Note on the field theory of neutrino mixing

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The possibility of constructing the Hilbert space of definite flavor neutrino states, especially a one-flavor neutrino state, is investigated in the theory with flavor-mixing mass terms in the Lagrangian. Reviewing the work of Blasone and Vitiello in detail, we clarify that even if we construct the Hilbert space of a definite flavor neutrino, the oscillation probabilities of neutrinos derived according to the usual way include arbitrary mass parameters. We examine the structure of the flavor neutrino propagator and show that the physical poles of the propagator coincide with mass eigenvalues of the mass matrix in the Lagrangian irrespective of such arbitrary parameters. This gives a possible way of escaping the arbitrariness. [S0556-2821(99)03809-6]

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I. INTRODUCTION

Since Pontecorvo [1] pointed out the possibility of neutrino oscillation and, in addition, the solar neutrino problem was proposed [2], the neutrino oscillation problem has been much investigated experimentally and theoretically. Indications in favor of neutrino oscillation from various kinds of experiments have been reported [3]. Regarding the theoretical aspect, several works have been published recently, especially on the field theoretical approach to neutrino oscillations [4–10]. One of the controversial points is how to define field theoretically one (anti)neutrino state with a definite flavor for deriving the neutrino oscillation formula. More definitely, the problem is how to define field theoretically the state such as $|\nu_e\rangle$ employed usually in the quantum mechanical treatment [1]. Whereas there exists the assertion that it is impossible to construct a Fock space of ''weak states'' [4], Blasone and Vitiello have given the opposite assertion by defining the creation and annihilation operators of definite flavor neutrinos [7]. They said that the constructed Hilbert space of definite flavor states is unitarily inequivalent to that of definite mass states and that the effect due to such an inequivalence can be observed in the low-energy experiment of neutrino oscillations.

The main purpose of the present paper is to investigate the field theory of neutrino mixing. The investigation consists of the following two topics. The first is to examine the problem of how to construct the Hilbert space of definite flavor states in connection with the work of Blasone and Vitiello [7]. The second is to examine the structures of the flavor-neutrino propagators in order to define the physical neutrino mass on the basis of a Green-function approach to the field theory with particle mixings [11]. The remaining part of the present paper is organized as follows. In Sec. II, we consider the problem of how to construct the Hilbert space of definite flavor neutrino states by employing creation operators of (anti)neutrinos with definite masses and flavors. After exploring in detail the logical structures of Ref. [7] on the construction of the Hilbert space of definite flavor states, we point out that the relation between creation and annihilation operators of the flavor neutrinos and those of the neutrino with definite masses utilized in Ref. [7] has no field theoretical basis; thus the conclusions drawn in Ref. [7] are unphysical. Section III is devoted to examining the structures of the flavor-neutrino propagators in the case of two flavors. (The three flavor case is explained in Appendix B.) This section is pedagogical and helpful to grasp the essence of the procedure as to how to diagonalize the propagator developed by Kaneko, Ohnuki, and Watanabe [11] in the time when various models with particle mixings had been discussed [12]. The last section is a summary and discussion, in which a remark is given on the formula of the neutrino oscillation probability.

We discuss exclusively the case of Dirac neutrinos with two flavors in the body of this paper.

II. TWO KINDS OF HILBERT SPACES

A. Summary of the approach developed by Blasone and Vitiello

Let us review briefly the work of Blasone and Vitiello [7]. Their starting point is the relation between two sets of neutrino fields $\{\nu_\sigma(x),\sigma=e,\mu\}$ and $\{\nu_i(x),i=1,2\}$ with definite flavors and masses, respectively. This relation is given by a Bogoliubov transformation expressed as

$$\begin{pmatrix}\nu_e(x)\\ \nu_\mu(x)\end{pmatrix}=G^{-1}(\theta;t)\begin{pmatrix}\nu_1(x)\\ \nu_2(x)\end{pmatrix}G(\theta;t)=\begin{pmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{pmatrix}\begin{pmatrix}\nu_1(x)\\ \nu_2(x)\end{pmatrix},\qquad(2.1)$$

where $G(\theta;t)$ is given by

$$G(\theta;t)=\exp\left[\theta\int d^3x[\nu_1^\dagger(x)\nu_2(x)-\nu_2^\dagger(x)\nu_1(x)]\right],\quad x^0=t.\qquad(2.2)$$

$\nu_1(x)$ and $\nu_2(x)$ are expanded as

$$\nu_i(x)=\frac{1}{\sqrt{V}}\sum_{\vec{k}r}\{u^r_{\vec{k},i}\alpha^r_{\vec{k},i}(t)e^{i\vec{k}\cdot\vec{x}}+v^r_{\vec{k},i}\beta^{r\dagger}_{\vec{k},i}(t)e^{-i\vec{k}\cdot\vec{x}}\},$$

$$i=1,2, \tag{2.3}$$

where $\alpha^r_{\vec{k},i}(t)=\alpha^r_{\vec{k},i}(0)e^{-i\omega_i t}$ and $\beta^r_{\vec{k},i}(t)=\beta^r_{\vec{k},i}(0)e^{-i\omega_i t}$ with $\omega_i=\sqrt{\vec{k}^2+m_i^2}$. Here $u^r_{\vec{k},i}$ and $v^r_{\vec{k},i}$ are solutions of the free Dirac equation in momentum space with definite spin r and mass m_i. Here we add the t dependence of the operators $G,\alpha^r_{\vec{k},i}$, and $\beta^r_{\vec{k},i}$ explicitly in the above equations to make the explanations clearer. $u^r_{\vec{k},i}$, and $\alpha^r_{\vec{k},i}(t)$, etc., are written for simplicity as $u^r_{k,i}$, $\alpha^r_{k,i}$, etc., in this subsection hereafter.

The Hilbert space $\mathcal{H}_{1,2}$ of definite mass states is constructed by operating $\alpha^{r\dagger}_{k,i}$'s and $\beta^{r\dagger}_{k,i}$'s on the vacuum $|0\rangle_{1,2}$, which satisfies

$$\begin{Bmatrix}\alpha^r_{k,j}\\ \beta^r_{k,j}\end{Bmatrix}|0\rangle_{1,2}=0,\quad {}_{1,2}\langle 0|0\rangle_{1,2}=1. \tag{2.4}$$

By using the inverse of Eq. (2.1), the generic matrix element ${}_{1,2}\langle a|\nu_1(x)|b\rangle_{1,2}$ is written as

$${}_{1,2}\langle a|\nu_1(x)|b\rangle_{1,2}={}_{1,2}\langle a|G(\theta;t)\nu_e(x)G^{-1}(\theta;t)|b\rangle_{1,2}, \tag{2.5}$$

where $|a\rangle_{1,2}$ is the generic element of $\mathcal{H}_{1,2}$. The authors of Ref. [7], Blasone and Vitiello (BV), considered that, since the field operators ν_σ and ν_i are defined on Hilbert spaces $\mathcal{H}_{e,\mu}$ and $\mathcal{H}_{1,2}$ respectively, $G^{-1}(\theta)$ maps $\mathcal{H}_{1,2}$ to $\mathcal{H}_{e,\mu}$ and in particular the flavor vacuum $|0\rangle_{e,\mu}$ is given by

$$|0\rangle_{e,\mu}=G(\theta;t=0)^{-1}|0\rangle_{1,2}. \tag{2.6}$$

Equation (2.6) suggests that $|0\rangle_{e,\mu}$ is a condensate of massive neutrino-antineutrino pairs and a coherent state. Since these two Hilbert spaces are orthogonal to each other in the infinite volume limit, BV performed all the computations at a finite volume V and put $V\to\infty$ at the end.

From Eq. (2.1) BV gave, as a kind of dynamical map,

$$u^r_{k,\sigma}\tilde{\alpha}^r_{k,\sigma}\equiv G^{-1}(\theta;t)u^r_{k,j}\alpha^r_{k,j}G(\theta;t),$$

$$v^{r*}_{k,\sigma}\tilde{\beta}^r_{k,\sigma}\equiv G^{-1}(\theta;t)v^{r*}_{k,j}\beta^r_{k,j}G(\theta;t),$$

$$(\sigma,j)=(e,1),(\mu,2). \tag{2.7}$$

In order to exhibit this dynamical map explicitly, BV redefined ''for convenience'' the quantities appearing on the left-hand side (LHS) of Eq. (2.7) as $u^{r\alpha}_{k,1}\alpha^r_{k,e}\equiv u^{r\alpha}_{k,e}\tilde{\alpha}^r_{k,e}$, etc., and got

$$\begin{pmatrix}\alpha^r_{k,\sigma}\\ \beta^{r\dagger}_{k,\sigma}\end{pmatrix}\equiv G^{-1}(\theta;t)\begin{pmatrix}\alpha^r_{k,j}\\ \beta^{r\dagger}_{k,j}\end{pmatrix}G(\theta;t). \tag{2.8}$$

In order to obtain the formulas of the neutrino oscillation, BV prepared the one-electron-neutrino state at time $t=0$ defined as

$$|\alpha^r_{k,\sigma}(t=0)\rangle\equiv\frac{1}{\mathcal{N}}\alpha^{r\dagger}_{k,\sigma}(t=0)|0\rangle_{1,2}$$

$$=\frac{1}{\mathcal{N}}[\cos\theta\,\alpha^{r\dagger}_{k,1}(t=0)|0\rangle_{1,2}$$

$$+U_k\sin\theta\,\alpha^{r\dagger}_{k,2}(t=0)|0\rangle_{1,2}], \tag{2.9}$$

where $U_k\equiv u^{r*}_{k,2}u^r_{k,1}=v^{r*}_{-k,2}v^r_{-k,1}$, $r=$helicity, the normalization factor $\mathcal{N}^2=1-|V_k|^2\sin^2\theta$, and $|V_k|^2=1-|U_k|^2$. [In Ref. [7], after somewhat troublesome calculations employing momentum-spin eigenfunctions, BV introduced substantially the helicity eigenfunctions instead of the spin ones. In order to see the essential point in Ref. [7], it is enough for us to employ the helicity-momentum eigenfunctions from the outset to derive Eq. (2.9).]

By taking the time evolution of Eq. (2.9) due to the free Hamiltonian of the $\nu_i(x)$ field as

$$|\alpha^r_{k,e}(t)\rangle=e^{iH_{1,2}t}|\alpha^r_{k,e}(0)\rangle, \tag{2.10}$$

the expectation values of the number operators

$$n_\sigma(k)\equiv\sum_r\alpha^{r\dagger}_{k,\sigma}(0)\alpha^r_{k,\sigma}(0) \tag{2.11}$$

with respect to the state $|\alpha^r_{k,e}(t)\rangle$ are given by

$$\langle\alpha^r_{k,e}(t)|n_e(k)|\alpha^r_{k,e}(t)\rangle=1-R_k\sin^2(2\theta)\sin^2\left(\frac{\Delta\omega}{2}t\right), \tag{2.12}$$

$$\langle\alpha^r_{k,e}(t)|n_\mu(k)|\alpha^r_{k,e}(t)\rangle=R_k\sin^2(2\theta)\sin^2\left(\frac{\Delta\omega}{2}t\right)$$

$$+\langle\alpha^r_{k,e}(0)|n_\mu(k)|\alpha^r_{k,e}(0)\rangle, \tag{2.13}$$

with $\Delta\omega=\sqrt{\vec{k}^2+m_1^2}-\sqrt{\vec{k}^2+m_2^2}$ and $R_k=|U_k|^2/(1-\sin^2\theta|V_k|^2)$. BV adopted Eqs. (2.12) and (2.13) as oscillation formulas. These oscillation formulas are different from the ordinary ones due to the factor R_k and the last term of Eq. (2.13). The factor R_k, which depends on k, m_1, and m_2, deviates from 1 due to the condensate and brings about a new effect to be detected in neutrino experiments at low energies. As to the last term of Eq. (2.13), we note that the sum of Eqs. (2.12) and (2.13) is not equal to 1. BV gave, as the normalization of the total probability,

$$\langle\alpha^r_{k,e}(t)|n_e(k)+n_\mu(k)|\alpha^r_{k,e}(t)\rangle-\langle\alpha^r_{k,e}(0)|n_\mu(k)|\alpha^r_{k,e}(0)\rangle$$

$$=1. \tag{2.14}$$

Here we point out two problems in the above argument. The first is that the meaning of the relations from Eqs. (2.6) to (2.8) is not clear. Although BV explained that the vacuum relation (2.6) is outwardly independent of the choice of the mapping (2.8), these two relations are in fact intimately connected to each other. This point will be examined in the next subsection. The other problem concerns the meaning of the second term of the RHS of Eq. (2.13) introduced to normalize the total probability. We will discuss this point in Sec. II C.

B. Reformulation

Now we reexamine the processes described above. For convenience notation is a little different from that in Ref. [7] which was employed above.

First of all, we consider the Lagrangian density expressed in terms of the Heisenberg field $\psi(x)$:

$$\mathcal{L}(x)=-\overline{\psi}(x)(\partial\!\!\!/+m)\psi(x)+\mathcal{L}_{int}(x). \tag{2.15}$$

When $\mathcal{L}_{int}(x)$ dose not include any derivative coupling, the equation of motion is written as

$$(\partial\!\!\!/+m)\psi(x)=\frac{\delta}{\delta\overline{\psi}(x)}\mathcal{L}_{int}(x)=J(x). \tag{2.16}$$

This Heisenberg field $\psi(x)$ can be expanded [13] in terms of helicity-momentum eigenfunctions as

$$\psi(x)=\frac{1}{\sqrt{V}}\sum_{\vec{k}r}\{u(kr)\alpha(kr;t)e^{i\vec{k}\cdot\vec{x}}+v(kr)\beta^{\dagger}(kr;t)e^{-i\vec{k}\cdot\vec{x}}\}$$
$$=\frac{1}{\sqrt{V}}\sum_{\vec{k}r}e^{i\vec{k}\cdot\vec{x}}\{u(kr)\alpha(kr;t)+v(-kr)\beta^{\dagger}(-kr;t)\}. \tag{2.17}$$

Here $\{u,v\}$ are the plane-wave eigenfunctions with mass μ and satisfy the following free Dirac equation:

$$(ik\!\!\!/+\mu)u(kr)=0,\quad (ik\!\!\!/-\mu)v(kr)=0,\quad k_0=\sqrt{\vec{k}^2+\mu^2}, \tag{2.18}$$

where $k\!\!\!/=\gamma^{\alpha}k_{\alpha}=\vec{\gamma}\vec{k}+\gamma^4 ik_0$ and $\gamma^{\alpha\dagger}=\gamma^{\alpha}$. (The helicity eigenfunctions are used for technical simplicity in the following, and their concrete forms are given in Appendix A [14].) The expansion coefficient operators in Eq. (2.17) are time dependent and satisfy the canonical commutation relations for the equal time, which are derived from the equal-time commutation relations $\{\psi_b(x),\psi^{\dagger}_{b'}(y)\}_{x_0=y_0}=\delta(\vec{x}-\vec{y})\delta_{bb'}$ and others $=0$. Note that one can choose any eigenfunctions with mass μ which is different from the mass m in the equation of motion (2.16). If $\psi(x)$ is a free field (i.e., $\mathcal{L}_{int}=0$) or an asymptotic field with physical mass m, we can take $\{u,v\}$ to be the plane-wave solutions of the free Dirac equations with $\mu=m$ and, at the same time,

$$\begin{pmatrix}\alpha(kr;t)\\ \beta(kr;t)\end{pmatrix}=\begin{pmatrix}\alpha(kr;0)\\ \beta(kr;0)\end{pmatrix}e^{-i\omega t},\quad \omega=\sqrt{\vec{k}^2+m^2}. \tag{2.19}$$

But generally, the time dependence of the operators is not so simple.

Returning to the neutrino case, we expand the neutrino field of mass eigenstate, which satisfies the free Dirac equation, as

$$\nu_j(x)=\frac{1}{\sqrt{V}}\sum_{\vec{k}r}e^{i\vec{k}\cdot\vec{x}}\{u_j(kr)\alpha_j(kr;t)+v_j(-kr)\beta^{\dagger}_j(-kr;t)\},\quad j=1,2, \tag{2.20}$$

where $\{u_j,v_j\}$ are the plane-wave eigenfunctions with mass m_j and satisfy the free Dirac equation (2.18) with $\mu=m_j$. The time dependence of the operator is expressed by Eq. (2.19) with $m=m_j$.

The neutrino field of flavor eigenstate is also expanded as

$$\nu_{\sigma}(x)=\frac{1}{\sqrt{V}}\sum_{\vec{k}r}e^{i\vec{k}\cdot\vec{x}}\{u_{\sigma}(kr)\alpha_{\sigma}(kr;t)+v_{\sigma}(-kr)\beta^{\dagger}_{\sigma}(-kr;t)\},\quad \sigma=e,\mu. \tag{2.21}$$

$\{u_{\sigma},v_{\sigma}\}$ are the plane-wave eigenfunctions with mass μ_{σ} and satisfy the free Dirac equation (2.18) with $\mu=\mu_{\sigma}$, but the time dependence of the creation and annihilation operators is not so simple as Eq. (2.19). We want to stress here that m_j's are the neutrino masses to be observed experimentally, while μ_{σ}'s are arbitrarily fixed. (The special mass symbol "μ_{σ}" is used to stress this arbitrariness.)

The relation between the two kind of creation and annihilation operators is not the same as the one between the field operators, Eq. (2.1). Generally we have

$$\begin{aligned}\begin{pmatrix}\alpha_{\sigma}(kr;t)\\ \beta^{\dagger}_{\sigma}(-kr;t)\end{pmatrix}&=\frac{1}{\sqrt{V}}\int d^3x e^{-i\vec{k}\cdot\vec{x}}\begin{pmatrix}\overline{u}_{\sigma}(kr)\\ \overline{v}_{\sigma}(-kr)\end{pmatrix}\gamma^4\nu_{\sigma}(x)\\ &=\frac{1}{\sqrt{V}}\int d^3x e^{-i\vec{k}\cdot\vec{x}}\begin{pmatrix}u^{*}_{\sigma}(kr)\\ v^{*}_{\sigma}(-kr)\end{pmatrix}\\ &\quad\times G^{-1}(\theta;t)\nu_j(x)G(\theta;t)\\ &=G^{-1}(\theta;t)\begin{pmatrix}\rho_{\sigma j}(k) & i\lambda_{\sigma j}(k)\\ i\lambda_{\sigma j}(k) & \rho_{\sigma j}(k)\end{pmatrix}\\ &\quad\times\begin{pmatrix}\alpha_j(kr;t)\\ \beta^{\dagger}_j(-kr;t)\end{pmatrix}G(\theta;t),\end{aligned} \tag{2.22}$$

where $(\sigma,j)=(e,1),(\mu,2)$:

$$\rho_{\sigma j}(k)\equiv u^{*}_{\sigma}(kr)u_j(kr)=v^{*}_{\sigma}(-kr)v_j(-kr)=\cos\left(\frac{\chi_{\sigma}-\chi_j}{2}\right),$$
$$i\lambda_{\sigma j}(k)\equiv u^{*}_{\sigma}(kr)v_j(-kr)=v^{*}_{\sigma}(-kr)u_j(kr)=i\sin\left(\frac{\chi_{\sigma}-\chi_j}{2}\right), \tag{2.23}$$

$$\cot\chi_{\sigma}=\frac{|k|}{\mu_{\sigma}},\quad \cot\chi_j=\frac{|k|}{m_j}.$$

There exists a unitary operator $I(\mu_\sigma;t)$ which realizes the transformation

$$\begin{pmatrix} \rho_{\sigma j}(k) & i\lambda_{\sigma j}(k) \\ i\lambda_{\sigma j}(k) & \rho_{\sigma j}(k) \end{pmatrix}\begin{pmatrix} \alpha_j(kr;t) \\ \beta_j^\dagger(-kr;t) \end{pmatrix} = I^{-1}(\mu_\sigma;t)\begin{pmatrix} \alpha_j(kr;t) \\ \beta_j^\dagger(-kr;t) \end{pmatrix} I(\mu_\sigma;t), \quad (2.24)$$

where

$$I(\mu_\sigma;t)=\prod_{\vec{k},r} \exp\Big\{ i\sum_{(\sigma,j)} \xi_{\sigma,j}(k)[\alpha_j^\dagger(kr;t)\beta_j^\dagger(-kr;t) + \beta_j(-kr;t)\alpha_j(kr;t)]\Big\}, \quad (2.25)$$

with $\xi_{\sigma,j}(k)=(\chi_\sigma-\chi_j)/2$, i.e., $\cos\xi_{\sigma j}(x)=\rho_{\sigma j}(x)$. Then Eq. (2.22) can be rewritten as

$$\begin{pmatrix} \alpha_\sigma(kr;t) \\ \beta_\sigma^\dagger(-kr;t) \end{pmatrix} = K^{-1}(\theta,\mu_\sigma;t)\begin{pmatrix} \alpha_j(kr;t) \\ \beta_j^\dagger(-kr;t) \end{pmatrix} K(\theta,\mu_\sigma;t), \quad (2.26)$$

where $K(\theta,\mu_\sigma;t)\equiv I(\mu_\sigma;t)G(\theta;t)$. The explicit matrix form of this transformation is given in Appendix A.

Next we introduce the mass and the flavor vacua $|0\rangle_m$ and $|0\rangle_f$ as

$$\begin{Bmatrix} \alpha_j(kr;t) \\ \beta_j(-kr;t) \end{Bmatrix}|0\rangle_m=0, \quad \begin{Bmatrix} \alpha_\sigma(kr;t) \\ \beta_\sigma(-kr;t) \end{Bmatrix}|0\rangle_f=0. \quad (2.27)$$

From Eq. (2.26), we obtain

$$0=\alpha_j(kr;t)|0\rangle_m=K(\theta,\mu_\sigma;t)\alpha_\sigma(kr;t)K^{-1}(\theta,\mu_\sigma;t)|0\rangle_m; \quad (2.28)$$

thus we see that the second condition of the vacuum of Eq. (2.27) is satisfied automatically when we define the vacuum $|0\rangle_f$ by

$$|0\rangle_f\equiv K^{-1}(\theta,\mu_\sigma;t)|0\rangle_m. \quad (2.29)$$

Because of the θ, μ_σ, and t dependence of $|0\rangle_f$, we denote the flavor vacuum as $|0\rangle_f=|0(\theta,\mu_\sigma;t)\rangle$ hereafter.

It is worthwhile to note that the vacuum relation (2.29) is given uniquely corresponding to the relation (2.26) for arbitrarily fixed $\{\mu_e,\mu_\mu\}$. If we choose that $\mu_e=m_1$ and $\mu_\mu=m_2$, then $I(\mu_\sigma;t)=1$. Equations (2.26) and (2.29) reduce to Eqs. (2.8) and (2.6), respectively, given by BV [7]:

$$\begin{pmatrix} \alpha_\sigma(kr;t) \\ \beta_\sigma^\dagger(-kr;t) \end{pmatrix}_{\mu_\sigma=m_j} = G^{-1}(\theta;t)\begin{pmatrix} \alpha_j(kr;t) \\ \beta_j^\dagger(-kr;t) \end{pmatrix} G(\theta;t) = \begin{pmatrix} \alpha_\sigma^{BV}(kr;t) \\ \beta_\sigma^{BV\dagger}(-kr;t) \end{pmatrix}, \quad (2.30)$$

$$|0(\theta,\mu_\sigma;t)\rangle_{\mu_\sigma=m_j}=G(\theta;t)^{-1}|0\rangle_m=|0(\theta;t)\rangle^{BV}.$$

Here we attach the index "BV" to the quantities corresponding to those employed by Blasone and Vittielo [7]. From Eq. (2.22) we obtain the relation between BV's operators and the general ones:

$$\begin{aligned}\begin{pmatrix} \alpha_\sigma(kr;t) \\ \beta_\sigma^\dagger(-kr;t) \end{pmatrix} &= \begin{pmatrix} \rho_{\sigma j}(k) & i\lambda_{\sigma j}(k) \\ i\lambda_{\sigma j}(k) & \rho_{\sigma j}(k) \end{pmatrix} G^{-1}(\theta;t) \times \begin{pmatrix} \alpha_j(kr;t) \\ \beta_j^\dagger(-kr;t) \end{pmatrix} G(\theta;t) \\ &= \begin{pmatrix} \rho_{\sigma j}(k) & i\lambda_{\sigma j}(k) \\ i\lambda_{\sigma j}(k) & \rho_{\sigma j}(k) \end{pmatrix}\begin{pmatrix} \alpha_\sigma^{BV}(kr;t) \\ \beta_\sigma^{BV\dagger}(-kr;t) \end{pmatrix} \\ &= J^{-1}(\mu_\sigma;t)\begin{pmatrix} \alpha_\sigma^{BV}(kr;t) \\ \beta_\sigma^{BV\dagger}(-kr;t) \end{pmatrix} J(\mu_\sigma;t),\end{aligned} \quad (2.31)$$

where

$$J(\mu_\sigma;t)=\prod_{\vec{k},r} \exp\Big\{ i\sum_{(\sigma,j)} \xi_{\sigma,j}(k)[\alpha_\sigma^{\dagger BV}(kr;t)\beta_\sigma^{\dagger BV}(-kr;t) + \beta_\sigma^{BV}(-kr;t)\alpha_\sigma^{BV}(kr;t)]\Big\}. \quad (2.32)$$

As easily seen from Eqs. (2.25), (2.30), and (2.32) we have $GJ=IG$.

From the reformulation described above, we see that, contrary to the choice of BV, there is no theoretical basis to choose special values of μ_σ's and then the μ_σ dependence is not removed from the formulas, such as Eqs. (2.26) and (2.29). Since any physical observable should have no μ_σ dependence, one may expect that the μ_σ dependence will disappear in calculated physical observables. To examine it, we calculate the oscillation probability in the next subsection in accordance with the line of BV [7].

C. Oscillation probability

Let us define the one-electron-neutrino state $|\alpha_e(kr;0)\rangle$ at the time $t=0$, in accordance with BV [7], by operating $\alpha_e^\dagger(kr;0)$ to $|0\rangle_m$. The initial condition $\langle\alpha_e(kr;0)|\alpha_e(kr;0)\rangle=1$ needs the normalization factor of the state as

$$\begin{aligned}|\alpha_e(kr;0)\rangle &= \frac{1}{\mathcal{N}_e(k)}\alpha_e^\dagger(kr;0)|0\rangle_m \\ &= \frac{1}{\mathcal{N}_e(k)}[\cos\theta\rho_{e1}(k)\alpha_1^\dagger(kr;0) + \sin\theta\rho_{e2}(k)\alpha_2^\dagger(kr;0)]|0\rangle_m,\end{aligned} \quad (2.33)$$

where $|\mathcal{N}_e(k)|^2=[\cos\theta\rho_{e1}(k)]^2+[\sin\theta\rho_{e2}(k)]^2$. Another initial condition $\langle\alpha_\mu(kr;0)|\alpha_e(kr;0)\rangle=0$ imposes the relation

$$\rho_{e1}(k)\rho_{\mu 1}(k)-\rho_{e2}(k)\rho_{\mu 2}(k)=0 \quad (2.34)$$

on the $\rho_{\sigma j}$ parameters. The expectation values of the number operators are

$$\langle \alpha_e(kr;t)|n_e(k;0)|\alpha_e(kr;t)\rangle = 1-\frac{[\rho_{e1}(k)\rho_{e2}(k)]^2}{\mathcal{N}_e(k)^2}\sin^2(2\theta)\sin^2\left(\frac{\Delta\omega}{2}t\right), \tag{2.35}$$

$$\begin{aligned}\langle \alpha_e(kr;t)|n_\mu(k;0)|\alpha_e(kr;t)\rangle &= \frac{1}{\mathcal{N}_e(k)^2}\left[\frac{1}{4}[\rho_{e1}(k)\rho_{\mu1}(k)-\rho_{e2}(k)\rho_{\mu2}(k)]^2 \right.\\ &\quad \left. +\rho_{e1}(k)\rho_{\mu1}(k)\rho_{e2}(k)\rho_{\mu2}(k)\sin^2(2\theta)\sin^2\left(\frac{\Delta\omega}{2}t\right)\right]\\ &\quad +(\sin^2\theta\lambda_{\mu1}^2+\cos^2\theta\lambda_{\mu2}^2),\end{aligned} \tag{2.36}$$

where $n_\sigma(k;t)\equiv\Sigma_r\alpha_\sigma^\dagger(kr;t)\alpha_\sigma(kr;t)$, $\sigma=e,\mu$. The sum of Eqs. (2.35) and (2.36) is not equal to 1. By defining the number operators $\overline{n}_\sigma(k;t)\equiv\Sigma_r\beta_\sigma^\dagger(kr;t)\beta_\sigma(kr;t)$ and $N_\sigma(k;t)\equiv n_\sigma(k;t)-\overline{n}_\sigma(k;t)$, we see that the equality

$$\langle \alpha_e(kr;t)|N_e(k;0)+N_\mu(k;0)|\alpha_e(kr;t)\rangle=1 \tag{2.37}$$

holds for any time $t\geqslant 0$, irrespectively of the values of μ_e and μ_μ.

Note that

$$\begin{aligned}\langle \alpha_e(kr;t)|\overline{n}_e(k;0)+\overline{n}_\mu(k;0)|\alpha_e(kr;t)\rangle|_{BV} &= \sin^2\theta|V_k|^2\\ &=\langle \alpha_e(kr;0)|n_\mu(k;0)|\alpha_e(kr;0)\rangle|_{BV}.\end{aligned} \tag{2.38}$$

Although BV introduced $\langle \alpha_e(kr;0)|n_\mu(k;0)|\alpha_e(kr;0)\rangle$ in Eq. (2.14) to normalize the total probability, Eq. (2.14) holds accidentally in the case of the BV choice $\mu_e=m_1$ and $\mu_\mu=m_2$. Such a term is essentially the contribution from the antineutrinos and the probability normalization is given by Eq. (2.37) in itself.

From the oscillation amplitudes

$$\begin{aligned}\mathcal{F}_{ee}(kr;t) &= {}_m\langle 0|\alpha_e(kr;0)\alpha_e^\dagger(kr;t)|0\rangle_m/\mathcal{N}_e(k)^2\\ &=\frac{1}{\mathcal{N}_e(k)^2}[(\cos\theta\rho_{e1})^2e^{i\omega_1 t}\\ &\quad +(\sin\theta\rho_{e2})^2e^{i\omega_2 t}],\\ \mathcal{F}_{\mu e}(kr;t) &= \langle \alpha_e(kr;0)|\alpha_\mu(kr;t)\rangle\\ &= {}_m\langle 0|\alpha_\mu(kr;0)\alpha_e^\dagger(kr;t)|0\rangle_m/\mathcal{N}_\mu(k)\mathcal{N}_e(k)\\ &=\frac{\cos\theta\sin\theta}{\mathcal{N}_\mu(k)\mathcal{N}_e(k)}(-\rho_{\mu1}\rho_{e1}e^{i\omega_1 t}\\ &\quad +\rho_{\mu2}\rho_{e2}e^{i\omega_2 t}),\end{aligned} \tag{2.39}$$

we can derive the oscillation probabilities, which are different from the oscillation formulas of BV [7], i.e., the expectation value of the number operators, as

$$|\mathcal{F}_{ee}(kr;t)|^2=1-\frac{(\rho_{e1}\rho_{e2})^2}{\mathcal{N}_e(k)^4}\sin^2(2\theta)\sin^2\left(\frac{\Delta\omega}{2}t\right), \tag{2.40}$$

$$\begin{aligned}|\mathcal{F}_{\mu e}(kr;t)|^2 &= \frac{\sin^2(2\theta)}{\mathcal{N}_\mu(k)^2\mathcal{N}_e(k)^2}\left[\frac{1}{4}(\rho_{e1}\rho_{\mu1}-\rho_{e2}\rho_{\mu2})^2 \right.\\ &\quad \left. +\rho_{e1}\rho_{e2}\rho_{\mu1}\rho_{\mu2}\sin^2\left(\frac{\Delta\omega}{2}t\right)\right]\\ &=\frac{(\rho_{e1}\rho_{e2})^2}{\mathcal{N}_e(k)^4}\sin^2(2\theta)\sin^2\left(\frac{\Delta\omega}{2}t\right),\end{aligned} \tag{2.41}$$

where the second equality in Eq. (2.41) is due to Eq. (2.34). (When we set $\rho_{e1}=\rho_{\mu2}=1$, $\rho_{e2}=\rho_{\mu1}=U_k$, $\lambda_{e1}=\lambda_{\mu2}=0$, and $\lambda_{e2}=-\lambda_{\mu1}=V_k$ corresponding to BV's choice, all the formulas above consistently tend to the corresponding ones in Refs. [7] and [10].) The total oscillation probability is equal to 1, i.e. $|\mathcal{F}_{ee}(kr;t)|^2+|\mathcal{F}_{\mu e}(kr;t)|^2=1$, but each oscillation probability $|\mathcal{F}_{ee}(kr;t)|^2$ or $|\mathcal{F}_{\mu e}(kr;t)|^2$ is μ_σ dependent through $\rho_{\sigma j}$'s. Thus the calculated oscillation probabilities seem to be unphysical.

There is another possibility to define the one-electron-neutrino state at the time $t=0$ as

$$|\alpha_e(kr;0)\rangle\equiv\alpha_e^\dagger(kr;0)|0(\theta,\mu_\sigma;0)\rangle. \tag{2.42}$$

In this case, the normalization of the state is automatically satisfied:

$$\langle \alpha_e(kr;0)|\alpha_e(kr;0)\rangle=1. \tag{2.43}$$

The oscillation amplitude derived from $\langle 0(\theta,\mu_\sigma;0)|\alpha_e(kr;0)\alpha_e^\dagger(kr;t)|0(\theta,\mu_\sigma;t)\rangle$ becomes the product of some number and the term ${}_m\langle 0|K'(\theta,\mu_\sigma;0)K'^{-1}(\theta,\mu_\sigma;t)|0\rangle_m$, where the prime on K means to exclude from $K(\theta,\mu_\sigma;t)$ the contribution of the momentum k and the helicity r. This vacuum expectation value becomes 0 as $V\to\infty$ as far as $t>0$, and then the oscillation amplitude is 0, while, as to the expectation values of the number operators with respect to the $|0(\theta,\mu_\sigma;t)\rangle$, we have nonvanishing values [10]. As easily confirmed, however, these expectation values are not free from the dependence on μ_σ's.

III. STRUCTURE OF THE NEUTRINO PROPAGATOR: THE CASE OF TWO FLAVORS

The naive way to convert the propagator of the flavor neutrino into the one of the mass eigenstates may be

$$\langle 0|T[\nu_\sigma(x)\overline{\nu_\rho}(y)]|0\rangle$$
$$=\langle 0|T[G^{-1}(\theta)\nu_i(x)G(\theta)G^{-1}(\theta)\overline{\nu_j}(y)G(\theta)]|0\rangle, \tag{3.1}$$

where $|0\rangle=|0\rangle_m$. It is not always clear whether the quantity of the LHS deserved to be called the propagator of the flavor neutrino. So we examine the pole structure of the neutrino propagator according to the diagonalizing procedure proposed by Kaneko, Ohnuki, and Watanabe [11], which had been developed many years ago as the field theory of particle mixture interactions.

The diagonalization of the flavor neutrino propagator in the three-flavor case can be examined along the same line as described below, which is given in Appendix B.

A. Starting Lagrangian

Let us consider the following Lagrangian density with a mutual transition between two neutrino fields specified by the flavor degrees of freedom $\sigma=e$ and μ:

$$\mathcal{L}(x)=-(\overline{\nu_e}(x)\ \ \overline{\nu_\mu}(x))(\partial\!\!\!/+M)\begin{pmatrix}\nu_e(x)\\ \nu_\mu(x)\end{pmatrix}+\mathcal{L}_{int}(x), \tag{3.2}$$

where

$$M=\begin{pmatrix}m_{ee} & m_{e\mu}\\ m_{\mu e} & m_{\mu\mu}\end{pmatrix},\qquad \partial\!\!\!/:=\gamma^\rho\partial_\rho=\vec{\gamma}\vec{\nabla}+\gamma^4\frac{1}{i}\frac{\partial}{\partial x^0},$$
$$(\gamma^\rho)^\dagger=\gamma^\rho. \tag{3.3}$$

$\mathcal{L}_{int}(x)$ includes the weak interaction and the Higgs one which remains after spontaneous symmetry breaking in the unified theory. As a result of $M^\dagger=M$, required from the Hermiticity of $\mathcal{L}(x)$, m_{ee} and $m_{\mu\mu}$ are real and $m^*_{e\mu}$ is equal to $m_{\mu e}$. The Hamiltonian is

$$\mathcal{H}_{e\mu}(x)=(\overline{\nu_e}(x)\ \ \overline{\nu_\mu}(x))(\vec{\gamma}\vec{\nabla}+M)\begin{pmatrix}\nu_e(x)\\ \nu_\mu(x)\end{pmatrix}-\mathcal{L}_{int}(x). \tag{3.4}$$

The eigenvalues of M are

$$m_{1(2)}=\frac{1}{2}\Big(m_{ee}+m_{\mu\mu}-(+)\sqrt{(m_{\mu\mu}-m_{ee})^2+4|m_{e\mu}|^2}\Big), \tag{3.5}$$

and $\mathcal{H}_{e\mu}(x)$ is expressed in diagonalized form as

$$\mathcal{H}_{e\mu}(x)=\sum \overline{\nu_j}(x)(\vec{\gamma}\vec{\nabla}+m_j)\nu_j(x)-\mathcal{L}_{int}(x). \tag{3.6}$$

For simplicity, we take $m_{e\mu}=m_{\mu e}$, derived from CP invariance; then we can take

$$\begin{pmatrix}\nu_e(k,r)\\ \nu_\mu(k,r)\end{pmatrix}=\begin{pmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{pmatrix}\begin{pmatrix}\nu_1(k,r)\\ \nu_2(k,r)\end{pmatrix}, \tag{3.7}$$

with

$$\tan\theta=\frac{1}{2m_{e\mu}}\Big[-(m_{\mu\mu}-m_{ee})+\sqrt{(m_{\mu\mu}-m_{ee})^2+4m^2_{e\mu}}\Big]. \tag{3.8}$$

We take $m_{\mu\mu}\geqslant m_{ee}\geqslant 0$ with no loss of generality; then $m_2\geqslant|m_1|$ and

$$m_1\geqslant 0\quad\text{for}\quad\sqrt{m_{ee}m_{\mu\mu}}\geqslant|m_{e\mu}|,$$
$$m_1<0\quad\text{for}\quad\sqrt{m_{ee}m_{\mu\mu}}<|m_{e\mu}|. \tag{3.9}$$

In the following calculations, it will be useful for us to employ the relations

$$m_{ee}=m_1(\cos\theta)^2+m_2(\sin\theta)^2,$$
$$m_{\mu\mu}=m_1(\sin\theta)^2+m_2(\cos\theta)^2, \tag{3.10}$$

$$m_{e\mu}=\sin\theta\cos\theta(-m_1+m_2),\qquad \tan(2\theta)=\frac{2m_{e\mu}}{m_{\mu\mu}-m_{ee}}; \tag{3.11}$$

further, we have

$$m_1m_2=m_{ee}m_{\mu\mu}-m^2_{e\mu}, \tag{3.12}$$

$$m_{ee}-m_1=-m_{\mu\mu}+m_2=m_{e\mu}\tan\theta, \tag{3.13}$$

$$m_{ee}-m_2=-m_{\mu\mu}+m_1=-m_{e\mu}\cot\theta. \tag{3.14}$$

B. Poles of the propagator matrix

The propagator expressed by Heisenberg fields and the corresponding vacuum is rewritten with the interaction fields as

$$S'_{\sigma\rho}(x-y):={}_H\langle 0|T[\nu^H_\sigma(x)\overline{\nu^H_\rho}(y)]|0\rangle_H$$
$$={}_I\langle 0|T[S\nu^I_\sigma(x)\overline{\nu^I_\rho}(y)]|0\rangle_I, \tag{3.15}$$

where $S\equiv T\exp(i\int d^4x\mathcal{L}_I)$ is the so-called Dyson S matrix. In the RHS of Eq. (3.15) we dropped the phase factor, which is irrelevant to the following considerations. $S'_{\sigma\rho}(x-y)$ can be calculated perturbatively by using the interaction representation.

The Fourier transform of the propagator (3.15), $S'_{\sigma\rho}(k\!\!\!/)=\int d^4x\exp(-ikx)S'_{\sigma\rho}(x)$, satisfies

$$S'_{\sigma\rho}=\delta_{\sigma\rho}S_\rho+\sum_\lambda S'_{\sigma\lambda}\Pi_{\lambda\rho}S_\rho, \tag{3.16}$$

where $S_\rho(k\!\!\!/)$ is the free propagator of the ν_ρ field. When we define the matrix $[f_{\sigma\rho}(k\!\!\!/)]$ to be

$$S'_{\sigma\rho}(k\!\!\!/)=[f(k\!\!\!/)^{-1}]_{\sigma\rho}, \tag{3.17}$$

we obtain

$$f_{\sigma\rho}(\not k)=\delta_{\sigma\rho}S_\rho(\not k)^{-1}-\Pi_{\sigma\rho}(\not k). \tag{3.18}$$

$[S'_{\sigma\rho}(\not k)]$ has two poles, determined by

$$\det[f_{\sigma\rho}(\not k)]=0. \tag{3.19}$$

Let us examine the pole structure of $S'_{\sigma\rho}$. We separate Hamiltonian (3.4) into two parts:

$$\mathcal{H}_{e\mu}(x)=\mathcal{H}^0_{e\mu}(x)+\mathcal{H}^{int}_{e\mu}(x)-\mathcal{L}_{int}(x), \tag{3.20}$$

$$\mathcal{H}^0_{e\mu}(x)=(\overline{\nu}_e(x)\ \ \overline{\nu}_\mu(x))\left(\vec{\gamma}\vec{\nabla}+\begin{pmatrix} m_{ee} & 0\\ 0 & m_{\mu\mu}\end{pmatrix}\right)\begin{pmatrix}\nu_e(x)\\ \nu_\mu(x)\end{pmatrix},$$

$$\mathcal{H}^{int}_{e\mu}(x)=(\overline{\nu}_e(x)\ \ \overline{\nu}_\mu(x))\begin{pmatrix} 0 & m_{e\mu}\\ m_{\mu e} & 0\end{pmatrix}\begin{pmatrix}\nu_e(x)\\ \nu_\mu(x)\end{pmatrix}.$$

Then the free propagator of the ν_ρ field corresponding to $\mathcal{H}^0_{e\mu}(x)$ is $S_\rho(\not k):=(-\not k+im_{\rho\rho}+\epsilon)^{-1}$, and we take the proper self-energy part to be $\Pi_{\sigma\rho}=-im_{\sigma\rho}$ as the contribu tion from

$$\nu_e \xrightarrow{\ \ m_{e\mu}\ \ \times\ \ } \nu_\mu .$$

We neglect here the weak and the Higgs interactions. (Properly speaking, one has to include the effect of Higgs interactions which is not always weak. We neglect this interaction owing to our ignorance of it.) We have

$$[f_{\sigma\rho}(\not k)]=\begin{pmatrix} -\not k+im_{ee} & im_{e\mu}\\ im_{e\mu} & -\not k+im_{\mu\mu}\end{pmatrix}. \tag{3.21}$$

As easily seen, $[S'_{\sigma\rho}(\not k)]$ has two poles at

$$\not k=im_j \quad \text{with} \quad m_j(j=1,2) \quad \text{given by} \quad \text{Eq. (3.5)}. \tag{3.22}$$

Therefore, the physical one-particle masses given as poles of $S'_{\sigma\rho}(\not k)$ are seen to coincide with the eigenvalues of the mass matrix M.

It should be noted that there is an arbitrariness in separating $\mathcal{H}(x)$ into the ''free'' and ''interaction'' parts. So it is worthy to make a remark on this point. We rewrite the Hamiltonian (3.4) as

$$\mathcal{H}_{e\mu}(x)=\tilde{\mathcal{H}}^0_{e\mu}(x)+\tilde{\mathcal{H}}^{int}_{e\mu}(x)-\mathcal{L}_{int}(x),$$

$$\tilde{\mathcal{H}}^0_{e\mu}(x)=(\overline{\nu}_e(x)\ \ \overline{\nu}_\mu(x))\left(\vec{\gamma}\vec{\nabla}+\begin{pmatrix} \tilde{m}_{ee} & 0\\ 0 & \tilde{m}_{\mu\mu}\end{pmatrix}\right)\begin{pmatrix}\nu_e(x)\\ \nu_\mu(x)\end{pmatrix}, \tag{3.23}$$

$$\tilde{\mathcal{H}}^{int}_{e\mu}(x)=(\nu_e(x)\ \ \overline{\nu}_\mu(x))\begin{pmatrix} \Delta_{ee} & m_{e\mu}\\ m_{e\mu} & \Delta_{\mu\mu}\end{pmatrix}\begin{pmatrix}\nu_e(x)\\ \nu_\mu(x)\end{pmatrix},$$

$$\Delta_{\sigma\sigma}:=m_{\sigma\sigma}-\tilde{m}_{\sigma\sigma}. \tag{3.24}$$

Then, instead of S_σ and $\Pi_{\rho\sigma}$ employed above, we use

$$\tilde{S}_\rho(\not k):=(-\not k+i\tilde{m}_{\rho\rho}+\epsilon)^{-1}, \quad [\tilde{\Pi}_{\rho\sigma}]:=\begin{pmatrix} -i\Delta_{ee} & -im_{e\mu}\\ -im_{e\mu} & -i\Delta_{\mu\mu}\end{pmatrix}. \tag{3.25}$$

Taking account of the contribution from $\tilde{\mathcal{H}}^{int}_{e\mu}(x)$ as before, we obtain

$$\delta_{\sigma\rho}\tilde{S}_\rho-\tilde{\Pi}_{\sigma\rho}=\delta_{\sigma\rho}S_\rho-\Pi_{\sigma\rho}=f_{\sigma\rho}, \tag{3.26}$$

which shows that the arbitrariness in defining $S_\rho(\not k)$ disappears in the physical one-particle masses.

C. Diagonalization of the pole part in the propagator

We examine the diagonalization of the pole part in the neutrino propagator $S'_{\sigma\rho}(\not k)$. Writing the cofactor corresponding to $f_{\sigma\rho}$ as $F_{\sigma\rho}$, we have

$$[S'_{\sigma\rho}]=[f_{\sigma\rho}]^{-1}=\frac{1}{\det[f]}[F_{\sigma\rho}]^T. \tag{3.27}$$

We define $f^{(j)}_{\sigma\rho}$ and $F^{(j)}_{\sigma\rho}$ to be the values of $f_{\sigma\rho}$ and $F_{\sigma\rho}$ at the pole m_j of $[S'_{\sigma\rho}]$, respectively. We have

$$\begin{aligned}[f^{(j)}_{\sigma\rho}][F^{(j)}_{\sigma\rho}]^T&=\begin{pmatrix} i(-m_j+m_{ee}) & im_{e\mu}\\ im_{e\mu} & i(-m_j+m_{\mu\mu})\end{pmatrix}\\ &\quad\times\begin{pmatrix} i(-m_j+m_{\mu\mu}) & -im_{e\mu}\\ -im_{e\mu} & i(-m_j+m_{ee})\end{pmatrix}\\ &=[-(m_j-m_{ee})(m_j-m_{\mu\mu})+m^2_{e\mu}]\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}\\ &=\det[f^j]\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}=0\end{aligned} \tag{3.28}$$

due to Eq. (3.13) and (3.14) [or in accordance with Eq. (3.19)].

Next we define

$$(\rho^{(j)})^{-1}:=\left.\frac{d\{\det[f_{\sigma\rho}(\not k)]\}}{d(-\not k)}\right|_{\not k=im_j}. \tag{3.29}$$

From Eq. (3.21) we obtain

$$\begin{aligned}\text{RHS of Eq. (3.29)}&=[-2\not k+i(m_{ee}+m_{\mu\mu})]_{\not k=im_j}\\ &=i(m_1-m_2)(-1)^j,\end{aligned} \tag{3.30}$$

leading to

$$\begin{pmatrix}\rho^{(1)}\\ \rho^{(2)}\end{pmatrix}=\begin{pmatrix}\dfrac{1}{i(m_2-m_1)}\\ \dfrac{1}{i(m_1-m_2)}\end{pmatrix}=\frac{s_\theta c_\theta}{im_{e\mu}}\begin{pmatrix}1\\ -1\end{pmatrix},$$

$$s_\theta\equiv\sin\theta, \quad c_\theta\equiv\cos\theta. \tag{3.31}$$

We introduce a set of new renormalized fields $\psi_j^R(x)$ and $\overline{\psi}_j^R(x)$, $j=1,2$, expressed as

$$\psi_\sigma(x)=\sum_j A_{\sigma j}\psi_j^R(x),\quad \overline{\psi}_\sigma(x)=\sum_j \overline{A}_{\sigma j}\overline{\psi}_j^R(x), \tag{3.32}$$

where the coefficients $A_{\sigma j}$'s are determined so that $\langle 0|T[\psi_i^R(x)\overline{\psi}_j^R(y)]|0\rangle$ has only one pole term like $\delta_{ij}/(-\not{k}+im_j)$; $\overline{A}_{\sigma j}$ is a complex conjugate to $A_{\sigma j}$. Thus, from

$$[S'(\not{k})]=\frac{\rho^{(1)}}{-\not{k}+im_1+\epsilon}F^{(1)T}+\frac{\rho^{(2)}}{-\not{k}+im_2+\epsilon}F^{(2)T} + (\text{nonpole contributions}), \tag{3.33}$$

the conditions of the renormalized field can be expressed as

$$\rho^{(1)}F^{(1)T}=A\begin{pmatrix}1&0\\0&0\end{pmatrix}A^\dagger,\quad \rho^{(2)}F^{(2)T}=A\begin{pmatrix}0&0\\0&1\end{pmatrix}A^\dagger, \tag{3.34}$$

i.e.,

$$\rho^{(j)}F^{(j)}_{\rho\sigma}=\overline{A}_{\rho j}A_{\sigma j}, \tag{3.35}$$

which leads to

$$\overline{\rho^{(j)}}\overline{F^{(j)}_{\rho\sigma}}=\overline{A}_{\sigma j}A_{\rho j}=\rho^{(j)}F^{(j)}_{\sigma\rho}. \tag{3.36}$$

Thus we have

$$\overline{A}_{\rho j}A_{\sigma j}=\frac{\overline{A}_{\rho j}A_{\mu j}\overline{A}_{\mu j}A_{\sigma j}}{A_{\mu j}\overline{A}_{\mu j}}=\begin{cases}\dfrac{\overline{\rho^{(j)}}\overline{F^{(j)}_{\mu\rho}}F^{(j)}_{\mu\sigma}}{F^{(j)}_{\mu\mu}},\\[2ex] \dfrac{\rho^{(j)}F^{(j)}_{\rho\mu}\overline{F^{(j)}_{\sigma\mu}}}{\overline{F^{(j)}_{\mu\mu}}}.\end{cases} \tag{3.37}$$

A possible solution is given by

$$A_{\sigma j}=\frac{|\rho^{(j)}|}{\sqrt{\rho^{(j)}F^{(j)}_{\mu\mu}}}F^{(j)}_{\mu\sigma}\omega,\quad |\omega|^2=1. \tag{3.38}$$

The concrete form of $[A_{\sigma j}]$ is expressed as

$$[A_{\sigma j}]=\left(\frac{|s_\theta c_\theta/m_{e\mu}|}{\sqrt{\dfrac{s_\theta c_\theta}{m_{e\mu}}(-m_1+m_{ee})}}\begin{pmatrix}-im_{e\mu}\\ i(-m_1+m_{ee})\end{pmatrix},\ \frac{|s_\theta c_\theta/m_{e\mu}|}{\sqrt{\dfrac{s_\theta c_\theta}{m_{e\mu}}(m_2-m_{ee})}}\begin{pmatrix}-im_{e\mu}\\ i(-m_2+m_{ee})\end{pmatrix}\right)\omega \tag{3.39}$$

$$=\frac{m_{e\mu}}{|m_{e\mu}|}i\omega\begin{pmatrix}-|c_\theta| & -|s_\theta|\\ |c_\theta|\dfrac{s_\theta}{c_\theta} & -|s_\theta|\dfrac{c_\theta}{s_\theta}\end{pmatrix}. \tag{3.40}$$

For $s_\theta,c_\theta,m_{e\mu}>0$ and $\omega=i$, we have

$$[A_{\sigma j}]=\begin{pmatrix}c_\theta & s_\theta\\ -s_\theta & c_\theta\end{pmatrix}. \tag{3.41}$$

This matrix is the same as the matrix which diagonalizes the mass matrix of neutrino in the Lagrangian (3.2).

From the construction, we see that the propagator

$$S^R_{ij}(\not{k}):=\text{Fourier transform of}\langle 0|T[\psi_i^R(x)\overline{\psi}_j^R(y)]|0\rangle$$

$$=\sum_{\sigma,\rho}B_{i\sigma}S'_{\sigma\rho}(\not{k})\overline{B}_{j\rho},\quad B:=A^{-1}, \tag{3.42}$$

has an one-pole term in the diagonal element, i.e.,

$$[S^R_{ij}(\not{k})]=\left[\sum_l\frac{1}{-\not{k}+im_l+\epsilon}\left(\sum_{\sigma,\rho}B_{i\sigma}\rho^{(l)}F^{(l)}_{\sigma\rho}(\not{k})\overline{B}_{j\rho}\right)+(\text{contribution from continuous spectra})\right]$$

$$=\begin{pmatrix}\dfrac{1}{-\not{k}+im_1+\epsilon} & 0\\ 0 & \dfrac{1}{-\not{k}+im_2+\epsilon}\end{pmatrix}+(\text{contribution from continuous spectra}). \tag{3.43}$$

When we write $A_{\sigma j}$ as $z^{1/2}_{\sigma j}$, i.e.,

$$[z^{1/2}_{\sigma j}]:=[A_{\sigma j}], \tag{3.44}$$

we can express $S'_{\sigma\rho}(\not{k})$ as

$$S'_{\sigma\rho}(\not k)=\sum_j \frac{z_{\sigma j}^{1/2}\overline{z}_{\rho j}^{1/2}}{-\not k+im_j+\epsilon}+\int d(\kappa^2)\frac{\lambda_{\sigma\rho}(\kappa,\not k)}{k^2+\kappa^2-i\epsilon}. \quad (3.45)$$

Note that the diagonalization procedure of the propagator $S'_{\sigma\rho}$ described above is somewhat different from that adopted by Kaneko *et al.* [11]. The authors of Ref. [11] considered the intermediate step by introducing a set of fields $\{\phi_j(x),\tilde{\phi}_j(x)\}$ as defined by

$$\psi_\sigma(x)=\sum_j A_{\sigma j}\phi_j(x),$$

$$\overline{\psi}_\sigma(x)=\sum_j A_{\sigma j}\tilde{\phi}_j(x), \quad (3.46)$$

and examined the pole-part diagonalization of

$$\tilde{S}_{ij}(\not k):=\text{Fourier transform of } \langle 0|T[\phi_i(x)\tilde{\phi}_j(y)]|0\rangle. \quad (3.47)$$

Such an intermediate procedure is not necessary in the present case due to dropping the effect of $\mathcal{L}_{int}(x)$.

IV. SUMMARY AND FINAL REMARKS

We have reconsidered the problem of constructing the two kinds of Hilbert space in the field theory with mixing among neutrinos. For that purpose we have mainly reexamined the paper of BV [7], in which it is asserted that the usual oscillation formulas of neutrinos receive certain modifications caused by the Bogoliubov transformation among the creation-annihilation operators with definite flavors and those with definite masses. After explaining the logical structure of Ref. [7], we gave a reformulation of it along the line of thought of BV [7], and made clear the problematic and dubious points of Ref. [7]. The fundamental viewpoint of our consideration is based on the following two points. The first is that any Heisenberg field operator, such as the flavor neutrino fields $\psi_e(x)$ and $\psi_\mu(x)$, is expanded in terms of a complete set of helicity-momentum eigenfunctions with an arbitrarily fixed mass μ_σ. The second is that any direct observable such as the oscillation formulas of neutrinos should have no dependence on such a mass μ_σ.

We have pointed out in Sec. II that, concerning the mapping (2.8) which corresponds to the case of a special choice $\mu_e=m_1$ and $\mu_\mu=m_2$, there is no theoretical reason which enforces us to adopt such a special mapping, and that various oscillation formulas such as those examined in Sec. II C cannot be thought of as correct. It seems worthwhile to stress that we cannot be free from μ_σ insofar as we try to prepare a one-flavor-neutrino state with definite momentum, $\alpha_\sigma^\dagger(kr;t)|0\rangle$ and $\beta_\sigma^\dagger(kr;t)|0\rangle$; therefore, it is necessary for us to derive the oscillation formula without recourse to such a state. One probable way may be obtained through the consideration with recourse to the neutrino propagator, as described in Sec. III. [The trial of Blasone, Henning, and Vitiello [10] using the neutrino propagator is still based on the one-flavor-neutrino state, $\alpha_\sigma^\dagger(kr;t)|0\rangle$ and $\beta_\sigma^\dagger(kr;t)|0\rangle$, and so seems not to be correct.] Considerations including such an approach are found in Refs. [4,6,8].

Here we give a remark on the preparation of the one-flavor-neutrino state, which is a certain refinement of an idea proposed by Sassaroli [9]. (The details are described in Appendix C.) The author of Ref. [9] prepared the one-flavor-neutrino state as

$$|\nu_\mu(k;0)\rangle=\sum_{r=\uparrow,\downarrow}[A_r\alpha_1^\dagger(kr;0)+B_r\alpha_2^\dagger(kr;0)]|0\rangle_m. \quad (4.1)$$

Determining A_r and B_r so as to satisfy the boundary conditions at the initial time, the usual oscillation formulas are reproduced. The preparation of the one-flavor-neutrino state, Eq. (4.1), is a mere assumption, and then it is a future problem whether Eq. (4.1) could be grounded on a field-theoretical approach.

In Sec. III and Appendix B we investigated the structure of the flavor-neutrino propagator by following the ‘‘diagonalization’’ procedure developed by Kaneko *et al.* [11]. A set of the renormalized fields $\{\psi_j^R(x),\overline{\psi}_j^R(x)\}$ can be defined as

$$\psi_\sigma(x)=\sum_j z_{\sigma j}^{1/2}\psi_j^R(x),\quad \overline{\psi}_\sigma(x)=\sum_j \overline{z}_{\sigma j}^{1/2}\overline{\psi}_j^R(x), \quad (4.2)$$

with $(z^{1/2})^\dagger(z^{1/2})=I$, so that the Fourier transform (FT) of $\langle 0|T[\psi_j^R(x)\overline{\psi}_j^R(y)]|0\rangle$, i.e., $S_{jj}^R(\not k)$, has a single one-pole term, and

$$S'_{\sigma\rho}(\not k)=\text{FT of }\langle 0|T[\psi_\sigma(x)\overline{\psi}_\rho(y)]|0\rangle$$
$$=\sum_j \frac{z_{\sigma j}^{1/2}\overline{z}_{\rho j}^{1/2}}{-\not k+im_j+\epsilon}+\int d(\kappa^2)\frac{\lambda_{\sigma\rho}(\kappa,\not k)}{k^2+\kappa^2-i\epsilon}. \quad (4.3)$$

The matrix $[z^{1/2}]$, which may be called the generalized z factor, has been shown to be essentially the same as that diagonalizing the mass matrix M in the starting Lagrangian with the neutrino mixing. The propagator $[S'_{\sigma\rho}(\not k)]$ should be independent of the choice of the mass parameters $\tilde{m}_{\rho\rho}$'s, that is, independent of the choice of the perturbative vacuum corresponding to the ‘‘free’’ Hamiltonians specified by the mass parameters $\tilde{m}_{\rho\rho}$'s. The result of Sec. III is consistent with this requirement.

Taking account of the considerations in Secs. II and III, it seems favorable for us to treat the neutrino-oscillation problem by utilizing the neutrino propagator. The works developed by Giunti *et al.* [5] and Grimus and Stockinger [8] seem instructive from our viewpoint.

On the construction of the related Hilbert spaces, we have shown explicitly in the critical examination of Ref. [7] that there is no physical basis selecting one-flavor Hilbert space from many-flavor Hilbert spaces which are mathematically allowed. In the superrelativistic case we are allowed to work with recourse to the Hilbert space of mass eigenstates [4], since the mass differences among neutrinos can be neglected in this energy region. Therefore, it seems to be an important task to derive field theoretically the oscillation formula in neutrino experiments at low energies, with recourse to the relation of field operators $\nu_\sigma = \Sigma_j A_{\sigma j} \nu_j(x)$. From this viewpoint, the works developed by Giunti *et al.* [5] and Grimus and Stockinger [8] are very suggestive, as mentioned above. It seems necessary for us to investigate in detail how to derive oscillation formulas reflecting real experimental situations on the basis of the field theory.

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APPENDIX A

Explicit forms of the plane-wave eigenfunctions $u(kr)$ and $v(kr)$, satisfying

$$(i\not{k}+m)u(kr)=0, \quad (-i\not{k}+m)v(kr)=0, \tag{A1}$$

are given in the Kramers representation of γ matrices (i.e., $\vec{\gamma}=-\rho_y\otimes\vec{\sigma}$, $\gamma^4=\rho_x\otimes I$, $\gamma_5=-\rho_z\otimes I$) [14] by

$$u(k\uparrow)=\begin{pmatrix} c\alpha \\ c\beta \\ s\alpha \\ s\beta \end{pmatrix}, \quad u(k\downarrow)=\begin{pmatrix} -s\beta^* \\ s\alpha^* \\ -c\beta^* \\ c\alpha^* \end{pmatrix}, \tag{A2}$$

$$v(k\uparrow)=\begin{pmatrix} s\beta^* \\ -s\alpha^* \\ -c\beta^* \\ c\alpha^* \end{pmatrix}, \quad v(k\downarrow)=\begin{pmatrix} c\alpha \\ c\beta \\ -s\alpha \\ -s\beta \end{pmatrix}. \tag{A3}$$

Here, $c=\cos(\chi/2)$, $s=\sin(\chi/2)$, $\cot\chi=|\vec{k}|/m$, $k_z=k\cos\vartheta$, $k_x+ik_y=k\sin\vartheta\, e^{i\phi}$, $\alpha=\cos(\vartheta/2)e^{-i\phi/2}$, $\beta=\sin(\vartheta/2)e^{i\phi/2}$. Here $u(kr)$ and $v(kr)$ are the eigenfunctions of helicity $\vec{s}\cdot\vec{k}/|\vec{k}|$, $\vec{s}=(I\times\vec{\sigma})/2$:

$$\frac{1}{k}(\vec{s}\cdot\vec{k})u(k\uparrow)=\frac{1}{2}u(k\uparrow), \quad \frac{1}{k}(\vec{s}\cdot\vec{k})u(k\downarrow)=-\frac{1}{2}u(k\downarrow),$$

$$\frac{1}{k}(\vec{s}\cdot-\vec{k})v(k\uparrow)=\frac{1}{2}v(k\uparrow), \quad \frac{1}{k}(\vec{s}\cdot-\vec{k})v(k\downarrow)=-\frac{1}{2}v(k\downarrow). \tag{A4}$$

The solutions of Eqs. (A1) with mass m_j are written as $u_j(kr)$ and $v_j(kr)$. We obtain

$$u_i^*(kr)u_j(ks)=v_i^*(-kr)v_j(-ks)=\rho_{ij}(k)\delta_{rs},$$

$$u_i^*(kr)v_j(-ks)=v_i^*(-kr)u_j(ks)=i\lambda_{ij}(k)\delta_{rs},$$

$$i,j=1,2,e,\mu, \tag{A5}$$

where $v_j(-kr):=v_j(pr)$ with $\vec{p}=-\vec{k}$, $p_0=k_{0j}=\sqrt{\vec{k}^2+m_j^2}$; $\rho_{ij}=\cos[(\chi_i-\chi_j)/2]$, $\lambda_{ij}=\sin[(\chi_i-\chi_j)/2]$ with $\cot\chi_j=|\vec{k}|/m_j$. We have

$$\sum_r \{u_j^b(kr)\cdot u_j^d(kr)^*+v_j^b(-kr)\cdot v_j^d(-kr)^*\}=\delta_{bd}. \tag{A6}$$

With this notation, the transformation of the creation and annihilation operators of flavor neutrinos into those of mass eigenstates, Eq. (2.26), is given by

$$\begin{pmatrix} \alpha_e(kr;t) \\ \alpha_\mu(kr;t) \\ \beta_e^\dagger(-kr;t) \\ \beta_\mu^\dagger(-kr;t) \end{pmatrix}=\mathcal{K}(\theta,k)\begin{pmatrix} \alpha_1(kr;t) \\ \alpha_2(kr;t) \\ \beta_1^\dagger(-kr;t) \\ \beta_2^\dagger(-kr;t) \end{pmatrix},$$

$$\mathcal{K}(\theta,k)=\begin{pmatrix} P(\theta,k) & i\Lambda(\theta,k) \\ i\Lambda(\theta,k) & P(\theta,k) \end{pmatrix}, \tag{A7}$$

with

$$P(\theta,k)=\begin{pmatrix} c_\theta\rho_{e1}(k) & s_\theta\rho_{e2}(k) \\ -s_\theta\rho_{\mu1}(k) & c_\theta\rho_{\mu2}(k) \end{pmatrix},$$

$$\Lambda(\theta,k)=\begin{pmatrix} c_\theta\lambda_{e1}(k) & s_\theta\lambda_{e2}(k) \\ -s_\theta\lambda_{\mu1}(k) & c_\theta\lambda_{\mu2}(k) \end{pmatrix}. \tag{A8}$$

$\mathcal{K}(\theta,k)$ is confirmed to be a unitary operator:

$$\mathcal{K}(\theta,k)\mathcal{K}(\theta,k)^\dagger=\mathcal{K}(\theta,k)^\dagger\mathcal{K}(\theta,k)=I. \tag{A9}$$

APPENDIX B

We examine the diagonalization of the flavor neutrino propagator in the three-flavor case along the same line of thought as given in Sec. III.

1. Three-flavor mixing mass matrix

The relevant Lagrangian density with mutual transitions among three-flavor neutrinos, e, μ, and τ, is written after taking account of the spontaneous symmetry breaking in the Higgs sector as

$$\mathcal{L}(x) = -(\overline{\nu_{eL}}(x)\ \overline{\nu_{\mu L}}(x)\ \overline{\nu_{\tau L}}(x))(\not\partial + M')\begin{pmatrix} \nu'_{eR}(x) \\ \nu'_{\mu R}(x) \\ \nu'_{\tau R}(x) \end{pmatrix} - (\overline{\nu'_{eR}}(x)\ \overline{\nu'_{\mu R}}(x)\ \overline{\nu'_{\tau R}}(x))(\not\partial + M'^{\dagger})\begin{pmatrix} \nu_{eL}(x) \\ \nu_{\mu L}(x) \\ \nu_{\tau L}(x) \end{pmatrix} + \mathcal{L}'_{int}(x), \tag{B1}$$

where $M' = [m'_{\sigma\rho}]$. ($\mathcal{L}'_{int}$ is assumed to include no bilinear terms and no derivative of the neutrino field.) We perform the unitary transformations

$$\begin{pmatrix} \nu_{eL}(x) \\ \nu_{\mu L}(x) \\ \nu_{\tau L}(x) \end{pmatrix} = V_L \begin{pmatrix} \nu_{1L}(x) \\ \nu_{2L}(x) \\ \nu_{3L}(x) \end{pmatrix},$$

$$\begin{pmatrix} \nu_{eR}(x) \\ \nu_{\mu R}(x) \\ \nu_{\tau R}(x) \end{pmatrix} = V_R \begin{pmatrix} \nu_{1R}(x) \\ \nu_{2R}(x) \\ \nu_{3R}(x) \end{pmatrix}, \tag{B2}$$

so that the mass matrix is diagonalized:

$$V_L^{\dagger} M' V_R = \begin{pmatrix} \mu_1 & 0 & \\ 0 & \mu_2 & 0 \\ 0 & 0 & \mu_3 \end{pmatrix} \text{ with real } \mu_j\text{'s}. \tag{B3}$$

We can arbitrarily use the right-handed neutrino field $\nu'_{\rho R}$ given by $\nu'_{\rho R}(x) = \Sigma_\sigma W_{\rho\sigma}\nu_{\sigma R}(x), W^{\dagger}W = I$, while the mass matrix M' (assumed to be $\det M' \neq 0$) is uniquely expressed as

$$M' = M \cdot U, \quad UU^{\dagger} = I, \quad M = [m_{\rho\sigma}], \tag{B4}$$

where M is Hermitian as well as positive definite. (The last means that all eigenvalues are positive.) Using the matrix V which diagonalizes M as

$$V^{\dagger} M V = \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix}, \quad m_j > 0, \quad VV^{\dagger} = I, \tag{B5}$$

we choose W to be

$$W = U^{\dagger}; \tag{B6}$$

then, by defining ν_{jL} and ν_{jR} as

$$\nu_{\sigma L/R}(x) := \sum_j v_{\sigma j}\nu_{jL/R}(x), \quad V = [v_{\sigma j}], \tag{B7}$$

the Lagrangian density (B1) is expressed as

$$\mathcal{L}(x) = -(\overline{\nu_{eL}}(x)\ \overline{\nu_{\mu L}}(x)\ \overline{\nu_{\tau L}}(x))(\not\partial + M)\begin{pmatrix} \nu_{eR}(x) \\ \nu_{\mu R}(x) \\ \nu_{\tau R}(x) \end{pmatrix} - \text{H.c.} + \mathcal{L}_{int}(x) = -(\overline{\nu_e}(x)\ \overline{\nu_\mu}(x)\ \overline{\nu_\tau}(x))(\not\partial + M)\begin{pmatrix} \nu_e(x) \\ \nu_\mu(x) \\ \nu_\tau(x) \end{pmatrix} + \mathcal{L}_{int}(x); \tag{B8}$$

the first term in the last line has the diagonal form $-\Sigma_{j=1}^3 \overline{\nu_j}(x)(\not\partial + m_j)\nu_j(x)$.

Similarly to Eq. (3.23), we write the Hamiltonian density as

$$\mathcal{H}(x) = \mathcal{H}^0(x) + \mathcal{H}^{int}(x) - \mathcal{L}_{int}(x),$$

$$\mathcal{H}^0(x) = (\overline{\nu_e}(x)\ \overline{\nu_\mu}(x)\ \overline{\nu_\tau}(x))(\vec{\gamma}\vec{\nabla} + \tilde{M}) \times \begin{pmatrix} \nu_e(x) \\ \nu_\mu(x) \\ \nu_\tau(x) \end{pmatrix}, \tag{B9}$$

$$\mathcal{H}^{int}(x) = (\overline{\nu_e}(x)\ \overline{\nu_\mu}(x)\ \overline{\nu_\tau}(x)) \times \begin{pmatrix} \Delta_{ee} & m_{e\mu} & m_{e\tau} \\ m_{\mu e} & \Delta_{\mu\mu} & m_{\mu\tau} \\ m_{\tau e} & m_{\tau\mu} & \Delta_{\tau\tau} \end{pmatrix} \times \begin{pmatrix} \nu_e(x) \\ \nu_\mu(x) \\ \nu_\tau(x) \end{pmatrix}, \tag{B10}$$

$$\tilde{M} = \begin{pmatrix} \tilde{m}_{ee} & 0 & 0 \\ 0 & \tilde{m}_{\mu\mu} & 0 \\ 0 & 0 & \tilde{m}_{\tau\tau} \end{pmatrix},$$

$$\Delta_{\rho\rho} = m_{\rho\rho} - \tilde{m}_{\rho\rho}. \tag{B11}$$

We give useful relations as follows:

$$m_{\rho\sigma} = \sum_{j=1}^{3} v_{\rho j}\overline{v}_{\sigma j} m_j, \quad \rho,\sigma = (e,\mu,\tau), \tag{B12}$$

$$V^{\dagger} = V^{-1} = \frac{1}{\det V}[u^T] \text{ with } u_{\sigma j} = \text{cofactor of } v_{\sigma j}, \tag{B13}$$

$$(\det V)\sum_j v_{\rho j}\overline{v}_{\sigma j}=(\det V)\delta_{\rho\sigma}$$

$$=\sum_j v_{\rho j}u_{\sigma j}, \qquad (B14)$$

$$(\det V)\sum_\rho \overline{v}_{\rho j}v_{\rho k}=(\det V)\delta_{jk}$$

$$=\sum_\rho u_{\rho j}v_{\rho k}, \qquad (B15)$$

$$\sum_j m_j=\sum_\rho m_{\rho\rho}, \qquad (B16)$$

$$\Pi_j m_j=\det M. \qquad (B17)$$

2. Pole structure of the Fourier transform of the neutrino propagator

We consider the Fourier transform of the neutrino propagator:

$$S'_{\sigma\rho}(k\!\!\!/)=\text{Fourier transform of } \langle 0|T[\nu_\sigma(x)\overline{\nu_\rho}(y)]|0\rangle, \qquad (B18)$$

where $\nu_\sigma(x)$ and $\overline{\nu_\rho}(y)$ are the unrenormalized Heisenberg operators appearing in the Hamiltonian (B9).

In the same way as described in Sec. III B, $S'_{\sigma\rho}(k\!\!\!/)$ satisfies

$$S'_{\sigma\rho}(k\!\!\!/)=\delta_{\sigma\rho}S_\rho(k\!\!\!/)+\sum_\lambda S'_{\sigma\lambda}(k\!\!\!/)\Pi_{\lambda\rho}(k\!\!\!/)S_\rho(k\!\!\!/),$$

$$\sigma,\rho=e,\mu,\tau, \qquad (B19)$$

with $S_\rho(k\!\!\!/)=(-k\!\!\!/+i\tilde{m}_{\rho\rho}+\epsilon)^{-1}$, and is expressed as

$$S'_{\sigma\rho}(k\!\!\!/)=[f(k\!\!\!/)^{-1}]_{\sigma\rho} \text{ with } f_{\sigma\rho}(k\!\!\!/)=\delta_{\sigma\rho}S_\rho(k\!\!\!/)^{-1}-\Pi_{\sigma\rho}(k\!\!\!/). \qquad (B20)$$

Dropping $\mathcal{L}_{int}(x)$, the proper self-energy part $\Pi_{\sigma\rho}$ is given as

$$\Pi_{\sigma\rho}=-i(M-\tilde{M})_{\sigma\rho}, \qquad (B21)$$

and we have

$$[f_{\sigma\rho}(k\!\!\!/)]=[\delta_{\sigma\rho}(-k\!\!\!/+i\tilde{m}_{\rho\rho})+i(M-\tilde{M})_{\sigma\rho}]$$

$$=\begin{pmatrix} -k\!\!\!/+im_{ee} & im_{e\mu} & im_{e\tau} \\ im_{\mu e} & -k\!\!\!/+im_{\mu\mu} & im_{\mu\tau} \\ im_{\tau e} & im_{\tau\mu} & -k\!\!\!/+im_{\tau\tau} \end{pmatrix}. \qquad (B22)$$

The physical one-particle masses are determined as three-poles obtained from

$$\det[f_{\sigma\rho}(k\!\!\!/)]=0. \qquad (B23)$$

From the form of Eq. (B22), we see that the arbitrariness in separating $\mathcal{H}^{int}(x)$ from the ''free'' part in Eq. (B10), i.e., the arbitrariness in defining $S_\rho(k\!\!\!/)$, disappears in the physical one-particle masses under the approximation (B21). These one-particle masses determined from Eq. (B23) with $f_{\sigma\rho}(k\!\!\!/)$ given by Eq. (B22) coincide with the eigenvalues $\{m_j, j=1,2,3\}$ of the mass matrix $M=[m_{\rho\sigma}]$.

3. Diagonalization of the pole part in the propagator

We follow the same procedure of the diagonalization as described in Sec. III C. Writing the cofactor of $f_{\sigma\rho}(k\!\!\!/)$ as $F_{\sigma\rho}(k\!\!\!/)$, we write $S'_{\sigma\rho}(k\!\!\!/)$ in the same form as Eq. (3.27); then, we obtain

$$\det[f_{\sigma\rho}]|_{k\!\!\!/=im_j}=0=\left[\sum_\lambda f^{(j)}_{\sigma\lambda}F^{(j)}_{\rho\lambda}\right]=\left[\sum_\lambda F^{(j)}_{\lambda\sigma}f^{(j)}_{\lambda\rho}\right]. \qquad (B24)$$

The explicit form of $\rho^{(j)}$ defined in the same way as Eq. (3.29) is written as

$$(\rho^{(j)})^{-1}=-(m_{ee}-m_j)(m_{\mu\mu}-m_j)-(m_{\mu\mu}-m_j)(m_{\tau\tau}-m_j)$$
$$-(m_{\tau\tau}-m_j)(m_{ee}-m_j)+m_{e\mu}m_{\mu e}+m_{\mu\tau}m_{\tau\mu}$$
$$+m_{e\tau}m_{\tau e}. \qquad (B25)$$

By employing Eqs. (B12) and (B13), we obtain, after some calculations,

$$(\rho^{(1)})^{-1}=-(m_1-m_2)(m_1-m_3),$$
$$(\rho^{(2)})^{-1}=-(m_2-m_1)(m_2-m_3),$$
$$(\rho^{(3)})^{-1}=-(m_3-m_1)(m_3-m_2). \qquad (B26)$$

As to $F^{(j)}_{\rho\tau}$'s, expressed from the definition as

$$F^{(j)}_{e\tau}=-m_{\mu e}m_{\tau\mu}+m_{\tau e}(m_{\mu\mu}-m_j),$$
$$F^{(j)}_{\mu\tau}=-m_{e\mu}m_{\tau e}+m_{\tau\mu}(m_{ee}-m_j),$$
$$F^{(j)}_{\tau\tau}=m_{\mu e}m_{e\mu}-(m_{ee}-m_j)(m_{\mu\mu}-m_j), \qquad (B27)$$

some calculations lead to

$$F^{(j)}_{\rho\tau}=\frac{\overline{v}_{\rho j}v_{\tau j}}{\rho^{(j)}}, \quad \rho=e,\mu,\tau, \quad j=1,2,3; \qquad (B28)$$

thus, we obtain

$$\frac{\rho^{(j)}}{F^{(j)}_{\tau\tau}}=\frac{(\rho^{(j)})^2}{|v_{\tau j}|^2}. \qquad (B29)$$

Next we define a set of new fields $\psi^R_j(x)$ and $\overline{\psi^R_j}(x)$, $j=1,2,3$, in the same way as Eq. (3.32). The condition for determining the matrix A is

$$\rho^{(j)}(F^{(j)})^T=AE^{(j)}A^\dagger, \qquad (B30)$$

with

$$E^{(1)}=\begin{pmatrix}1&0&0\\0&0&0\\0&0&0\end{pmatrix},\quad E^{(2)}=\begin{pmatrix}0&0&0\\0&1&0\\0&0&0\end{pmatrix},$$

$$E^{(3)}=\begin{pmatrix}0&0&0\\0&0&0\\0&0&1\end{pmatrix}.$$

The above equation leads to

$$\overline{A}_{\rho j}A_{\sigma j}=\rho^{(j)}F^{(j)}_{\rho\sigma}=\frac{\overline{A}_{\rho j}A_{\tau j}\overline{A}_{\tau j}A_{\sigma j}}{\overline{A}_{\tau j}A_{\tau j}}=\frac{\rho^{(j)}\overline{F}^{(j)}_{\tau\rho}F^{(j)}_{\tau\sigma}}{F^{(j)}_{\tau\tau}};\tag{B31}$$

therefore, noting Eq. (B29) we are allowed to take

$$A_{\sigma j}=\sqrt{\frac{\rho^{(j)}}{F^{(j)}_{\tau\tau}}}F^{(j)}_{\tau\sigma}\omega=\frac{\omega|\rho^{(j)}|}{|v_{\tau j}|}F^{(j)}_{\tau\sigma},\quad |\omega|^2=1.\tag{B32}$$

Employing the concrete forms (B28) of $F^{(j)}_{\tau\sigma}=\overline{F}^{(j)}_{\sigma\tau}$, we obtain

$$\begin{pmatrix}A_{e1}\\A_{\mu 1}\\A_{\tau 1}\end{pmatrix}=\frac{\omega|\rho^{(1)}|\overline{v}_{\tau 1}}{\rho^{(1)}|v_{\tau 1}|}\begin{pmatrix}v_{e1}\\v_{\mu 1}\\v_{\tau 1}\end{pmatrix}=\epsilon_1\begin{pmatrix}v_{e1}\\v_{\mu 1}\\v_{\tau 1}\end{pmatrix},\tag{B33}$$

$$\begin{pmatrix}A_{e2}\\A_{\mu 2}\\A_{\tau 2}\end{pmatrix}=\frac{\omega|\rho^{(2)}|\overline{v}_{\tau 2}}{\rho^{(2)}|v_{\tau 2}|}\begin{pmatrix}v_{e2}\\v_{\mu 2}\\v_{\tau 2}\end{pmatrix}=\epsilon_2\begin{pmatrix}v_{e2}\\v_{\mu 2}\\v_{\tau 2}\end{pmatrix},\tag{B34}$$

$$\begin{pmatrix}A_{e3}\\A_{\mu 3}\\A_{\tau 3}\end{pmatrix}=\frac{\omega|\rho^{(3)}|\overline{v}_{\tau 3}}{\rho^{(3)}|v_{\tau 3}|}\begin{pmatrix}v_{e3}\\v_{\mu 3}\\v_{\tau 3}\end{pmatrix}=\epsilon_3\begin{pmatrix}v_{e3}\\v_{\mu 3}\\v_{\tau 3}\end{pmatrix},\tag{B35}$$

where

$$\epsilon_j:=\frac{\omega|\rho^{(j)}|\overline{v}_{\tau j}}{\rho^{(j)}|v_{\tau j}|}.\tag{B36}$$

By choosing the order as $m_3>m_2>m_1$, we have

$$\rho^{(1)}<0,\quad \rho^{(2)}>0,\quad \rho^{(3)}<0;\tag{B37}$$

then,

$$\epsilon_j=(-1)^j\frac{\omega v_{\tau j}}{|\overline{v}_{\tau j}|}.\tag{B38}$$

Thus the form of the matrix A satisfies the unitary condition, i.e.,

$$AA^\dagger=A^\dagger A=I,\tag{B39}$$

and is essentially the same as V which diagonalizes the mass matrix M:

$$A=[A_{\rho j}]=V\cdot E\quad\text{with}\quad E=\begin{pmatrix}\epsilon_1&0&0\\0&\epsilon_2&0\\0&0&\epsilon_3\end{pmatrix},\tag{B40}$$

$$A^\dagger MA=\begin{pmatrix}m_1&0&0\\0&m_2&0\\0&0&m_3\end{pmatrix}.\tag{B41}$$

$S^R_{ij}(\not k)$, the Fourier transform of $\langle 0|T[\psi^R_i(x)\overline{\psi}^R_j(y)]|0\rangle$, is now written as

$$\begin{aligned}[S^R_{ij}(\not k)]&=A^\dagger S'(\not k)A\\&=\left[\sum_l\frac{1}{-\not k+im_1+\epsilon}\left(\sum_{\sigma\rho}\overline{A}_{\sigma i}\rho^{(l)}F^{(l)}_{\sigma\rho}A_{\rho j}\right)\right]\\&\quad+(\text{contribution from continuous spectra})\\&=\begin{pmatrix}\frac{1}{-\not k+im_1+\epsilon}&0&0\\0&\frac{1}{-\not k+im_2+\epsilon}&0\\0&0&\frac{1}{-\not k+im_3+\epsilon}\end{pmatrix}\\&\quad+(\text{contribution from continuous spectra}),\end{aligned}\tag{B42}$$

and as to $S'_{\sigma\rho}(\not k)$ we obtain the same form as given by Eq. (3.45).

APPENDIX C

We consider a process of one ν_μ production at the time $t=0$ and detection of ν_σ at the time $t>0$. We assume that this initial ν_μ state is so prepared that it can be expressed by

$$|\nu_\mu(k;0)\rangle=\sum_{r=\uparrow,\downarrow}[A_r\alpha_1^\dagger(kr;0)+B_r\alpha_2^\dagger(kr;0)]|0\rangle_m,\tag{C1}$$

where A_r and B_r are to be determined in accordance with the experimental situation of preparing the initial state $|\nu_\mu;0\rangle$; $r=$helicity. Now we examine the probability amplitudes of finding one left-handed flavor neutrino ν_σ at a space-time point $(\vec{x},t)$ through some charged-current weak interaction for detection. We define these amplitudes as

$$\Phi_\sigma(\vec{x},t)\equiv{}_m\left\langle 0\left|\frac{1+\gamma_5}{2}\nu_\sigma(\vec{x},t)\sum_{r=\uparrow,\downarrow}[A_r\alpha_1^\dagger(kr;0)+B_r\alpha_2^\dagger(kr;0)]\right|0\right\rangle_m,\quad \sigma=e,\mu. \tag{C2}$$

In the Kramers representation of γ matrices explained in Appendix A, we have

$$\frac{1+\gamma_5}{2}u_j(k\uparrow)=s_j\begin{pmatrix}0\\0\\\chi^{(+)}(k)\end{pmatrix},\quad \chi^{(+)}(k)\equiv\begin{pmatrix}\alpha(k)\\\beta(k)\end{pmatrix},$$

$$\frac{1+\gamma_5}{2}u_j(k\downarrow)=c_j\begin{pmatrix}0\\0\\\chi^{(-)}(k)\end{pmatrix},\quad \chi^{(-)}(k)\equiv\begin{pmatrix}-\beta^*(k)\\\alpha^*(k)\end{pmatrix},$$

$$j=1,2, \tag{C3}$$

where $s_j=\sin(\chi_j/2)$, $c_j=\cos(\chi_j/2)$ with $\cot\chi_j=|\vec{k}|/m_j$; $\chi^{(+)}(k)$ and $\chi^{(-)}(k)$ are the eigenfunctions of $(\vec{\sigma}\cdot\vec{k})/k$ with the eigenvalues $+1$ and -1, respectively. Thus, Φ_σ's are rewritten as

$$\Phi_e(\vec{x},t)=\frac{e^{i\vec{k}\cdot\vec{x}}}{\sqrt{V}}\left[(A_\uparrow c_\theta s_1e^{-i\omega_1t}+B_\uparrow s_\theta s_2e^{-i\omega_2t})\times\begin{pmatrix}0\\0\\\chi^{(+)}(k)\end{pmatrix}+(A_\downarrow c_\theta c_1e^{-i\omega_1t}+B_\downarrow s_\theta c_2e^{-i\omega_2t})\times\begin{pmatrix}0\\0\\\chi^{(-)}(k)\end{pmatrix}\right], \tag{C4}$$

$$\Phi_\mu(\vec{x},t)=\frac{e^{i\vec{k}\cdot\vec{x}}}{\sqrt{V}}\left[(-A_\uparrow s_\theta s_1e^{-i\omega_1t}+B_\uparrow c_\theta s_2e^{-i\omega_2t})\times\begin{pmatrix}0\\0\\\chi^{(+)}(k)\end{pmatrix}+(-A_\downarrow s_\theta c_1e^{-i\omega_1t}+B_\downarrow c_\theta c_2e^{-i\omega_2t})\begin{pmatrix}0\\0\\\chi^{(-)}(k)\end{pmatrix}\right]. \tag{C5}$$

We require the following boundary conditions:

$$\mathrm{BC}\langle 1\rangle,\quad \Phi_e(\vec{x},t=0)=0, \tag{C6}$$

$$\mathrm{BC}\langle 2\rangle,\quad \int d^3x|\Phi_\mu(\vec{x},t=0)|^2=1. \tag{C7}$$

From BC$\langle 1\rangle$, we obtain

$$A_\uparrow=-\frac{s_\theta s_2}{c_\theta s_1}B_\uparrow,\quad A_\downarrow=-\frac{s_\theta c_2}{c_\theta c_1}B_\downarrow. \tag{C8}$$

Under these constraints, BC$\langle 2\rangle$ leads to

$$[|B_\uparrow|^2s_2^2+|B_\downarrow|^2c_2^2]/c_\theta^2=1. \tag{C9}$$

The probabilities of finding ν_e and ν_μ at time t, defined by

$$P(\nu_\mu\to\nu_\mu;t)\equiv\int d^3x|\Phi_\mu(\vec{x},t)|^2, \tag{C10}$$

$$P(\nu_\mu\to\nu_e;t)\equiv\int d^3x|\Phi_e(\vec{x},t)|^2, \tag{C11}$$

are given by

$$P(\nu_\mu\to\nu_\mu;t)=\frac{1}{c_\theta^2}[|B_\uparrow|^2s_2^2+|B_\downarrow|^2c_2^2]\times\left[1-\sin^2(2\theta)\sin^2\left(\frac{\Delta\omega}{2}t\right)\right]=1-\sin^2(2\theta)\sin^2\left(\frac{\Delta\omega}{2}t\right)\quad\text{due to Eq. (C9)}, \tag{C12}$$

$$P(\nu_\mu\to\nu_e;t)=s_\theta^2[|B_\uparrow|^2s_2^2+|B_\downarrow|^2c_2^2]4\sin^2\left(\frac{\Delta\omega}{2}t\right)=\sin^2(2\theta)\sin^2\left(\frac{\Delta\omega}{2}t\right)\quad\text{due to Eq. (C9)}, \tag{C13}$$

which coincide with the oscillation formulas usually employed. Note that, in general, the prepared state $|\nu_\mu(k;0)\rangle$, Eq. (C1), cannot be set equal to the state

$$\sum_r C_r\alpha_\mu^\dagger(kr;0)|0\rangle_m/\mathcal{N}_\mu(k). \tag{C14}$$

Because if we set it so, Eqs. (C8) lead to

$$\frac{s_2}{s_1}=\frac{\rho_{\mu1}}{\rho_{\mu2}},\quad \frac{c_2}{c_1}=\frac{\rho_{\mu1}}{\rho_{\mu2}}, \tag{C15}$$

by taking into account of Eqs. (A7) and Eq. (A8), and (C15) does not hold for $m_1\neq m_2$, i.e., as far as the neutrino mixing exists.

The above derivation of the usual oscillation formula, which amounts to a certain refinement of the idea proposed by Sassaroli [9], is essentially based on the assumption of the initial state, Eq. (C1). Although it may be necessary for us to establish the physical basis of this assumption, the above derivation gives us a clue to the task of how to define a one-flavor-neutrino state $|\nu_\mu(k;0)\rangle$ under the condition

$\nu_\sigma(x)=\Sigma_j A_{\sigma j}\nu_j(x)$. In the super relativistic case, the same linear transformation as that for $\{\nu_\sigma(x),\nu_j(x)\}$,

$$\begin{pmatrix}\alpha_\sigma(kr;t)\\ \beta_\sigma(kr;t)\end{pmatrix}=\sum_j A_{\sigma j}\begin{pmatrix}\alpha_j(kr;t)\\ \beta_j(kr;t)\end{pmatrix}, \tag{C16}$$

is obtained and we have, from Eq. (C8),

$$A_{\uparrow(\downarrow)}=-\frac{s_\theta}{c_\theta}B_{\uparrow(\downarrow)} \quad \text{due to} \quad \frac{s_2}{s_1}\rightarrow 1, \quad \frac{c_2}{c_1}\rightarrow 1, \tag{C17}$$

which is consistent with

$$|\nu_\mu(kr;0)\rangle=\alpha_\mu^\dagger(kr;0)|0\rangle_m. \tag{C18}$$

[1] B. Pontecorvo, Zh. Eksp. Teor. Fiz. **33**, 549 (1958) [Sov. Phys. JETP **6**, 429 (1958); Z. Maki, M. Nakagawa, and S. Sakata, Prog. Theor. Phys. **28**, 870 (1962); See also V. Gribov and B. Pontecorvo, Phys. Lett. **28B**, 493 (1969); S. M. Bilenky and B. Pontecorvo, *ibid.* **61B**, 248 (1976); Phys. Rep. **41**, 225 (1978).

[2] P. Davis and D. Evans, in Proceedings of the Seminar on Active Process on the Sun and the Problem of Solar Neutrino, Leningrad, 1976 (unpublished); See the monograph written by Bilenky and Pontecorvo referred to in [1] and the Super-Kamiokande Collaboration, Y. Fukuda *et al.*, Phys. Rev. Lett. **81**, 1562 (1998).

[3] As to the related references, see, for example, the references cited in the paper of S. M. Bilenky, C. Giunti, and W. Grimus, Phys. Rev. D **57**, 1920 (1998).

[4] C. Giunti, C. W. Kim, and U. W. Lee, Phys. Rev. D **45**, 2414 (1992).

[5] C. Giunti, C. W. Kim, J. A. Lee, and U. W. Lee, Phys. Rev. D **48**, 4310 (1993).

[6] J. Rich, Phys. Rev. D **48**, 4318 (1993).

[7] M. Blasone and G. Vitiello, Ann. Phys. (N.Y.) **244**, 283 (1995); **249**, 363(E) (1996); see also E. Alfinito, M. Blasone, A. Iorio, and G. Vitiello, Phys. Lett. B **362**, 91 (1995).

[8] W. Grimus and P. Stockinger, Phys. Rev. D **54**, 3414 (1996).

[9] E. Sassaroli, hep-ph/9609476; hep-ph/9805480.

[10] M. Blasone, P. A. Henning, and G. Vitiello, hep-th/9803157; M. Blasone, G. Vitiello, and P. A. Henning, hep-ph/9807370; M. Blasone, hep-ph/9810329.

[11] T. Kaneko, Y. Ohnuki, and K. Watanabe, Prog. Theor. Phys. **30**, 521 (1963).

[12] K. Fujii, Nuovo Cimento **44**, 722 (1964). See also the references cited therein.

[13] H. Umezawa, in *Lectures of Nuclear Physics*, edited by S. Kikuchi *et al.* (Kyoritsu Schuppan, Tokyo, 1963), Vol. 12.

[14] E.g., E. Cornaldesi, Nucl. Phys. **7**, 305 (1958).