Vacuum energy cancellation in a nonsupersymmetric string

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We present a nonsupersymmetric orbifold of type II string theory and show that it has a vanishing cosmological constant at the one and two loop levels. We argue heuristically that the cancellation may persist at higher loops. [S0556-2821(99)04808-0]

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I. INTRODUCTION

One of the most intriguing and puzzling pieces of data is the (near-)vanishing of the cosmological constant Λ [1]. Unbroken supersymmetry would ensure that perturbative quantum corrections to the vacuum energy vanish [in the absence of a U(1) D term] due to cancellations between bosonic and fermionic degrees of freedom. However, although both bosons and fermions appear in the low-energy spectrum, they are not related by supersymmetry and this mechanism for cancelling Λ is not realized.

Because string theory (M theory) is a consistent quantum theory which incorporates gravity, it is interesting (and necessary) to see how string theory copes with the cosmological constant. In a perturbative string framework, because the string coupling g_{st} (the dilaton) is dynamical, the quantum vacuum energy constitutes a potential for it. So the issue of turning on a nontrivial string coupling is related to the form of the vacuum energy in string theory.

In this paper we present a class of perturbative string models in which supersymmetry is broken at the string scale but perturbative quantum corrections to the cosmological constant cancel. We begin with a simple mechanism that ensures the (trivial) vanishing of the 1-loop vacuum energy (as well as certain tadpoles and mass renormalizations). We then compute the (spin-structure-dependent part of the) 2-loop partition function and demonstrate that it vanishes. This requires some analysis of world sheet gauge-fixing conditions, modular transformations, and contributions from the boundaries of moduli space. Examination of the general form of higher-loop amplitudes suggests that they similarly may cancel and we next present this argument. We are unable to rigorously generalize our 2-loop calculation to higher loops at this point because of the complications of higher-genus moduli space. We hope to be able to make the higher-genus result more precise by using an operator formalism as will become clearer in the text, though we leave that for future work.

In addition we discuss how this model may fit into the framework [2] relating conformal fixed lines or points in quantum field theory to vanishing dilaton potentials or isolated minima of the dilaton potential in string theory. This provides hints as to where to look for more general models with vanishing Λ. In particular we will be interested in models without the tree level Bose-Fermi degeneracy that we have here, as well as models in which the dilaton is stabilized. We should note in this regard that instead of working in four-dimensional (4D) perturbative string theory as we do here, we could consider the same class of models in 3D string theory and consider the limit of large g_{st}. If the appropriate D-brane bound states exist in this theory to provide Kaluza-Klein modes of an M theoretic fourth dimension, one could obtain in this way 4D M theory vacua with vanishing cosmological constant and no dilaton (in this way similar to the scenario of [3], but here without the need for 3D supersymmetry).

We understand that a complementary set of models has been found in the free fermionic description [4]. We would like to thank Zurab Kakushadze for pointing out (and fixing) an error in our original model.

II. NON-ABELIAN ORBIFOLDS AND THE 1-LOOP COSMOLOGICAL CONSTANT

Consider the world sheet path integral formulation of orbifold compactifications [5]. In general one mods out by a discrete symmetry group of the 10-dimensional string theory. This group involves rotations of the left and right-moving worldsheet scalars $X^{\mu}_{L,R}$ and fermions $\psi^{\mu}_{L,R}$ as well as shifts of the scalars $X^{\mu}_{L,R}$. Here $\mu=1,\ldots,10$ is a spacetime $SO(9,1)$ vector index. The worldsheet path integral at a given loop order h splits up into a sum over different twist structures, in which the fields are twisted by orbifold group elements in going around the various cycles of the genus-h Riemann surface Σ_h. These twists must respect the homology relation

$$\prod_{i=1}^{h} a_i b_i a_i^{-1} b_i^{-1} = 1, \tag{2.1}$$

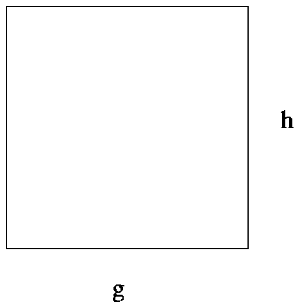

FIG. 1. Torus twisted by elements (g,h).

where a_i and b_i are the canonical 1-cycles on Σ_h. In particular, at genus 1, one sums over pairs (g,h) of *commuting* orbifold space group elements g and h (see Fig. 1).

In considering nonsupersymmetric orbifolds, this suggests an interesting class of models. Consider orbifolds in which no commuting pair of group elements breaks all the supersymmetry (i.e., projects out all of the gravitinos), but in which the full group does break all the supersymmetry. At the one-loop level, each contribution to the path integral then effectively preserves some supersymmetry and therefore vanishes. This is a formal way of encoding the fact that the spectrum for this type of model will have Bose-Fermi degeneracy at all mass levels (though no supersymmetry). So the one-loop partition function, as well as appropriate tadpoles, mass renormalizations, and three-point functions, are uncorrected.

We will discuss the following specific model.[1] Let us start with type II string theory compactified on a square torus $T^6 \sim (S^1)^6$ at the self-dual radius $R = l_s$ (where $l_s = \sqrt{\alpha'}$ is the string length scale). Consider the asymmetric orbifold generated by the elements f and g:

S^1	f	g
1	$(-1,s)$	$(s,-1)$
2	$(-1,s)$	$(s,-1)$
3	$(-1,s)$	$(s,-1)$
4	$(-1,s)$	$(s,-1)$
5	$(s^2,0)$	(s,s)
6	(s,s)	$(0,s^2)$
	$(-1)^{F_R}$	$(-1)^{F_L}$

We have indicated here how each element acts on the left and right moving Ramond-Neveu-Schwarz (RNS) degrees of freedom of the superstring. Here s refers to a shift by $R/2$. So for example f reflects the left-moving fields $X_L^{1\cdots 4}, \psi_L^{1\cdots 4}$ and shifts $X_R^{1\cdots 4}$ by $R/2$, X_L^5 by R, and $X^6 = \frac{1}{2}(X_L^6 + X_R^6)$ by $R/2$. In addition it includes an action of $(-1)^{F_R}$ which acts with a (-1) on all spacetime spinors coming from right-moving worldsheet degrees of freedom.

[1]Other similar models can be constructed, some of which do not actually require the group to be non-Abelian to get 1-loop cancellation [6,4,7].

This can be thought of as discrete torsion [8]: in the right-moving Ramond sector the f-projection has the opposite sign from what it would have without the $(-1)^{F_R}$ action. Similarly the above table indicates the action of the generator g on the world sheet fields. This orbifold satisfies level-matching and the necessary conditions derived in [8,9] for higher-loop modular invariance (we do not know if these conditions are sufficient).

There are several features to note about the spectrum of this model. First, it is not supersymmetric. In particular, f projects out all the gravitinos with spacetime spinor quantum numbers coming from the right-movers. Similarly g projects out the gravitinos with left-moving spacetime spinor quantum numbers. Because of the shifts included in our orbifold action, there are no massless states in twisted sectors, so in particular no supersymmetry returns in twisted sectors. Second, the model is nonetheless Bose-Fermi degenerate. In particular the massless spectrum has 32 bosonic and 32 fermionic physical states.

In addition to the spectrum of perturbative string states there is a D-brane spectrum in this theory which one can analyze along the lines of [10]. This will be of interest in placing this example in a more general context in the final section.

Our orbifold group elements satisfy the following algebraic relations:

$$f_g = g f T_L^{-1} T_R, \quad f T_L^q = T_L^{-q} f, \quad g T_R^q = T_R^{-q} g, \tag{2.2}$$

where T_L denotes a shift by R on $X_L^{1\cdots 4}$ and T_R denotes a shift by R on $X_R^{1\cdots 4}$. Clearly also f commutes with T_R and g commutes with T_L.

The first relation in Eq. (2.2) tells us that f and g do not commute in the orbifold space group. Therefore at the one loop level they never both appear as twists (f,g) in the partition function (i.e., we cannot twist by f on the a-cycle and by g on the b-cycle). Furthermore we can check that no commuting pair of elements break all the supersymmetry. In order to break the supersymmetry we would need pairs of the form $(f T_L^a T_R^b, g T_L^c T_R^d)$ or $(f T_L^{\tilde{a}} T_R^{\tilde{b}}, f g T_L^{\tilde{c}} T_R^{\tilde{d}})$, for arbitrary integers $a,b,c,d,\tilde{a},\tilde{b},\tilde{c},\tilde{d}$. (We could also have the latter form with f interchanged with g but these are isomorphic.) By using the relations (2.2) we see that neither pair of elements commutes:

$$(f T_L^a T_R^b)(g T_L^c T_R^d) = (g T_L^c T_R^d)(f T_L^a T_R^b) T_L^{2c+1} T_R^{1-2b}. \tag{2.3}$$

So there is no choice of integers a,b,c,d for which the two elements commute in the space group of the orbifold. Similarly

$$(f T_L^{\tilde{a}} T_R^{\tilde{b}})(f g T_L^{\tilde{c}} T_R^{\tilde{d}}) = (f g T_L^{\tilde{c}} T_R^{\tilde{d}})(f T_L^{\tilde{a}} T_R^{\tilde{b}}) T_L^{2\tilde{c}-2\tilde{a}-1} T_R^{1-2\tilde{b}}. \tag{2.4}$$

So at the one loop level, there will not be any contribution to the partition function.

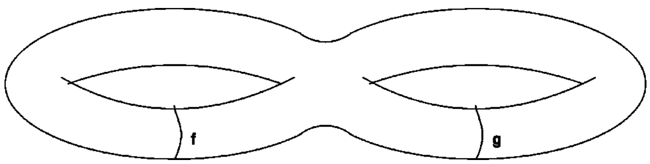

FIG. 2. Basic twist structure at genus 2.

III. THE 2-LOOP VACUUM ENERGY

At two loops the orbifold algebra itself does not automatically ensure the cancellation of the partition function. Let us denote the canonical basis of 1-cycles by $2h$-dimensional vectors $(a_1,\ldots,a_h;\ b_1,\ldots,b_h)$. At genus two, we run into twist structures like $(1,1;f,g)$ around the canonical cycles.

In Fig. 2 we indicate the *cuts* in the diagram in a given twist structure—here the fields are twisted in going around the b-cycles, as in doing so they pass through the indicated cuts. In particular this diagram involves both f and g twists, and therefore has the information about the full supersymmetry breaking of the model. Is there reason to believe the vacuum energy might nonetheless cancel? Heuristically, the following argument suggests that we should indeed expect a cancellation. Consider evaluating the diagram of Fig. 2 near the factorization limit in which the diagram looks like a propagator tube connecting two tori. Because of the homology relations, in this twist structure the intermediate state in this propagator is untwisted. The diagram thus becomes a sum over products of tadpoles of untwisted propagating states (weighted by e^{-mT} where m is the mass of the state and T gives the length of the tube). Each term is a tadpole of the untwisted state in the g-twisted theory times a tadpole of the untwisted state in the f-twisted theory. The contour deformation arguments of [11] imply that these tadpoles vanish. In order to make this rigorous one needs to see explicitly that unphysical states decouple properly (which only has to happen after summing over all twist structures). In what follows we will provide an explicit computation of the 2-loop contribution and verify that it vanishes.

A. Back to 1-loop

In order to appreciate the relevant mechanism, it is worth returning momentarily to the 1-loop (supersymmetric) contribution $(1,f)$ (see Fig. 3).

This contribution must vanish by supersymmetry, but it is instructive to observe how the spin structure sum works in this case before going on to our 2-loop diagram. The amplitude is

$$\mathcal{A}_1=\int \frac{d^2\tau}{(\operatorname{Im}\tau)^2}\operatorname{Tr}(q^{L_0}\overline{q}^{\overline{L_0}}f), \tag{3.1}$$

where $q=e^{2\pi i\tau}$ and L_0 and $\overline{L_0}$ are the usual Virasoro zero mode generators. Let us consider the spin-structure dependent piece of this amplitude. As explained in [12], the determinants for the world sheet Dirac operators acting on the RNS fermions are proportional to theta functions. The θ-function is defined (for general genus h) by

$$\theta[\alpha,\beta](z|\tau)=\sum_n e^{[\pi i(n+\alpha)^t\tau(n+\alpha)+2\pi i(n+\alpha)(z+\beta)]}. \tag{3.2}$$

Here $z\in\mathbb{C}^h/(\mathbb{Z}^h+\tau\mathbb{Z}^h)$ and τ is the period matrix of the Riemann surface, defined in terms of the canonical basis of holomorphic 1-forms ω_i by $\oint_{a_j}\omega_i=\delta_{ij}$ and $\oint_{b_j}\omega_i=\tau_{ij}$. The characteristics α,β encode the spin structure [13], i.e., the boundary conditions of the fermions around the a and b cycles respectively of the Riemann surface. So for example if $\alpha_1=1/2$ (respectively 0), the corresponding fermion has *periodic* (respectively *antiperiodic*) boundary conditions around the a_1 cycle.

The integrand of the 1-loop amplitude (3.1) is proportional to

$$\mathcal{A}_1\propto\sum_{\alpha,\beta}\eta_{\alpha,\beta}\theta^2[\alpha,\beta](0|\tau)\,\theta^2\left[\alpha,\beta+\frac{1}{2}\right](0|\tau), \tag{3.3}$$

where $\eta_{\alpha,\beta}$ are the phases encoding the Gliozzi-Scherk-Olive (GSO) projection. The first θ^2 factor comes from the left-moving RNS fermions $\psi_L^{1\cdots 4}$ and the second θ^2 factor comes from the other four transverse left-moving fermions $\psi_L^{5\cdots 8}$. The symmetry between these two factors will play an important role for us. Let us consider first the terms in the sum (3.3) with $\alpha=1/2$. This describes left-moving Ramond-sector states propagating in the loop, as the left-moving fermions ψ_L are periodic around the a-cycle. Because we have an f-twist around the b-cycle, half the ψ_L^μ are periodic around the b-cycle and half are antiperiodic around the b-cycle for each value of β in the sum. Thus in each $\alpha=1/2$ term half the RNS fermions have zero modes, so these terms identically vanish.

Let us now consider the terms with $\alpha=0$, which describe left-moving Neveu-Schwarz states propagating in the loop. These give

$$\sum_{\beta=0,1/2}\eta_{0,\beta}\theta^2[0,\beta](0|\tau)\,\theta^2[0,\beta+1/2](0|\tau). \tag{3.4}$$

Note that both terms in this sum have the same functional form $(\theta^2[0,1/2](0|\tau)\,\theta^2[0,0](0|\tau))$. The only issue left is then the relative phase between them. The sum over β is simply the GSO projection on the states propagating around the b-cycle. Let us normalize $\eta_{0,0}$ to 1. Then $\eta_{0,1/2}=-1$.

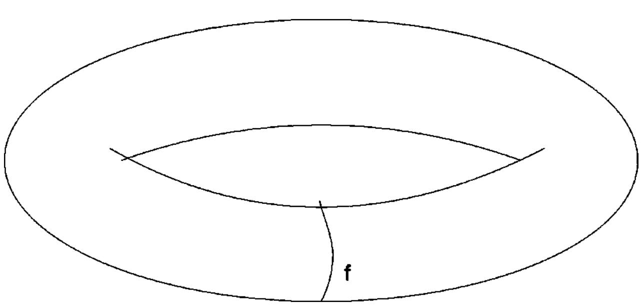

FIG. 3. One-loop diagram with an f twist on the b cycle.

This follows from the fact that *in the NS sector* the GSO projection operator is $1-(-1)^F$. This encodes the fact that we must project onto odd fermion number in the superstring in order to project out the tachyon which would otherwise come from the vacuum at the $-1/2$ mass level. So our integrand is

$$(1-1)\,\theta^2[0,0]\,\theta^2[0,1/2]=0. \tag{3.5}$$

B. 2-loops with supersymmetry

In order to proceed to the 2-loop computation, we must consider various subtleties arising in string loop computations for strings with world sheet supersymmetry. (See for example [14,15,16] for reviews with some references.) Let us begin by briefly reviewing some of the issues in the supersymmetric case. We will work in the RNS formulation; for discussion of the supersymmetric case in Green-Schwarz language see for example [17].

In performing the Polyakov path integral at genus h, we must integrate over all the world sheet fields including the world sheet metric $\hat{h}$ and gravitino χ. This infinite dimensional space is reduced to a finite dimensional space of (super-)moduli by dividing out the diffeomorphisms and local supersymmetry transformations. There are $3h-3$ complex bosonic moduli τ and $2h-2$ complex supermoduli ζ. At genus $h=2$ we can take the gravitino to have delta-function support on the world sheet for even spin structures [18]. (For odd spin structures the amplitude vanishes as a result of the integration of fermionic zero modes.)

We will review the supersymmetric cancellation at 2 loops. As explained for example in [16,19,14], the type II string path integral can be written as

$$\int_{\mathcal{SM}_h} d\mu_0 \int [dB\, dC\, dX] e^{-S} \prod_{r=1}^{6h-6} (\eta_r, B) \prod_{a=1}^{4h-4} \delta((\eta_a, B)). \tag{3.6}$$

Here B,C denote the b,β and c,γ ghosts, where (b,c) are the spin-$(2,-1)$ conformal ghosts and (β,γ) are the spin-$(3/2,-1/2)$ superconformal ghosts. X denotes the matter fields and η_r and η_a are Beltrami differentials relating the metric and gravitino to the moduli and supermoduli (in essence, they determine the way in which superdiffeomorphism invariance is gauge-fixed). In components,

$$(\eta_r, B)=\int \eta_{r\bar{z}}^{z} b_{zz}+\int \eta_{r\bar{z}}^{+}\beta_{z+}+\int \eta_{rz}^{\bar{z}} b_{\bar{z}\bar{z}}+\int \eta_{rz}^{-}\beta_{\bar{z}-}, \tag{3.7}$$

$$(\eta_a, B)=\int \eta_{a\bar{z}}^{z} b_{zz}+\int \eta_{a\bar{z}}^{+}\beta_{z+}+\int \eta_{az}^{\bar{z}} b_{\bar{z}\bar{z}}+\int \eta_{az}^{-}\beta_{\bar{z}-}. \tag{3.8}$$

As explained, e.g., in [20,16], we can write the path integral measure on supermoduli space in terms of a fixed measure on moduli space

$$d\mu_0 = d\mu[\mathrm{sdet}(\eta,\Phi)]^{-1}[\mathrm{sdet}(\Phi,\Phi)]^{1/2}. \tag{3.9}$$

Here $d\mu$ is a fixed measure on the supermoduli space $\mathcal{SM}_h$, integrated over a fixed domain independent of the beltrami differentials. Φ contains the $3h-3$ holomorphic and $3h-3$ antiholomorphic 2-differentials (b ghost zero mode wave functions) and the $2h-2$ holomorphic and $2h-2$ antiholomorphic 3/2-differentials (β ghost zero modes).

After choosing delta-function support for the world sheet gravitinos, and integrating out the supermoduli, one obtains a correlation function of picture changing operators [19]

$$:e^{\phi}T_F:=c\partial\xi+\frac{1}{2}e^{\phi}\psi^{\mu}\partial X^{\mu}-\frac{1}{4}\partial\eta e^{2\phi}b-\frac{1}{4}\partial(\eta e^{2\phi}b) \tag{3.10}$$

and other ghost insertions

$$\sum_{\alpha,\beta,twists}\int d\mu[\mathrm{sdet}(\eta,\Phi)]^{-1}[\mathrm{sdet}(\Phi,\Phi)]^{1/2}[dX][dB] \times[dC]e^{-S}(\hat{\eta},b)^{6h-6}\xi(x_0)\prod_{a=1}^{2h-2}:e^{\phi}T_F(z_a): \times\prod_{a=2h-3}^{4h-4}:e^{\bar{\phi}}\bar{T}_F(z_a):\bar{\xi}(\bar{y}_0). \tag{3.11}$$

The superconformal ghosts $\beta=\partial\xi e^{-\phi}$, $\gamma=\eta e^{\phi}$ are defined in terms of spin-0 and spin-1 fermions ξ,η and a scalar ϕ [11]. The spin-0 fermion ξ has a zero mode on the surface which is absorbed by the insertion of $\xi(x_0)$ in Eq. (3.11). There is an anomaly in the ghost number U(1) current which requires insertions of operators with total ghost number $2h-2$ to get a nonvanishing result. The correlation functions (3.11) can be evaluated using the formulas derived in, e.g., [21,22].

We will now fix the gauge for the gravitinos by making a definite choice of points $z_{1,2}$. As explained in [14], the choice of points must be taken in such a way that the gauge slice chosen is transverse to the gauge transformations. It must also respect modular invariance of the amplitude [19,14]. Ultimately, we will be interested in a gauge choice for which $z_1,z_2\to\Delta_\gamma$, where Δ_γ is a divisor corresponding to an odd spin structure γ, that is a point where a holomorphic 1/2-differential has a zero. As explained in [14], this choice (which amounts to putting the insertions at one of the branch points in a hyperelliptic description of the surface) satisfies transversality. It was argued in [23,24] that despite earlier worries [14], this choice is also consistent with modular invariance. The modular invariance is not manifest in the description in terms of θ-functions, as the calculation of correlation functions on the Riemann surface [21] involve a choice of reference spin structure δ. Having to choose a spin structure naively appears to violate modular invariance. Had we chosen a different reference spin structure δ', we would have shifted the arguments of our theta functions by elements $n+m\tau$ of the Jacobian lattice. Such a shift introduces a τ-dependent phase multiplying the θ-function—the θ-functions transform as sections of line bundles over the Jacobian torus. These phases must cancel out of the properly defined integrand, and in [23] this was demonstrated explicitly for certain (nonvanishing) 2-loop contributions.

We need to consider the (left-moving) spin-structure-dependent pieces of the correlation function, the poles arising from the spin-structure-independent local behavior of the picture changing correlator, and the behavior of the determinant (3.9) in this gauge. According to [19] we have the following contributions to the spin-structure-dependent pieces of the 2-loop partition function. The matter part of T_F contributes

$$\sum_{\delta} \langle \delta|\gamma\rangle \frac{\theta[\delta]^4(0)\,\theta[\delta](z_1-z_2)}{\theta[\delta](z_1+z_2-2\Delta_\gamma)}. \tag{3.12}$$

Here $\delta\equiv(\alpha,\beta)$ encodes the spin structure of the various contributions and $\langle\delta|\gamma\rangle=e^{4\pi i(\alpha\gamma_2-\beta\gamma_1)}$ encodes the Gliozzi-Scherk-Olive (GSO) phases [25]. Here the arguments $\Sigma p-\Sigma q$ in terms of p and q which are sets of points on the Rieman surface is shorthand for the Jacobi vector $\Sigma\int_{p_0}^{p}\omega_i-\Sigma\int_{p_0}^{q}\omega_i$.

Let us first, following [23], take $z_1+z_2=2\Delta_\gamma$, that is place z_1+z_2 at a divisor corresponding to the canonical class, without setting $z_1=z_2$. The contribution (3.12) then simplifies to

$$\sum_{\delta} \langle \delta|\gamma\rangle \theta[\delta]^3(0)\,\theta[\delta](z_1-z_2)=4\,\theta[\gamma]^4(z_1-\Delta_\gamma), \tag{3.13}$$

where in the last step we have used a Riemann identity. The Riemann vanishing theorem then implies that this vanishes identically as a function of z_1 [23]. Thus in this case whatever poles arise as $z_1\to z_2$, the identical zero from the spin structure sum cancels it.

Now turning to the ghost piece of the correlation function of picture-changing operators, one obtains contributions isomorphic to Eq. (3.13) as well as

$$\omega_i(z_1)\,\frac{\theta[\delta]^5(0)\,\partial_i\theta[\delta](2z_2-2\Delta_\gamma)}{\theta^2[\delta](z_1+z_2-2\Delta_\gamma)}. \tag{3.14}$$

Here ω_i are the canonical basis of holomorphic one-forms on the Riemann surface, satisfying $\int_{a_i}\omega_j=\delta_{ij}$ and $\int_{b_i}\omega_j=\tau_{ij}$ where τ is the period matrix for the surface. Again simplifying this by first taking $z_1+z_2=2\Delta_\gamma$ we obtain

$$\sum_{\delta} \langle \delta|\gamma\rangle \partial_{z_1}(\theta[\delta]^3(0)\,\theta[\delta](z_1-z_2)) =4\partial_{z_1}(\theta[\gamma]^4(z_1-\Delta_\gamma)). \tag{3.15}$$

Because the right-hand side of this expression is a derivative of 0 (by the Riemann vanishing theorem), it vanishes identically. Again any poles from the picture changing operator product expansions (OPEs) are irrelevant [23].

C. (Non-)superstring perturbation theory

In an orbifold model, one can consider separately different twist structures, and analyze the fundamental domain of the modular group that preserves a given twist structure. In general there are an infinite number of contributions coming from different choices of bosonic shifts. In Sec. IV we will analyze the twist structure of Fig. 2 (with no additional bosonic shifts) and see that the resulting modular group acts freely on τ. In this situation, the choice of a branch point for $z_{1,2}$ is manifestly modular invariant; the possible obstruction to modular invariance discussed in [19,14] does not arise, as there are no orbifold points in the moduli space. We also analyze in Sec. IV the boundary contributions and see that they vanish. One can show that with arbitrary additional shifts (respecting the homology relation of the Riemann surface) there are still no orbifold points in the moduli space.

We will analyze the twist structure $(1,1,f,g)$ (it will later be shown why this is the only twist structure which needs to be analyzed). The f twist affects the characteristics of some of the θ functions (arising from twisted fields) by shifting them by (0,0,1/2,0)—we shall denote this as a shift by $\frac{1}{2}L$. κ will be defined as $\gamma+(0,0,0,1/2)$, and we choose γ such that both γ and κ are odd.

The correlation function of the matter part of the picture changing operators breaks into two contributions. The terms involving $\langle\psi^i\partial X^i(z_1)\psi^i\partial X^i(z_2)\rangle$ with $i=5,\ldots,10$ give

$$\sum_{\delta} \langle \kappa|\delta\rangle \frac{\theta[\delta](0)^2\,\theta[\delta+\frac{1}{2}L](0)^2\,\theta[\delta](z_1-z_2)}{\theta[\delta](z_1+z_2-2\Delta_\gamma)} \times\left(p_i^\mu\omega^i(z_1)p_j^\mu\omega^j(z_2)\,\frac{1}{E(z_1,z_2)} +\frac{6}{E(z_1,z_2)^2}\,\partial_{z_1}\partial_{z_2}\log E(z_1,z_2)\right)\det(\Phi_a^{3/2}(z_b)). \tag{3.16}$$

Upon setting $z_1+z_2=2\Delta_\gamma$ we can cancel the denominator against one factor in the numerator to get

$$\sum_{\delta} \langle \kappa|\delta\rangle \theta[\delta](0)\,\theta[\delta](z_1-z_2)\,\theta\left[\delta+\frac{1}{2}L\right](0)^2 =4\,\theta[\kappa]\left(\frac{1}{2}(z_1-z_2)\right)\theta\left[\kappa+\frac{1}{2}L\right]\left(\frac{1}{2}(z_1-z_2)\right) \tag{3.17}$$

for the spin-dependent piece of this correlator. Because κ is an odd spin structure, this vanishes like $(z_1-z_2)^2$. As $z_1\to z_2$ the determinant factor (3.9) produces another zero: plugging in the delta function η_a we obtain

$$[\mathrm{sdet}(\eta,\Phi)]^{-1}\propto\det(\eta_a,\Phi_b^{3/2})=\det(\Phi_b^{3/2}(z_a)). \tag{3.18}$$

Here $\Phi_b^{3/2}$, $b=1,2$ form a basis of holomorphic 3/2-differentials. As the z_a approach each other, the determinant (3.18) goes to zero, so all in all Eq. (3.16) has a $(z_1-z_2)^3$

multiplying the prime forms. However, since the prime-forms are yielding poles as $z_1 \to z_2$, it remains to check that there are no finite pieces in Eq. (3.16).

Note that $E(z_1,z_2)$ goes like z_1-z_2 as $z_1 \to z_2$. Thus, the terms proportional to $1/E(z_1,z_2)^2$ times the loop momenta clearly vanish in the limit, since there is only a second order pole from the prime forms which cannot cancel the third order zero we found from the spin structure sum and the superdeterminant. This leaves the term which goes like $[1/E(z_1,z_2)^2]\partial_{z_\perp}\partial_{z_2}\log E(z_1,z_2)$. Using the fact that $E(z_1,z_2)$ has a Taylor expansion of the form

$$E(z_1,z_2) \sim \sum_{n=0}^{\infty} c_n (z_1-z_2)^{2n+1} \tag{3.19}$$

as $z_1 \to z_2$, one sees that this combination of prime forms has an expansion

$$\frac{1}{E(z_1,z_2)^2}\partial_{z_1}\partial_{z_2}\log E(z_1,z_2) \sim \sum_{n=-2}^{\infty} d_n (z_1-z_2)^{2n}. \tag{3.20}$$

On the other hand, the determinant factor is an *odd* function of z_1-z_2 with an expansion of the form

$$\det(\Phi_a^{3/2}(z_b)) \sim \sum_{m=0}^{\infty} e_m (z_1-z_2)^{2m+1} \tag{3.21}$$

while the sum over spin structures (3.17) is an even function with a second order zero at $z_1=z_2$. From these facts, it is easy to see that the full expression (3.16) has an expansion of the form

$$\sum_{j=0}^{\infty} f_j (z_1-z_2)^{2j-1} \tag{3.22}$$

as $z_1 \to z_2$.

Examining Eq. (3.22), we see that there are no finite contributions as $z_1 \to z_2$ and there is a (gauge artifact) pole as $z_1 \to z_2$; in fact this pole receives contributions from the various matter and ghost correlators proportional to the matter or ghost central charges, and hence cancels once all of the terms are taken into account (since $c_{tot}=c_{matter}+c_{ghost}=0$). We will see this explicitly once we compute the remaining matter and ghost contributions.

The second type of matter correlator arises from contracting the $\psi^i\partial X^i(z_1)\psi^i\partial X^i(z_2)$ with $i=1,\dots,4$. This leads to a contribution

$$\sum_{\delta}\langle\kappa|\delta\rangle\frac{\theta[\delta](0)^3\theta\left[\delta+\frac{1}{2}L\right](0)\theta\left[\delta+\frac{1}{2}L\right](z_1-z_2)}{\theta[\delta](z_1+z_2-2\Delta_\gamma)} \times\left(p_i^\mu\omega^i(z_1)p_j^\mu\omega^j(z_2)\frac{1}{E(z_1,z_2)^2} + \frac{4}{E(z_1,z_2)^2}\partial_{z_1}\partial_{z_2}\log E(z_1,z_2)\right) \times\det(\Phi_a^{3/2}(z_b)). \tag{3.23}$$

Choosing $z_1+z_2=2\Delta_\gamma$, the spin sum in Eq. (3.23) simplifies to

$$\sum_{\delta}\langle\kappa|\delta\rangle\theta[\delta](0)^2\theta\left[\delta+\frac{1}{2}L\right](0)\theta\left[\delta+\frac{1}{2}L\right](z_1-z_2) \tag{3.24}$$

which, after applying a Riemann identity, becomes

$$4\theta[\kappa]\left(\frac{1}{2}(z_1-z_2)\right)^2\theta\left[\kappa+\frac{1}{2}L\right]\left(\frac{1}{2}(z_1-z_2)\right)^2. \tag{3.25}$$

So in fact after summing over spin structures this looks the same as the spin sum of the first type of matter contribution (3.17). Again, it vanishes like $(z_1-z_2)^2$ as $z_1 \to z_2$.

Now, the argument for the cancellation proceeds as it did for the first type of matter contribution. The terms involving only the $1/E(z_1,z_2)^2$ multiplying loop momenta only have a second order pole, which cannot cancel the third order zero coming from the determinant times the spin structure sum (3.25). The terms involving higher inverse powers of the prime forms lead to a simple pole (which cancels after summing over matter and ghosts, as it is proportional to the total central charge) and no finite contributions.

Next, let us consider the terms in the correlator of picture changing operators coming from the ghost part of the world sheet supercurrent. These terms take the form

$$\left\langle -\frac{1}{4}c\partial\xi(z_1)(2\partial\eta e^{2\phi}b+\eta\partial e^{2\phi}b+\eta e^{2\phi}\partial b)(z_2)\right\rangle + \left\langle -\frac{1}{4}(2\partial\eta e^{2\phi}b+\eta\partial e^{2\phi}b+\eta e^{2\phi}\partial b)(z_1)c\partial\xi(z_2)\right\rangle. \tag{3.26}$$

There are three types of terms that arise [19]. We are in the twist structure $(1,1,f,g)$. As in the matter sector, the f twist affects the characteristics of the θ-functions arising in the world sheet correlation functions and determinants. We will denote the shift in the characteristic, which is (0,0,1/2,0), as $\frac{1}{2}L$. The first type of contribution is

$$\sum_{\delta}\langle\kappa|\delta\rangle\frac{\theta[\delta](0)^3\theta\left[\delta+\frac{1}{2}L\right](0)^2\theta[\delta](2z_2-2\Delta_\gamma)\theta\left(z_1-z_2+\sum w-3\Delta\right)}{\theta[\delta](z_1+z_2-2\Delta_\gamma)^2E(z_1,z_2)^3}$$
$$\times\det(\Phi_a(z_b))\frac{\prod E(z_1,w)}{\prod E(z_2,w)}\partial_{z_1}\log\left(\frac{\prod E(z_1,w)}{E(z_1,z_2)^5\sigma(z_1)}\right)+(z_1\leftrightarrow z_2). \tag{3.27}$$

The second is

$$\sum_{\delta}\langle\kappa|\delta\rangle\frac{\theta[\delta](0)^3\theta\left[\delta+\frac{1}{2}L\right](0)^2\omega_i(z_1)\partial^i\theta[\delta](2z_2-2\Delta_\gamma)\theta\left(z_1-z_2+\sum w-3\Delta\right)}{\theta[\delta](z_1+z_2-2\Delta_\gamma)^2E(z_1,z_2)^3}\det(\Phi_a(z_b))\frac{\prod E(z_1,w)}{\prod E(z_2,w)}+(z_1\leftrightarrow z_2). \tag{3.28}$$

The third is

$$\sum_{\delta}\langle\kappa|\delta\rangle\frac{\theta[\delta](0)^3\theta\left[\delta+\frac{1}{2}L\right](0)^2\theta[\delta](2z_2-2\Delta_\gamma)\omega_i(z_1)\partial^i\theta\left(z_1-z_2+\sum w-3\Delta\right)}{\theta[\delta](z_1+z_2-2\Delta_\gamma)^2E(z_1,z_2)^3}\det\Phi_a(z_b)\frac{\prod E(z_1,w)}{\prod E(z_2,w)}+(z_1\leftrightarrow z_2). \tag{3.29}$$

Setting $z_1+z_2=2\Delta_\gamma$ and doing the spin structure sum we find for the spin-structure-dependent pieces of contributions (3.27) and (3.29):

$$\sum_{\delta}\langle\kappa|\delta\rangle\theta[\delta](0)\theta\left[\delta+\frac{1}{2}L\right](0)^2\theta[\delta](2z_2-2\Delta_\gamma)$$
$$=\theta[\kappa](z_2-\Delta_\gamma)^2\theta\left[\kappa+\frac{1}{2}L\right](z_2-\Delta_\gamma)^2$$
$$\sim(z_1-z_2)^2+c_4(z_1-z_2)^4+\cdots \tag{3.30}$$

for some constant c_4 where in the last line we expanded the result in a Taylor expansion around $z_1=z_2$. For contribution (3.28) we get

$$\sum_{\delta}\langle\kappa|\delta\rangle\theta[\delta](0)\theta\left[\delta+\frac{1}{2}L\right](0)^2\omega_i(z_1)\partial^i\theta[\delta](2z_2-2\Delta_\gamma)$$
$$=\partial_{z_1}\left(\theta[\kappa]\left(\frac{1}{2}(z_1-z_2)\right)^2\theta\left[\kappa+\frac{1}{2}L\right]\left(\frac{1}{2}(z_1-z_2)\right)^2\right)$$
$$\sim(z_1-z_2)+b_3(z_1-z_2)^3+\cdots. \tag{3.31}$$

As for the matter contributions, although the spin structure sums give vanishing contributions, they multiply singularities arising from the prime forms $E(z_1,z_2)$ and we must analyze the potential finite terms in the Taylor expansion. Let us consider first Eq. (3.27). There are two types of contributions here. After doing the spin structure sum as above the first takes the form

$$-5\frac{\partial_1E(z_1,z_2)}{E(z_1,z_2)^4}[z_{12}^2+c_4z_{12}^4+\cdots][z_{12}+e_3z_{12}^3+\cdots]$$
$$\times\frac{\prod E(z_1,w)}{\prod E(z_2,w)}+(z_1\leftrightarrow z_2), \tag{3.32}$$

where we denote z_1-z_2 by z_{12}. Here the second factor comes from the spin structure sum, the third from the Taylor expansion of the determinant about $z_1=z_2$ (where e_3 is some constant). We should emphasize what is meant here by $(z_1\leftrightarrow z_2)$. We are computing a correlation function of picture changing operators. The ghost piece of this correlator has the form (3.26). So for example the second term in Eq. (3.26) corresponds to the term denoted $z_1\leftrightarrow z_2$ in Eq. (3.32). So in particular the second term involves interchanging the operators in the ghost correlator, without changing z_1 to z_2 in the determinant factor. The first and fourth factors involving the prime forms encode the physical poles and zeroes of the correlator. The leading singularity from the prime forms here comes from the $1/z_{12}^4$ term in the expansion of the prime form factors. Therefore only the leading term in the Taylor expansion of the spin structure sum and determinant factors potentially survive (so we can ignore the terms proportional to c_4 or e_3, which give fifth-order zeros). Similarly expanding the prime forms $E(z_1,z_2)$ gives a subleading term with only a $1/z_{12}^2$ pole, which is cancelled by the third order zero coming from the leading piece of the spin structure sum times determinant.

Putting the factors together, we see that the leading piece is a simple pole in z_{12}. The first three factors in Eq. (3.32) are the same in the term with $z_1 \leftrightarrow z_2$. When we include the term with $z_1 \leftrightarrow z_2$, they multiply the prime form factor $[\Pi E(z_1,w)/\Pi E(z_2,w)]+[\Pi E(z_2,w)/\Pi E(z_1,w)]$. This is even under $z_1 \leftrightarrow z_2$. In our Taylor expansion it therefore becomes of the form $O(1)+f_2 z_{12}^2+\dots$, and only the first term contributes. Therefore in Taylor expanding the contribution (3.32), we get a pole piece plus higher order terms which vanish in the limit $z_1 \to z_2$. In particular, no finite pieces survive. What is the interpretation of the pole piece? It is proportional to the ghost central charge, and precisely cancels the pole piece coming from the matter contribution.

The second type of contribution in Eq. (3.27) takes the form

$$\frac{1}{E(z_1,z_2)^3}[z_{12}^2+\cdots][z_{12}+\cdots] \times \frac{\prod E(z_1,w)}{\prod E(z_2,w)} \partial_1 \log\left(\frac{\prod E(z_1,w)}{\sigma(z_1)} \right), \tag{3.33}$$

where the $\cdots$ denote terms which vanish automatically as $z_1 \to z_2$. The leading pole from the prime forms here is cubic. Before including the $z_1 \leftrightarrow z_2$ term there is a finite piece obtained by multiplying this times the third order zero obtained from the spin structure sum and determinant factors. The spin structure sum is even under the interchange of z_1 and z_2 in this case, and as discussed above the determinant factor is the same in both terms. The factor $1/E(z_1,z_2)^3$ does change sign between the two terms, however. So when we add the $(z_1 \leftrightarrow z_2)$ term the contribution cancels.

Let us now consider the contribution (3.28). This gives a contribution of the form

$$\frac{1}{E(z_1,z_2)}[z_{12}+\cdots][z_{12}+\cdots] \times \left(\frac{\prod E(z_1,w)}{\prod (z_2,w)} + \frac{\prod E(z_2,w)}{\prod E(z_1,w)} \right). \tag{3.34}$$

Here similarly to the above analysis we took into account the relative sign of the two contributions in Eq. (3.26) and included the $z_1 \leftrightarrow z_2$ contribution. The last factor here is even under interchange of z_1 and z_2, so its Taylor expansion is of the form $1+h_2 z_{12}^2+\cdots$ for some constant h_2. The leading contribution here is a simple pole, and there is no finite contribution.

Unlike the previous simple poles we have encountered, the pole encountered here does not cancel with the other matter and ghost contributions [it is *not* one of the pieces which would have contributed to the c/z^4 pole in the operator product expansion (OPE) of picture changing operators before accounting for spin structure sums and determinant factors]. However, on general grounds we expect such gauge artifact poles to constitute total derivatives on moduli space. Otherwise the invariance of the path integral on gauge slice would be lost. In this case, we can argue for that conclusion as follows. The pole we are discussing receives a $1/(z_1-z_2)^3$ contribution from the prime forms which is softened to $1/(z_1-z_2)^2$ by the theta function zero (and then to a simple pole by the determinant factor). In the OPE of picture changing operators, the $1/(z_1-z_2)^2$ divergence is multiplied by the stress-energy tensor, which gives a derivative with respect to the metric and therefore the moduli. The term we are finding is part of this total derivative. In the gauge we have chosen, it is the only nonvanishing piece (the other pieces vanish even before integration over the moduli space). However, since there cannot be gauge artifact poles, we expect it to integrate to zero (which one can argue for by analyzing the boundary contributions, as we will do later).

Finally let us consider the last ghost contribution (3.29). This contribution takes the form

$$\frac{1}{E(z_1,z_2)^3}[z_{12}^2+\cdots][z_{12}+\cdots]\frac{\prod E(z_1,w)}{\prod E(z_2,w)}+(z_1 \leftrightarrow z_2). \tag{3.35}$$

In this contribution before including the $z_1 \leftrightarrow z_2$ contribution there is a potential finite term from the third order pole multiplying a third order zero in z_{12}. Here again, in the limit $z_1 \to z_2$ every factor except the first is the same in the two terms. The first factor $1/E(z_1,z_2)^3$ has the opposite sign in the two terms. Thus again after including the $z_1 \leftrightarrow z_2$ term the contribution cancels.

IV. BOUNDARY CONTRIBUTIONS

In the previous section, we studied the two loop diagram with twists by f and g going around the $b_{1,2}$ cycles, i.e., with twist structure $(1,1,f,g)$. We saw that the computation yields a *vanishing integrand* if we make a very specific choice of insertion points for the picture-changing operators: $e^{\phi}T_F$:. Since the answer should be independent of the choice of these insertion points, this seems to imply that the two loop vacuum energy vanishes.

However, under a change of the choice of insertion points, it can be shown that the computation changes by a total derivative [19,26]

$$\int_{\mathcal{F}} \partial\omega, \tag{4.1}$$

where $\mathcal{F}$ is the appropriate fundamental domain of integration for the computation. Therefore, one must worry about contributions arising at the boundary of $\mathcal{F}$ [14].

A. The fundamental domain

What is the fundamental domain $\mathcal{F}$ for this computation? At genus two, the Teichmuller space is given very explicitly in terms of the Siegel upper half space of 2×2 matrices:

$$\mathcal{H}_2=\{\tau_{2\times 2}:\quad \tau^{tr}=\tau,\quad \mathrm{Im}\,\tau>0\}.$$

τ is the period matrix of the genus two surface. The modular group at genus two is $G=\mathrm{Sp}(4,\mathbb{Z})$. The moduli space can then be constructed by taking the quotient of $\mathcal{H}_2$ by G. One must also remove the modular orbit of the diagonal matrices.

For our computation, on the other hand, we have twists $(1,1,f,g)$ about the (a_1,a_2,b_1,b_2) cycles of the surface. Therefore, we need to integrate the correlator of the picture changing operators over $\mathcal{F}=\mathcal{H}_2/\tilde{G}$, where $\tilde{G}$ is the subgroup of $\mathrm{Sp}(4,\mathbb{Z})$ which preserves the twist structure $(1,1,f,g)$.

It is easy to see that the allowed matrices are the ones that act on the homology (a_1,a_2,b_1,b_2) like

$$\begin{pmatrix} a & b & 0 & 0\\ c & d & 0 & 0\\ x & y & 1 & 0\\ z & w & 0 & 1\end{pmatrix}. \tag{4.2}$$

Denoting the 2×2 blocks as $\begin{pmatrix} A & B\\ C & D\end{pmatrix}$ we must impose

$$A^{tr}C=C^{tr}A,\quad B^{tr}D=D^{tr}B,\quad A^{tr}D-C^{tr}B=1 \tag{4.3}$$

which is just the requirement that Eq. (4.2) is in $\mathrm{Sp}(4,\mathbb{Z})$. This further restricts the allowed matrices (4.2) to be of the form

$$\begin{pmatrix} 1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ x & y & 1 & 0\\ y & w & 0 & 1\end{pmatrix}. \tag{4.4}$$

Now, if $\begin{pmatrix} A & B\\ C & D\end{pmatrix}$ acts on the homology, then the action on the period matrix τ is given by $\begin{pmatrix} D & C\\ B & A\end{pmatrix}$—in other words,

$$\tau\rightarrow(D\tau+C)(B\tau+A)^{-1}. \tag{4.5}$$

So from the allowed actions on the homology (4.4), we see that the identifications to be made on the period matrices are

$$\begin{pmatrix} \tau_1 & \tau_{12}\\ \tau_{12} & \tau_2\end{pmatrix}\rightarrow\begin{pmatrix} \tau_1+x & \tau_{12}+y\\ \tau_{12}+y & \tau_2+w\end{pmatrix}. \tag{4.6}$$

In addition, positivity of $\mathrm{Im}\,\tau$ requires that

$$\mathrm{Im}\,\tau_{1,2}>0,\quad (\mathrm{Im}\,\tau_{12})^2<\mathrm{Im}\,\tau_1\,\mathrm{Im}\,\tau_2. \tag{4.7}$$

The constraints (4.6) and (4.7) together yield the correct fundamental domain $\mathcal{F}\subset\mathcal{H}_2$ for our computation. $\tau_{1,2}$ live on strips with real part between $(-1/2,1/2)$ and positive imaginary part, while τ_{12} has real part between $(-1/2,1/2)$ and imaginary part bounded above and below by the second inequality in Eq. (4.7). Also, we must recall that in describing the moduli space of Riemann surfaces in terms of $\mathcal{H}_2$, we had to delete the modular orbit of diagonal matrices, yielding an additional boundary at $\tau_{12}\rightarrow 0$.

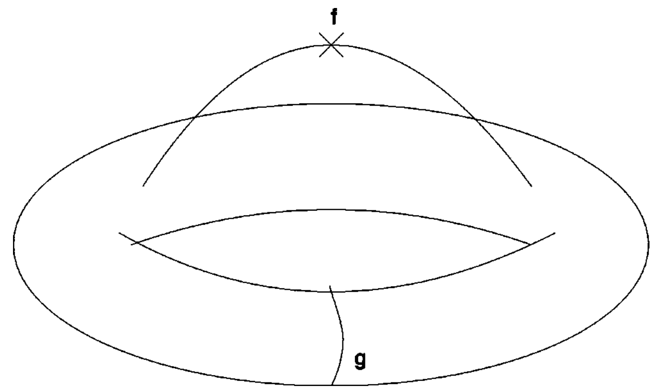

FIG. 4. Picture of boundary (1).

B. The boundaries

Now that we have determined $\mathcal{F}$, we can look for boundaries where the total derivative (4.1) might give a contribution after integration by parts. There are in fact three boundaries in $\mathcal{F}$. We will examine each of these boundaries in turn, and argue that no boundary contribution exists.

(1) τ_1 or $\tau_2\rightarrow i\infty$. In this limit, one of the handles degenerates to a semicircle glued on to the ''fat'' handle at two points (i.e., a homology cycle collapses) (see Fig. 4). It was argued in [14] that in such a limit, no boundary contribution exists in theories without physical tachyons. Our theory has no physical tachyons, so we will receive no contribution from this boundary.

(2) $\tau_{12}\rightarrow 0$. In this limit, the genus two surface degenerates into two tori connected by a very long, thin tube (see Fig. 5). Only massless physical states propagate in this tube [14], and in this limit the genus two vacuum amplitude is related to a sum of products of one loop tadpoles for the massless states.

The relevant one loop tadpoles are computed on tori with twists $(1,f)$ or $(1,g)$ around the (a,b) cycles. Now, the f and g twist alone preserve $d=4$, $\mathcal{N}=2$ supersymmetry. So, there are no one loop tadpoles for states in the f or g twisted theory. This implies that the genus two diagram vanishes in this limit.

(3) $\mathrm{Im}\,\tau_{1,2}\rightarrow 0$ or $(\mathrm{Im}\,\tau_{12})^2\rightarrow(\mathrm{Im}\,\tau_1)(\mathrm{Im}\,\tau_2)$. To see the vanishing in this limit, we recall that the integrand for the vacuum amplitude contains a factor of $e^{-S(X)}$, i.e., the action for map from the genus two surface to spacetime. The relevant maps (given the f and g twists about the b cycles of the surface) wind around the X_5 and X_6 directions of spacetime. This yields a contribution to the action which goes like

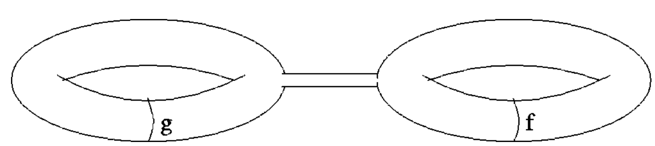

FIG. 5. Picture of boundary (2).

$$S \simeq \frac{R^2}{\alpha'} \left\{ \frac{\mathrm{Im}\,\tau_1 + \mathrm{Im}\,\tau_2}{\mathrm{Im}\,\tau_1\,\mathrm{Im}\,\tau_2 - (\mathrm{Im}\,\tau_{12})^2} \right\}, \tag{4.8}$$

where R is the radius of the X_5 and X_6 circles [27,28]. Now, positivity of Im τ comes to the rescue: If Im $\tau_1 \to 0$ at fixed Im τ_2, then the second inequality in Eq. (4.7) implies that $S \to \infty$; if Im $\tau_{1,2} \to 0$, one can prove that the denominator in Eq. (4.8) vanishes as the square of the numerator (once again using positivity of Im τ), so $S \to \infty$; if Im $\tau_{1,2}$ are fixed and $(\mathrm{Im}\,\tau_{12})^2$ approaches Im τ_1 Im τ_2, it is obvious that the action diverges.

The upshot is that the $e^{-S(X)}$ in the integrand vanishes quickly enough at this boundary to rule out any contributions.

C. Cases with shifts

In additional to the amplitude $(1,1,f,g)$ which knows about the supersymmetry breaking at genus two, there are other genus 2 amplitudes with $(1,1,f,g)$ twists on the world sheet fermions around the (a_1,a_2,b_1,b_2) cycles but with additional shifts acting on the bosonic fields. In fact we show in Sec. V that this is (up to modular transformations) the full set of supersymmetry breaking diagrams that we need to consider at genus two.

In each twist structure we will find that the spin-structure dependent part of the vacuum amplitude vanishes. This leaves the issue of possible boundary contributions. After summing over the various twist structures, we know the genus two vacuum energy can be written as an integral over $\mathcal{M}_2$, the moduli space of genus two Riemann surfaces. The possible boundary contributions (after we compactify $\mathcal{M}_2$) will come from boundaries of type (1) and (2) in Sec. IV B (where a handle collapses or the surface degenerates into two surfaces of lower genus connected by a long, thin tube). Hence, if we can argue that with arbitrary twist and shift structures on the *a,b* cycles the vacuum amplitude vanishes at boundaries of type (1) and (2), we will be done.

As we will discuss in Sec. V, up to additional shifts on various cycles the possible structures (which break all of the supersymmetry) are basically $(1,1,f,g)$, (f,g,g,f) and (f,fg,fg,f) (up to possible exchanges of the role of f and g). Since we could use modular transformation to relate these to $(1,1,f,g)$ twist structure on the fermions, the spin-structure dependent piece of the amplitude vanishes in each of these cases. In addition, each of these vanishes at boundaries of type (1) because there are no physical tachyons. This leaves the analysis of boundary (2).

Any amplitude with $(1,1,f,g)$ twists on the fermions, regardless of additional shifts, vanishes at boundary (2) because it can be written as a product of tadpoles in the $\mathcal{N}=2$ supersymmetric f and g orbifolds (as in Sec. IV B). On the other hand, the amplitude with twist (f,g,g,f) would naively yield a product of one loop tadpoles in a nonsupersymmetric theory. However, it turns out that the state propagating on the tube between the first and second handle must be a massive state because it must be twisted to be emitted from the ''subtorus'' with (f,g) twist on its (a,b) cycles. Since only massive states can run in the tube, there is no contribution at the boundary of moduli space (where the tube becomes infinitely long). A similar discussion applies to the (f,fg,fg,f) twist structure with arbitrary shifts.

V. TWISTS AT GENUS $h \geq 2$

A priori on a genus h Riemann surface, one needs to consider any combination of twists on the various cycles a_i, b_i for $i=1,\ldots,h$ consistent with the relation

$$a_1 b_1 a_1^{-1} b_1^{-1} \cdots a_h b_h a_h^{-1} b_h^{-1} = 1. \tag{5.1}$$

In this section, we will argue that in fact using modular transformations one can greatly reduce the kinds of twist structures that one needs to consider.

For our considerations, we do not need to worry about twists that preserve some of the spacetime supersymmetry at genus h (for instance, twists only by f around various cycles). The real concern will be sets of twists around different cycles which break the full spacetime supersymmetry. We will now show that, up to inducing shifts on the world sheet bosons around some cycles, one *only* has to consider f and g twists on the b_{h-1} and b_h cycles with no twists on any other cycles. Any twist which breaks all of the spacetime supersymmetry can be brought to this canonical $(1,\ldots,1,f,g)$ form by modular transformations.

Since in this section we will be ignoring the possible shifts on bosons around various cycles (we are only interested in the *f,g* action on fermions), we can use relations like

$$f^2 = g^2 = 1, \quad fg = gf \tag{5.2}$$

which are true for the action on fermions (but only true in the full model up to shifts in the space group).

A. Genus $h=2$

We will show that all twists of interest can be taken to the $(1,\ldots,f,g)$ form in several steps. First, consider genus two surfaces. The modular group Sp(4,$\mathbb{Z}$) is generated by

$$D_{a_1} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad D_{a_2} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix}, \tag{5.3}$$

$$D_{b_1} = \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad D_{b_2} = \begin{pmatrix} 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \tag{5.4}$$

$$D_{a_1^{-1}a_2} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 1 & 1 & 0 \\ 1 & -1 & 0 & 1 \end{pmatrix}, \tag{5.5}$$

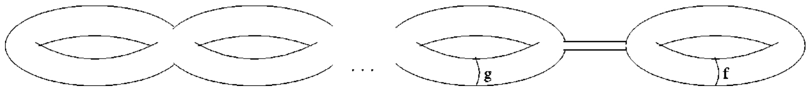

FIG. 6. One sums over states in the channel between the f and g twisted handles.

which are simply the Dehn twists about the various cycles of the genus two surface, acting on the homology (a_1,a_2,b_1,b_2).[2]

We now consider genus $h=2$ twists which are not of the canonical $(1,1,f,g)$ form but which break all the supersymmetry.

(1) First, take the cases where no ''subtorus'' has twists which break the full supersymmetry [i.e., no f,g twists on dual (a,b) cycles]. Then, by using $\mathrm{SL}(2,\mathbb{Z})\subset \mathrm{Sp}(4,\mathbb{Z})$ transformations which act on the (a_1,b_1) and (a_2,b_2) cycles, one can arrange to have twists only on the b cycles, so the twist structure is (1,1,*,*). Then the only cases we need to worry about are $(1,1,f,fg)$ and $(1,1,g,fg)$. One can easily see that $(1,1,f,fg)$ is mapped by D_{b_1} to $(f,1,f,fg)$ and then by $D_{a_1^{-1}a_2}$ to $(f,1,1,g)$. That in turn is SL(2,$\mathbb{Z}$) equivalent to $(1,1,f,g)$. A similar manipulation works for the $(1,1,g,fg)$ case.

(2) Second, consider the case where there *are* twists on some ''subtorus'' that break the full supersymmetry. Examples are (f,g,g,f) and (f,fg,fg,f). Now, for instance, (f,g,g,f) can be mapped by $D_{a_1^{-1}a_2}$ to (f,g,g,f) which is equivalent [using SL(2,$\mathbb{Z}$) transformations on both subtori] to $(1,1,f,g)$. One can similarly reduce (f,fg,fg,f) and other analogous structures to the canonical form. [Recall that in this discussion we are ignoring extra bosonic shifts that make the twist structures considered here consistent with Eq. (5.1)].

So, we find that *all* supersymmetry breaking twists at genus $h=2$ can be mapped by the modular group to $(1,1,f,g)$ (up to shifts on world sheet bosons). This is important because our vanishing at $h=2$ was for the spin structure dependent part of precisely this twist structure, and is independent of any shifts on world sheet bosons.

B. Genus $h>2$

We now argue that at arbitrary genus, one can reduce all supersymmetry breaking twist structures to $(1,\ldots,1,f,g)$ using modular transformations. We will need to use three important facts: (1) Among the elements of $\mathrm{Sp}(2h,\mathbb{Z})$ there are matrices that allow one to permute the different ''subtori'' (sets of conjugate a, b cycles) of the genus h surface; (2) in order to satisfy Eq. (5.1), there must exist an *even* number of ''subtori'' with twists on the (a_i,b_i) cycles that break all the supersymmetry. (3) using $\mathrm{Sp}(4,\mathbb{Z})\subset \mathrm{Sp}(2h,\mathbb{Z})$ one can map

$$(1,1,f,f)\rightarrow(1,1,f,1), \tag{5.6}$$

i.e., one can group like twists on neighboring b cycles onto a single b cycle.

Putting together our $h=2$ result with facts (1)–(3) above, we see that at genus $h>2$ the only twist structure we need to consider is $(1,1,\ldots,1,f,g)$. To prove this, we simply work on genus 2 subsurfaces [using Sp(4,$\mathbb{Z}$) subgroups of the modular group] to reduce everything to f or g twists on b cycles, and then use (1) and (3) to simplify to a single f and g twist.

VI. COMMENTS ON HIGHER LOOP VANISHING

Once we have put the twists on our genus h surface Σ_h into the canonical $(1,\ldots,1,f,g)$ form, we can provide a rough physical argument for the vanishing. This section is very heuristic; it would be nice to make these arguments more precise.

The argument involves supersymmetry. One can think of Σ_h in terms of a genus $h-1$ surface Σ_{h-1} (with a g projection on one cycle) connected to an extra handle (holding the f projection) by a nondegenerate tube (on which massive or massless string states may propagate) (see Fig. 6).

This suggests that one rewrite the diagram as

$$\Sigma_s\ \ (s\ \text{ tadpole on }\ \Sigma_{h-1})\times e^{-M_sT}$$
$$\times(s\ \text{ tadpole on }\ f\text{-projected handle}), \tag{6.1}$$

where the sum runs over possible intermediate physical states s of mass M_s, and the tube has length T. In this way of thinking about it, the diagram vanishes because even for massive string states, the tadpoles at genus $h-1$ in the g projected theory and at genus one in the f projected theory should vanish (as those theories are both 4D $\mathcal{N}=2$ supersymmetric).

Toward a (perturbative) symmetry argument

The world sheet arguments for the perturbative vanishing of the cosmological constant in supersymmetric string theories used contour deformations of the spacetime supercurrents crucially [29,19,15]. Thus one could see the constraint of spacetime supersymmetry through world sheet current algebra.

In our theory, of course, the spacetime supercurrents are projected out, which is to say that they have monodromy around the twist fields in the orbifold. On the Riemann surface with a given twist structure [as in Fig. 6], these operators pick up phases upon traversing the cuts in the diagram.

Let us consider the argument of [29] in this context. One splits open one handle of the surface (say the handle with the f-cut), and rewrites the propagating state V as $\oint_{a_f} S_{+-}V'$ for some operator V'. (So in particular V' describes a boson if the original state was fermionic in spacetime and vice versa.) Here S_{+-} refers to a would-be spacetime supercurrent with eigenvalues $+1$ and under f and -1 under g. The cycle a_f is

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[2]There are also inhomogeneous terms that shift the characteristics of the theta functions coming from the fermion determinants under such a modular transformation; these lead to a change of spin structure but do not change the orbifold twist structure.

the a-cycle on the handle with the f cut. Without the g cut, one can deform the contour integral around the rest of the Riemann surface and turn the fermion loop into a boson loop (or vice versa) with a cancelling sign. With the g-cut, however, one is left with a remainder contribution of the form

$$2 \oint_{a_f} dx \oint_{a_g} dy \langle S_{+-}(x) S_{+-}(y) \rangle. \qquad (6.2)$$

The direct calculation of this contribution could be done at genus 2 in a similar way to the partition function calculation we presented in the previous sections (using the correlation functions of [21]). As before it would be hard to then generalize this computation to higher genus precisely, due to our lack of explicit understanding of the moduli space (and the problem of choosing a consistent gauge slice for the world sheet gravitino). It would be nice to understand if there is some simple topological reason that this remainder must vanish at arbitrary genus.

VII. RELATION TO AdS-CFT CORRESPONDENCE

There has been a great deal of recent work on the fascinating conjectures relating conformal field theories in various dimensions to string theory in anti–de Sitter (AdS) backgrounds [30,31]. It was argued in [2] that certain nonsupersymmetric instances of this correspondence could lead to the discovery of nonsupersymmetric string backgrounds with vanishing cosmological constant. The predictions of new fixed lines at the level of the leading large-N theory based on the correspondence [2] were verified directly through remarkable cancellations in perturbative diagrams [32] in a host of models that could be constructed quite systematically [33]. Here, we review and elaborate on the idea of extending predictions of the correspondence to finite-N fixed lines, and explain how the class of orbifolds we have discussed in this paper may be realizations.

The correspondences between 4D CFTs and string backgrounds have (in the 't Hooft limit)

$$\frac{\alpha'}{R^2}\,\text{expansion} \rightarrow \text{expansion in } (g_{YM}^2 N)^{-1/2}, \qquad (7.1)$$

$$g_{st} \;\text{expansion} \rightarrow \text{expansion in } g_{YM}^2 = \frac{1}{N}. \qquad (7.2)$$

In cases where one has a *nonsupersymmetric* fixed line (to all orders in $1/N$ as well as $g_{YM}^2 N$) realized on branes in string theory, we would obtain by this correspondence a stable nonsupersymmetric string vacuum which exists at arbitrary values of the coupling g_{st}.

The equation of motion for the dilaton $g_{st} = e^{\phi}$ is

$$(-g)^{1/2} \partial_\mu (\sqrt{(-g)} g^{\mu\nu} \partial_\nu \phi) = -\frac{\partial V}{\partial \phi}, \qquad (7.3)$$

where $V(\phi)$ is the dilaton potential and g is the AdS metric. Thus one finds that, according to the AdS conformal field theory (CFT) correspondence, a conformal fixed point in the dual field theory implies a minimum of the bulk vacuum energy with respect to the g_{st}. Consequently, stability of the spacetime background for arbitrary dilaton vacuum expectation value (VEV) $\langle \phi \rangle$ (which is implied by the existence of a nonsupersymmetric fixed line as above) would imply that there is no g_{st} dependent vacuum energy.

Nonsupersymmetric theories with vanishing β-function at leading order in $1/N$ but nonzero β-functions at subleading orders [2,33,32] should provide a concrete testing ground for ideas relating the holographic principle [34] to the cosmological constant [35]. In particular, one would generically expect perturbative contributions to the cosmological constant (dilaton tadpole), which are related (via a possibly nontrivial map) to beta functions in the boundary field theory. In perturbative string theory these contributions come in generically at the order of the supersymmetry breaking scale, which is the string scale in these models. In general, the AdS-CFT correspondence relates perturbative string corrections to $1/N$ corrections in the boundary QFT. Therefore the perturbative string corrections should be encoded in the boundary theory in a way consistent with the holographic reduction in the number of degrees of freedom.

Dualities between field theory fixed lines (which exists to all orders in $1/N$) and nonsupersymmetric string backgrounds would have different consequences in the different AdS_d dualities. In the orbifolds $\mathrm{AdS}_5 \times (S^5/\Gamma)$, the duality would imply that the effective *ten-dimensional* cosmological constant vanishes. In the large $g_{YM}^2 N$ limit, we expect from Eq. (7.1) that the AdS and orbifolded sphere also each become flat. However, in this limit the spacetime theory regains supersymmetry away from the fixed loci of Γ, so this does not yield a nonsupersymmetric theory in the bulk of spacetime.

However, there are cases where the large N limit *could* yield a nonsupersymmetric theory in bulk with vanishing cosmological constant. Consider for instance dualities between type II B strings on $\mathrm{AdS}_2 \times S^2 \times (T^6/\Gamma)$ and conformally invariant quantum mechanical systems (some supersymmetric instances of such dualities were conjectured in [30]). In these cases, going through the analogous arguments we would be talking about the effective *four-dimensional* cosmological constant. In the large N limit, $\mathrm{AdS}_2 \times S^2 \rightarrow \mathbb{R}^4$ while the size of the T^6/Γ remains *fixed* (it does not decompactify). Thus, if we break supersymmetry on the internal space we might be able to find examples of the AdS-CFT correspondence which predict vanishing 4D cosmological constant in a bulk nonsupersymmetric theory. This provides a strong motivation for understanding conformally invariant quantum mechanical systems with ''fixed lines'' (corresponding to the spacetime g_{st}). In particular, at least naively a quantum mechanical model which is classically conformal will not develop a β-function since there are no ultraviolet divergences (though one may need to worry about IR problems).

We have two comments about trying to find models in this way via the AdS-CFT correspondence.

(1) The $\mathrm{AdS}_2 \times S^2$ geometries of interest arise as the near horizon limits of Reissner-Nordstrom black holes. In nonsupersymmetric situations, where π_1 of the compactification is typically small, it can be very difficult to find stable black

holes of this sort by wrapping branes. In part this is because one often finds a 4D effective Lagrangian of the form

$$\mathcal{L}=\int d^4x\ \phi F_{\mu\nu}F^{\mu\nu}+\partial^{\mu}\phi\partial_{\mu}\phi+\cdots, \tag{7.4}$$

where F is the field strength for the U(1) gauge field under which the Reissner-Nordstrom black hole carries charge, and ϕ is some scalar field. Then, the equation of motion for ϕ becomes

$$\partial^{\mu}\partial_{\mu}\phi\sim F_{\mu\nu}F^{\mu\nu} \tag{7.5}$$

and this forces ϕ to have a nontrivial profile in the black hole solution which breaks the AdS isometry.

In order to get around problems of stability and of the existence of scalars with linear couplings to F^2, it is useful to start with models containing very few scalars. Asymmetric orbifolds are one natural source of such models. Starting with configurations of wrapped branes invariant under the orbifold group, one can obtain Reissner-Nordstrom black holes in asymmetric orbifolds such as the one we have studied here. One can then predict vanishing cosmological constant based on the conformally invariant family of quantum mechanical systems living on the boundary of the near-horizon geometry, as in the argument above. It is intriguing that this rather indirect argument relates the problem of fixing moduli to the cosmological constant problem. It would be nice to understand the constraints more systematically.

(2) The orbifold we have been discussing not only has

$$\Lambda_{1\text{-}loop}=\Lambda_{2\text{-}loop}=\cdots=0 \tag{7.6}$$

but also has

$$\Lambda_{1\text{-}loop}=\int d^2m_i 0,\quad \Lambda_{2\text{-}loop}=\int d^6m_i 0,\ \ldots. \tag{7.7}$$

That is, the vacuum amplitudes vanish *point by point* on the moduli space of Riemann surfaces (within a particular twist structure: it will vanish point-by-point on all twist structures if one also acts on the gauge-fixing choice with the relevant modular transformations). This vanishing integrand reflects the exceptionally simple spectrum of our theories (Bose-Fermi degeneracy, etc.).

In more general examples that might come out of nonsupersymmetric versions of the AdS-CFT correspondence as above, we would expect Eq. (7.6) to hold (since the conformal quantum mechanics has a fixed line, and the dilaton VEV is arbitrary). However, there is no reason to expect Eq. (7.7) to hold in general examples. It would be nice to find an example where, e.g., $\Lambda_{1\text{-}loop}$ vanishes but not point-by-point (as in Atkin-Lehner symmetry [36]).

It would be very interesting to find similar models with a more realistic low energy spectrum. In addition, we could potentially find nonsupersymmetric models in 4D with no dilaton by finding 3D string models satisfying our conditions and taking $g_{st}\rightarrow\infty$.

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