates should be dominantly supplied by nonfactorizable dynamical contributions of various hadrons in the present approach. The $D^0\to\overline{K}^0\phi$ amplitude is dominantly given by a sum of its ETC term describing contributions of multihadron intermediate states and the $\overline{K}^0$ meson pole amplitude [31]. The observed suppression of the $D_s^+\to\pi^+\rho^0$ suggests that a hybrid pseudoscalar meson (π_H) with a mass very close to m_{D_s} and with a rather narrow width exists (a recently observed pseudo scalar hybrid meson $\pi(1800)$ [32] may be assigned to this one although its mass $\sim$1.8 GeV is not sufficiently close to m_{D_s}) and that its pole contribution cancel a sum of the ETC term and the pion pole amplitude for this decay [33]. A strange component (K_H) belonging to the same multiplet as π_H does not disturb our good result on the $D^0\to\overline{K}^0\phi$ decay. A recent observation of the $D_s^+\to\pi^+\omega$ with a small rate [34], $B(D_s^+\to\omega\pi^+)=(2.7\pm1.2)\times10^{-3}$, suggests that an isotriplet hybrid meson with $J^{PC}=1^{+-}$ exists but is not very close to m_{D_s} (probably much lower than m_{D_s} as expected [16]) or couples very weakly to $\pi\omega$. Quasi-two-body decays of charm mesons including factorizable contributions will be investigated more extensively elsewhere.

Finally, hadronic weak interactions of charm mesons are intimately related to hadron spectroscopy. More informations of hadron spectroscopy will be needed to find a more quantitative solution to the puzzles in hadronic weak interactions.

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TABLE III. (a) Ideally mixed scalar $[qq][\overline{qq}]$ mesons with $C=0$. (b) Ideally mixed scalar $[qq][\overline{qq}]$ mesons with $C=1$.

S	$I=1$	$I=\frac{1}{2}$	$I=0$	Mass(GeV)
		(a)		
1		$\hat{\kappa}$		0.90
		$\hat{\kappa}^*$		1.60
0			$\hat{\sigma}$	0.65
			$\hat{\sigma}^*$	1.45
	$\hat{\delta}^s$		$\hat{\sigma}^s$	1.10
	$\hat{\delta}^{s*}$		$\hat{\sigma}^{s*}$	1.80
		(b)		
S	$I=1$	$I=\frac{1}{2}$	$I=0$	Mass(GeV)
1	$\hat{F}_I$		$\hat{F}_0$	(2.4)
	$\hat{F}_I^*$		$\hat{F}_0^*$	(3.2)
0		$\hat{D}$		(2.2)
		$\hat{D}^*$		(3.0)
		$\hat{D}^s$		(2.6)
		$\hat{D}^{s*}$		(3.4)

owes to them the decomposition of the effective weak Hamiltonian into factorizable and nonfactorizable parts.

APPENDIX A: FOUR-QUARK MESONS

Scalar four-quark mesons with charm quantum number $C=0$ and 1 are listed, where S and I denote the strangeness and isospin (see Tables III and IV). Particles with superscript s contain an $s\bar{s}$ pair. Mass values of noncharm ($C=0$) mesons are given in Ref. [19]. Masses of four-quark charm ($C=1$) mesons estimated by using the quark counting (with $m_u=m_d$, $\Delta m_s=m_s-m_u=0.2$ GeV, $\Delta m_c=m_c-m_u=1.5$ GeV and the predicted mass values of $C=0$ mesons mentioned above) are given between parentheses ( ). Particles containing double $(s\bar{s})$ pairs and $(c\bar{c})$ pair(s) are dropped since they do not contribute in this paper.

APPENDIX B: ASYMPTOTIC MATRIX ELEMENTS OF $\tilde{H}_w$

Constraints on asymptotic matrix elements of nonfactorizable weak Hamiltonian $\tilde{H}_w$ have been previously derived by using an algebraic method based on commutation relations between charges and currents (and hence the effective weak Hamiltonian) and, then, by counting all possible connected quark-line diagrams. We here summarize a part of them which are useful in this paper. We here describe the Cabibbo-angle favored $(\Delta C=-1,\Delta S=-1)$ and suppressed $(\Delta C=-1,\Delta S=0)$ weak Hamiltonians as $\tilde{H}_w(-,-)$ and $\tilde{H}_w(-,0)$, respectively, for convenience' sake.

(i) Constraints on asymptotic ground-state-meson matrix elements of $\tilde{H}_w$:

TABLE IV. (a) Ideally mixed scalar $(qq)(\overline{qq})$ mesons with $C=0$. (b) Ideally mixed scalar $(qq)(\overline{qq})$ mesons with $C=1$.

S	$I=2$	$I=\frac{3}{2}$	$I=1$	$I=\frac{1}{2}$	$I=0$	Mass(GeV)
			(a)			
2			E_{KK}			1.55
			E_{KK}^*			2.10
1		$E_{\pi K}$		C_K		1.35
		$E_{\pi K}^*$		C_K^*		1.95
				C_K^s		1.75
				C_K^{s*}		2.20
0	$E_{\pi\pi}$		C_π		C	1.15
	$E_{\pi\pi}^*$		C_π^*		C^*	1.80
			C_π^s		C^s	1.55
			C_π^{s*}		C^{s*}	2.10
			(b)			
S	$I=2$	$I=\frac{3}{2}$	$I=1$	$I=\frac{1}{2}$	$I=0$	Mass(GeV)
2				E_{KF}		(3.1)
				E_{KF}^*		(3.7)
1			$E_{\pi F}$		C_F	(2.9)
			$E_{\pi F}^*$		C_F^*	(3.5)
					C_F^s	(3.3)
					C_F^{s*}	(3.9)
0		$E_{\pi D}$		C_D		(2.7)
		$E_{\pi D}^*$		C_D^*		(3.3)
				C_D^s		(3.1)
				C_D^{s*}		(3.7)
-1			$E_{\bar{K}D}$			(2.9)
			$E_{\bar{K}D}^*$			(3.5)

$$\langle \bar{K}^0|\tilde{H}_w(-,-)|D^0\rangle = -\langle \pi^+|\tilde{H}_w(-,-)|D_s^+\rangle,$$

$$\begin{aligned}\langle \bar{K}^{*0}|\tilde{H}_w(-,-)|D^0\rangle &= -\langle \rho^+|\tilde{H}_w(-,-)|D_s^+\rangle \\ &= \langle \bar{K}^0|\tilde{H}_w(-,-)|D^{*0}\rangle \\ &= -\langle \pi^+|\tilde{H}_w(-,-)|D^{*0}\rangle \\ &= (\sqrt{2}k_0/h)\langle \bar{K}^0|\tilde{H}_w(-,-)|D^0\rangle,\end{aligned} \tag{B1}$$

$$\langle \pi^+|\tilde{H}_w(-,0)|D^+\rangle = -\langle K^+|\tilde{H}_w(-,0)|D_s^+\rangle,$$

$$\begin{aligned}\langle \pi^+|\tilde{H}_w(-,0)|D^{*+}\rangle &= -\langle K^+|\tilde{H}_w(-,0)|D_s^{*+}\rangle \\ &= \langle \rho^+|\tilde{H}_w(-,0)|D^+\rangle \\ &= -\langle K^{*+}|\tilde{H}_w(-,0)|D_s^+\rangle \\ &= (\sqrt{2}k_0/h)\langle \pi^+|\tilde{H}_w(-,0)|D^+\rangle,\end{aligned} \tag{B2}$$

where $k_0=\sqrt{\frac{1}{2}}h$ with $h=\langle\rho^0|A_{\pi^+}|\pi^-\rangle$ has been obtained by using an algebraic method [35]. It will be understood more intuitively since all the external states in the above matrix elements of $\tilde{H}_w$ are of helicity $=0$ states of the ground-state $\{q\bar{q}\}_0$ mesons and the difference of spins will be not very important in the IMF.

(ii) Constraints on asymptotic matrix elements of $\tilde{H}_w$ between the ground-state-meson and $[qq][\overline{qq}]$ meson states:

$$\langle\hat{\kappa}^{*0}|\tilde{H}_w(-,-)|D^0\rangle=-\langle\delta^{s*+}|\tilde{H}_w(-,-)|D_s^+\rangle=(k_a^*/2A_a^*)\langle\overline{K^0}|H_w(-,-)|D^0\rangle,$$

$$\begin{aligned}\langle\overline{K^0}|\tilde{H}_w(-,-)|\hat{D}^{*0}\rangle&=-\langle\overline{K^0}|\tilde{H}_w(-,-)|\hat{D}^{s*+}\rangle\\&=\sqrt{\frac{1}{2}}\langle\pi^+|\tilde{H}_w(-,-)|\hat{F}_I^{*+}\rangle\\&=-\sqrt{\frac{1}{2}}\langle\pi^0|\tilde{H}_w(-,-)|\hat{F}_I^{*0}\rangle\\&=(\tilde{k}_a^*/2A_a^*)\langle\overline{K^0}|\tilde{H}_w(-,-)|D^0\rangle,\end{aligned}$$

$$\langle\pi^+|\tilde{H}_w(-,-)|\hat{F}^{*+}\rangle=0, \tag{B3}$$

$$\begin{aligned}\langle\hat{\delta}^{s*+}|\tilde{H}_w(-,0)|D^+\rangle&=\sqrt{2}\langle\hat{\delta}^{s*0}|\tilde{H}_w(-,0)|D^0\rangle=\langle\hat{\sigma}^*|\tilde{H}_w(-,0)|D^0\rangle\\&=-\sqrt{2}\langle\hat{\sigma}^{s*}|\tilde{H}_w(-,0)|D^0\rangle=(k_a^*/2A_a^*)\langle\pi^+|\tilde{H}_w(-,0)|D^+\rangle,\end{aligned}$$

$$\begin{aligned}-\sqrt{2}\langle\pi^+|\tilde{H}_w(-,0)|\hat{D}^{*+}\rangle&=-2\langle\pi^0|\tilde{H}_w(-,0)|\hat{D}^{*0}\rangle=\langle K^+|\tilde{H}_w(-,0)|\hat{F}_I^{*+}\rangle\\&=\langle K^0|\tilde{H}_w(-,0)|\hat{F}_I^{*0}\rangle=(\tilde{k}_a^*/\sqrt{2}A_a^*)\langle\pi^+|\tilde{H}_w(-,0)|D^+\rangle,\end{aligned}$$

$$\langle K^+|\tilde{H}_w(-,0)|\hat{F}^{*+}\rangle=0, \tag{B4}$$

where A_a^* is the invariant matrix element of axial charge defined by $A_a^*=-\frac{1}{2}\langle\hat{\kappa}^{*+}|A_{\pi^+}|K^0\rangle$.

(iii) Constraints on asymptotic matrix elements of $\tilde{H}_w$ between the ground-state-meson and $(qq)(\overline{qq})$ meson states:

$$\begin{aligned}\sqrt{\frac{3}{2}}\langle E_{\pi K}^{*+}|\tilde{H}_w(-,-)|D^+\rangle&=\left(\frac{3}{\sqrt{2}}\right)\langle E_{\pi K}^{*0}|\tilde{H}_w(-,-)|D^0\rangle\\&=3\langle C_K^{*0}|\tilde{H}_w(-,-)|D^0\rangle=\sqrt{3}\langle C_\pi^{s*+}|\tilde{H}_w(-,-)|D_s^+\rangle\\&=(k_s^*/A_s^*)\langle\overline{K^0}|\tilde{H}_w(-,-)|D^0\rangle,\end{aligned}$$

$$\begin{aligned}-\sqrt{\frac{3}{2}}\langle\pi^+|\tilde{H}_w(-,-)|E_{\pi F}^{*+}\rangle&=\sqrt{\frac{3}{2}}\langle\pi^0|\tilde{H}_w(-,-)|E_{\pi F}^{*0}\rangle\\&=-\left(\frac{3}{\sqrt{2}}\right)\langle\overline{K^0}|\tilde{H}_w(-,-)|E_{\pi D}^{*0}\rangle=\sqrt{\frac{3}{2}}\langle K^-|\tilde{H}_w(-,-)|E_{\pi D}^{*-}\rangle\\&=3\langle\overline{K^0}|\tilde{H}_w(-,-)|C_D^{*0}\rangle=-\sqrt{3}\langle\overline{K^0}|\tilde{H}_w(-,-)|C_D^{s*0}\rangle\\&=-\sqrt{\frac{3}{2}}\langle K^+|\tilde{H}_w(-,-)|E_{KF}^{*+}\rangle=(\tilde{k}_s^*/A_s^*)\langle\overline{K^0}|\tilde{H}_w(-,-)|D^0\rangle,\end{aligned} \tag{B5}$$

$$\langle\pi^+|\tilde{H}_w(-,-)|C_F^{*+}\rangle=0.$$

$$\sqrt{3}\langle E^{*+}_{\pi\pi}|\tilde{H}_w(-,0)|D^+\rangle = \left(\frac{3}{\sqrt{2}}\right)\langle E^{*0}_{\pi\pi}|\tilde{H}_w(-,0)|D^0\rangle$$
$$= \sqrt{3}\langle C^{s*+}_{\pi}|\tilde{H}_w(-,0)|D^+\rangle = \sqrt{6}\langle C^{s*0}_{\pi}|\tilde{H}_w(-,0)|D^0\rangle$$
$$= -\sqrt{3}\langle C^{*+}_{\pi}|\tilde{H}_w(-,0)|D^+\rangle = -3\langle C^{*}|\tilde{H}_w(-,0)|D^0\rangle$$
$$= \sqrt{6}\langle C^{s*}|\tilde{H}_w(-,0)|D^0\rangle$$
$$= -\left(\frac{3}{\sqrt{2}}\right)\langle E^{*+}_{\pi K}|\tilde{H}_w(-,0)|D^+_s\rangle = -3\langle C^{*+}_{K}|\tilde{H}_w(-,0)|D^+_s\rangle$$
$$= (k^*_s/A^*_s)\langle \pi^+|\tilde{H}_w(-,0)|D^+\rangle, \tag{B6}$$

$$\left(\frac{3}{2\sqrt{2}}\right)\langle \pi^+|\tilde{H}_w(-,0)|E^{*+}_{\pi D}\rangle = 3\langle \pi^0|\tilde{H}_w(-,0)|E^{*0}_{\pi D}\rangle$$
$$= -\sqrt{\frac{3}{2}}\langle \pi^-|\tilde{H}_W(-,0)|E^{*-}_{\pi D}\rangle$$
$$= -3\langle \pi^+|\tilde{H}_w(-,0)|C^{*+}_{D}\rangle$$
$$= -3\sqrt{2}\langle \pi^0|\tilde{H}_w(-,0)|C^{*0}_{D}\rangle = (\tilde{k}^*_s/A^*_s)\langle \pi^+|\tilde{H}_w(-,0)|D^+\rangle,$$

$$\langle C^{*0}_{\pi}|\tilde{H}_w(-,0)|D^0\rangle = \langle K^+|\tilde{H}_w(-,0)|E^{*+}_{\pi F}\rangle = 0,$$

where A^*_s is the invariant matrix element of axial charge defined by $A^*_s = \langle C^{*+}_K|A_{\pi^+}|K^0\rangle$.

In the above, k^*_a and $\tilde{k}^*_a$(k^*_s and $\tilde{k}^*_s$) are not generally equal to each other. However, use of the commutation relation, $[[\tilde{H}_w(-,-),V_{D^-_s}],V_{D^-_s}] = [[\tilde{H}_w(-,-),V_{\pi^-}],V_{\pi^-}]$, with asymptotic $SU_f(4)$ symmetry [or an $SU_f(4)$ extension of the nonet symmetry in the flavor $SU_f(3)$ with respect to asymptotic matrix elements of charges] leads to $k^*_a = \tilde{k}^*_a$ (and $k^*_s = \tilde{k}^*_s$).

Matrix elements of $\tilde{H}_w(-,-)$ and $\tilde{H}_w(-,0)$ can be related to each other, for example, as

$$\langle \pi^+|\tilde{H}_w(-,-)|D^+_s\rangle = \frac{V_{cs}}{V_{cd}}\langle \pi^+|\tilde{H}_w(-,0)|D^+\rangle, \quad \text{etc.} \tag{B7}$$

by using the commutation relations $[\tilde{O}_\pm(-,-),V_{K^0}] = \tilde{O}_\pm(-,0)$ and $[\tilde{O}_\pm(-,0),V_{\bar{K}^0}] = 2\tilde{O}_\pm(-,-)$, where $\tilde{O}_\pm(-,-)$ and $\tilde{O}_\pm(-,0)$ are four quark operators in $\tilde{H}_w(-,-)$ and $\tilde{H}_w(-,0)$, respectively, and contributions of the QCD induced penguin term have been neglected.

APPENDIX C: NONFACTORIZABLE AMPLITUDES

We here list hard pseudoscalar-meson amplitudes as the nonfactorizable ones which include the ETC term describing continuum contributions and the surface term containing pole contributions of the ground-state $\{q\bar{q}\}_0$, scalar $[qq][\overline{qq}]$ and $(qq)(\overline{qq})$ mesons and a glueball. They are revised from the ones given in Ref. [8] in which the amplitudes involved some misprints and the insufficient parametrization of phases of the ETC terms.

(i) Cabibbo-angle-favored decays:

$$M_{\text{non-f}}(D^+\to\pi^+\bar{K}^0) \simeq -\frac{i}{\sqrt{2}f_\pi}\langle \pi^+|\tilde{H}_w|D^+_s\rangle\Bigg\{\left(1-\frac{f_\pi}{f_K}\right)e^{i\delta_3} + \left[\left(\frac{m_D^2-m_K^2}{m_{D^*}^2-m_K^2}\right) - \left(\frac{m_D^2-m_\pi^2}{m_{D^*_s}^2-m_\pi^2}\right)\left(\frac{f_\pi}{f_K}\right)\right]k_0$$
$$+\left[\left(\frac{m_D^2-m_K^2}{m_{\hat{D}^*}^2-m_K^2}\right) - \left(\frac{m_D^2-m_\pi^2}{m_{\hat{F}^*_I}^2-m_\pi^2}\right)\left(\frac{f_\pi}{f_K}\right)\right]k^*_a + \left[2\left(\frac{m_D^2-m_K^2}{m_D^2-m_{E^*_{\pi K}}^2}\right) + 2\left(\frac{m_D^2-m_\pi^2}{m_D^2-m_{E^*_{\pi K}}^2}\right)\left(\frac{f_\pi}{f_K}\right)\right.$$
$$\left.-\left(\frac{m_D^2-m_K^2}{m_{E^*_{\pi D}}^2-m_K^2}\right) - \left(\frac{m_D^2-m_\pi^2}{m_{E^*_{\pi F}}^2-m_\pi^2}\right)\left(\frac{f_\pi}{f_K}\right)\right]k^*_s\Bigg\}, \tag{C1}$$

$$M_{\text{non-f}}(D^0\to\pi^+K^-)\simeq\frac{i}{\sqrt{2}f_\pi}\langle\pi^+|\tilde{H}_w|D_s^+\rangle\Bigg\{-\frac{1}{3}\left(1-\frac{f_\pi}{f_K}\right)e^{i\delta_3(\pi\bar{K})}+\frac{1}{3}\left(4-\frac{f_\pi}{f_K}\right)e^{i\delta_1(\pi\bar{K})}$$
$$+\left[\left(\frac{m_D^2-m_K^2}{m_D^2-m_{K^*}^2}\right)-\left(\frac{m_D^2-m_\pi^2}{m_D^2-m_{K^*}^2}\right)\left(\frac{f_\pi}{f_K}\right)+\left(\frac{m_D^2-m_\pi^2}{m_{D_s^*}^2-m_\pi^2}\right)\left(\frac{f_\pi}{f_K}\right)\right]k_0$$
$$+\left[\left(\frac{m_D^2-m_K^2}{m_D^2-m_{\hat{\kappa}^*}^2}\right)+\left(\frac{m_D^2-m_\pi^2}{m_D^2-m_{\hat{\kappa}^*}^2}\right)\left(\frac{f_\pi}{f_K}\right)-\left(\frac{m_D^2-m_\pi^2}{m_{\hat{F}_I^*}^2-m_\pi^2}\right)\left(\frac{f_\pi}{f_K}\right)\right]k_a^*$$
$$-\left[\left(\frac{m_D^2-m_K^2}{m_D^2-m_{E_{\pi K}^*}^2}\right)+\left(\frac{m_D^2-m_\pi^2}{m_D^2-m_{E_{\pi K}^*}^2}\right)\left(\frac{f_\pi}{f_K}\right)-2\left(\frac{m_D^2-m_K^2}{m_{E_{\pi D}^*}^2-m_K^2}\right)+\left(\frac{m_D^2-m_\pi^2}{m_{E_{\pi F}^*}^2-m_\pi^2}\right)\left(\frac{f_\pi}{f_K}\right)\right]k_s^*\Bigg\},\qquad\text{(C2)}$$

$$M_{\text{non-f}}(D^0\to\pi^0\bar{K}^0)\simeq-\frac{i}{\sqrt{2}f_\pi}\langle\pi^+|\tilde{H}_w|D_s^+\rangle\sqrt{\frac{1}{2}}\Bigg\{\frac{2}{3}\left(1-\frac{f_\pi}{f_K}\right)e^{i\delta_3(\pi\bar{K})}+\frac{2}{3}\left(2-\frac{1}{2}\frac{f_\pi}{f_K}\right)e^{i\delta_1(\pi\bar{K})}$$
$$+\left[\left(\frac{m_D^2-m_K^2}{m_D^2-m_{K^*}^2}\right)-\left(\frac{m_D^2-m_\pi^2}{m_D^2-m_{K^*}^2}\right)\left(\frac{f_\pi}{f_K}\right)+\left(\frac{m_D^2-m_K^2}{m_{D^*}^2-m_K^2}\right)\right]k_0$$
$$+\left[\left(\frac{m_D^2-m_K^2}{m_D^2-m_{\hat{\kappa}^*}^2}\right)+\left(\frac{m_D^2-m_\pi^2}{m_D^2-m_{\hat{\kappa}^*}^2}\right)\left(\frac{f_\pi}{f_K}\right)+\left(\frac{m_D^2-m_K^2}{m_{\hat{D}^*}^2-m_K^2}\right)-2\left(\frac{m_D^2-m_\pi^2}{m_{\hat{F}_I^*}^2-m_\pi^2}\right)\left(\frac{f_\pi}{f_K}\right)\right]k_a^*$$
$$+\left[\left(\frac{m_D^2-m_K^2}{m_D^2-m_{E_{\pi K}^*}^2}\right)+\left(\frac{m_D^2-m_\pi^2}{m_D^2-m_{E_{\pi K}^*}^2}\right)\left(\frac{f_\pi}{f_K}\right)+\left(\frac{m_D^2-m_K^2}{m_{E_{\pi D}^*}^2-m_K^2}\right)-2\left(\frac{m_D^2-m_\pi^2}{m_{E_{\pi F}^*}^2-m_\pi^2}\right)\left(\frac{f_\pi}{f_K}\right)\right]k_s^*\Bigg\},\qquad\text{(C3)}$$

$$M_{\text{non-f}}(D_s^+\to K^+\bar{K}^0)\simeq-\frac{i}{\sqrt{2}f_K}\langle\pi^+|\tilde{H}_w|D_s^+\rangle\Bigg\{e^{i\delta_2(K\bar{K})}+\left(\frac{m_{D_s}^2-m_K^2}{m_{D^*}^2-m_K^2}\right)k_0$$
$$+\left[2\left(\frac{m_{D_s}^2-m_K^2}{m_{D_s}^2-m_{\hat{\delta}^{s*}}^2}\right)-\left(\frac{m_{D_s}^2-m_K^2}{m_{\hat{D}^{s*}}-m_K^2}\right)\right]k_a^*+2\left[\left(\frac{m_{D_s}^2-m_K^2}{m_{D_s}^2-m_{C_\pi^s}^2}\right)-\left(\frac{m_{D_s}^2-m_K^2}{m_{E_{KF}}^2-m_K^2}\right)\right]k_s^*\Bigg\}.\qquad\text{(C4)}$$

(ii) Cabibbo-angle suppressed decays:

$$M_{\text{non-f}}(D^0\to\pi^+\pi^-)\simeq\frac{i}{\sqrt{2}f_\pi}\langle\pi^+|\tilde{H}_w|D^+\rangle\Bigg\{e^{i\delta_0(\pi\pi)}+\left(\frac{m_D^2-m_\pi^2}{m_{D^*}^2-m_\pi^2}\right)k_0+\left[2\left(\frac{m_D^2-m_\pi^2}{m_D^2-m_{\hat{\sigma}^*}^2}\right)-\left(\frac{m_D^2-m_\pi^2}{m_{\hat{D}^*}^2-m_\pi^2}\right)\right]k_a^*$$
$$-\left[2\left(\frac{m_D^2-m_\pi^2}{m_D^2-m_{E_{\pi\pi}}^2}\right)-\left(\frac{m_D^2-m_\pi^2}{m_{E_{\pi D}}^2-m_\pi^2}\right)\right]k_s^*-\frac{2}{Z}\left(\frac{m_D^2-m_\pi^2}{m_D^2-m_{S^*}^2}\right)k_g\Bigg\},\qquad\text{(C5)}$$

$$M_{\text{non-f}}(D^0\to\pi^0\pi^0)\simeq\frac{i}{\sqrt{2}f_\pi}\langle\pi^+|\tilde{H}_w|D^+\rangle\sqrt{\frac{1}{2}}\Bigg\{e^{i\delta_0(\pi\pi)}+\left(\frac{m_D^2-m_\pi^2}{m_{D^*}^2-m_\pi^2}\right)k_0+\left[2\left(\frac{m_D^2-m_\pi^2}{m_D^2-m_{\hat{\sigma}^*}^2}\right)-\left(\frac{m_D^2-m_\pi^2}{m_{\hat{D}^*}^2-m_\pi^2}\right)\right]k_a^*$$
$$+\left[2\left(\frac{m_D^2-m_\pi^2}{m_D^2-m_{E_{\pi\pi}}^2}\right)-\left(\frac{m_D^2-m_\pi^2}{m_{E_{\pi D}}^2-m_\pi^2}\right)\right]k_s^*-\frac{2}{Z}\left(\frac{m_D^2-m_\pi^2}{m_D^2-m_{S^*}^2}\right)k_g\Bigg\},\qquad\text{(C6)}$$

$$M_{\text{non-f}}(D^+\to\pi^+\pi^0)\simeq\frac{i}{\sqrt{2}f_\pi}\langle\pi^+|\tilde{H}_w|D^+\rangle\Bigg\{\left[2\left(\frac{m_D^2-m_\pi^2}{m_D^2-m_{E_{\pi\pi}}^2}\right)-\left(\frac{m_D^2-m_\pi^2}{m_{E_{\pi D}}^2-m_\pi^2}\right)\right]\sqrt{2}k_s^*\Bigg\},\qquad\text{(C7)}$$

$$M_{\text{non-f}}(D^0\to K^+K^-)\simeq -\frac{i}{\sqrt{2}f_K}\langle\pi^+|\tilde{H}_w|D^+\rangle\Bigg\{e^{i\delta_2(K\bar{K})}+\left(\frac{m_D^2-m_K^2}{m_{D_s^*}^2-m_K^2}\right)k_0+\left[2\left(\frac{m_D^2-m_K^2}{m_D^2-m_{\hat{\sigma}^{s*}}^2}\right)-\frac{m_D^2-m_K^2}{m_{\hat{F}_I^*}^2-m_K^2}\right]k_a^*$$
$$-\left[2\left(\frac{m_D^2-m_K^2}{m_D^2-m_{C^{s*}}^2}\right)-2\left(\frac{m_D^2-m_K^2}{m_{E^*_{\bar{K}D}}^2-m_K^2}\right)+\left(\frac{m_D^2-m_K^2}{m_{C_F^*}^2-m_K^2}\right)\right]k_s^*+2\left(\frac{m_D^2-m_\pi^2}{m_D^2-m_{S^*}^2}\right)k_g\Bigg\}, \tag{C8}$$

$$M_{\text{non-f}}(D^0\to K^0\bar{K}^0)\simeq \frac{i}{\sqrt{2}f_K}\langle\pi^+|\tilde{H}_w|D^+\rangle\Bigg\{\left[\left(\frac{m_D^2-m_K^2}{m_{E_{\bar{K}D}}^2-m_K^2}\right)-2\left(\frac{m_D^2-m_K^2}{m_{E_{\pi F}}^2-m_K^2}\right)\right]k_s^*-2\left(\frac{m_D^2-m_K^2}{m_D^2-m_{S^*}^2}\right)k_g\Bigg\}, \tag{C9}$$

$$M_{\text{non-f}}(D^+\to K^+\bar{K}^0)\simeq -\frac{i}{\sqrt{2}f_K}\langle\pi^+|\tilde{H}_w|D^+\rangle\Bigg\{e^{i\delta_2(K\bar{K})}+\left(\frac{m_D^2-m_K^2}{m_{D_s^*}^2-m_K^2}\right)k_0+\left[2\left(\frac{m_D^2-m_K^2}{m_D^2-m_{\hat{\delta}^{s*}}^2}\right)-2\left(\frac{m_D^2-m_K^2}{m_{\hat{F}_0^*}^2-m_K^2}\right)\right.$$
$$\left.+\left(\frac{m_D^2-m_K^2}{m_{\hat{F}_I^*}^2-m_K^2}\right)\right]k_a^*-\left[2\left(\frac{m_D^2-m_K^2}{m_D^2-m_{C_\pi^{s*}}^2}\right)-\left(\frac{m_D^2-m_K^2}{m_{E^*_{\bar{K}D}}^2-m_K^2}\right)+\left(\frac{m_D^2-m_K^2}{m_{C_F^*}^2-m_K^2}\right)\right]k_s^*\Bigg\}, \tag{C10}$$

$$M_{\text{non-f}}(D_s^+\to \pi^+K^0)\simeq \frac{i}{\sqrt{2}f_\pi}\langle\pi^+|\tilde{H}_w|D^+\rangle\Bigg\{\left[2\left(\frac{f_\pi}{f_K}\right)-1\right]e^{i\delta_1(\pi K)}$$
$$-\left[\left(\frac{m_{D_s}^2-m_K^2}{m_{D_s}^2-m_{K*}^2}\right)-\left(\frac{m_{D_s}^2-m_\pi^2}{m_{D_s}^2-m_{K*}^2}\right)\left(\frac{f_\pi}{f_K}\right)-\left(\frac{m_{D_s}^2-m_\pi^2}{m_{D*}^2-m_\pi^2}\right)\left(\frac{f_\pi}{f_K}\right)\right]k_0$$
$$+\left[\left(\frac{m_{D_s}^2-m_K^2}{m_{D_s}^2-m_{\hat{\kappa}*}^2}\right)+\left(\frac{m_{D_s}^2-m_\pi^2}{m_{D_s}^2-m_{\hat{\kappa}*}^2}\right)\left(\frac{f_\pi}{f_K}\right)-2\left(\frac{m_{D_s}^2-m_K^2}{m_{\hat{F}_I^*}^2-m_K^2}\right)+\left(\frac{m_{D_s}^2-m_\pi^2}{m_{\hat{F}*}^2-m_\pi^2}\right)\left(\frac{f_\pi}{f_K}\right)\right]k_a^*$$
$$-\left[\left(\frac{m_{D_s}^2-m_K^2}{m_{D_s}^2-m_{E^*_{\pi K}}^2}\right)+\left(\frac{m_{D_s}^2-m_\pi^2}{m_{D_s}^2-m_{E^*_{\pi K}}^2}\right)\left(\frac{f_\pi}{f_K}\right)-2\left(\frac{m_{D_s}^2-m_K^2}{m_{E^*_{\pi F}}^2-m_K^2}\right)+\left(\frac{m_{D_s}^2-m_\pi^2}{m_{C_D^{s*}}^2-m_\pi^2}\right)\left(\frac{f_\pi}{f_K}\right)\right]k_s^*\Bigg\}, \tag{C11}$$

$$M_{\text{non-f}}(D_s^+\to \pi^0K^+)\simeq \frac{i}{\sqrt{2}f_\pi}\langle\pi^+|\tilde{H}_w|D^+\rangle\sqrt{\frac{1}{2}}\Bigg\{\left[2\left(\frac{f_\pi}{f_K}\right)-1\right]e^{i\delta_1(\pi K)}$$
$$-\left[\left(\frac{m_{D_s}^2-m_K^2}{m_{D_s}^2-m_{K*}^2}\right)-\left(\frac{m_{D_s}^2-m_\pi^2}{m_{D_s}^2-m_{K*}^2}\right)\left(\frac{f_\pi}{f_K}\right)-\left(\frac{m_{D_s}^2-m_\pi^2}{m_{D*}^2-m_\pi^2}\right)\left(\frac{f_\pi}{f_K}\right)\right]k_0$$
$$+\left[\left(\frac{m_{D_s}^2-m_K^2}{m_{D_s}^2-m_{\hat{\kappa}*}^2}\right)+\left(\frac{m_{D_s}^2-m_\pi^2}{m_{D_s}^2-m_{\hat{\kappa}*}^2}\right)\left(\frac{f_\pi}{f_K}\right)-2\left(\frac{m_{D_s}^2-m_K^2}{m_{\hat{F}_I^*}^2-m_K^2}\right)+\left(\frac{m_{D_s}^2-m_\pi^2}{m_{\hat{F}*}^2-m_\pi^2}\right)\left(\frac{f_\pi}{f_K}\right)\right]k_a^*$$
$$+\left[\left(\frac{m_{D_s}^2-m_K^2}{m_{D_s}^2-m_{E^*_{\pi K}}^2}\right)+\left(\frac{m_{D_s}^2-m_\pi^2}{m_{D_s}^2-m_{E^*_{\pi K}}^2}\right)\left(\frac{f_\pi}{f_K}\right)-\left(\frac{m_{D_s}^2-m_\pi^2}{m_{C_D^{s*}}^2-m_\pi^2}\right)\left(\frac{f_\pi}{f_K}\right)\right]k_s^*\Bigg\}. \tag{C12}$$

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