Combining *CP* asymmetries in $B \rightarrow K\pi$ decays

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We prove an approximate relation, to leading order in dominant terms, between CP-violating rate differences in $B^0/\overline{B}^0 \rightarrow K^\pm \pi^\mp$ and $B^\pm \rightarrow K^\pm \pi^0$. We show how data from these two processes may be combined in order to enhance the significance of a nonzero result. [S0556-2821(99)05909-3]

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Up to now, CP violation has only been observed in the mixing of neutral K meson states [1]. Thus, it remains to be confirmed that CP violation in the kaon system arises from phases in the Cabibbo-Kobayashi-Maskawa matrix [2] describing weak charge-changing transitions of quarks. Such evidence can be provided by B meson decays, in which the standard model predicts sizable CP asymmetries between partial rates of B mesons and their corresponding antiparticles [3]. Model-dependent calculations of CP asymmetries in B decays to a pair of charmless pseudoscalar mesons have been carried out by a large number of authors [4].

In the present paper we study relations between direct CP asymmetries in $B \rightarrow K\pi$ decays, following from a model-independent hierarchy among various contributions to decay amplitudes. Observation of three of these decays, $B^0 \rightarrow K^+\pi^-$, $B^+ \rightarrow K^+\pi^0$ and $B^+ \rightarrow K^0\pi^+$, combining processes with their charge conjugates, was reported recently by CLEO [5,6]. The number of events in these modes is 43, 38, and 12, respectively. We shall show in this note that while each individual measurement is unlikely to provide a significant nonzero asymmetry measurement at present levels of statistics, the rate differences in the first two processes are expected to be related:

$$\Gamma(B^0 \rightarrow K^+\pi^-) - \Gamma(\overline{B^0} \rightarrow K^-\pi^+) \simeq 2[\Gamma(B^+ \rightarrow K^+\pi^0) - \Gamma(B^- \rightarrow K^-\pi^0)]. \quad (1)$$

With present statistics and with an estimate of the maximum asymmetry (46%) possible in the standard model, the combined sample of $K^\pm\pi^\mp$ and $K^\pm\pi^0$ events is sufficiently large to display a suitably averaged asymmetry of up to four standard deviations. A somewhat more conservative estimate follows from considering rate differences.

In order to study $B \rightarrow K\pi$ decays, we will employ a diagrammatic approach based on flavor SU(3) [7]. Since we concentrate on strangeness-changing processes, the major part of our analysis will only require isospin symmetry. SU(3) symmetry and SU(3) breaking effects [8] will be introduced when relating these processes to corresponding strangeness-conserving $B \rightarrow \pi\pi$ decays. The decomposition of decay amplitudes in terms of flavor flow topologies is [9]

$$\begin{aligned}
-A(B^0 \rightarrow K^+\pi^-) &= \left(P + \frac{2}{3}P^c_{EW}\right) + (T) \\
&= B_{1/2} - A_{1/2} - A_{3/2}, \\
-\sqrt{2}A(B^+ \rightarrow K^+\pi^0) &= \left(P + P_{EW} + \frac{2}{3}P^c_{EW}\right) + (T + C + A) \\
&= B_{1/2} + A_{1/2} - 2A_{3/2}, \\
A(B^+ \rightarrow K^0\pi^+) &= \left(P - \frac{1}{3}P^c_{EW}\right) + (A) \\
&= B_{1/2} + A_{1/2} + A_{3/2}, \\
\sqrt{2}A(B^0 \rightarrow K^0\pi^0) &= \left(P - P_{EW} - \frac{1}{3}P^c_{EW}\right) - (C) \\
&= B_{1/2} - A_{1/2} + 2A_{3/2}. \quad (2)
\end{aligned}$$

On the right-hand-sides we also include an equivalent decomposition in terms of isospin amplitudes [10], where A and B are $\Delta I = 1$ and $\Delta I = 0$ amplitudes and subscripts denote the isospin of $K\pi$. This equivalence is implied by the relations

$$\begin{aligned}
B_{1/2} &= \left(P + \frac{1}{6}P^c_{EW}\right) + \frac{1}{2}(T + A), \\
A_{1/2} &= \left(\frac{1}{3}P_{EW} - \frac{1}{6}P^c_{EW}\right) \\
&\quad + \left(-\frac{1}{6}T + \frac{1}{3}C + \frac{1}{2}A\right), \\
A_{3/2} &= -\frac{1}{3}(P_{EW} + P^c_{EW}) - \frac{1}{3}(T + C). \quad (3)
\end{aligned}$$

The terms in the first parentheses of Eqs. (1) and (2), a QCD penguin (P), an electroweak penguin (P_{EW}) and a

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color-suppressed electroweak penguin (P^c_{EW}) amplitude, carry each a weak phase $\arg(V^*_{tb}V_{ts})=-\pi$. The other three terms, tree (T), color-suppressed (C) and annihilation (A) amplitudes, carry a different weak phase $\arg(V^*_{ub}V_{us})=\gamma$.

We will assume a hierarchy among amplitudes carrying the same weak phase [9]

$$|P|\gg|P_{EW}|\gg|P^c_{EW}|, \tag{4}$$

$$|T|\gg|C|\gg|A|, \tag{5}$$

where a roughly common hierarchy factor of about 0.2 describes the ratio of sequential amplitudes. The hierarchy between penguin amplitudes is based on QCD and electroweak loop factors and is supported by model calculations of short distance operator matrix elements [11]. Very recently this hierarchy was shown to follow from model-independent considerations [12]. The hierarchy between C and T is taken from short distance QCD corrections and phenomenology of $B\to\overline{D}\pi$ decays [13]. The measured rates of color-suppressed processes, such as $B^0\to\overline{D}^0\pi^0$ [14], show that rescattering effects do not enhance C to the level of T. This will also be assumed to be the case for $B\to K\pi$. Finally, the hierarchy between A and C follows essentially from a f_B/m_B factor in A relative to T [15]. Several authors [16,17] have noted recently that the last assumption, $|A|\ll|C|$, can be spoiled by rescattering effects (from intermediate states mediated by T) through soft annihilation or up-quark penguin topologies. We will therefore leave open the possibility that $|A|\sim|C|$. The possibility that $|A|$ can be as large as $|T|$, implied by some model-dependent calculations [18], will be excluded. We consider it unlikely in view of existing limits on rescattering in $B^0\to K^+K^-$ [16].

Interference between amplitudes carrying different weak phases and different strong phases leads to CP rate differences between the processes in Eqs. (1) and their charge conjugates. Such interference involves the product of the magnitudes of the amplitudes appearing in the first parenthesis with the amplitudes in the second parenthesis, a sine factor of their relative weak phase and a sine of the relative strong phase. Thus, all the contributions are proportional to $\sin\gamma$, whereas the strong phase difference is generally unknown and may depend on the product. We denote by $2\vec{P}\vec{T}$ the interference between P and T contributing to $\Delta(K^+\pi^-)\equiv\Gamma(B^0\to K^+\pi^-)-\Gamma(\overline{B^0}\to K^-\pi^+)$, and use similar notations for other interference terms and other CP rate differences. One then finds for the B-$\overline{B}$ rate differences the following expressions, where terms are written in decreasing order using Eqs. (3) and (4), and the smallest terms are neglected:

$$\Delta(K^+\pi^-)=2\vec{P}\vec{T}+\frac{4}{3}\vec{P}^c_{EW}\vec{T}, \tag{6}$$

$$\Delta(K^+\pi^0)=\vec{P}\vec{T}+\vec{P}_{EW}\vec{T}+\vec{P}\vec{C}+\vec{P}\vec{A}+\vec{P}_{EW}\vec{C}+\frac{2}{3}\vec{P}^c_{EW}\vec{T}+\cdots,$$

$$\Delta(K^0\pi^+)=2\vec{P}\vec{A}+\cdots,$$

$$\Delta(K^0\pi^0)=-\vec{P}\vec{C}+\vec{P}_{EW}\vec{C}+\frac{1}{3}\vec{P}^c_{EW}\vec{C}.$$

We note that, in the absence of electroweak penguin amplitudes, one finds [19]

$$\Delta(K^+\pi^-)+\Delta(K^0\pi^+)=2\Delta(K^+\pi^0)+2\Delta(K^0\pi^0). \tag{7}$$

However, this relation is spoiled by electroweak penguin contributions.

Comparing the four rate differences, we see that the dominant terms of the form $\vec{P}\vec{T}$ appear only in the first two rate differences, leading at this order to the relation

$$\Delta(K^+\pi^-)\approx 2\Delta(K^+\pi^0). \tag{8}$$

The next-to-leading terms correcting this relation are $\vec{P}_{EW}\vec{T}$ and $\vec{P}\vec{C}$. The first term can be shown to lead to a negligible rate difference. The argument is based on a property of the $A_{3/2}$ amplitude, which was shown recently [20] to consist of $T+C$ and electroweak penguin contributions with approximately equal strong phases. Using this property we conclude that

$$\vec{P}_{EW}\vec{T}+\vec{P}_{EW}\vec{C}+\vec{P}^c_{EW}\vec{T}+\vec{P}^c_{EW}\vec{C}\approx 0, \tag{9}$$

or, to leading order, that $\vec{P}_{EW}\vec{T}\approx 0$. Since the term $\vec{P}\vec{C}$ is the only next-to-leading correction to Eq. (8), this equality is expected to hold to about 20%.

Using the hierarchy $|A|\ll|C|$, it has often been assumed that the rate difference $\Delta(K^0\pi^+)$ is extremely small. However, recently it was argued [16,17] that rescattering effects may enhance A to the level of C, thus leading to a CP asymmetry in this process at a level of 10%. This would imply that the term $\vec{P}\vec{A}$ appearing in both $\Delta(K^0\pi^+)$ and $\Delta(K^+\pi^0)$ is next-to-leading and may be comparable to $\vec{P}\vec{C}$. In this case Eq. (8) could be violated by up to about 40%, and a better approximation becomes

$$\Delta(K^+\pi^-)+\Delta(K^0\pi^+)\approx 2\Delta(K^+\pi^0). \tag{10}$$

A way of gauging the importance of the $\vec{P}\vec{A}$ term would be by measuring a nonzero value for $\Delta(K^0\pi^+)$. The dominant correction to the approximate relation (10) is the term $-2\vec{P}\vec{C}$ which is contributed by $2\Delta(K^0\pi^0)$ on the right-hand side of Eq. (7).

In order to study relative asymmetries, let us first discuss the rates themselves. To leading order, all four $B\to K\pi$ processes are dominated by the (gluonic) penguin terms P in Eq. (1), and their rates as well as their charge-conjugates rates are expected to satisfy the relations

$$\Gamma(B^+\to K^0\pi^+)=\Gamma(B^0\to K^+\pi^-)=2\Gamma(B^+\to K^+\pi^0)$$
$$=2\Gamma(B^0\to K^0\pi^0). \tag{11}$$

The next-to-leading corrections to these equalities are interference terms of the form $2\,\mathrm{Re}(PT^*)$ and $2\mathrm{Re}(PP^*_{EW})$. To first order in small quantities, the $B\to K\pi$ rates satisfy the sum rule [21]

$$2\Gamma(B^+\to K^+\pi^0)+2\Gamma(B^0\to K^0\pi^0)$$
$$=\Gamma(B^+\to K^0\pi^+)+\Gamma(B^0\to K^+\pi^-), \qquad (12)$$

which may be used to anticipate a small $B^0\to K^0\pi^0$ rate.

The first order terms modify the rate equalities Eq. (11). The difference between $2\mathcal{B}(B^+\to K^+\pi^0)$ and $\mathcal{B}(B^+\to K^0\pi^+)$ is given by two terms, $2\,\mathrm{Re}(PT^*)+2\,\mathrm{Re}(PP^*_{EW})$, which can imply a factor as large as about two between these branching ratios. In fact, the present central values measured for these branching ratios differ by a factor of 2.1 [6], which can be used [20] to place model-independent bounds on the weak phase γ. The rate difference of the two processes involving charged kaons, to which Eq. (8) applies, is given to this order by

$$2\Gamma(B^+\to K^+\pi^0)-\Gamma(B^0\to K^+\pi^-)=2\,\mathrm{Re}(PP^*_{EW}), \qquad (13)$$

where the correction term is expected to be no larger than about 40% of each of these rates. Since a similar approximation applies to the rate difference (or a better one in case of small rescattering effects), the asymmetries of these two processes can differ by about a factor of two. In general, with no other information about the above interference terms, the equality between rate differences Eq. (8) holds to a better approximation than the equality of corresponding asymmetries.

In order to estimate the CP asymmetries in $B^0/\overline{B^0}\to K^\pm\pi^\mp$ and $B^\pm\to K^\pm\pi^0$, one must know the ratio $|T/P|$. Using previous data [5] we have shown [22] that this ratio is smaller than one, representing another hierarchy factor of about 0.2. Let us update information about P and T using more recent data [6]. We will quote squares of amplitudes in terms of decay rates.

The most straightforward way of obtaining $|P|$ is from the observed CP-averaged branching ratio [6]

$$\mathcal{B}(B^\pm\to K^0/\overline{K^0}\pi^\pm)=(14\pm5\pm2)\times10^{-6}, \qquad (14)$$

since there are no first order corrections to P in these processes even when $|A|$ is as large as $|C|$. Using the value [6] $\tau(B^+)=(1.65\pm0.04)\times10^{-12}\,\mathrm{s}$, we find $|P|^2=\Gamma(B^\pm\to K^0/\overline{K^0}\pi^\pm)=(8.5\pm3.3)\times10^6\ \mathrm{s}^{-1}$.

An estimate of $|T|$ is more uncertain at this time. While CLEO has quoted upper limits at 90% confidence level:

$$\mathcal{B}(B^0/\overline{B^0}\to\pi^+\pi^-)<8.4\times10^{-6},$$
$$\mathcal{B}(B^\pm\to\pi^\pm\pi^0)<16\times10^{-6}, \qquad (15)$$

their data imply signals for these decays with significance of 2.9 and 2.3 standard deviations, respectively. Taking these signals seriously, we may obtain from the reported event rates and efficiencies the branching ratios

$$\mathcal{B}(B^0/\overline{B^0}\to\pi^+\pi^-)=(3.7^{+2.0}_{-1.7})\times10^{-6}, \qquad (16)$$

$$\mathcal{B}(B^\pm\to\pi^\pm\pi^0)=(5.9^{+3.2}_{-2.7})\times10^{-6}. \qquad (17)$$

While destructive interference between tree and penguin amplitudes in $B^0\to\pi^+\pi^-$ and/or constructive interference between tree and color-suppressed or electroweak penguin amplitudes in $B^+\to\pi^+\pi^0$ may lead to $\mathcal{B}(B^0\to\pi^+\pi^-)<2\mathcal{B}(B^+\to\pi^+\pi^0)$ [7], we shall ignore such effects as in Ref. [22]. Thus, using an SU(3) relation between $B\to K\pi$ and $B\to\pi\pi$, taking $\tau(B^0)=(1.56\pm0.04)\times10^{-12}\,\mathrm{s}^{-1}$, and introducing SU(3) breaking through f_K/f_π [7,8], we have the two independent estimates

$$|T|^2_{B^0}=\left[\frac{V_{us}}{V_{ud}}\frac{f_K}{f_\pi}\right]^2\Gamma(B^0/\overline{B^0}\to\pi^+\pi^-)=(1.8\pm0.9)\times10^5\ \mathrm{s}^{-1}, \qquad (18)$$

$$|T|^2_{B^\pm}=2\left[\frac{V_{us}}{V_{ud}}\frac{f_K}{f_\pi}\right]^2\Gamma(B^\pm\to\pi^\pm\pi^0)=(5.4\pm2.7)\times10^5\ \mathrm{s}^{-1}, \qquad (19)$$

whose average is $|T|^2=(2.16\pm0.85)\times10^5\ \mathrm{s}^{-1}$. When combined with our estimate for $|P|^2$, and assuming an additional 20% error from neglecting a penguin amplitude in $B^0\to\pi^+\pi^-$ and a color-suppressed amplitude in $B^+\to\pi^+\pi^0$, this leads to $|T/P|=0.160\pm0.054$, or $|T/P|<0.23$ at 90% confidence level. A more precise determination of this ratio requires more statistics. We will assume its preliminary value. A slightly larger value of $|T+C|/|P|=0.24\pm0.06$ was estimated in Refs. [20] and [23].

As we have shown, CP asymmetries in $B^0/\overline{B^0}\to K^\pm\pi^\mp$ and $B^\pm\to K^\pm\pi^0$ are equal to each other, to leading order in $|T/P|$, $|P_{EW}/P|$ and $|C/T|$, and are given by $2|T/P|\sin\gamma\sin\delta$, where δ is the strong phase difference between T and P. This phase is generally unknown, but could be substantial. While the tree amplitudes are expected to factorize, thus showing little evidence for rescattering effects, the penguin amplitude obtains a large contribution from a so-called charming penguin term [24], involving long distance effects of rescattering from charm-anticharm intermediate states. It is therefore conceivable that δ could attain a large value, such that $\sin\delta\sim1$. The values of γ allowed at present [25] include those around 90° obeying $\sin\gamma\sim1$. We therefore conclude that an interesting range of asymmetry measurements includes the value $2|T/P|$ which we found to be 0.32 ± 0.11 to leading order, or less than 46% at 90% confidence level.

To first approximation, CP asymmetries in the processes $B^0/\overline{B^0}\to K^\pm\pi^\mp$ and $B^\pm\to K^\pm\pi^0$ are equal. Averaging them leads to a statistically more significant result than measuring them separately. Denoting by N_n, A_n and N_c, A_c the number of events and asymmetries in $B^0/\overline{B^0}\to K^\pm\pi^\mp$ and $B^\pm\to K^\pm\pi^0$, respectively, one may define an averaged asymmetry

$$A_{\mathrm{av}}\equiv\frac{N_nA_n+N_cA_c}{N_{\mathrm{tot}}}, \qquad (20)$$

where $N_{\rm tot}$ is the total number of $K^{\pm}\pi^0$ and $K^{\pm}\pi^{\mp}$ events. It is easy to show that, under the assumption of equal asymmetries, the total number of events $N_{\rm tot}$ required to observe this asymmetry at the n-standard-deviation level of significance does not exceed $N_{\rm tot}=(n/A_{\rm av})^2$. Thus, for $|A_{\rm max}|=0.46$ and $N_{\rm tot}=43+38=81$ events, one could see a signal as large as four standard deviations, whereas the maximum signals based on N_n and N_c separately would not be expected to exceed 3σ. Backgrounds and particle misidentification will degrade these estimates somewhat.

As mentioned, in general the approximate equality of CP rate differences in the processes $B^0/\overline{B^0}\to K^{\pm}\pi^{\mp}$ and $B^{\pm}\to K^{\pm}\pi^0$ is expected to hold to a better accuracy than the equality of corresponding asymmetries. Thus, we shall estimate the errors on the separate quantities

$$(\vec{P}\vec{T})_n \equiv \Delta(K^+\pi^-)/2, \quad (\vec{P}\vec{T})_c \equiv \Delta(K^+\pi^0) \tag{21}$$

and on their average, and compare these with the maximum possible value of

$$\vec{P}\vec{T} = 2|P||T|\sin(\delta_T-\delta_P)\sin\gamma. \tag{22}$$

The most conservative estimate is based directly on the experimental errors on sums of rates for particles and antiparticles, which one may show are equal to the errors on rate *differences*: $\delta\Delta(K^+\pi^-)=\delta\Gamma_n$, $\delta\Delta(K^+\pi^0)=\delta\Gamma_c$, where

$$\Gamma_n \equiv \Gamma(B^0\to K^+\pi^-)+\Gamma(\overline{B^0}\to K^-\pi^+),$$
$$\Gamma_c \equiv \Gamma(B^+\to K^+\pi^0)+\Gamma(B^-\to K^-\pi^0). \tag{23}$$

[In practice $\delta\Delta(K^+\pi^-)$ may exceed $\delta\Gamma_n$ if there is a kinematic ambiguity between $K^+\pi^-$ and $K^-\pi^+$ final states.] Using branching ratios

$$(1/2)[\mathcal{B}(B^0\to K^+\pi^-)+\mathcal{B}(\overline{B^0}\to K^-\pi^+)] = (14\pm 3\pm 1)\times 10^{-6}, \tag{24}$$

$$(1/2)[\mathcal{B}(B^+\to K^+\pi^0)+\mathcal{B}(\overline{B^-}\to K^-\pi^0)] = (15\pm 4\pm 3)\times 10^{-6}, \tag{25}$$

and the B^0 and B^+ lifetimes mentioned above, we find

$$\Gamma_n=(17.9\pm 4.1)\times 10^6\ {\rm s}^{-1}, \quad \Gamma_c=(18.2\pm 6.1)\times 10^6\ {\rm s}^{-1}. \tag{26}$$

These rates [which should obey $\Gamma_n/2=\Gamma_c$ if P were the only amplitude present, as noted in Eq. (11)] lead to individual errors on $\vec{P}\vec{T}$ of

$$\delta(\vec{P}\vec{T})_n=\delta\Gamma_n/2=2.0\times 10^6\ {\rm s}^{-1},$$

$$\delta(\vec{P}\vec{T})_c=\delta\Gamma_c=6.1\times 10^6\ {\rm s}^{-1}, \tag{27}$$

and a combined error of

$$\delta(\vec{P}\vec{T})_{\rm comb}=1.9\times 10^6\ {\rm s}^{-1}. \tag{28}$$

The maximum value of $\vec{P}\vec{T}$ is

$$|\vec{P}\vec{T}|_{\rm max}=2\sqrt{|P|^2|T|^2}=(2.71\pm 0.75)\times 10^6\ {\rm s}^{-1}, \tag{29}$$

based on the estimates of $|P|^2$ and $|T|^2$ given above. Comparing the error (28) with the value (29), one sees that reduction of the error by a factor of anywhere from about $\sqrt{3}$ to 3 could permit a non-zero observation of $\vec{P}\vec{T}$ at the 3σ level.

In the ideal case in which fractional rate errors scale as $1/\sqrt{N}$, Eq. (27) would be replaced by

$$\delta(\vec{P}\vec{T})_n=1.36\times 10^6\ {\rm s}^{-1}, \quad \delta(\vec{P}\vec{T})_c=2.95\times 10^6\ {\rm s}^{-1}, \tag{30}$$

leading to a combined error of $\delta(\vec{P}\vec{T})_{\rm comb}=1.24\times 10^6\ {\rm s}^{-1}$, sufficient to demonstrate a 3σ effect if $\vec{P}\vec{T}$ were at the upper limit of its allowed range.

To conclude, we have shown that to leading order in small quantities it makes sense to combine the CP-violating rate differences in the decays $B^0\to K^+\pi^-$ and $B^+\to K^+\pi^0$. Whereas the identification of the flavor of charged secondaries in $B^0/\overline{B^0}\to K^{\pm}\pi^{\mp}$ decays requires good particle identification in order to avoid a kinematic ambiguity involving $\pi\leftrightarrow K$ interchange, no such ambiguity afflicts the decays $B^{\pm}\to K^{\pm}\pi^0$. The averaged rate difference can be large enough in the standard model that it would be detectable at present levels of sensitivity.

Note added in proof. A recent analysis by Neubert [26] [see especially his Fig. 8(a)] supports our claim of a strong correlation between $A_{CP}(K^+\pi^0)$ and $A_{CP}(K^+\pi^-)$, where $A_{CP}(f)\equiv[\Gamma(f)-\Gamma(\bar{f})]/[\Gamma(f)+\Gamma(\bar{f})]$.

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