Embedding phenomenological quark-lepton mass matrices into SU(5) gauge models

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We construct phenomenological quark-lepton mass matrices based on an S_3 permutation symmetry in a manner fully compatible with SU(5) grand unification. The Higgs particles we need are **5**, **45** and their conjugates. The model gives a charge $-1/3$ quark versus a charged-lepton mass relation, and also a good fit to mass-mixing relations for the quark sector, as well as an attractive mixing pattern for the lepton sector, explaining a large mixing angle between ν_μ and ν_τ, and either a large or small $\nu_e-\nu_\mu$ mixing angle, depending on the choice of couplings, consistent with the currently accepted solutions to the solar neutrino problem. [S0556-2821(99)00813-9]

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Predictions of the quark-lepton mass spectrum are the least successful aspect of unified gauge theories. In a classical SU(5) grand unified theory (GUT) with the simplest choice of Higgs scalars, we obtain mass relations between the charge $-1/3$ quarks (referred to simply as down quarks) and charged leptons, $m_d=m_e$, $m_s=m_\mu$ and $m_b=m_\tau$. While the last of these relations agrees with experiment [1], the other two are far from reality. Georgi and Jarlskog (GJ) [2] have shown that the mass degeneracy of d/e and s/μ can be lifted by introducing **45**-plet Higgs particles to give at the GUT scale,

$$m_d=3m_e, \quad m_s=(1/3)m_\mu, \quad m_b=m_\tau, \tag{1}$$

in reasonable agreement with experiment. No prediction, however, has been given within the SU(5) framework to the charge-2/3 quark spectrum and hence to quark mixing which is usually described in terms of the Cabibbo-Kobayashi-Maskawa (CKM) matrix. By extending the unifying group to SO(10), one may relate the charge-2/3 quark (referred to as the up quark) mass to the Dirac mass of neutrinos and, also, the quark-mixing angles to the lepton-mixing angles. The prediction, however, is too tight, and does not seem to be consistent with experiment (e.g., [3]), or else the Higgs boson content becomes very complicated [4].

There is an empirical approach, successful in giving the quark mass-mixing relation at a phenomenological level. It usually assumes some *ad hoc* symmetry imposed on the mass matrices, as advocated first by Fritzsch [5]. Several simple representations of the quark mass matrices have been known that give mass-mixing relations in fair agreement with experiment [6]. This approach also successfully applies to understanding the relation between lepton mixing and neutrino mass [7] indicated by the solar neutrino problem [8] and by the atmospheric neutrino experiment [9]. In Ref. [10] we have proposed specific quark-lepton mass matrices based on $S(3)_L\times S(3)_R$ symmetry, or the so-called "democratic" principle [11], and small symmetry-breaking terms, which lead to attractive mixing patterns for neutrinos.

One of the problems with the "phenomenological matrix approach" is that consistency with the unified gauge model is unclear. If one straightforwardly imposes compatibility with a gauge model, on the other hand, we are usually led to unwanted relations for quark and lepton masses as remnants of prototype gauge models.

In this paper we show that there exists a successful matrix model based on the S_3 symmetry approach, which is fully compatible with SU(5) GUT's and at the same time gives predictions for quark and lepton masses and their mixings that agree with experiment [12]. The extension to supersymmetric SU(5) is trivial. The compatibility with SU(5) reduces the arbitrariness of the matrices, but also inspires us to modify them for the quark sector [13], which brings the predicted quark mixing in good agreement with experiment, especially for the (2,3) sector of flavor. For the neutrino sector all results presented in Ref. [10] are retained.

We start with the observation that the phenomenological matrices given in [13] (for quarks) and [10] (for leptons) are compatible with the Yukawa couplings in the presence of a **5**-plet Higgs particles of SU(5); the introduction of a **45**-plet, which is necessary to lift unwanted mass degeneracy, requires only a minimum modification of the symmetry-breaking matrix for the up-quark sector. We write the Higgs coupling

$$\begin{aligned}\mathcal{L}_{\rm Yukawa}=&\,Y(5_H)_{Uij}\mathbf{10}_i\mathbf{10}_j\mathbf{5}_H+Y(45_H)_{Uij}\mathbf{10}_i\mathbf{10}_j\mathbf{45}_H\\&+Y(5_H^*)_{D/Eij}\mathbf{5}_i^*\mathbf{10}_j\mathbf{5}_H^*+Y(45_H^*)_{D/Eij}\mathbf{5}_i^*\mathbf{10}_j\mathbf{45}_H^*\\&+\kappa(5_H5_H)_{\nu ij}\mathbf{5}_i^*\mathbf{5}_j^*\frac{\mathbf{5}_H\mathbf{5}_H}{M_R},\end{aligned}\tag{2}$$

where boldface symbols with subscript H denote

Higgs scalars of a specified multiplet, and those with subscript i or j (refer to flavor) are SU(5) matter fields, $\mathbf{5}_i=(d_1^c,d_2^c,d_3^c,e^-,\nu_e)_{Li}$ and $\mathbf{10}_j=(u_1^c,\ldots,u_1^c,\ldots,d_i^c,\ldots,e^+)_{Lj}$. We write the down-quark and charged-lepton sectors as D/E as they are unified. The last term of Eq. (2) is an effective neutrino coupling where the neutrino is taken to be of the Majorana type. We suppose that it is induced from heavy Majorana right-handed neutrinos N_i [SU(5) singlet] with mass M_R; hence $\mathbf{45}_H\mathbf{45}_H$ does not couple to give an effective mass in Eq. (2). We suppose that the main part of the mass term arises from $\mathbf{5}_H$; $\mathbf{45}_H$ gives only perturbations.

We postulate that the main part of the mass matrices is S_3 permutation symmetry invariant, i.e., $S_3^{\mathbf{10}}\times S_3^{\mathbf{5}}$ in our context (**10** and **5** refer to representations of fermions). The choice of the Yukawa coupling matrix $Y(5_H^*)_{D/Eij}$ is then unique:

$$Y=\begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix}. \qquad (3)$$

Two matrices are allowed for $Y(5_H)_{Uij}$ from the invariance under $S_3^{\mathbf{10}}\times S_3^{\mathbf{5}}$:

$$\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix},\quad \begin{bmatrix}0&1&1\\1&0&1\\1&1&0\end{bmatrix}. \qquad (4)$$

We take the 1:1 combination of the two to give form (3) for simplicity and for agreement with the democracy argument as in [11,13]. We also assume for $\kappa(5_H5_H)_{ij}$ the first matrix of (4) for the reason explained in [10]. We remark that it is easy to construct a seesaw mechanism consistent with this matrix [14].

We break $S_3^{\mathbf{10}}\times S_3^{\mathbf{5}}$ symmetry with an extra Yukawa coupling to $\mathbf{5}_H^*$ and a coupling to $\mathbf{45}_H^*$ for down-quark and charged-lepton sectors, and $\mathbf{5}_H$ and $\mathbf{45}_H$ for the up-quark sector. Namely, our mass matrices are

$$M_D=[Y(5_H^*)_D+aY(45_H^*)_D]\langle\phi_{5_H}^*\rangle, \qquad (5)$$

$$M_E=[Y(5_H^*)_D-3aY(45_H^*)_D]\langle\phi_{5_H}^*\rangle, \qquad (6)$$

$$M_U=[Y(5_H)_U+bY(45_H)_U]\langle\phi_{5_H}\rangle, \qquad (7)$$

where $a=\langle\phi_{45_H}^*\rangle_D/\langle\phi_{5_H}^*\rangle_D$ and $b=\langle\phi_{45_H}\rangle_U/\langle\phi_{5_H}\rangle_U$. Or, more explicitly, we write

$$M_D=\frac{K}{3}\left(\begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix}+\begin{bmatrix}-\epsilon_D&0&0\\0&\epsilon_D&0\\0&0&\delta_D\end{bmatrix}\right) \qquad (5')$$

for the down-quark sector. At this level the symmetry-breaking term of Eq. (5′) may be $\mathbf{5}^*$ or $\mathbf{45}^*$. The Higgs particle leading to symmetry breaking is either a new Higgs particle that develops a small vacuum expectation value or the same Higgs particle, giving the main mass term, but with small Yukawa couplings. We do not distinguish these two possibilities here.

For the charged-lepton sector,

$$M_E=\frac{K}{3}\left(\begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix}+\begin{bmatrix}-\epsilon_L&0&0\\0&\epsilon_L&0\\0&0&\delta_L\end{bmatrix}\right). \qquad (6')$$

If all symmetry-breaking terms in the second matrices of Eqs. (5′) and (6′) come from $\mathbf{5}_{\mathbf{H}}^*$ (i.e., $a=0$), we are led to unwanted down-quark–charged-lepton mass degeneracy. This problem can be avoided by assuming that δ elements are generated from the coupling to a $\mathbf{45}^*{}_H$-plet Higgs scalar, while ϵ terms are from $\mathbf{5}^*{}_H$. We have then

$$\epsilon_D=\epsilon_L=\epsilon,\quad \delta_D=-\delta_L/3=\delta, \qquad (8)$$

with real ϵ and δ [15]. We obtain

$$m_b\simeq K(1+\delta/9),\qquad m_s\simeq 2K\delta/9,$$
$$m_d\simeq -K\epsilon^2(1+\delta)/6\delta, \qquad (9)$$

$$m_\tau\simeq K(1-3\delta/9),\qquad m_\mu\simeq -2K\delta/3,$$
$$m_e\simeq K\epsilon^2(1-3\delta)/18\delta, \qquad (10)$$

after diagonalization of the matrices. The masses of Eqs. (9) and (10) satisfy the SU(5) GJ mass relation (1) when $\delta\ll 1$. Now all parameters in the down-quark and charged-lepton sectors are determined solely by e,μ, and τ masses.

If we take the same form as Eq. (5′) also for the up-quark sector [13], i.e., the symmetry-breaking term necessarily limited to $\mathbf{5}^*$, we are led to $V_{23}\simeq 0.015$ compared with experiment 0.036–0.042, whereas V_{12} is successfully predicted. We note, however, in our scheme (5)–(7) that the breaking terms for up- and down-quark sectors may not necessarily be the same form. In fact, a $\mathbf{45}_H$ Higgs representation, when coupled to $\mathbf{10}_i\times\mathbf{10}_j$, should give matrix elements different from Eq. (5′). That is, it gives rise to flavor off-diagonal elements, rather than diagonal due to the antisymmetric nature of $\mathbf{45}_H$. Therefore, we take, for Eq. (7),

$$M_U=\frac{K'}{3}\left(\begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix}+\begin{bmatrix}-\epsilon_U&0&\delta_U\\0&\epsilon_U&\delta_U\\-\delta_U&-\delta_U&0\end{bmatrix}\right), \qquad (11)$$

where δ_U comes from $\mathbf{45}_H$. Here, δ_U in the (1,3) matrix element may generally differ from that in (2,3), or simply it may even vanish. We take a parallelism with the down-quark sector that $\mathbf{45}_H^*$ couples to the third generation matrix elements: we found that the simple choice taken here gives resulting mixing angles in agreement with experiment.

For this mass matrix the mass eigenvalues are (δ_U and ϵ_U being real)

TABLE I. Input quark-lepton mass parameters and the prediction of our model (mass in MeV units). ''Expt.'' means the experimental vale expected at the GUT energy scale, as given by a two-loop analysis of [16]. The input masses given here are used to predict the CKM matrix (15) and the lepton mixing matrix (17).

	m_e	m_μ	m_τ	m_d	m_s	m_b	m_u	m_c	m_t
Expt.	0.325	68.60	1171	1.3 ± 0.2	$26.5^{+3.4}_{-3.7}$	1000 ± 40	1.0 ± 0.2	302^{+25}_{-27}	$129^{+196}_{-40}\times10^3$
Pred.	input	input	input	0.67	24.4	1120	input	input	input

$$m_t \simeq K'(1-2\delta_U^2/9), \quad m_c \simeq 2K'\delta_U^2/9, \quad m_u \simeq -K'\epsilon_U^2/6\delta_U^2, \tag{12}$$

and the resulting quark mixing angles read

$$V_{12} \simeq \sqrt{m_s/m_d} - \sqrt{m_u/m_c} \tag{13}$$

and

$$V_{23} \simeq m_s/\sqrt{2}m_b - \sqrt{m_c/m_t}. \tag{14}$$

The explicit analytical expression is more complicated for V_{13}, and we do not bother the reader by writing it here.

Let us now carry out a numerical analysis. The parameters for the down-quark and charged-lepton sectors are fixed by m_e, m_μ, m_τ to be $K=1.13$ GeV, $\epsilon=0.019$, and $\delta=-0.093$. Here, we should use masses at the GUT energy scale, and our input parameters are taken from the two-loop calculations of Ref. [16] as presented in Table I, where we also compare predicted down-quark masses with those expected at GUT mass scale. The agreement of the prediction with ''experiment'' is good, though m_d is somewhat smaller [17]. For the up-quark sector we take $K'=129$ GeV, $\epsilon_U=-0.00072$, and $\delta_U=-0.103$ to fit the central values of u,c,t masses in Table I.

The resulting mixing matrix is

$$|V_{\rm quark}| = \begin{bmatrix} 0.975 & 0.220 & 0.0086 \\ 0.220 & 0.975 & 0.036 \\ 0.016 & 0.033 & 0.999 \end{bmatrix}, \tag{15}$$

which is compared with the experimental values $|V_{12}| = 0.217–0.224, |V_{23}| = 0.036-0.042$, and $|V_{13}| = 0.002–0.005$. We emphasize that excellent agreement is achieved with experiment for V_{23}, whereas a factor of 2 disagreement has been taken to be a problem with the democratic matrix [13]. Our solution seems to be the simplest among others [6]. While the predicted V_{13} is somewhat larger than experiment, we do not pursue this problem further here [we can bring V_{13} in good agreement with experiment without creating a conflict with our principles, if ϵ_D is put on the (1,3) and (3,1) components, in addition to (1,1) and (2,2), of Eq. (5′)].

The Majorana neutrino coupling to a Higgs boson is taken freely from the other sectors, since therein $\mathbf{5}_i^*\mathbf{5}_j^*$ does not appear. The only requirement is that the matrix should respect $S_3^{\mathbf{10}}\times S_3^{\mathbf{5}}$ in its main part, and we take the first matrix of (4). We may take the $S_3^{\mathbf{10}}\times S_3^{\mathbf{5}}$-breaking terms to be

$$M_\nu^{(1)} = \begin{bmatrix} 0 & \epsilon_\nu & 0 \\ \epsilon_\nu & 0 & 0 \\ 0 & 0 & \epsilon'_\nu \end{bmatrix} \quad \text{or} \quad M_\nu^{(1)} = \begin{bmatrix} -\epsilon_\nu & 0 & 0 \\ 0 & \epsilon_\nu & 0 \\ 0 & 0 & \epsilon'_\nu \end{bmatrix}. \tag{16}$$

This is exactly the same as the model we have presented in [10], and the lepton (neutrino) mixing matrix reads

$$|V_l| = \begin{bmatrix} 0.998 & 0.045 & 0.050 \\ 0.066 & 0.613 & 0.787 \\ 0.005 & 0.789 & 0.614 \end{bmatrix} \quad \text{or}$$

$$|V_l| \simeq \begin{bmatrix} 0.737 & 0.674 & 0.050 \\ 0.386 & 0.479 & 0.787 \\ 0.555 & 0.562 & 0.614 \end{bmatrix}, \tag{17}$$

for the two alternatives given in Eqs. (16). The second mixing matrix virtually agrees with the one given by Fritzsch and Xing [7] derived from different principles. The mass eigenvalues are $K_\nu \pm \epsilon_\nu$ and $K_\nu + \epsilon'_\nu$, hence describing three neutrinos almost degenerate in mass. We note that the mixing matrices are predominantly determined by charged lepton masses; neutrino masses change the elements little. The case with Eqs. (17) describes large mixing for $\nu_\mu - \nu_\tau, \sin^2 2\theta_{\mu\tau} \simeq 0.93$, or the ν_μ survival fraction of 54% in the atmospheric neutrino experiment, and small mixing for $\nu_e - \nu_\mu (\sin^2 2\theta_{e\mu} \simeq 8\times10^{-3})$, consistent with the small angle solution of the Mikheyev-Smirnov-Wolfenstein (MSW) explanation for the solar neutrino problem [8]. The case with Eqs. (17) predicts large mixing for both $\nu_\mu - \nu_\tau$ and $\nu_e - \nu_\mu$, the latter being consistent with either the MSW large angle solution or mixing angle required in solar neutrino oscillations in vacuum, independent of the neutrino mass difference squared. The (1,3) elements of Eqs. (17) come out to be consistent with the CHOOZ experiment [18], which yields roughly $<0.2–0.3$ for this element when the mass of ν_τ is in the range Super-Kamiokande indicates.

It is interesting to note a hierarchical symmetry-breaking structure in our matrix. The **45** breaking terms are of the order of $\delta_U \simeq \delta_D \simeq 1/10$. The magnitude of **5** breaking represented by $\epsilon_U, \epsilon_D, \epsilon_\nu$, and ϵ'_ν is significantly smaller, and it is $<1/100$. The symmetry breaking in the neutrino sector, where only **5** breaking is relevant, is much smaller than that in the other sectors, and the neutrino masses appear as almost degenerate.

In conclusion, we have shown that one can successfully embed phenomenological quark-lepton mass matrices obtained by the democracy principle into the SU(5) scheme. The choice of matrices is not yet unique, but the interlocking of the two principles tightly constrains the allowed form, and reduces the number of parameters of the model; the matrix for charged leptons is no longer independent of that for down quarks. In addition we have found the matrices in better agreement with experiment after tweaking to reconcile with the SU(5) than other empirical matrices constructed without referring to gauge models. The approach reconciling empirical matrices with gauge theory as we have done in this paper might perhaps give a guiding principle to understand the Higgs sector of gauge theories, which otherwise appears too arbitrary.

Note added. we have learned that Mohapatra and Nussinov [19] have recently proposed a gauge model for S_3 symmetric mass matrices embedded into $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$.

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[14] Let us assume that heavy N_i transform as $\mathbf{5}^*$ under $S_3^{\mathbf{5}}$. Then the Yukawa coupling $Y(\mathbf{5}_H)_{Nij} N_i \mathbf{5}_j^* \mathbf{5}_H$ may be taken as $Y(\mathbf{5}_H)_{Nij} \propto \delta_{ij}$. We may also assume that the Majorana mass matrix $M_{Rij} N_i N_j$ takes an $S_3^{\mathbf{5}}$-invariant form $M_{Rij} \propto \delta_{ij}$. Integration of N_i then gives $\kappa(5_H 5_H)_{\nu ij} \propto \delta_{ij}$. If M_{Rij} is given by $\Sigma(\mathbf{24})^2/M_{\rm pl}$, we have $M_{Rij} \sim 10^{14}$ GeV for the SUSY unification scale $\langle \Sigma(\mathbf{24}) \rangle \sim 2\times 10^{16}$ GeV; in this case we expect $m_\nu \sim O(1\ \text{eV})$.

[15] Our matrix elements are assumed to be all real for simplicity of the model. It is possible to introduce phases. In fact, there is a well-known mismatch between down quark masses and the Cabibbo angle with natural relation (14): if experimental values are put into the right-hand side of Eq. (14), V_{12} somewhat deviates from experiment, and this problem is usually cured by introducing phases (see Fritzsch [5]). Our model also inherits this problem. If we introduce phases for all matrix elements, we can give the right value for m_d and also fit V_{12}. V_{13} changes very little and V_{23} can be larger than our prediction (discussed below) by up to 50% (hence the real phase is favored for δ). In this paper we do not dare to pursue precise agreement with experiment, keeping the simplicity of the model, since the introduction of phases modifies the model only at a quantitative level.

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