Electromagnetic properties of hadrons via the *u*-*d* mass difference and direct photon exchange

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We demonstrate that a u-d quark mass difference (which we estimate to be about 4 MeV) within quantum loops can reproduce the effects of the Coleman-Glashow electromagnetic tadpole operator. [S0556-2821(99)04109-0]

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I. INTRODUCTION

It is now well understood that the neutron is heavier than the proton because the down quark is heavier than the up quark, which more than compensates the positive electromagnetic charge energy of the proton. An earlier, alternative interpretation of the idea is due to Coleman and Glashow [1] who express this as an electromagnetic Hamiltonian operator

$$H_{em}=H_{JJ}+H^3_{tad}, \tag{1}$$

which contains an intrinsic ''tadpole'' term transforming as the third component of isospin [λ_3 in an SU(3) context]; the H_{JJ} piece corresponds to the first order in the α contribution due to photon exchange, which can be estimated in various ways and is predominantly positive. By using the group theoretical properties of H^3_{tad} one can thereby correlate many mass differences and magnetic moments of hadrons within a multiplet.

In this paper we shall try to ascribe the tadpole term to the u-d constituent quark mass difference and therefore obtain a more dynamical picture of its origin. This will involve evaluating the *differences* of certain ''bubble graphs'' plus vacuum ''tadpole'' diagrams for hadron self-energies, in addition to virtual photon emission or absorption graphs. Apart from the vacuum diagrams, these bubble graph differences contain logarithmic infinities, requiring regularization. (The results are actually insensitive to the regularization and do not depend on momentum routing.) In any case, as needed, we will at various points make use of simple quark chiral symmetry ideas [2], such as the Goldberger-Treiman relation (for $f_\pi\simeq 93$ MeV and $g=2\pi/\sqrt{3}$)

$$f_\pi g=(m_u+m_d)/2\equiv\hat{m}\simeq 337\ \text{MeV}, \tag{2}$$

which correspond to ''gap equations'' in a dynamical context, in order to handle logarithmic infinities which sometimes arise. [The magnitude (2) also arises in the context of magnetic moments [3].] We will see that this quark interpretation of the Coleman-Glashow tadpole works quite well and provides a more fundamental description of the electromagnetic properties of hadrons.

However, before starting, it is as well to remind ourselves about the sort of magnitudes that we are chasing. The p-n mass difference of about -1.3 MeV gives a first indication: since virtual photon exchange provides the positive terms

$$\langle p|H_{JJ}|p\rangle\simeq 1.2\ \text{MeV},\quad \langle n|H_{JJ}|n\rangle\simeq 0, \tag{3}$$

we may deduce [4]

$$\langle p|H^3_{tad}|p\rangle-\langle n|H^3_{tad}|n\rangle\simeq -2.5\ \text{MeV}. \tag{4}$$

The naive conclusion is therefore that $m_d-m_u\sim 3$ MeV. In the same vein, the observed $\Sigma^- - \Sigma^+$ mass splitting of 8 MeV suggests that

$$m_d-m_u\simeq(m_{\Sigma^-}-m_{\Sigma^+})/2\simeq 4\ \text{MeV}, \tag{5}$$

since the H_{JJ} contribution, due alone to magnetic moments, is small; it also accords with the SU(3) symmetry prediction [5] $(H^3_{tad})_{\Sigma^+-\Sigma^-}\simeq 3(H^3_{tad})_{p-n}\simeq -7.5$ MeV. Finally, we may extract a crude value for m_d-m_u from the observed K^0-K^+ mass difference, since both $d\bar{s}$ and $u\bar{s}$ have the same quark structure; thus,

$$m_d-m_u\simeq m_{K^0}-m_{K^+}\simeq 4\ \text{MeV}.$$

We will review expectations for the u-d mass difference in the next section. Then in Sec. III we shall study pseudoscalar meson electromagnetic mass splittings from the perspective of QED and quark loop diagrams. A similar approach is applied to vector mesons in Sec. IV, with particular emphasis on $\Delta I=1$, ω-ρ^0 mixing. We will find that all cases associated with logarithmic divergences are governed by a universal (tadpole) scale of about $-5200\ \text{MeV}^2$, which is determined by a d-u mass difference of about 4 MeV. There we also offer some comparisons with other group-theoretical approaches.

II. CONSTITUENT QUARK *D*-*U* MASS DIFFERENCE

The nonstrange and strange constituent quark masses are very roughly given by $\hat{m}\simeq m_\rho/2\sim 380$ MeV, $m_s\simeq m_\phi/2\sim 510$ MeV, respectively. Similar results follow by examining the baryon masses but more accurate scales are found from the Goldberger-Treiman relation (2) and its analogue $f_K g=(m_s+\hat{m})/2$. Since experiments [6] give $f_K/f_\pi\simeq 1.22$, one deduces that $m_s/\hat{m}\simeq 1.44$, leading to the mass scales

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(within a few MeV), $\hat{m}\simeq 340$ MeV, $m_s\simeq 490$ MeV.

To obtain the still smaller u-d mass difference, it is necessary to look at electromagnetic (EM) mass differences between baryons, as indicated above, which suggest that $m_d-m_u\sim 4$ MeV. A similar inference can be drawn from the pseudoscalar mass differences: first, we note that H_{tad} does not contribute to the π^+-π^0 mass difference; second, we take it that the H_{JJ} term gives a comparable magnitude both to π^+-π^0 and K^+-K^0 squared masses. It follows that the tadpole contribution to $m_{K^+}^2-m_{\pi^+}^2$ is determined by [4] $\Delta m_K^2-\Delta m_\pi^2\simeq -5200$ MeV^2. Because we are ascribing this part to the underlying quarks, we may roughly equate it to (as we shall confirm in different guise in the next section) $2(m_u-m_d)m_K$, whereupon one deduces $m_d-m_u\sim 4$ MeV. Dashen's EM PCAC (partial conservation of axial vector coupling) theorems [7] are in conformity with this result.

Another source of information is the amazingly accurate hyperfine splitting quark model [8] which predicts the constituent mass difference $m_d-m_u\simeq 6$ MeV, but in conjunction with a (baryon) quark mass scale of $\hat{m}\simeq 363$ MeV. Since this is rather greater than the previous scale by a factor of about 10%, one is inclined to reduce the Isgur value of m_d-m_u to 5.5 MeV or less. The Particle Data Group tables [6] provide yet another source, but for the current quark mass difference; they say that $(m_d-m_u)_{current}$ hovers around 5 MeV with an error of about 2 MeV. A more global approach due to Lichtenberg [9] finds that the constituent d-u quark mass difference exceeds 4.1 MeV. Given all these clues, we expect that constituent mass difference $m_d-m_u\simeq 4$–5.5 MeV will be fairly close to the truth.

Let us use the magnitude of H_{tad}^3 to come to some conclusions about the magnitude of the H_{JJ} piece for various baryons and thereby estimate the strong interaction cutoff scale. The group-theoretical factors ensuing from the Coleman-Glashow operator H_{tad}^3 and the overall scale, estimated in Ref. [10], provide the figures

$$(H_{tad}^3)_{p-n}\simeq \frac{2}{3}(H_{tad}^3)_{\Sigma^0-\Sigma^-}\simeq \frac{1}{3}(H_{tad}^3)_{\Sigma^+-\Sigma^-} \simeq \frac{1}{2}(H_{tad}^3)_{\Xi^0-\Xi^-}\simeq -2.5\ \text{MeV}. \tag{6}$$

Concentrating on the proton, we deduce that $(H_{JJ})_p\simeq 1.2$ MeV in order to give the observed $n-p$ mass difference. Thus, neglecting magnetic moment contributions to the fermion self-energy (which never exceed about 0.3 MeV) and using the standard QED result

$$(H_{JJ})_p\simeq \Sigma(m)=\frac{3\alpha}{2\pi}\left[\ln\left(\frac{\Lambda}{m_p}\right)+\frac{1}{4}\right]\simeq 1.2\ \text{MeV}, \tag{7}$$

TABLE I. SU(2) mass splittings for octet baryons, in MeV.

Baryons	H_{JJ}	H_{tad}^3	Total	Data
m_p-m_n	1.2	-2.5	-1.3	-1.29
$m_{\Sigma^0}-m_{\Sigma^-}$	-1.0	-3.8	-4.8	-4.9 ± 0.1
$m_{\Sigma^+}-m_{\Sigma^-}$	-0.3	-7.5	-7.8	-8.1 ± 0.1
$m_{\Xi^0}-m_{\Xi^-}$	-1.1	-5.0	-6.1	-6.4 ± 0.6

we require a strong interaction cutoff $\Lambda\simeq 1.05$ GeV. This is a reasonable magnitude since it comes from vector-meson-dominated intermediate states and we will be adopting similar values subsequently to estimate the photon exchange contributions to meson masses. But in any case the picture looks rather good for baryons when one also includes [10] the smaller magnetic contributions, as one can see from Table I.

III. PSEUDOSCALAR MESON MASS DIFFERENCES

We ascribe the SU(2) differences to photon exchange and the d-u quark mass disparity in intermediate loops in order to see if we can arrive at the same sort of estimate as the group-theoretical tadpole method. Turning first to the pions, it is readily established that the quark loop diagrams give (see Fig. 1), at zero external momentum [11],

$$(m_{\pi^+}^2-m_{\pi^0}^2)_{q\ loops}=-8iN_cg^2\int \bar{d}^4p\frac{p^2-m_um_d}{(p^2-m_u^2)(p^2-m_d^2)} +4iN_cg^2\int \bar{d}^4p\left[\frac{1}{p^2-m_u^2}+\frac{1}{p^2-m_d^2}\right]. \tag{8}$$

[We have dropped the isospin-1 $a_0(980)$ vacuum tadpole contribution, because a_0 does not couple to pion pairs.] The result (8) equals $(m_d-m_u)^2$ and may be neglected, in agreement with group-theoretical symmetry arguments [1]. Not so the photon exchange contribution [12] to the charged pion, which is quadratically divergent in QED and, via dispersion relations, may be estimated to equal

$$(m_{\pi^\pm}^2)_{JJ}\simeq \frac{\alpha}{2\pi}\{\Lambda^2+m_\pi^2[\ln(\Lambda^2/m_\pi^2)-1]\}. \tag{9}$$

Since it is identified with $\Delta m_\pi^2\simeq 1260$ MeV^2 experimentally, we require a cutoff $\Lambda\simeq 1.02$ GeV, rather close to the $p-n$ cutoff, used earlier. This is an encouraging sign.

Next we consider the kaons. Here, neither the quark bubble nor the a_0 tadpole graph is negligible and we need, as ever, the photon exchange contribution

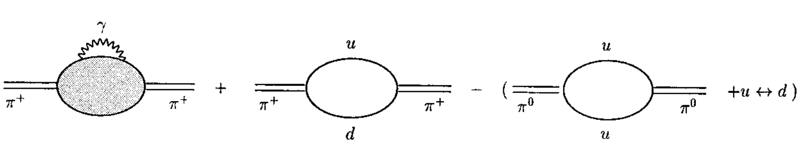

FIG. 1. Quark loop plus photon exchange contributions to $m_{\pi^+}-m_{\pi^-}$.

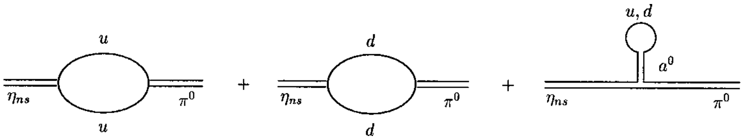

FIG. 2. Quark loop contributions to the $\pi-\eta_{ns}$ transition element.

$$(m^2_{K^\pm})_{JJ}\simeq\frac{\alpha}{2\pi}\{\Lambda^2+m_K^2[\ln(\Lambda^2/m_K^2)-1]\}\simeq 1420\ \mathrm{MeV}^2, \tag{10}$$

again using a cutoff of about 1.05 GeV. The near agreement between Eqs. (9) and (10) is compatible with Dashen's PCAC result [7]. The difference between quark loop contributions (see Fig. 2) is

$$\begin{aligned}(m^2_{K^+}-m^2_{K^0})_{q\ loops}&\\ =-8iN_cg^2\int&\frac{\bar{d}^4p}{(p+k)^2-m_s^2}\\ &\times\left[\frac{p\cdot(p+k)-m_sm_u}{p^2-m_u^2}-\frac{p\cdot(p+k)-m_sm_d}{p^2-m_d^2}\right]\\ &+\frac{8igg_{a_0KK}N_c}{m^2_{a_0}}\int\bar{d}^4p\left[\frac{m_u}{p^2-m_u^2}-\frac{m_d}{p^2-m_d^2}\right].\end{aligned} \tag{11}$$

Using the logarithmic divergent gap equation for the $s\bar{u}$ system [13], namely, $f_Kg=(m_s+\hat{m})/2$ or

$$1=-4iN_cg^2\int\frac{\bar{d}^4p}{(p^2-m_s^2)(p^2-\hat{m}^2)}, \tag{12}$$

and the regularization-insensitive identity [13]

$$\int\frac{\bar{d}^4p}{p^2-m^2}=\int\frac{m^2\bar{d}^4p}{(p^2-m^2)^2}+\frac{im^2}{16\pi^2},$$

we are able to estimate the zero-momentum finite expression [14]

$$\begin{aligned}(\Delta m_K^2)_{q\ loops}&\equiv(m^2_{K^+}-m^2_{K^0})_{q\ loops}\\ &\simeq(m_u-m_d)[2(2\hat{m}-m_s)\\ &\quad-3\hat{m}^2(\hat{m}-m_s)/2(\hat{m}^2+m_s^2)]\\ &\quad+8g_{a_0KK}(m_u-m_d)\hat{m}^2/gm^2_{a_0},\end{aligned} \tag{13}$$

remembering that $g^2N_c=4\pi^2$. Since the experimental magnitude is $(\Delta m_K^2)=(\Delta m_K^2)_{JJ}+(\Delta m_K^2)_{q\ loops}\simeq-3960\ \mathrm{MeV}^2$, we require that $(\Delta m_K^2)_{q\ loops}\simeq-5380\ \mathrm{MeV}^2$. To make further progress we require some knowledge about g_{a_0KK}. On the one hand, we have the chiral estimate [13] $g_{a_0KK}=(m^2_{a_0}-m_K^2)/2f_K\simeq 3140$ MeV, and on the other hand, U(3) symmetry says that $g_{a_0KK}=g_{\sigma\pi\pi}/2=m_\sigma^2/2f_\pi\simeq 2550$ MeV; probably the true value lies somewhere in between, say, $g_{a_0KK}\simeq 2700$ MeV with a possible error of 200 MeV. Substituting this into Eq. (13), we are led to the value $m_d-m_u\simeq 5380/1320\simeq 4.1$ MeV, which is quite reasonable.

The same idea can be used to estimate the nonstrange electromagnetic transition amplitude $M_{\pi\eta_{ns}}=\langle\pi^0|H_{em}|\eta_{ns}\rangle$. In this case we do not have to worry about photon exchange and the surviving one-loop diagrams are given in Fig. 3. Here one finds

$$\begin{aligned}M_{\pi\eta_{ns}}=&-4ig^2N_c\int\bar{d}^4p\left[\frac{1}{p^2-m_u^2}-\frac{1}{p^2-m_d^2}\right]\\ &-\frac{8iN_cgg_{a_0\pi\eta_{ns}}}{m^2_{a_0}}\int\bar{d}^4p\left[\frac{m_u}{p^2-m_u^2}-\frac{m_d}{p^2-m_d^2}\right].\end{aligned}$$

We are on much firmer ground now if we claim that $g_{a_0\pi\eta_{ns}}=2g_{a_0KK}=m_\sigma^2/f_\pi\simeq 5140$ MeV, since this just relies on U(2) symmetry. Using the gap equation and $m_\sigma=2\hat{m}$, we arrive at the clean result $M_{\pi\eta_{ns}}=2\hat{m}(m_u-m_d)+16\hat{m}^3(m_u$

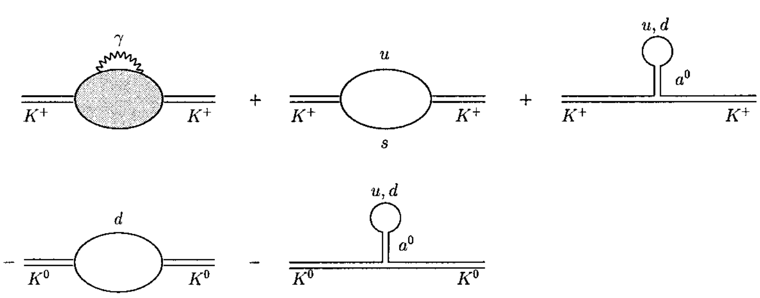

FIG. 3. Quark loop plus photon exchange contributions to $m_{K^0}-m_{K^+}$.

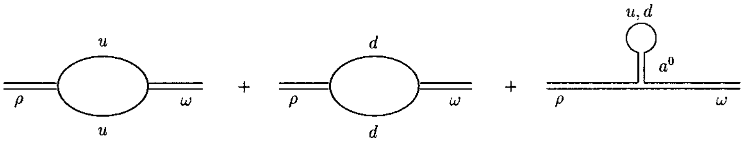

FIG. 4. Quark loop contributions to the $\rho-\omega$ transition element.

$-m_d)/m_{a_0}^2 \simeq 1310(m_u-m_d)$, and thereby can predict the characteristic value $M_{\pi\eta_{ns}} \simeq -5240\ (\mathrm{MeV})^2$, for $m_d-m_u \simeq 4$ MeV.

IV. VECTOR MESON MASS DIFFERENCES

Next we turn to the vector mesons and the all-important coupling between the $I=0,\omega$ and the $I=1,\rho$. The calculations are even simpler in this case. First, we have the photon exchange term, which comes out cleanly near the vector mass shell $k^2=m_\rho m_\omega$ as

$$(H_{JJ})_{\omega\rho}(k)=e^2k^2/(g_\rho g_\omega)\simeq 640\ \mathrm{MeV}^2, \tag{14}$$

since the leptonic rates give $g_\rho/e \simeq 16.6$ and $g_\omega/e \simeq 56.3$ via vector meson dominance. Then we have the QED-like bubble polarization tensor term

$$\Pi_{\mu\nu}=(-k^2g_{\mu\nu}+k_\mu k_\nu)\Pi(k^2,m_q^2)g_\rho^2/4, \tag{15}$$

where, in a first approximation, we have used the U(2) symmetry coupling constants, $g_{\rho^0uu}=-g_{\rho^0dd}=g_{\omega uu}=g_{\omega dd}=g_\rho$. The polarization function, being [15]

$$\Pi(k^2,m^2)=-8iN_c\int_0^1 d\alpha\int\frac{\alpha(1-\alpha)\overline{d}^4p}{[p^2-m^2+k^2\alpha(1-\alpha)]^2}, \tag{16}$$

the difference between the u and d quark contributions is easily found, because it is finite. At $k^2=m_\rho^2\simeq m_\omega^2$ one obtains

$$\begin{aligned}\Pi(k^2,m_u^2)-\Pi(k^2,m_d^2) &= -16iN_c(m_d^2-m_u^2)\int_0^1\alpha(1-\alpha)d\alpha\\ &\quad\times\int\frac{\overline{d}^4p}{[p^2+k^2\alpha(1-\alpha)]^3}=\frac{N_c(m_d^2-m_u^2)}{2\pi^2k^2}.\end{aligned} \tag{17}$$

Hence, via the inverse propagator $\Delta^{-1}_{\mu\nu}(k)=k_\mu k_\nu-k^2g_{\mu\nu}+\Pi_{\mu\nu}(k)=-g_{\mu\nu}(k^2-m^2)+k_\mu k_\nu$ terms, one sees that $-k^2\Pi(k^2)$ has the significance of a squared mass. To this bubble contribution must be added the a_0 tadpole contribution (see Fig. 4 for the sum of all graphs), which equals

$$-\frac{4iN_cg_{a_0\omega\rho}}{m_{a_0}^2}\int\overline{d}^4p\left[\frac{m_u}{p^2-m_u^2}-\frac{m_d}{p^2-m_d^2}\right].$$

Let us invoke SU(4) spin-flavor symmetry and set $g_{a_0\pi\eta_{ns}}=g_{a_0\rho\omega}$ in order to progress the evaluation; in this manner we estimate the a_0 tadpole contribution to equal that of the $\pi-\eta_{ns}$ transition, namely, $16\hat{m}^3(m_u-m_d)/m_{a_0}^2$. Altogether, one deduces

$$(H_{q\ loops})_{\omega\rho}=g_\rho^2N_c(m_u^2-m_d^2)/8\pi^2+16\hat{m}^3(m_u-m_d)/m_{a_0}^2. \tag{18}$$

The right-hand side can be estimated by using the experimental value $g_\rho=5.03$ [2,16]; one finds that both terms in Eq. (18) contribute almost equally, the result being $(H_{q\ loops})_{\omega\rho}\simeq 1310(m_u-m_d)\simeq -5240\ \mathrm{MeV}^2$, for $m_d-m_u \simeq 4$ MeV. Adding it to the photon exchange contribution, we end up with

$$(H_{em})_{\omega\rho}=(H_{JJ})_{\omega\rho}+(H_{q\ loops})_{\omega\rho}\simeq -4600\ \mathrm{MeV}^2. \tag{19}$$

This agrees reasonably well with the magnitude $(H_{em})_{\omega\rho}\simeq -4520\ \mathrm{MeV}^2$, derived experimentally from the $\omega\to\rho^0\to 2\pi$ rate [17,18].

We can extend these ideas to other mixings like $\rho J/\psi$, $\rho\Upsilon$, but such calculations are sensitive to the amount of admixture of nonstrange mesons in the heavy meson states. Indeed the experimental rates for J/ψ and Υ to two pions and two kaons directly measure the admixtures—and they are very small. Thus we are unable to test properly our quark loop hypothesis in those cases.

The calculations above confirm that one can view the Coleman-Glashow tadpole piece as equivalent to a d-u mass difference of about 4 MeV in the context of a quark model. In this way we achieve a more fundamental picture of the electromagnetic properties of hadrons, when we combine the quark mass difference effect with standard photon emission and absorption.

To conclude this paper, we compare ρ-ω mixing with other methods of estimation. First, there is the method based on the Coleman-Glashow tadpole [19]:

$$(H^3_{tad})_{\omega\rho}=-\langle 0|H^3_{tad}|a_0^0\rangle g_{a_0\omega\rho^0}/m_{a_0}^2\equiv -f_{a_0}m_{a_0}^2/f_\pi. \tag{20}$$

For $f_{a_0}\simeq 0.5$ MeV, this gives $(H^3_{tad})_{\omega\rho}\simeq -5200\ \mathrm{MeV}^2$. Second, one may use SU(3) symmetry to connect this matrix element with the K^* masses:

$$(H^3_{tad})_{\omega\rho}\simeq \Delta m_{K^*}^2-\Delta m_\rho^2\simeq -5130\ \mathrm{MeV}^2. \tag{21}$$

Third, one can apply fully fledged SU(6) symmetry to equate Eq. (21) with

$$(H^3_{tad})_{\omega\rho}\simeq \Delta m_K^2-\Delta m_\pi^2\simeq -5220\ \mathrm{MeV}^2. \tag{22}$$

In no case is there any striking discrepancy. It might be possible also to generalize the argument to heavier mesons such as D, D_s, D_c, and B; this would require strong faith in mass extrapolations and we have not been brave enough to try that. In this connection it is worth recalling the result of Lane and Weinberg [20], based on phenomenological chiral Lagrangians, which yields $m_d - m_u \sim 4.5$ MeV and which predicts $m_{D^+} - m_{D^0} \simeq 6.7$ MeV.

As a parting note, observe that whether we use the Coleman-Glashow tadpole or regard it as the effect of an EM quark loop, one is always making contact with data in the (low-energy) s channel. Alternatively, Harari [21] invoked crossing and duality to convert the view into the (high-energy) t channel. By studying superconvergent relations, he identified the $\Delta I = 1$ tadpole with a subtraction constant in the t channel, associated with the ρ trajectory, and thereby justified the Coleman-Glashow procedure.

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