

Duality of nonsupersymmetric large N gauge theories

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Starting from Seiberg's electric-magnetic duality for supersymmetric QCD, we construct dual pairs of nonsupersymmetric gauge theories. This is accomplished by first taking the large N limit of supersymmetric QCD and its dual partner and then performing a special "orbifold projection" recently introduced by Kachru and Silverstein. We argue that in the large N limit the two projected theories remain dual. The nonsupersymmetric gauge theories which can be studied in this fashion have nonsupersymmetric field content, chiral fermions, and exactly massless scalar matter. [S0556-2821(99)01810-X]

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I. INTRODUCTION

In recent years many examples of four-dimensional supersymmetric gauge theories have been found to exhibit non-Abelian electric-magnetic duality [1,2]. The existence of a weakly coupled dual description of the infrared physics of a strongly coupled gauge theory allows one to obtain a wealth of information on the low energy dynamics practically for free.

Clearly, it is very interesting to explore if electric-magnetic duality also occurs for nonsupersymmetric non-Abelian gauge theories. Unfortunately, only little progress in this direction has been made. Several groups have introduced soft supersymmetry breaking to supersymmetric dual pairs of theories in an attempt to deform away from the supersymmetric limit in a controlled fashion. However, intrinsic to this approach is that once the scale of supersymmetry breaking is increased to be near the scale of the strong dynamics, non-calculable corrections become large and make predictions impossible. Other attempts at guessing nonsupersymmetric dualities based on 't Hooft anomaly matching [3] or intuition from string theory [4] led to a few candidates of nonsupersymmetric duals. However, in the absence of decisive consistency checks, these dualities remain suspect at best.

In this paper we propose a new approach to generating nonsupersymmetric duals. This approach employs a new technique which is motivated from recent advances in string theory. Namely, Madacena [5] conjectured that four-dimensional $\mathcal{N}=4$ superconformal $SU(N)$ gauge theory is dual to supergravity on five-dimensional anti-de Sitter space in the limit of large N . Kachru and Silverstein [6] extended the conjecture to theories with less supersymmetry by orbifolding the supergravity theory and determining the corresponding "orbifold" field theory. Following up on this idea [7] gave a general recipe on how to generate "orbifold projected" daughter field theories from the $\mathcal{N}=4$ supersymmetric parent theory. Using string perturbation techniques, they further proved the following statement to all orders in perturbation theory: *the correlation functions of the "orbifold" daughter theories are identical to corresponding correlation functions in the parent theory in the limit of large N .*

This statement will be central to our derivation of nonsupersymmetric duality. Finally, Bershadsky and Johansen [8] showed how the above statement can be proved in the context of large N field theory with no reference to string theory. They also noted that the "orbifold projection" technique can be applied more generally to large N field theories with less than $\mathcal{N}=4$ supersymmetry.

The precise definition of orbifolding will be given in Sec. III. Roughly, the procedure is to identify a discrete global symmetry of the parent field theory. One then also embeds this discrete symmetry into the gauge group by using a special representation of the discrete group. Now "orbifolding" simply means to eliminate all the fields of the parent theory which are not invariant under the discrete symmetry. The daughter theories interactions are inherited from the parent theory by keeping all terms of the Lagrangian which only involve daughter fields.

In this paper we apply "orbifolding" to supersymmetric QCD and its electric-magnetic dual to generate new nonsupersymmetric duals. The idea is simple: "orbifolding" takes a large N parent field theory and generates a daughter field theory whose correlation functions are identical to the parent's. The daughters are also large N gauge theories, but typically have less or no supersymmetry. Applying identical orbifolds to both the electric and magnetic theory of Seiberg's supersymmetric QCD generates two daughters. Since the infrared properties of the parents were related by duality, we conclude that the daughters must also be related by duality. Note that we have "derived" nonsupersymmetric duality from Seiberg's supersymmetric duality.

A possible pitfall which could invalidate the derivation is that Seiberg's duality might not capture large N physics. This is possible because the infrared limit in which the duality is expected to hold might not commute with the large N limit. This would be the case if supersymmetric QCD includes states with masses of order Λ_{QCD}/N which are not included in Seiberg's dual, but which become massless in the limit of large N . Such a situation does occur in QCD where the mass of the η' goes to zero at large N [9]. Such incompatibility of limits has also been argued to occur near the monopole points of $\mathcal{N}=2$ theories [10]. In Sec. II of this paper, we study this concern by checking $\mathcal{N}=1$ supersymmetric infrared results against large N predictions in two cases: the gluino condensate for supersymmetric Yang-Mills theory

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[11] and the strength of meson couplings in supersymmetric QCD in the limit of large numbers of colors and flavors. We find agreement with large N expectations in both cases and no evidence for new massless states. Nevertheless, noncompatibility of the infrared and large N limits remains a worry and a more thorough investigation in future work would be desirable.

The new duals which we will derive in this paper are very far from being pairs of generic nonsupersymmetric field theories. A typical orbifold daughter and its dual has a product gauge group of the form $SU(d_1 N) \times SU(d_2 N) \times \cdots \times SU(d_n N)$ with gauge couplings given by

$$\left(\frac{g}{\sqrt{d_1}}, \frac{g}{\sqrt{d_2}}, \dots, \frac{g}{\sqrt{d_n}} \right). \quad (1.1)$$

Here d_i are integers of order 1 and g is related to the running gauge coupling of the parent theory. The daughters are often chiral, and they typically have exactly massless scalars in addition to the massless fermions. That the scalars are kept massless to all orders in perturbation theory is clearly a remnant of the supersymmetry in the parent theory which would disappear if—for example—we allowed the ratios of couplings to differ from their finely tuned values in Eq. (1.1).

It would be very exciting if the results of this paper could be extended to finite N . There are two avenues to pursue: one could try to orbifold parents with higher supersymmetry so that the daughters still retain some supersymmetry; then, supersymmetric nonrenormalization theorems can be used to argue that the exact results continue to hold at finite N . An example of such applications is the derivation of the Seiberg-Witten curves for the Coulomb branch of certain $\mathcal{N}=1$ gauge theories [12] by orbifolding $\mathcal{N}=2$ theories [13]. Another interesting avenue to pursue would be to use the non-supersymmetric large N duality as a starting point to study the finite N theory in the $1/N$ expansion. Going to finite N would introduce corrections to the duality of order $1/N$, but one might still find a qualitatively correct picture. At finite N one would expect the scalars to obtain a mass and possibly also some chiral symmetry breaking. It is conceivable that the symmetries and consistency with the large N limit turn out to be sufficiently constraining to allow such a qualitative analysis.

Finally, it would be interesting to see if the orbifolds considered in this paper can be related to brane configurations in string theory or supergravity. One would expect to learn more about both the field theories and string theory from such an embedding.

The remainder of this paper is structured as follows. In Sec. II we compare some of the exact supersymmetric results to large N expectations. In Sec. III we introduce the method of deriving a daughter field theory from a parent theory via “orbifolding.” We also prove—following [8]—that the correlators of the daughters are trivially related to corresponding parent theory correlators. In Sec. IV we apply the orbifold projection technique to Seiberg’s electric-magnetic dual pair to obtain an $SU(N)^3$ chiral nonsupersymmetric gauge theory and its $SU(F-N)^3$ dual. We also perform a few consistency

checks on this new duality, but we leave further exploration of the duality and more extensive consistency checks to future work.

II. LARGE N SUPERSYMMETRIC QCD

In this section we compare expectations of the large N expansion [14] with exact infrared solutions to supersymmetric QCD due to Seiberg and many others [2]. In the examples we discuss, we find agreement between the two techniques. This result is not obvious because one might have worried that the large N theory contains states whose masses scale as Λ_{QCD}/N^k for some $k>0$ and become arbitrarily light in the large N limit. These states would be important to the infrared (IR) physics of the large N limit. However, in the framework of the exact supersymmetric results, these states are excluded from the effective IR Lagrangian because their masses are of order Λ_{QCD} . Thus the two methods can give completely different results if the large N limit does not commute with the IR limit taken by Seiberg and collaborators. This problem appears to arise near the monopole points in the Coulomb phase of $\mathcal{N}=2$ supersymmetric theories as discussed in [10]. As we will show in this section, the limits do appear to commute for supersymmetric QCD, allowing us to use both techniques interchangeably. This result will be crucial to the arguments of Secs. III and IV.

A. Gaugino condensate for supersymmetric glue

In this section we compare large N expectations for supersymmetric QCD with no flavors with the exact results. The contents of this subsection are not new and can be found in the literature [11,15–17]. We start with the Lagrangian in the customary normalization

$$\begin{aligned} \mathcal{L} = & -\frac{1}{4g_h^2} \int d^2\theta W_\alpha W^\alpha + \text{H.c.} = -\frac{1}{4g^2} (F_{\mu\nu})^2 + \frac{1}{g^2} \bar{\lambda} i \not{D} \lambda \\ & + \frac{i\theta}{32\pi^2} F_{\mu\nu} \tilde{F}^{\mu\nu}. \end{aligned} \quad (2.1)$$

The gauge fields A_μ and gaugino λ transform in the adjoint representation of the gauge group $SU(N)$, and a trace over $SU(N)$ indices is implied. The holomorphic gauge coupling g_h satisfies a simple renormalization group equation which terminates at one loop, and it is convenient to define the renormalization group invariant holomorphic scale Λ_h as

$$\Lambda_h^{3N} = M^{3N} e^{-8\pi^2/g_h^2(M)}. \quad (2.2)$$

Here $g_h^2(M)$ is the holomorphic coupling evaluated at the renormalization scale M . It is related to the conventional coupling g of the Lagrangian with canonically normalized kinetic terms via the Shifman-Vainshtein equation

$$\text{Re} \left(\frac{8\pi^2}{g_h^2} \right) = \frac{8\pi^2}{g^2} + N \ln g^2. \quad (2.3)$$

The large N limit of the theory is obtained by taking N to infinity while keeping

$$\bar{g}^2 = g^2 N \quad (2.4)$$

fixed. In terms of the rescaled coupling $\bar{g}$, the Lagrangian takes the form

$$\mathcal{L} = \frac{N}{\bar{g}^2} \left[-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \bar{\lambda} i \not{D} \lambda \right], \quad (2.5)$$

where a chiral rotation of the gaugino field has been performed to set $\theta=0$. In this form the large N counting is performed easily¹ with the usual well-known results. For the purpose of counting N 's, gluinos can be treated identically as gluons. The leading vacuum-to-vacuum amplitude is of order N^2 and arises from planar diagrams with arbitrary numbers of gluon and gluino loops. The expectation value of any operator which is defined as a trace of elementary fields is of order N . We therefore expect that the gluino condensate is of order N . Let us check this prediction explicitly using the exact result [1,11]

$$\langle \text{tr } \lambda \lambda \rangle = e^{2\pi i k/N} \Lambda_h^3. \quad (2.6)$$

To continue, we need to determine the N scaling of Λ_h , which is obtained by exponentiating Eq. (2.3):

$$\text{Re}[\Lambda_h^3] = M^3 \frac{1}{g^2} e^{-8\pi^2/g^2 N} = M^3 \frac{N}{\bar{g}^2} e^{-8\pi^2/\bar{g}^2} = N \Lambda^3. \quad (2.7)$$

The last equality defines Λ , which is both independent of N and renormalization group invariant. Λ (and not Λ_h) is the scale where the theory becomes strongly coupled and where one expects hadron masses. Thus we find that the gluino condensate scales like N as predicted. Note that with canonically normalized gluino fields the condensate scales as N^2 . Assuming confinement, the large N expansion further predicts that glueballs and their superpartners are weakly interacting with effective couplings of order $1/N$. Unfortunately, we cannot check this prediction because the massive hadron spectrum is beyond the reach of our exact infrared techniques.

B. Supersymmetric QCD with flavors

In this section we consider supersymmetric QCD with N colors and F flavors in the large N limit. There are two qualitatively different large N limits to consider: (i) large N with F constant, in which case quark loops are suppressed because there are less quarks than gluons to run in loops, or (ii) large N with F/N constant, in which case quark loops are just as

important as gluon loops. The first alternative is the large N limit which is most often applied to QCD phenomenology. We will take F large as well because we are interested in comparing large N expectations with the results of Seiberg's duality which only exists for $F > N$.

The Lagrangian of supersymmetric QCD with flavors is

$$\mathcal{L} = \frac{N}{\bar{g}^2} \left[-\frac{1}{4} \int d^2\theta W_\alpha W^\alpha + \text{H.c.} \right. \\ \left. + \int d^4\theta (Q_i^\dagger e^V Q_i + \bar{Q}_i e^V \bar{Q}_i^\dagger) \right], \quad (2.8)$$

where Q and $\bar{Q}$ are the quark chiral superfields, $i = 1, \dots, F$ is a flavor index, and we have scaled rescaled all fields such as to move the dependence on the gauge coupling $g^2 = \bar{g}^2/N$ into the overall factor. It is convenient to perform the N counting in component form before integrating out the auxiliary fields. Every propagator scales as $1/N$, every vertex as N , loops of color lines as N , and loops of flavor lines contribute $F \sim N$. It is then easy to see that the leading vacuum-to-vacuum amplitudes are of order N^2 and are given by planar diagrams with arbitrary numbers of loops of adjoints and fundamentals. If one assumes confinement, one can estimate the interaction strength of the confined degrees of freedom. For example, for properly normalized ‘‘mesons’’ $M = QQ$ one finds

$$\langle M_1 \cdots M_k \rangle \sim N^{1-k/2}. \quad (2.9)$$

Thus mesons interact weakly with effective coupling $N^{-1/2}$. To compare this result with expectations from Seiberg's duality, we should keep in mind that we have assumed confinement. Consequently, we should only expect to find agreement if the composite ‘‘mesons’’ are appropriate low energy degrees of freedom. One might also learn something interesting by studying baryons which are more difficult to treat in the large N expansion; we leave this for future work.

The dual of supersymmetric QCD with N colors and F flavors is an $SU(F-N)$ gauge theory with F flavors of dual quarks. There is also a fundamental ‘‘meson’’ which is coupled to the dual quarks in the superpotential $W = \mu^{-1} M q \bar{q}$, where μ is the scale which appears in the matching between the fundamental meson of the dual and the composite meson of the electric theory, $M = QQ$. We now wish to take the large N limit in the dual as well. We take large N and F with fixed ratio, but we also need to determine how to scale the dual gauge and Yukawa couplings.

We can fix the scaling of the dual gauge coupling by assuming that the magnetic theory has a sensible large N expansion. This determines that $\tilde{g}^2 = \bar{g}^2 N$. Demanding that loops with internal mesons M do not destroy the expansion and that the meson Yukawa coupling does not become irrelevant at large N also fixes the large N behavior of $\mu \sim N^{1/2}$.

Now we can compare with our prediction from the electric theory. There we found that the mesons are weakly coupled with coupling of order $N^{-1/2}$ by assuming that the theory confines, that is, by assuming that the mesons are

¹A lucid introduction to large N counting can be found in [18]. Note that the usual large N arguments only apply to all orders in perturbation theory, but it is expected that they continue to hold nonperturbatively. The results of this section confirm this expectation.

proper infrared degrees of freedom. From the dual description, we see that the mesons are the correct infrared degrees of freedom for $N < F < 3/2N$ and that their coupling is $\mu^{-1} \sim N^{-1/2}$ as predicted.

III. “ORBIFOLD” PROJECTION

In this section we discuss a projection technique which allows one to generate pairs of field theories which have identical large N correlation functions. The theories which can be related in this way generically have differing amounts of supersymmetry, and often chiral theories are related to nonchiral ones. One starts with a “parent” theory and projects to a “daughter” theory by eliminating all fields which are not invariant under a specifically chosen discrete global symmetry of the parent Lagrangian. Even though the technique is more general, we will limit ourselves to $SU(N)$ gauge theories with matter in the adjoint and fundamental representations. In this section we first discuss the technique of “orbifolding” in general and then present three explicit examples [and in Sec. IV we discuss the example of $SU(3N)$ orbifold by $\mathbb{Z}_3$ in detail].

The orbifolding technique was first introduced in the context of Maldacena’s conformal field-theory–string-theory duality [5] by Kachru and Silverstein [6] and was further developed and formalized in [7]. We will largely follow and expand on the work of Bershadsky and Johansen [8] who rederived the results of [6,7] from the perturbative large N expansion in field theory. Before describing the projection technique, we recall a few basic group theory facts.

A. Group theory: Regular representation and projectors

A discrete finite group $G = \{g_1, g_2, \dots, g_\Gamma\}$ with group multiplication $\circ$ is associative, closed under multiplication, has a unique identity element, and there exists a unique inverse for each group element. The *regular representation* of

a group is given by Γ -dimensional matrices γ^a , which are defined by $g_a \circ g_i = g_j (\gamma^a)_{ji}$. Here the superscript $a = 1, \dots, \Gamma$ labels group elements, and there is no distinction between lower and upper indices on the group elements. The identity g_1 is the $\Gamma \times \Gamma$ unit matrix $\mathbb{1}_\Gamma$ in this representation. Using the fact that $g_a \circ g_b \neq g_b$ unless $a = 1$, one immediately obtains that all group elements except the identity element are traceless:

$$\text{Tr } \gamma^a = \Gamma \delta_1^a. \quad (3.1)$$

The regular representation is reducible: it can be shown that the decomposition contains each irreducible representation R_l of the group with multiplicity equal to its dimension $d_l = \dim(R_l)$. Note that this property of the regular representation implies the well-known identity $\sum_l (d_l)^2 = \Gamma$. In a convenient basis, the γ^a are of the form

$$\gamma^a = \begin{pmatrix} (r_1^a) & & & & \\ & (r_2^a) & & & \\ & & \ddots & & \\ & & & \overbrace{(r_l^a)}^{d_l \text{ times}} & \\ & & & & \ddots \\ & & & & & (r_l^a) \\ & & & & & & \ddots \end{pmatrix}. \quad (3.2)$$

In the next section we will assign Γ N -dimensional vectors [corresponding to the gauge indices of an $SU(\Gamma N)$ gauge group] to transform under an N -fold copy of the regular representation. These transformation matrices have the general form

$$\gamma_N^a = \begin{pmatrix} \overbrace{\begin{pmatrix} (r_1^a) & & \\ & \ddots & \\ & & (r_1^a) \end{pmatrix}}^{d_1 N \text{ times}} & & \overbrace{\begin{pmatrix} (r_2^a) & & \\ & \ddots & \\ & & (r_2^a) \end{pmatrix}}^{d_2 N \text{ times}} & & \\ & & & \ddots & \\ & & & & \ddots \end{pmatrix}. \quad (3.3)$$

The fundamental Q and adjoint A of $SU(\Gamma N)$ then transform as

$$Q \rightarrow \gamma_N^a Q, \quad A \rightarrow \gamma_N^a A (\gamma_N^a)^\dagger. \quad (3.4)$$

We will be interested in the components of the matrix A which are invariant under all such transformations. They are easily determined by applying Schur's lemma, which states that a matrix which commutes with all elements of an irreducible representation is a multiple of the unit matrix. We find that the invariant components of A are located in blocks on the diagonal, transforming as an adjoint of $SU(d_1 N) \times SU(d_2 N) \times \cdots \times SU(d_n N) \subset SU(\Gamma N)$:

$$\begin{pmatrix} A_1 \otimes \mathbb{1}_{d_1} & & & \\ & A_2 \otimes \mathbb{1}_{d_2} & & \\ & & \ddots & \\ & & & A_n \otimes \mathbb{1}_{d_n} \end{pmatrix}. \quad (3.5)$$

We will also have use for a projector onto invariants of the group. It is defined as

$$P_R = \frac{1}{\Gamma} \sum_{a=1}^{\Gamma} r^a, \quad (3.6)$$

where the r^a are representation matrices of the (not necessarily irreducible) representation R . Using

$$r^b P_R = \frac{1}{\Gamma} \sum_{a=1}^{\Gamma} r^b r^a = \frac{1}{\Gamma} \sum_{c=1}^{\Gamma} r^c = P_R, \quad (3.7)$$

it is easy to show that $P^2 = P$ and that $P = 1$ in the trivial representation, whereas $P = 0$ in all other irreducible representations. Thus, when acting on a column vector transforming in the representation R , the projector extracts the invariant components.

Finally, recall that one can form tensor product representations $S \otimes T$ with representation matrices $s^a \otimes t^a$. The projector onto invariants in such a tensor product representation is

$$P_{S \otimes T} = \frac{1}{\Gamma} \sum_{a=1}^{\Gamma} s^a \otimes t^a. \quad (3.8)$$

B. How to “orbifold” a field theory

Using the regular representation and the projector defined in the last subsection, we now defined the “orbifold” projection which takes us from a large N parent theory to a daughter theory. We consider a theory with gauge group $SU(\Gamma N)$. We define the action of the discrete group G on all the fields of the theory by the following procedure: assign the gauge indices to N -fold copies of the regular representation as in Eq. (3.3). A vector or an adjoint then transforms as in Eq. (3.4). Furthermore, we can also embed the discrete group in the global symmetries of the theory. If the size of a factor of the global symmetry group grows with N , we assign the global group index to a multiple of the regular representation as well. Global groups under which the fields trans-

form only in finite dimensional representations can be assigned to arbitrary representations of G . The orbifolded theory is obtained by simply deleting from the parent theory the fields which are not invariant under the action of each element of G . The Lagrangian of the daughter theory is obtained from the parent Lagrangian by keeping the interactions which involve only invariant fields and discarding all others. In the next subsection we will prove that the correlators of these daughter theories as defined here have the very special property that their correlators are identical to the correlators of their parents (after a rescaling of the gauge coupling constants by a group theoretical factor). Before moving on, it is probably useful to demonstrate the orbifolding procedure on three examples.

1. Example (i)

First, consider $SU(\Gamma N)$ pure gauge theory. The gauge indices are transformed by multiplication with γ_N^a as defined in Eq. (3.3). Gluons in the adjoint of $SU(\Gamma N)$ transform as $A_\mu \rightarrow \gamma_N^a A_\mu (\gamma_N^a)^\dagger$. The invariant gluons in A_μ form an adjoint representation of $SU(d_1 N) \times SU(d_2 N) \times \cdots \times SU(d_n N) \subset SU(\Gamma N)$. Thus the daughter theory consists of n decoupled $SU(d_i N)$ pure Yang-Mills theories. The interactions of the daughter theory are obtained from the parent theory by taking the parent Lagrangian and removing all terms which involve fields that were projected out. In this case it is easy to see that the resulting Lagrangian describes a product of decoupled Yang-Mills theories. There is a small subtlety regarding the Yang-Mills couplings of the $SU(d_i N)$ factors: they are not all equal to the couplings of the parent theory. To see this recall that the i th invariant component of A_μ multiplies a d_i -dimensional unit matrix [Eq. (3.5)]. The traces over colors in the Lagrangian therefore include also traces over these unit matrices $\mathbb{1}_{d_i}$, which each yield a factor of d_i . Therefore, the projected theory has overall factors of d_i in front of the Lagrangians for each of the Yang-Mills factors. By redefining couplings and rescaling fields, we can remove these factors. The end results is that $SU(\Gamma N)$ with gauge coupling g is projected to $SU(d_1 N) \times SU(d_2 N) \times \cdots \times SU(d_n N)$ with gauge couplings

$$(g_1, g_2, \dots, g_n) = \left(\frac{g}{\sqrt{d_1}}, \frac{g}{\sqrt{d_2}}, \dots, \frac{g}{\sqrt{d_n}} \right). \quad (3.9)$$

Note that these couplings are not renormalization group invariant, but that ratios of couplings are invariant in the limit of large N . The statement that daughter and parent have identical correlation functions in the large N limit turns out to be trivial for this example. The statement here is that correlators of $SU(d_i N)$ gauge theory with coupling $g/\sqrt{d_i}$ are identical to correlators of $SU(\Gamma N)$ with coupling $g/\sqrt{\Gamma}$. We knew that already since the large N limit only depends on the combination $g^2 N$ in either case. However, this example can be viewed as a useful consistency check on the arguments which we present in the next section.

2. Example (ii)

Consider, as a second example, $\mathcal{N}=1$ supersymmetric $SU(\Gamma N)$ Yang-Mills theory. This theory has a global $U(1)_R$ symmetry which rotates the gluino field by a phase.² We can now embed the discrete group G into the R symmetry such that the gluinos transform in a one-dimensional representation R_l of G in addition to their transformation from the gauge indices. The gauge bosons do not carry R charge and transform according to their gauge charge only $A_\mu \rightarrow \gamma_N^a A_\mu (\gamma_N^a)^\dagger$. Gluinos carry both gauge and R charge; they transform as $\lambda \rightarrow r_l^a \gamma_N^a \lambda (\gamma_N^a)^\dagger$, where r_l^a is a transformation matrix in a one-dimensional representation. In other words, r_l^a is a phase. The invariant gluons are the same as in the first example. Therefore, the gauge group of the daughter is again $SU(d_1 N) \times SU(d_2 N) \times \cdots \times SU(d_n N)$. The gluinos are more interesting. Depending on which representation R_l we choose, different components of λ survive the projection. In the special case where R_l is taken to be the trivial representation, the gluinos are projected identically to the gluons and we obtain n decoupled $\mathcal{N}=1$ supersymmetric $SU(d_i N)$ gauge theories. In the more interesting case where R_l is chosen to be nontrivial, we find a nonsupersymmetric daughter. Consider, for example, the discrete group $\mathbb{Z}_\Gamma$ and pick the representation $R = \{r^a = e^{2\pi i a/\Gamma}, \text{ for } a = 1, \dots, \Gamma\}$. With this choice the invariant gluinos are in $N \times N$ blocks which are shifted to the right of the diagonal. The resulting nonsupersymmetric theory has chiral fermion field content

$SU(N)$	$SU(N)$	$SU(N)$	$\cdots$	$SU(N)$	
$\square$	$\bar{\square}$	1	$\cdots$	1	
1	$\square$	$\bar{\square}$	$\cdots$	1	
1	1	$\square$	$\cdots$	1	
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	
$\bar{\square}$	1	1	$\cdots$	$\square$	(3.10)

The interactions are determined by $SU(N)^\Gamma$ gauge invariance with the gauge couplings of all the $SU(N)$ factors identified. The overall normalization of the gauge couplings is identical to the normalization in the original theory because the irreducible representations of $\mathbb{Z}_\Gamma$ are all one dimensional. So the d_i which appear in Eq. (3.9) are all equal to 1. A convenient way of representing the matter content of this theory and similar orbifold theories is to use “moose” [19,20] or “quiver” diagrams [21]. The moose diagram corresponding to Eq. (3.10) is given in Fig. 1.

The result we have obtained for this theory is much less trivial. We have related the chiral nonsupersymmetric theory, Eq. (3.10), to $N=1$ supersymmetric yang-Mills theory, a theory about whose vacuum structure we know a lot more.

²In quantum theory the R symmetry is broken by instantons to $\mathbb{Z}_{2\Gamma N}$. Strictly speaking, we therefore need to embed our global symmetry into this subgroup. Similar comments apply to $U(1)_R$ and axial $U(1)$ in the next example.

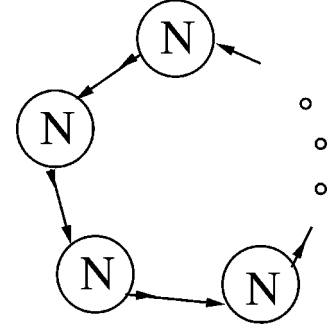


FIG. 1. Diagram corresponding to an $SU(N) \times SU(N) \times \cdots \times SU(N)$ gauge theory with fermions in chiral representations. Each circle represents a gauge group factor, and straight lines with arrows correspond to fermion representations. Outgoing arrows indicate a fundamental, whereas ingoing arrows denote antifundamentals. For more details on this graphical notation and its application to duality, see [20].

3. Example (iii)

Consider, as a third and final example, $\mathcal{N}=1$ supersymmetric QCD with ΓN colors and ΓF flavors in the limit of large N and F and F/N fixed. This model requires a straightforward generalization of the orbifold procedure described in [8]. In addition to using the regular representation to act on gauge indices, we also embed the discrete group into the $SU(\Gamma F)_L \times SU(\Gamma F)_R$ global symmetry using an F -fold copy of the regular representation. We define the action of the $SU(\Gamma F)$'s such that all fields of supersymmetric QCD transform with one upper and one lower index. Therefore, fundamentals of $SU(\Gamma N)$ are antifundamentals of $SU(\Gamma F)$ and vice versa. Having only fields with one upper and one lower index is not necessary, but makes the presentation more transparent.

The theory also has $U(1)_R$, baryon number, and axial $U(1)$ symmetries. As in the previous example, some of these symmetries are anomalous so that we are really only dealing with a discrete subgroup. We will nevertheless continue to refer to these subgroups as $U(1)$ whenever the context makes things clear. We define a “canonical” $U(1)_R$ such that gluinos have charge 1 and squarks are chargeless; supersymmetry then determines the charge of quarks to be -1 . In order to perform the projection, we assign axial $U(1)$ and baryon number to the trivial representation of G , and transform R charged fields in a one-dimensional representation as in the previous example. We will use Q , $\bar{Q}$ and Ψ , $\bar{\Psi}$ for the scalar and fermionic components of the quark superfields transforming in the fundamental and antifundamental representations of $SU(\Gamma N)$ color. The fields then transform as follows under G : gluons and gluinos transform as before, $A_\mu \rightarrow \gamma_N^a A_\mu (\gamma_N^a)^\dagger$, $\lambda \rightarrow r_l^a \gamma_N^a \lambda (\gamma_N^a)^\dagger$. Squarks transform as $Q \rightarrow \gamma_N^a Q (\gamma_F^a)^\dagger$ and $\bar{Q} \rightarrow (\gamma_N^a)^\dagger \bar{Q} \gamma_F^a$. And finally quarks transform as $\Psi \rightarrow (r_l^a)^\dagger \gamma_N^a \Psi (\gamma_F^a)^\dagger$ and $\bar{\Psi} \rightarrow (r_l^a)^\dagger (\gamma_N^a)^\dagger \bar{\Psi} \gamma_F^a$.

Choosing as in the second example above the discrete group $G = \mathbb{Z}_\Gamma$ and representation $R = \{e^{2\pi i k/\Gamma}, \text{ for } k = 1, \dots, \Gamma\}$ for the embedding into $U(1)_R$, we get the matter content depicted in Fig. 2. The couplings of the projected theory are equal to each other because all of the d_i in Eq.

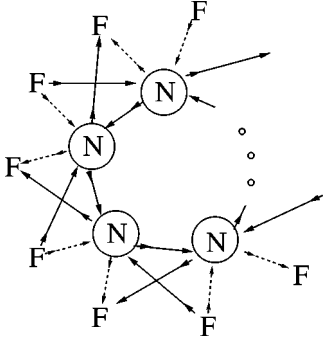


FIG. 2. Diagram corresponding to an $SU(N)^k$ gauge theory with $SU(F)^k \times SU(F)^k$ flavor group. Each solid line with arrows indicates a fermion representation, while a dashed line indicates a scalar. The arrows indicate the representation under the gauge or flavor group; outgoing arrows stand for fundamentals and ingoing arrows for antifundamentals.

(3.9) are equal to 1. The special case of $SU(3N) \rightarrow SU(N) \times SU(N) \times SU(N)$ is discussed in detail in Sec. IV.

C. Large N correlators of parent and daughter theories

In this section we apply and extend the arguments of Bershadsky and Johansen [8] who proved that all correlators of the daughter theories are given by corresponding correlators of the parent theory after a trivial rescaling of the couplings. The practical application of this result is that one can calculate a correlator of the (possibly non-supersymmetric) daughter theory by instead doing the calculation in the full supersymmetric parent theory and rescale the coupling constants in the end:

$$\mathcal{M}_{\text{daughter}}\left(g_i^2 = \frac{g^2}{d_i}\right) = \mathcal{M}_{\text{parent}}\left(\frac{g^2}{\Gamma}\right). \quad (3.11)$$

Here $\mathcal{M}_{\text{daughter}}$ is a correlator of the daughter theory with some number of external fields. It depends on the various coupling constants g_i of the daughter theory which have special values as determined by the orbifold projection procedure (see Sec. III B 1).

We will first show that correlators between the daughter and parent are related at *one loop* by evaluating the color and flavor traces for a typical one-loop diagram in both theories explicitly. Then we extend the proof to all orders by simply iterating the one-loop result. For most of this section, we will follow the proof in [8], which applies only to theories with fields in adjoint representations. But as already predicted in [8], the proof can easily be generalized, which is what we present here for the case of bifundamental matter fields, the case relevant for “orbifolding” supersymmetric QCD.

In the proof we will use ’t Hooft’s double-line notation [14]. In this notation the flow of color and flavor indices through a Feynman diagram is made explicit. A propagator for a field in the fundamental is represented by a single line $\rightarrow$ with an arrow indicating the direction of the index flow. For simplicity, we will limit ourselves to theories with adjoints and bifundamentals. We choose the representations such that all fields carry one fundamental and one antifunda-

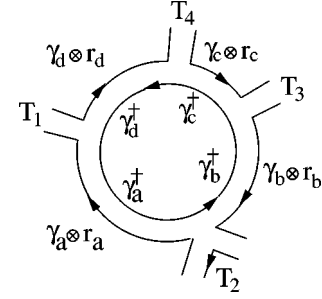


FIG. 3. Typical one-loop diagram with projectors.

mental index, so that they are always represented by two lines with oppositely oriented arrows $\rightleftharpoons$. This orientation of the arrows is important for the arguments below.³ Furthermore, to simplify the N counting, we only consider interaction vertices with color and flavor indices contracted in a single trace. Such a vertex cannot be written as a product of two separately color and flavor invariant operators.

Before presenting the chain of arguments which proves the claim made above, we list two general properties of large N diagrams which are used in the proof. (i) In the large N limit, the perturbation series is dominated by planar diagrams with arbitrary numbers of loops.⁴ Loops of adjoint or bifundamental fields contribute equal N factors as we are considering $F \sim N$. (ii) All external (double) lines should be attached to a single index loop. This is true because the insertion of external lines into a loop breaks the index flow in the loop which costs a factor of N . Putting external fields on more than one loop would cost additional powers of N . Thus at leading order in $1/N$ all external fields attach to a single quark loop which we can choose to be the boundary of the diagram without loss of generality.

We now prove that Eq. (3.11) holds at one loop by showing that it holds for simply connected one-loop diagrams contributing to $\mathcal{M}$. A typical planar one-loop diagram in the daughter theory with five external fields is shown in Fig. 3. All the propagators in the diagram correspond to fields of the daughter theory, but we can rewrite the diagram entirely as a diagram involving fields of the parent theory with appropriate projectors inserted. The external lines are restricted to daughter theory fields, but we can rewrite them as parent theory fields and insert a projector as in Eq. (3.6), $P = (1/\Gamma) \sum \gamma_a \otimes r_a \otimes \gamma_a^\dagger$, which projects onto the daughter fields. Here γ_a and $\gamma_a^\dagger$ act on the indices represented by the double lines (color and flavor), whereas the r_a act on the remaining indices which transform nontrivially under G . The internal propagators are also for fields of the projected

³More general applications are possible as well. For example, theories with symmetric and/or antisymmetric tensors might be interesting to study. Some care is required in extending the following arguments to theories with such tensors as their index flow corresponds to two lines with the same orientation.

⁴We will not discuss gauge choices and associated ghosts. The large N arguments as well as the orbifold projection are easily extended with no fundamental changes to diagrams with ghosts.

theory, but again we can instead use propagators of the parent theory and insert a projector P with every propagator. The interaction vertices $T_1, \dots, T_4$ are all invariant under G , and since all the lines coming into each vertex are projected, the interaction vertices can also be replaced by vertices from the parent theory. We have now written the diagram entirely in terms of parent theory propagators and vertices by adding projectors for each internal propagator and external line. Note that by rewriting the diagrams in this way we relate the daughter theory with couplings $g_i = g/\sqrt{d_i}$ to the parent theory with coupling g . (See the discussion in Sec. III B 1.)

The projectors are represented in Fig. 3 by the $\gamma^\dagger$ and $\gamma \otimes r$. The $\gamma^\dagger$ act on the innermost index line, whereas the γ 's act on the exterior one. The r 's act on indices of T , which are not displayed in the double line notation. For example, in the $\mathcal{N}=4$ supersymmetric theories considered in [6–8] the r 's act on $SU(4)_R$ indices. Finally, the indices a, b, c, d are all summed from 1 to Γ and the diagram has an overall factor of $1/\Gamma^4$ from the projectors on internal propagators.⁵

It will be the goal of the next few paragraphs to show that this daughter theory diagram is equal to $1/\Gamma$ times the corresponding parent theory diagram (which is obtained by leaving out all the projectors).

First, we remark that the diagram factors into an overall numerical “group theory factor,” which we call $\mathcal{A}$, and the complicated rest of the diagram with all its spin and momentum dependence. We define the “group theory diagram” to contain the traces over color, flavor, and all other internal indices as well as the coupling constants. Thus we can represent the Feynman diagram as a product of two diagrams: one “group theory diagram,” which contains the internal index structure and coupling constants for which we use 't Hooft's double line notation, and a “stripped diagram,” which has been stripped of all internal group structure, but whose propagators carry the momentum and spin dependence of the original Feynman diagram. The difference between the daughter and parent theory diagrams is entirely in the internal index structure. We are interested in the difference between daughter and parent diagrams for which the “stripped diagram” is irrelevant. Thus from now on we will only be concerned with the “group theory diagram” and forget all momentum and spin dependence. It should be clear that the factorization into a “group theory diagram” and a “stripped diagram” is general and continues to hold for all higher loop diagrams as well. We will use this fact when we generalize to all orders.

⁵Note that we have been careless by not designating which index lines are flavor or color lines. This will be reflected in the equations to come by an ambiguity over which representations γ and r matrices are in. For example, the γ could be either in an N - or F -fold copy of the regular representation, depending on whether they are inserted on a color or flavor index loop. The difference is that closed flavor loops give factors of ΓF , whereas color loops give ΓN . We will ultimately only be interested in the ratio of amplitudes in the daughter theory to amplitudes in the parent theory. In the ratio the difference between color and flavor lines drops out.

We can now explicitly write the amplitude corresponding to the “group theory diagram” of Fig. 3:

$$\mathcal{A} = g^5 \frac{1}{\Gamma^4} \sum_{a,b,c,d=1}^{\Gamma} \text{Tr}[\gamma_a^\dagger \gamma_b^\dagger \gamma_c^\dagger \gamma_d^\dagger] \text{Tr}[T_1(\gamma_d \otimes r_d) T_4(\gamma_c \otimes r_c) \times T_3(\gamma_b \otimes r_r) T_2(\gamma_a \otimes r_a)]. \quad (3.12)$$

The traces simply follow from following the index lines in the direction of the arrows and contracting all matrices which we encounter along the way in a big trace. This is simple for the inner loop, but for the outer loop, we must also include T tensors which come from the interaction vertices. The T 's carry a number of indices for each double line entering the vertices; for each double line, there are two color or flavor indices and one index that the r 's act on. The g^5 prefactor comes from the coupling constants at the interaction vertices. In general, there can be different coupling constants for different types of vertices. We generically denote them all by g . Again, the difference will not matter in the end. What is important is that we define the couplings such that an n field vertex has a coupling g^{n-2} . This prescription is important for the rescaling of couplings which needs to be done in the end. Note that this assignment is automatic for gauge interactions, is consistent with supersymmetry, and is required for the existence of a nontrivial large N limit.

To evaluate amplitude Eq. (3.12), recall that the γ 's are in multiples of the regular representation whose generators are traceless except for the identity generator. Therefore the diagram vanishes unless

$$\gamma_a^\dagger \gamma_b^\dagger \gamma_c^\dagger \gamma_d^\dagger = \mathbb{1}_{\Gamma N}. \quad (3.13)$$

We can use this fact to eliminate one of the sums, say, over d , and set $\gamma_d = \gamma_a^\dagger \gamma_b^\dagger \gamma_c^\dagger$. Since the γ 's form a faithful representation of G , Eq. (3.13) also holds for group elements $g_a^{-1} g_b^{-1} g_c^{-1} g_d^{-1} = 1$. Thus we also have $r_d = r_a^\dagger r_b^\dagger r_c^\dagger$. This leaves us with

$$\mathcal{A} = g^5 \frac{1}{\Gamma^4} \sum_{a,b,c=1}^{\Gamma} \text{Tr}[\mathbb{1}_{\Gamma N}] \text{Tr}[T_1(\gamma_a^\dagger \otimes r_a^\dagger)(\gamma_b^\dagger \otimes r_b^\dagger)(\gamma_c^\dagger \otimes r_c^\dagger) \times T_4(\gamma_c \otimes r_c) T_3(\gamma_b \otimes r_b) T_2(\gamma_a \otimes r_a)]. \quad (3.14)$$

In the next paragraph we prove that the $(\gamma \otimes r)$ can be commuted through the T 's; we use this fact here to simplify Eq. (3.14) by moving all the T 's to the right of the $(\gamma \otimes r)$'s. Then we can annihilate all $\gamma \otimes r$ with $\gamma^\dagger \otimes r^\dagger$ and obtain

$$\mathcal{A} = g^5 \frac{1}{\Gamma^4} \sum_{a,b,c=1}^{\Gamma} \text{Tr}[\mathbb{1}_{\Gamma N}] \text{Tr}[T_4 T_3 T_2 T_1]. \quad (3.15)$$

The sums have all become trivial and give factors of Γ , and we finally have

$$\mathcal{A} = g^5 N \text{Tr}[T_4 T_3 T_2 T_1]. \quad (3.16)$$

This should be compared with the amplitude of the parent theory which is almost identical: it has the same trace $\text{Tr}[T_4 T_3 T_2 T_1]$ and coupling constants g_{parent}^5 , but it has a

factor of ΓN from the closed loop. The two amplitudes differ only by a factor of Γ , which we can absorb into the coupling constant g^2 . After the rescaling the loop counting parameter $\bar{g}^2$ is the same in both theories, $d_i N \times g^2 / d_i$ in the daughter and $\Gamma N \times g^2 / \Gamma$ in the parent. Thus we have shown that Eq. (3.11) holds at one loop.

Before we get to the generalization to all loops, we need to justify that we were allowed to commute T 's with $\gamma \otimes r$'s. To do this we first put back explicit indices on the vertices $T^{LRIJK} \dots$. The L and R indices stand collectively for all the indices of the internal propagators of the loop which enter into the vertex from the left or right. The $IJK \dots$ stand for the external lines, one index for each external line. Now recall that the interaction vertices of the parent theory are G invariant. Thus transforming all indices of $T^{LRIJK} \dots$ with an element $g_a \in \Gamma$ leaves it invariant. Doing such a transformation introduces a $\gamma_a \otimes r_a \otimes \gamma_a^\dagger$ on each line emerging from T . On the external lines these matrices can be absorbed into the projectors P using Eq. (3.7). We then have

$$T^{LRIJK} \dots = (\gamma_a^\dagger \otimes r_a^{(L)} \otimes \gamma_a)_L T^{LRIJK} \dots (\gamma_a \otimes (r_a^\dagger)^{(R)} \otimes \gamma_a^\dagger)_R, \quad (3.17)$$

where it is implied that the external lines $LRIJK \dots$ are all projected, and the $\dagger$'s on the r_a are chosen to conform with the conventions used above. It is obvious from the double line notation (see Fig. 3) that one of the γ 's is directly contracted with the $\gamma^\dagger$ and we can annihilate them. So we finally find

$$(\gamma_a^\dagger \otimes (r_a^\dagger)^{(L)})_L T^{LRIJK} \dots = T^{LRIJK} \dots ((r_a^\dagger)^{(R)} \otimes \gamma_a^\dagger)_R, \quad (3.18)$$

which shows that we can commute the $\gamma_a \otimes r_a$ past the T 's as claimed. Note that in the process the matrix $\gamma_a \otimes r_a$ changes from the representation appropriate to act on the propagator to the left of T in the loop to the one appropriate for the propagator on the right.

The generalization of the derivation of Eq. (3.11) to all order is very simple. The leading large N diagrams are planar with arbitrary numbers of color and flavor loops. Again, a general planar diagram can be split into a numerical group theory factor which we represent by a double-line diagram times a factor containing all the spin and momentum information. The second factor can be ignored because it drops out in the ratio of daughter to parent. To compute the ‘‘group theory diagram’’ (see Fig. 4 for a typical three-loop example) in the daughter theory, we proceed exactly as before; we rewrite the diagram entirely in terms of parent theory propagators and vertices with projectors. Then we choose one of the internal index loops and shrink it to an effective vertex by calculating the trace for this loop. Note that this calculation is identical to the calculation we just did for the one-loop derivation. We obtain a new diagram with one fewer loop and a new effective vertex with an effective coupling. As before, we find that the effective vertex is the same as what we would have obtained if we had calculated in the parent theory with a coupling constant rescaled by $1/\sqrt{\Gamma}$. We now iterate this procedure by shrinking loop after loop. Each time we find that we get the same answer as in the parent

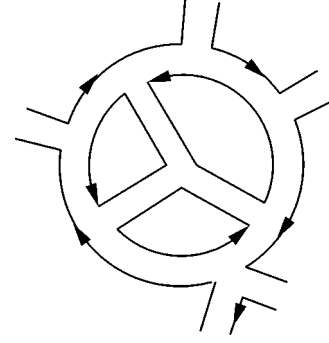


FIG. 4. Typical planar three-loop diagram.

theory with the rescaled coupling. This completes the derivation of Eq. (3.11) to all orders in perturbation theory.

One might worry that the quality of correlation functions of daughter and parent theories, Eq. (3.11), could be spoiled by nonperturbative effects, which would then invalidate the dualities we derive. This worry about nonperturbative effects is not special to our situation, and we take the successes of large N expansions in general as evidence that large N perturbative arguments also apply nonperturbatively. The agreement of the nonperturbative gluino condensate calculation with large N expectations (see Sec. II) corroborates this argument, and we will assume that Eq. (3.11) continues to hold nonperturbatively.

IV. EXAMPLE: CHIRAL $SU(N) \times SU(N) \times SU(N)$

In this section we construct our explicit example of a nonsupersymmetric dual pair. We begin by reviewing Seiberg's electric-magnetic duality. Then we apply the orbifold projection to obtain the field content and interactions of the nonsupersymmetric dual pair. We conclude with a few comments and perform consistency checks on the duality.

A. Parent theory duality

As parent theory, choose $N=1$ supersymmetric QCD with $3N$ colors and $3F$ flavors. The nonanomalous global symmetries of this ‘‘electric’’ theory are summarized in the following table:

	$SU(3N)$	$SU(3F)$	$SU(3F)$	B	R
Q	$\square$	$\bar{\square}$	1	$\frac{1}{N}$	$\frac{F-N}{F}$
$\bar{Q}$	$\bar{\square}$	1	$\square$	$-\frac{1}{N}$	$\frac{F-N}{F}$

(4.1)

where Q and $\bar{Q}$ denote the quark superfields. We will always denote the scalar component of a chiral superfield with the same letter as the superfield. Thus for the squarks we will also use Q and $\bar{Q}$, and we will use Ψ and $\bar{\Psi}$ for the fermionic components of the quark superfields. The vector superfield which contains the gluons A also contains an adjoint of fermions, the gluinos λ .

The two nonanomalous $U(1)$ symmetries can be determined by assigning charges to the three fermion fields $\lambda, \Psi, \bar{\Psi}$ subject to the $SU(3N)^2 U(1)$ anomaly cancellation constraint. The charges of the scalars are then determined by supersymmetry. All the global symmetries except the R symmetry commute with supersymmetry, implying that boson and fermion components of the superfields have identical charges. The R charges of bosons and fermions differ because the superspace coordinate θ transforms with charge one.

Supersymmetric QCD has an IR dual with $3\tilde{N} \equiv 3(F - N)$ dual colors and $3F$ flavors. The fields of this “magnetic” theory transform as

	$SU(3\tilde{N})$	$SU(3F)$	$SU(3F)$	B	R
q	$\bar{\square}$	$\square$	1	$\frac{1}{\tilde{N}}$	$\frac{N}{F}$
$\bar{q}$	$\square$	1	$\bar{\square}$	$-\frac{1}{\tilde{N}}$	$\frac{N}{F}$
m	1	$\bar{\square}$	$\square$	0	$2\frac{F-N}{F}$

(4.2)

Here $q, \bar{q}$ are the dual quark superfields with fermion components $\psi, \bar{\psi}$, and m is the meson superfield; it is related by duality to the composite field QQ of the electric theory. The fermionic components of m will be denoted by χ . The magnetic theory also has a tree level superpotential $W = mqq$.

Two discrete subgroups of the global and gauge symmetries given above will be important for the projection which we perform in the next subsection. Both correspond to continuous $U(1)_R$ symmetries in the classical limit, but are broken to Z_{3N} and $Z_{3(F-N)}$ by instantons in the quantum theory. The charges of the various fields under the two symmetries are given in the following table:

	λ	Q	$\bar{Q}$	Ψ	$\bar{\Psi}$	$\tilde{\lambda}$	q	$\bar{q}$	ψ	$\bar{\psi}$	m	χ
Z_{3N}	1	1	1	0	0	1	0	0	-1	-1	2	1
$Z_{3(F-N)}$	1	0	0	-1	-1	1	1	1	0	0	0	-1

(4.3)

Note that the role of the two symmetries is exchanged under duality.

There are a number of consistency checks which have been performed on this duality. One check is that there exists an operator map between gauge-invariant chiral operators which is consistent with all the global symmetries:

$$\begin{aligned} QQ &\leftrightarrow m, \\ Q^N &\leftrightarrow q^{F-N}, \\ \bar{Q}^N &\leftrightarrow \bar{q}^{F-N}. \end{aligned}$$

The global anomalies of both theories match, both theories have the same moduli space of vacua, and finally the duality is consistent under deformations of the theory.

In the following subsections we perform orbifold projections on both electric and magnetic theories. At leading order in large N , we expect the orbifold theories’ correlators to be given by their supersymmetric parent theories’ modulo the rescaling of the coupling constants. Since the parent theories are dual in the IR, we conclude that the daughters are also dual.

B. Projecting the electric theory

The projection is carried out by identifying a Z_3 subgroup of the global symmetries and then projecting out all the fields which are not Z_3 invariant. As explained in Sec. III, the Z_3 symmetry is taken to act as a multiple of the regular representation on the $SU(3N)$ and $SU(3F)$ indices. In addition, it may also be mixed with any of the finite global symmetries. For example, it could be acting trivially on all global charges; it could be embedded into baryon number B or the R symmetry, or into one of the two discrete symmetries. If the projection involves a subgroup of the R symmetry, fermions and bosons are projected differently and supersymmetry is broken.

I will first describe the action on $SU(3N)$ and $SU(3F)$ and then discuss the different possibilities for the finite global charges. $SU(3N)$ and $SU(3F)$ indices transform in multiples of the regular representation. The regular representation of Z_3 is given by the matrices

$$\left\{ \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega^2 \end{pmatrix}, \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{pmatrix} \right\},$$

$$\omega = e^{2\pi i/3}. \quad (4.4)$$

It suffices to check invariance under the second group element above because it generates the group. We take an $SU(3N)$ index to transform by multiplication with a matrix in an N -fold copy of the regular representation. For the second group element this $3N \times 3N$ matrix is

$$\Omega_{3N} = \begin{pmatrix} \mathbb{1}_N & | & & | \\ \hline - & - & - & - & - \\ & | & \omega \mathbb{1}_N & | & \\ \hline - & - & - & - & - \\ & | & & | & \omega^2 \mathbb{1}_N \end{pmatrix}, \quad (4.5)$$

where $\mathbb{1}_N$ is the $N \times N$ unit matrix. The gluons in the adjoint representation of $SU(3N)$ then transform as

$$A_\mu \rightarrow \Omega_{3N} A_\mu \Omega_{3N}^\dagger. \quad (4.6)$$

It is clear that only the block-diagonal components of A_μ are invariant. Thus the projection breaks the $SU(3N)$ to its $SU(N) \times SU(N) \times SU(N)$ subgroup.

The projection of the matter fields depends on the chosen embedding of the Z_3 into the global symmetries. As a warm-up, we first discuss the case where the only Z_3 transformation comes from the gauge and $SU(3F)$ indices:

In this case supersymmetry will be unbroken. Thus gauginos are projected identically to gauge bosons. The quarks and squarks both transform as $3N, 3F$ under $SU(3N) \times SU(3F)$. Transforming the fundamental color index with Ω_{3N} and the antifundamental flavor index with the $3F \times 3F$ dimensional $Q_{3F}^\dagger$, we find that the invariant fields are $N \times F$ blocks on the diagonal of Q :

$$\begin{pmatrix} Q_1 & | & & | \\ \hline - & - & - & - \\ & | & Q_2 & | \\ \hline - & - & - & - \\ & | & & | \\ & & & Q_3 \end{pmatrix}. \quad (4.7)$$

Similarly, only the block-diagonal components of $\bar{Q}$ survive the projection. The interactions of the daughter theory are obtained from the parent theory by simply dropping any interaction terms involving non- Z_3 invariant fields.

The result is three completely decoupled identical copies of supersymmetric QCD with gauge groups $SU(N)$ and fla-

vor groups $SU(F) \times SU(F)$. It is clear that we will not gain any new insight into the dynamics of supersymmetric QCD by studying this projection since it just relates supersymmetric QCD to itself modulo a rescaling of the number of colors and flavors.

We will show in the next subsection that the dual theory is also simply projected down to three independent copies of $SU(F-N)$ gauge theory. Thus we see that the projection commutes with duality in this case.

Let us now discuss a nontrivial projection which breaks supersymmetry. We embed the Z_3 in the discrete Z_{3N} R symmetry. In addition to the transformation from the regular representation, each field is multiplied by ω to the power of its Z_{3N} charge [see Eq. (4.3)]. For example, the gauginos transform with an additional factor of ω compared to the gauge bosons $\lambda \rightarrow \omega \Omega_{3N} \lambda \Omega_{3N}^\dagger$. The invariant gauginos in $N \times N$ block notation are

$$\begin{pmatrix} & \lambda_1 & \\ & & \lambda_2 \\ \lambda_3 & & \end{pmatrix}. \quad (4.8)$$

Squarks and quarks transform as $Q \rightarrow \omega \Omega_{3N} Q \Omega_{3F}^\dagger$ and $\Psi \rightarrow \Omega_{3N}^\dagger \Psi \Omega_{3F}$. All matter fields which survive the projection are given in the following table.

	$SU(N)$	$SU(N)$	$SU(N)$	$SU(F)$	$SU(F)$	$SU(F)$	$SU(F)$	$SU(F)$	$SU(F)$	B	R
λ_1	$\square$	$\bar{\square}$								0	1
λ_2		$\square$	$\bar{\square}$							0	1
λ_3	$\bar{\square}$		$\square$							0	1
Q_1	$\square$				$\bar{\square}$					$\frac{1}{N}$	$\frac{F-N}{F}$
Q_2		$\square$				$\bar{\square}$				$\frac{1}{N}$	$\frac{F-N}{F}$
Q_3			$\square$	$\bar{\square}$						$\frac{1}{N}$	$\frac{F-N}{F}$
$\bar{Q}_1$	$\bar{\square}$								$\square$	$-\frac{1}{N}$	$\frac{F-N}{F}$
$\bar{Q}_2$		$\bar{\square}$					$\square$			$-\frac{1}{N}$	$\frac{F-N}{F}$
$\bar{Q}_3$			$\bar{\square}$					$\square$		$-\frac{1}{N}$	$\frac{F-N}{F}$
Ψ_1										$\frac{1}{N}$	$-\frac{N}{F}$
Ψ_1	$\square$			$\bar{\square}$						$\frac{1}{N}$	$-\frac{N}{F}$
Ψ_2		$\square$			$\bar{\square}$					$\frac{1}{N}$	$-\frac{N}{F}$
Ψ_3			$\square$			$\bar{\square}$				$\frac{1}{N}$	$-\frac{N}{F}$

$$\begin{array}{c|c} \bar{\Psi}_1 & \bar{\square} \\ \bar{\Psi}_2 & \bar{\square} \\ \bar{\Psi}_3 & \bar{\square} \end{array} \quad \left| \quad \begin{array}{c} \square \\ \square \\ \square \end{array} \right.$$

The Lagrangian of the orbifolded theory contains the usual kinetic terms for all the fields with gauge covariant derivatives. In addition, there are Yukawa and scalar couplings of the form

$$\mathcal{L} \sim \sum_i \left[Q_i^\dagger \lambda_i \Psi_{i+1} + \bar{Q}_i^\dagger \lambda_{i-1} \bar{\Psi}_{i-1} + \sum_a \left| Q_i^- t^a Q_i - \bar{Q}_i^\dagger t^a \bar{Q}_i \right|^2 \right], \quad (4.9)$$

where t^a are generators of the $SU(N)$ algebra; gauge and flavor indices are traced over. In canonical normalization for all fields, the scalar couplings are equal to g^2 and gauge and Yukawa couplings are g .

C. Projecting the magnetic theory and duality

To orbifold the magnetic theory, we again assign gauge indices to transform in the regular representation.⁶ In order to decide how to treat global indices, we need to copy exactly what we did in the electric theory. The magnetic theory has the same global symmetries as the electric theory, which uniquely fixes the projection.

Performing the projection as described in the previous

$$\begin{array}{c|c} \square & -\frac{1}{N} \quad -\frac{N}{F} \\ \square & -\frac{1}{N} \quad -\frac{N}{F} \\ \square & -\frac{1}{N} \quad -\frac{N}{F} \end{array}$$

subsection, we find—not very surprisingly—that the dual gauge group is projected as

$$SU(3\tilde{N}) \rightarrow SU(\tilde{N}) \times SU(\tilde{N}) \times SU(\tilde{N}).$$

The projection on gluinos and quarks depends on our choice of embedding for $U(1)_R$. Recall the warm-up example where we embedded the $\mathbb{Z}_3$ trivially into $U(1)_R$. Doing the same in the dual also preserves supersymmetry; thus, we have gluinos in the same representations as gluons and dual quarks are projected similarly to electric quarks, Eq. (4.7). The new ingredient is the meson field m : it also simply gets projected to three blocks on the diagonal. Thus we find three decoupled copies of $SU(\tilde{N})$ theories with dual quarks and meson fields for each. This is obviously the dual of three copies of supersymmetric QCD with gauge group $SU(N)$. Thus we find no surprises; the orbifold projection applied to the electric-magnetic $SU(3N) \leftrightarrow SU(3(F-N))$ dual pair yielded 3 times an $SU(N) \rightarrow SU(F-N)$ pair.

Much more interesting is the nontrivial projection discussed in the electric theory. Again, the dual gauge group gets projected to $SU(\tilde{N}) \times SU(\tilde{N}) \times SU(\tilde{N})$. The projection of the matter fields is also straightforward using the charge assignments given in Eqs. (4.2) and (4.3):

	$SU(N)$	$SU(\tilde{N})$	$SU(\tilde{N})$	$SU(F)$	$SU(F)$	$SU(F)$	$SU(F)$	$SU(F)$	$SU(F)$	B	R
$\tilde{\lambda}_1$	$\square$	$\bar{\square}$								0	1
$\tilde{\lambda}_2$		$\square$	$\bar{\square}$							0	1
$\tilde{\lambda}_3$	$\bar{\square}$		$\square$							0	1
q_1	$\bar{\square}$			$\square$						$\frac{1}{\tilde{N}}$	$\frac{N}{F}$
q_2		$\bar{\square}$			$\square$					$\frac{1}{\tilde{N}}$	$\frac{N}{F}$
q_3			$\bar{\square}$			$\square$				$\frac{1}{\tilde{N}}$	$\frac{N}{F}$
$\bar{q}_1$	$\square$						$\bar{\square}$			$-\frac{1}{\tilde{N}}$	$\frac{N}{F}$
$\bar{q}_2$		$\square$						$\bar{\square}$		$-\frac{1}{\tilde{N}}$	$\frac{N}{F}$
$\bar{q}_3$			$\square$						$\bar{\square}$	$-\frac{1}{\tilde{N}}$	$\frac{N}{F}$

⁶Note that since the regular representation is real, it does not matter whether we assign fundamental or antifundamental indices to transform in the regular representation.

ψ_1	$\bar{\square}$		$\square$		$\frac{1}{\tilde{N}}$	$\frac{N-F}{F}$
ψ_2		$\bar{\square}$		$\square$	$\frac{1}{\tilde{N}}$	$\frac{N-F}{F}$
ψ_3			$\bar{\square}$	$\square$	$\frac{1}{\tilde{N}}$	$\frac{N-F}{F}$
$\bar{\psi}_1$	$\square$				$-\frac{1}{\tilde{N}}$	$\frac{N-F}{F}$
$\bar{\psi}_2$		$\square$			$-\frac{1}{\tilde{N}}$	$\frac{N-F}{F}$
$\bar{\psi}_3$			$\square$		$-\frac{1}{\tilde{N}}$	$\frac{N-F}{F}$
m_1			$\bar{\square}$		0	$2\frac{F-N}{F}$
m_2				$\bar{\square}$	0	$2\frac{F-N}{F}$
m_3			$\bar{\square}$		0	$2\frac{F-N}{F}$
χ_1				$\bar{\square}$	0	$1-2\frac{N}{F}$
χ_2			$\bar{\square}$		0	$1-2\frac{N}{F}$
χ_3			$\bar{\square}$	$\square$	0	$1-2\frac{N}{F}$

This theory has a slightly more complicated Lagrangian than the electric theory due to the remains of the superpotential term. We find

$$\begin{aligned} \mathcal{L} \sim \sum_i \left[q_i^\dagger \tilde{\lambda}_{i-1} \psi_{i-1} + \bar{q}_i^\dagger \tilde{\lambda}_i \bar{\psi}_{i+1} + \sum_a \left| q_i^\dagger t^a q_i - \bar{q}_i^\dagger t^a \bar{q}_i \right|^2 \right. \\ \left. + \psi_i m_i \bar{\psi}_i + q_i \chi_{i+2} \bar{\psi}_i + \psi_i \chi_{i-1} \bar{q}_i + |q_i \bar{q}_i|^2 + |q_i m_{i-1}|^2 \right. \\ \left. + |\bar{q}_i m_{i+1}|^2 \right]. \end{aligned} \quad (4.10)$$

Note that this dual is weakly coupled in the infrared for $F \leq \frac{3}{2}N$. Completely analogous to the parent theory duality, this duality also exchanges strong and weak coupling.

We postpone performing a full set of consistency checks on our proposed duality to future work. Here we just give the map of gauge invariants containing only scalars:

$$Q_i \bar{Q}_i \rightarrow m_i,$$

$$Q_i^N \rightarrow q_{i+1}^{F-N},$$

$$\bar{Q}_i^N \rightarrow \bar{q}_{i-1}^{F-N}.$$

This map is necessary to perform a comparison of the flat directions in the two theories.

It is straightforward to compare the anomalies in both electric and magnetic theories. All the anomalies for the global symmetries given in the tables agree. This also follows from our derivation of the dual pair as follows: the anomalies mentioned above are all calculated with a planar triangle diagram with global currents at the vertices. According to the general arguments presented in the previous section, these diagrams are related to the corresponding anomaly diagrams in the parent theories (supersymmetric QCD and its dual) via rescaling by a factor of $\Gamma=3$. Since the anomalies matched in the electric and magnetic descriptions of supersymmetric QCD and since the global group of the daughter theories is a subgroup of the global symmetries of the parents, the anomalies must match here as well. Note that this derivation makes it clear that the anomalies also match for finite N , which might be of significance for attempts to continue the proposed duality to finite N .

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