Unconstrained Hamiltonian formulation of $SU(2)$ gluodynamics

A. M. Khvedelidze*
Bogoliubov Theoretical Laboratory, Joint Institute for Nuclear Research, Dubna, Russia

H.-P. Pavel
Fachbereich Physik der Universität Rostock, D-18051 Rostock, Germany

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$SU(2)$ Yang-Mills field theory is considered in the framework of the generalized Hamiltonian approach and the equivalent unconstrained system is obtained using the method of Hamiltonian reduction. A canonical transformation to a set of adapted coordinates is performed in terms of which the Abelianization of the Gauss law constraints is trivialized and the pure gauge degrees of freedom drop out from the Hamiltonian after projection onto the constraint shell. For the remaining gauge invariant fields two representations are introduced where the three fields which transform as scalars under spatial rotations are separated from the three rotational fields. An effective low energy nonlinear sigma model type Lagrangian is derived which out of the six physical fields involves only one of the three scalar fields and two rotational fields summarized in a unit vector. Its possible relation to the effective Lagrangian proposed recently by Faddeev and Niemi is discussed. Finally the unconstrained analog of the well-known nonnormalizable ground state wave functional which solves the Schrödinger equation with zero energy is given and analyzed in the strong coupling limit.
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I. INTRODUCTION

One of the main issues in the Hamiltonian formulation of Yang-Mills theories is to find the projection from the phase space of canonical variables constrained by the non-Abelian Gauss law to the ''smaller'' phase space of unconstrained gauge invariant coordinates only. Dealing with the problem of the elimination of the pure gauge degrees of freedom two approaches exist, the perturbative and the nonperturbative one, with complementary features. The conventional perturbative gauge fixing method works successfully for the description of high energy phenomena, but fails in applications in the infrared region. The correct nonperturbative reduction of gauge theories [1–15], on the other hand, leads to representations for the unconstrained Yang-Mills systems, which are valid also in the low energy region, but unfortunately up to now have been rather complicated for practical calculations. The guideline of these investigations is the search for a representation of the gauge invariant variables which are suitable for the description of the infrared limit of Yang-Mills theory. To get such a representation for the unconstrained system we are following the Dirac generalized Hamiltonian formalism [16–18] using the method of Hamiltonian reduction ([19–21] and references therein) instead of the conventional gauge fixing approach [22]. In previous work [23] it was demonstrated that for the case of the mechanics of spatially constant $SU(2)$ Dirac-Yang-Mills fields an unconstrained Hamiltonian can be derived which has a simple practical form. The elimination of the gauge degrees of freedom has been achieved by performing a canonical transformation to new adapted coordinates, in terms of which the Abelianization of the Gauss law constraints is trivialized, and then carrying out the projection onto the constraint shell. The obtained unconstrained system then describes the dynamics of a symmetric second rank tensor under spatial rotations. The main-axis-transformation of this symmetric tensor allowed us to separate the gauge invariant variables into scalars under ordinary space rotations and into ''rotational'' degrees of freedom. In this final form the physical Hamiltonian can be quantized without operator ordering ambiguities.

In this work we shall generalize our approach from non-Abelian Dirac-Yang-Mills mechanics [23] to field theory. We shall give a Hamiltonian formulation of classical $SU(2)$ Yang-Mills field theory entirely in terms of gauge invariant variables, and separate these into scalars under ordinary space rotations and into ''rotational'' degrees of freedom. It will be shown that this naturally leads to their identification as fields with ''nonrelativistic spin-2 and spin-0.'' Furthermore the separation into scalar and rotational degrees of freedom will turn out to be very well suited for the study of the infrared limit of unconstrained Yang-Mills theory. We shall obtain an effective low energy theory involving only two of the three rotational fields and one of the three scalar fields, and shall discuss its possible relation to the effective soliton Lagrangian proposed recently in [24]. Finally we shall analyze the well-known exact, but nonnormalizable, solution [25] of the functional Schrödinger equation with zero energy in the framework of the unconstrained formulation of $SU(2)$ Yang-Mills theory.

The outline of the article is as follows. In Sec. II we present the Hamiltonian reduction of $SU(2)$ Yang-Mills field theory. We perform the canonical transformation to a new set of adapted coordinates, Abelianize the Gauss law constraints, and achieve the unconstrained description for $SU(2)$ Yang-Mills theory. In Sec. III two representations for the physical field in terms scalars and rotational degrees are described. Section IV is devoted to the study of the infrared

*Permanent address: Tbilisi Mathematical Institute, 380093, Tbilisi, Georgia.

limit of unconstrained gluodynamics. In Sec. V the well-known nonnormalizable solution of the functional Schrödinger equation with zero energy is analyzed in our unconstrained formulation of $SU(2)$ Yang-Mills theory. Finally, in Sec. VI, we give our conclusions. In the Appendix we list several formulas for nonrelativistic spin-0, spin-1 and spin-2 used in the text.

II. REDUCTION OF GAUGE DEGREES OF FREEDOM

The degenerate character of the conventional Yang-Mills action for $SU(2)$ gauge fields $A_\mu^a(x)$

$$\mathcal{S}[A]:=-\frac{1}{4}\int d^4x F^a_{\mu\nu}F^{a\mu\nu},$$

$$F^a_{\mu\nu}:=\partial_\mu A^a_\nu-\partial_\nu A^a_\mu+g\epsilon^{abc}A^b_\mu A^c_\nu, \tag{2.1}$$

requires the use of the generalized Hamiltonian approach [16]. From the definition of the canonical momenta $P_0^a := \partial L/\partial(\partial_0 A_0^a)$, $E_{ai}:=\partial L/\partial(\partial_0 A_{ai})$ it follows, that the phase space spanned by the variables (A_0^a,P_0^a), (A_{ai},E_{ai}) is restricted by the three primary constraints $P_0^a(x)=0$. According to the Dirac procedure in this case the evolution of the system is governed by the total Hamiltonian containing three arbitrary functions $\lambda_a(x)$:

$$H_T:=\int d^3x\Bigg[\frac{1}{2}(E^2_{ai}+B^2_{ai}(A)) - A_0^a(\partial_i E_{ai}+g\epsilon_{abc}A_{bi}E_{ci})+\lambda_a(x)P_0^a(x)\Bigg], \tag{2.2}$$

where $B_{ai}(A):=\epsilon_{ijk}(\partial_j A_{ak}+\frac{1}{2}g\epsilon_{abc}A_{bj}A_{ck})$ is the non-Abelian magnetic field. From the conservation of the primary constraints $P_0^a=0$ in time one obtains the non-Abelian Gauss law constraints

$$\Phi_a:=\partial_i E_{ai}+g\epsilon_{abc}A_{bi}E_{ci}=0. \tag{2.3}$$

Although the total Hamiltonian (2.2) depends on the arbitrary functions $\lambda_a(x)$ it is possible to extract the dynamical variables which have uniquely predictable dynamics. Furthermore they can be chosen to be free of any constraints. Such an extracted system with predictable dynamics without constraints is called unconstrained.

The non-Abelian character of the secondary constraints,

$$\{\Phi_a(x),\Phi_b(y)\}=g\epsilon_{abc}\Phi_c(x)\delta(x-y), \tag{2.4}$$

is the main obstacle for the corresponding projection to the unconstrained phase space. For Abelian constraints Ψ_α $(\{\Psi_\alpha,\Psi_\beta\}=0)$ the projection to the reduced phase space can be simply achieved in the following two steps. One performs a canonical transformation to new variables such that part of the new momenta P_α coincide with the constraints Ψ_α. After the projection onto the constraint shell, i.e. putting in all expressions $P_\alpha=0$, the coordinates canonically conjugate to the P_α drop out from the physical quantities. The remaining canonical pairs are then gauge invariant and form the basis for the unconstrained system. For the case of non-Abelian constraints (2.4) it is clearly impossible to find such a canonical basis only via canonical transformation. The way to avoid this difficulty is to replace the set of non-Abelian constraints (2.4) by a new set of Abelian constraints which describe the constraint surface in phase space. This Abelianization procedure reduces the problem to the Abelian case. There are several methods of Abelianization of constraints (see e.g. [20,21] and references therein).

A. Canonical transformation and Abelianization of the Gauss law constraints

The problem of Abelianization is considerably simplified when studied in terms of coordinates adapted to the action of the gauge group. The knowledge of the $SU(2)$ gauge transformations U for the gauge potentials $A_\mu:=A_\mu^a\tau_a/2$ (τ_a Pauli matrices)

$$A_\mu\to A'_\mu=U(x)\left(A_\mu+\frac{i}{g}\partial_\mu\right)U^{-1}(x), \tag{2.5}$$

which leave the Yang-Mills action (2.1) invariant, directly promts us with the choice of adapted coordinates by using the following point transformation to the new set of Lagrangian coordinates q_j $(j=1,2,3)$ and the six elements $Q_{ik}=Q_{ki}$ $(i,k=1,2,3)$ of the positive definite symmetric 3×3 matrix Q:

$$A_{ai}(q,Q):=O_{ak}(q)Q_{ki}-\frac{1}{2g}\epsilon_{abc}(O(q)\partial_i O^T(q))_{bc}, \tag{2.6}$$

where $O(q)$is an orthogonal 3×3 matrix parametrized by the q_i.[1] In the following we shall show that in terms of these variables the non-Abelian Gauss law constraints (2.3) only depend on the q_i and their conjugate momenta p_i and after Abelianization become $p_i=0$. The unconstrained variables Q and their conjugate P are gauge invariant, i.e. commute with the Gauss law, and represent the basic variables for all observable quantities.[2] The transformation (2.6) induces a point

[1]In the strong coupling limit the representation (2.6) reduces to the so-called polar representation for arbitrary quadratic matrices for which the decomposition can be proven to be well-defined and unique (see for example [26]). In the general case we have the additional second term which takes into account the inhomogeneity of the gauge transformation and Eq. (2.6) has to be regarded as a set of partial differential equations for the q_i variables. The uniqueness and regularity of the suggested transformation (2.6) depends on the boundary conditions imposed. In the present work the uniqueness and regularity of the change of coordinates is assumed as a reasonable conjecture without search for the appropriate boundary conditions.

[2]The freedom to use other canonical variables in the unconstrained phase space corresponds to another fixation of the six variables Q in the representation (2.6). This observation clarifies the connection with the conventional gauge fixing method. We shall discuss this point in forthcoming publications (see also Ref. [5]).

canonical transformation linear in the new canonical momenta P_{ik} and p_i. Using the corresponding generating functional depending on the old momenta and the new coordinates,

$$F_3[E;q,Q]:=\int d^3z E_{ai}(z) A_{ai}(q(z),Q(z)), \qquad (2.7)$$

one can obtain the new canonical momenta p_i and P_{ik}

$$p_j(x):=\frac{\delta F_3}{\delta q_j(x)}=-\frac{1}{g}\Omega_{jr}(D_i(Q)O^T E)_{ri}, \qquad (2.8)$$

$$P_{ik}(x):=\frac{\delta F_3}{\delta Q_{ik}(x)}=\frac{1}{2}(E^T O+O^T E)_{ik}. \qquad (2.9)$$

Here

$$\Omega_{ji}(q):=\frac{i}{2}\mathrm{Tr}\left(O^T(q)\frac{\partial O(q)}{\partial q_j}J_i\right), \qquad (2.10)$$

with the 3×3 matrix generators of $SO(3)$, $(J_i)_{mn}:=i\epsilon_{min}$, and the corresponding covariant derivative $D_i(Q)$ in the adjoint representation

$$(D_i(Q))_{mn}:=\delta_{mn}\partial_i - ig(J_k)_{mn}Q_{ki}. \qquad (2.11)$$

A straightforward calculation based on the linear relations (2.8) and (2.9) between the old and the new momenta leads to the following expression for the field strengths E_{ai} in terms of the new canonical variables

$$E_{ai}=O_{ak}(q)[P_{ki}+g\epsilon_{kis}{}^*D^{-1}_{sl}(Q)[\mathcal{S}_l-(\Omega^{-1}p)_l]]. \qquad (2.12)$$

Here ${}^*D^{-1}$ is the inverse of the matrix operator

$${}^*D_{ik}(Q):=-i(D_m(Q)J_m)_{ik}, \qquad (2.13)$$

and

$$\mathcal{S}_k(x):=\epsilon_{klm}(PQ)_{lm}-\frac{1}{g}\partial_l P_{kl}. \qquad (2.14)$$

Using the representations (2.6) and (2.12) one can easily convince oneself that the variables Q and P make no contribution to the Gauss law constraints (2.3)

$$\Phi_a=-gO_{as}(q)\Omega^{-1}_{sj}(q)p_j=0. \qquad (2.15)$$

Here and in Eq. (2.12) we assume that the matrix Ω is invertible. The equivalent set of Abelian constraints is

$$p_a=0. \qquad (2.16)$$

They are Abelian due to the canonical nature of the new variables.

B. The Hamiltonian in terms of unconstrained fields

After having rewritten the model in terms of the new canonical coordinates and after the Abelianization of the Gauss law constraints, the construction of the unconstrained Hamiltonian system is straightforward. In all expressions we can simply put $p_a=0$. In particular, the Hamiltonian in terms of the unconstrained canonical variables Q and P can be represented by the sum of three terms

$$H[Q,P]=\frac{1}{2}\int d^3x\left[\mathrm{Tr}(P)^2+\mathrm{Tr}(B^2(Q))+\frac{1}{2}\vec{E}^2(Q,P)\right]. \qquad (2.17)$$

The first term is the conventional quadratic ''kinetic'' part and the second the ''magnetic potential'' term which is the trace of the square of the non-Abelian magnetic field

$$B_{sk}:=\epsilon_{klm}\left(\partial_l Q_{sm}+\frac{g}{2}\epsilon_{sbc}Q_{bl}Q_{cm}\right). \qquad (2.18)$$

It is intersting that after the elimination of the pure gauge degrees of freedom the magnetic field strength tensor is the commutator of the covariant derivatives (2.11) $F_{ij}=[D_i(Q),D_j(Q)]$.

The third, nonlocal term in the Hamiltonian (2.17) is the square of the antisymmetric part of the electric field (2.12), $E_s:=(1/2)\epsilon_{sij}E_{ij}$, after projection onto the constraint surface. It is given as the solution of the system of differential equations[3]

$${}^*D_{ls}(Q)E_s=g\mathcal{S}_l, \qquad (2.19)$$

with the derivative ${}^*D_{ls}(Q)$ defined in Eq. (2.13). Note that the vector $\mathcal{S}_i(x)$, defined in Eq. (2.14), coincides up to divergence terms with the spin density part of the Noetherian angular momentum, $S_i(x):=\epsilon_{ijk}A_{aj}E_{ak}$, after transformation to the new variables and projection onto the constraint shell.[4] The solution $\vec{E}$ of the differential equation (2.19) can be expanded in a $1/g$ series $E_s=\Sigma_{n=0}^{\infty}E_s^{(n)}$. The zeroth order term is

$$E_s^{(0)}=\gamma^{-1}_{sk}\epsilon_{klm}(PQ)_{lm}, \qquad (2.21)$$

with $\gamma_{ik}:=Q_{ik}-\delta_{ik}\ \mathrm{Tr}(Q)$, and the first order term is determined as

$$E_s^{(1)}:=\frac{1}{g}\gamma^{-1}_{sl}[(\mathrm{rot}\vec{E}^{(0)})_l-\partial_k P_{kl}] \qquad (2.22)$$

from the zeroth order term. The higher terms are then obtained by the simple recurrence relations

[3]We remark that for the solution of this equation we need to impose boundary conditions only on the physical variables Q, in contrast to Eq. (2.6) for which boundary conditions only for the unphysical variables q_i are needed.

[4]Note that the presence of this divergence term destroys the $so(3)$ algebra of densities due to the presence of Schwinger terms

$$\{\mathcal{S}_i(x),\mathcal{S}_j(y)\}=\epsilon_{ijk}\mathcal{S}_k(x)\delta(x-y)+\epsilon_{ijs}P_{sk}(x)\partial_k^x\delta(x-y), \qquad (2.20)$$

but maintains the value of spin and its algebra if one neglects the surface terms.

$$E_s^{(n+1)} := \frac{1}{g}\gamma_{sl}^{-1}(\mathrm{rot}\vec{E}^{(n)})_l . \tag{2.23}$$

One easily recognizes in these expressions the conventional definition of the covariant curl operation [27] in terms of the covariant derivative

$$\mathrm{curl}\ \ S(e_i, e_j) := \langle \nabla_{e_i} S, e_j\rangle - \langle \nabla_{e_j} S, e_i\rangle,$$

calculated in the basis $e_i := (\gamma^{1/2})_{ij}\partial_j$ and $\gamma_{ij} := \langle e_i, e_j\rangle$ with the corresponding connection $\nabla_{e_i} e_j = \Gamma^l_{ij} e_l$, e.g.

$$E^{(1)}_{ij} = \mathrm{curl}\ \ S(e_i, e_j). \tag{2.24}$$

III. THE UNCONSTRAINED HAMILTONIAN IN TERMS OF SCALAR AND ROTATIONAL DEGREES OF FREEDOM

In the previous section we have obtained the unconstrained Hamiltonian system in terms of physical fields represented by a positive definite symmetric matrix Q. The initial gauge fields A_i transformed as vectors under spatial rotations. We now would like to study the transformation properties of the corresponding reduced matrix field Q. For systems possessing some rigid symmetry it is well known to be very useful for practical calculations to pass to a coordinate basis such that a subset of the variables is invariant under the action of the symmetry group. In this section we shall therefore carry out the explicit separation of the rotational degrees of freedom, which vary under rotations, from the scalars.

A. Transformation properties of the unconstrained fields under space rotations

In order to search for a parametrization of the unconstrained variables in Yang-Mills theory adapted to the action of the group of spatial rotations we shall study the corresponding transformation properties of the field Q. The total Noetherian angular momentum vector for $SU(2)$ gluodynamics is

$$I_i = \epsilon_{ijk}\int d^3x\left(E_{aj}A_{ak} + x_k E_{al}\frac{\partial A_{al}}{\partial x^j}\right). \tag{3.1}$$

After elimination of the gauge degrees of freedom it reduces to

$$I_i = \int d^3x\,\epsilon_{ijk}((PQ)_{jk} + x_k\ \mathrm{Tr}(P\partial_j Q)), \tag{3.2}$$

where surface terms have been neglected.

Under infinitisimal rotations in 3-dimensional space, $\delta x_i = \omega_{ij}x_j$, generated by formula (3.2), the physical field Q transforms as

$$\delta_\omega Q_{ij} = \epsilon_{smn}\omega_{mn}\{Q_{ij}, I_s\} = \omega_{mn}(S^{mn}Q)_{ij} + \text{orbital part transf.} \tag{3.3}$$

with the matrices

$$(S)^{mn}_{(il)(sj)} := (\delta_{il}\delta^m_j\delta^n_s + \delta^m_i\delta^n_l\delta_{sj}) - (m \leftrightarrow n), \tag{3.4}$$

which describe the $SO(3)$ rotations of a 3-dimensional second rank tensor field

$$Q'_{ik} = R_{il}(\omega)R_{km}(\omega)Q_{lm}. \tag{3.5}$$

It is well known that any symmetric second rank tensor can be decomposed into its irreducible components, one spin-0 and the five components of a spin-2 field by extraction of its trace [28]. On the other hand it can be diagonalized via a main-axis-transformation, which corresponds to a separation of the diagonal fields, which are invariant under rotations, from the rotational degrees of freedom. In the following paragraphs we shall investigate both representations and their relation to each other.

B. The unconstrained Hamiltonian in terms of spin-2 and spin-0 fields

As shown in the preceeding paragraph the six independent elements of the matrix field Q can be represented as a mixture of fields with nonrelativistic spin-2 and spin-0. In order to put the theory into a more transparent form explicitly showing its rotational invariance, it is useful to perform a canonical transformation to the corresponding spin-2 and spin-0 fields as new variables. To achieve this let us decompose the symmetric matrix Q into the irreducible representations of the $SO(3)$ group

$$Q_{ij}(x) = \frac{1}{\sqrt{2}}Y_A(x)T^A_{ij} + \frac{1}{\sqrt{3}}\Phi(x)I_{ij}, \tag{3.6}$$

with the field Φ proportional to the trace of Q as spin-0 field and the 5-dimensional spin-2 vector $\mathbf{Y}(x)$ with components Y_A labeled by its value of spin along the z- axis, $A = \pm 2, \pm 1, 0$.[5] I is the 3×3 unit matrix and the five traceless 3×3 basis matrices $\mathbf{T}_A$ are listed in the Appendix.

The momenta $P_A(x)$ and $P_\Phi(x)$ canonical conjugate to the fields $Y_A(x)$ and $\Phi(x)$ are the components of the corresponding expansion for the P variable

$$P_{ij}(x) = \frac{1}{\sqrt{2}}P_A(x)T^A_{ij} + \frac{1}{\sqrt{3}}P_\Phi(x)I_{ij}. \tag{3.7}$$

For the magnetic field B we obtain the expansion

[5] Everywhere in the article 3-dimensional vectors are topped by an arrow and their Cartesian and spherical components are labeled by small Latin and Greek letters respectively, while the 5-dimensional spin-2 vectors are written in boldface and their ''spherical'' components labeled by capital Latin letters. For the lowering and raising of the indices of 5-dimensional vectors the metric tensor $\eta_{AB} = (-1)^A\delta_{A,-B}$ is used.

$$B_{ij}(x)=\frac{1}{\sqrt{2}}H_A(x)T^A_{ij}+\frac{1}{\sqrt{2}}h_\alpha(x)J^\alpha_{ij}+\frac{1}{\sqrt{3}}b(x)I_{ij}, \tag{3.8}$$

with the components

$$H_A:=\frac{1}{2}c^{(2)}_{AB\gamma}\partial^\gamma Y^B+\frac{g}{\sqrt{3}}\left(\frac{1}{\sqrt{2}}{}^*Y_A-\Phi Y_A\right), \tag{3.9}$$

$$h_\alpha:=\frac{1}{2}d^{(1)}_{\alpha\beta C}\partial^\beta Y^C+\sqrt{\frac{2}{3}}\partial_\alpha\Phi, \tag{3.10}$$

$$b:=\frac{g}{\sqrt{3}}\left(\frac{1}{2}\mathbf{Y}^2-\Phi^2\right), \tag{3.11}$$

in terms of the structure constants $c^{(2)}_{AB\gamma}$ and $d^{(1)}_{\alpha\beta C}$ of the algebra of the spin-1 matrices J_α ($\alpha=\pm 1,0$) and the spin-2 matrices $\mathbf{T}_A$, listed in the Appendix, and another five-dimensional vector

$${}^*Y_A:=d^{(2)}_{ABC}Y^BY^C, \tag{3.12}$$

with constants $d^{(2)}_{ABC}$ given explicitly in the Appendix. Finally we obtain the reduced Hamiltonian in terms of spin-2 and spin-0 field components

$$H[\mathbf{P},\mathbf{Y},P_\Phi,\Phi]:=\frac{1}{2}\int d^3x(\mathbf{P}^2(x)+\vec{E}^2(x)+P^2_\Phi(x)+\mathbf{H}^2(x)+\vec{h}^2(x)+b^2(x)), \tag{3.13}$$

with expressions (3.9) for the magnetic field components and the antisymmetric part $\vec{E}$ of the electric field given by Eqs. (2.21) – (2.23), expressing Q and P in terms of $\mathbf{Y}$, Φ and $\mathbf{P}$, P_Φ via Eqs. (3.6) and (3.7). In order to discuss the transformation properties of the spin-2 fields $\mathbf{Y}$ under spatial rotations we rewrite the angular momentum vector (3.2) in terms of the fields $\mathbf{Y},\mathbf{P}$ and Φ,P_Φ

$$I_i=S_i+\epsilon_{ijk}\int d^3x\ \ x_j(P_\Phi\partial_k\Phi+P_A\partial_kY^A), \tag{3.14}$$

with the spin part

$$S_i=i(\mathbf{J}_i)_A{}^BY^AP_B. \tag{3.15}$$

Here the three 5×5 matrices $\mathbf{J}_i$ are the elements of the $so(3)$ algebra. They are shown explicitly in the Appendix. The I_i generate the transformation of the 5-dimensional vector $\mathbf{Y}$ under infinitisimal rotations in 3-dimensional space $\delta x_i=\epsilon_{ijk}\omega_kx_j$

$$\delta_\omega Y_A=\omega_k\{Y_A,S_k\}=-i\omega(\mathbf{J}_k)_A{}^BY_B. \tag{3.16}$$

For finite spatial rotations $R(\omega)$ we therefore have

$$Y'_A=D_{AB}(\omega)Y_B, \tag{3.17}$$

with the well-known 5-dimensional spin-2 D-functions [28] related to the 3×3 orthogonal matrix $R(\omega)$ via

$$D_{AB}(\omega)=\tfrac{1}{2}\mathrm{Tr}(R(\omega)\mathbf{T}_AR^T(\omega)\mathbf{T}_B). \tag{3.18}$$

The transformation rule (3.17) is in accordance with Eq. (3.5).

Note that for a complete investigation of the transformation properties of the reduced matrix field Q under the whole Poincaré group one should also include the Lorentz transformations. But we shall limit ourselves here to the isolation of the scalars under spatial rotations and can treat Q in terms of ''nonrelativistic spin-0 and spin-2 fields'' in accordance with the conclusions obtained in the work [3]. The study of the nonlinear representations of the whole Poincaré group in terms of the unconstrained variables will be the subject of further investigation.

C. Separation of scalar and rotational degrees of freedom

In this paragraph we would like to introduce a parametrization of the 5-dimensional $\mathbf{Y}$ field in terms of three Euler angles and two variables which are invariant under spatial rotations. The transformation property (3.17) prompts us with the parametrization

$$Y_A(x)=D_{AB}(\chi(x))M^B(x), \tag{3.19}$$

in terms of the three Euler angles $\chi_i=(\phi,\theta,\psi)$ and some 5-vector $\mathbf{M}$. The special choice

$$\mathbf{M}(\rho,\alpha)=\rho\left(-\frac{1}{\sqrt{2}}\sin\alpha,0,\cos\alpha,0,-\frac{1}{\sqrt{2}}\sin\alpha\right) \tag{3.20}$$

corresponds to the main-axis-transformation of the original symmetric 3×3 matrix field $Q(x)$,

$$Q(x)=R^T(\chi(x))Q_{\mathrm{diag}}(\phi_1(x),\phi_2(x),\phi_3(x))R(\chi(x)), \tag{3.21}$$

with the $D(\chi)$ related to $R(\chi)$ via Eq. (3.18) and the rotational invariant variables Φ,ρ,α related to the diagonal elements ϕ_i via[6]

$$\phi_1:=\frac{1}{\sqrt{3}}\Phi+\sqrt{\frac{2}{3}}\rho\cos\left(\alpha+\frac{2\pi}{3}\right),$$

$$\phi_2:=\frac{1}{\sqrt{3}}\Phi+\sqrt{\frac{2}{3}}\rho\cos\left(\alpha+\frac{4\pi}{3}\right),$$

[6]Similar variables have been used as density and deformation variables in the collective model of Bohr in nuclear physics [29] and as a parametrization for the square of the eigenvalues of the rotational invariant part of the gauge field by [30] in the representation proposed in [8].

$$\phi_3 := \frac{1}{\sqrt{3}}\Phi + \sqrt{\frac{2}{3}}\rho \cos\alpha. \tag{3.22}$$

As mentioned in the first part of the paper, the matrix Q is symmetric positive definite. The variables ϕ_i are therefore positive

$$\phi_i \geqslant 0, \quad i=1,2,3, \tag{3.23}$$

and the domain of definition for the variables α and ρ can correspondingly be taken as

$$0 \leqslant \rho \leqslant \sqrt{2}\Phi, \qquad \alpha \leqslant \frac{\pi}{3}. \tag{3.24}$$

The main-axis transformation of the symmetric second rank tensor field Q therefore induces a parametrization of the five spin-2 fields Y^A in terms of the three rotational degrees of freedom, the Euler angles $\chi_i = (\psi,\theta,\phi)$, which describe the orientation of the ''intrinsic frame,'' and the two invariants ρ and α represented by the 5-vector $\mathbf{M}$. As the three scalars under spatial rotations we can hence use either ρ, α, and the spin-0 field Φ, or the three fields ϕ_i $(i=1,2,3)$.

In the following we shall use the main-axis representation (3.21). The momenta π_i and p_{χ_i}, canonical conjugate to the diagonal elements ϕ_i and the Euler angles χ_i, can easily be found using the generating function

$$\begin{aligned} F_3[\phi_i,\chi_i;P] &:= \int d^3x \ \ \mathrm{Tr}(QP) \\ &= \int d^3x \ \ \mathrm{Tr}(R^T(\chi) Q_{\mathrm{diag}}(\phi) R(\chi) P) \end{aligned} \tag{3.25}$$

as

$$\begin{aligned} \pi_i(x) &= \frac{\partial F_3}{\partial \phi_i(x)} = \mathrm{Tr}(P R^T \overline{\alpha}_i R), \\ p_{\chi_i}(x) &= \frac{\partial F_3}{\partial \chi_i(x)} = \mathrm{Tr}\left(\frac{\partial R^T}{\partial \chi_i} R [PQ - QP]\right). \end{aligned} \tag{3.26}$$

Here $\overline{\alpha}_i$ are the diagonal matrices with the elements $(\overline{\alpha}_i)_{lm} = \delta_{li}\delta_{mi}$. Together with the off-diagonal matricies $(\alpha_i)_{lm} = |\epsilon_{ilm}|$ they form an orthogonal basis for symmetric matrices, shown explicitly in the Appendix. The original physical momenta P_{ik} can then be expressed in terms of the new canonical variables as

$$P(x) = R^T(x)\left(\sum_{s=1}^{3} \pi_s(x)\overline{\alpha}_s + \frac{1}{2}\sum_{s=1}^{3} \mathcal{P}_s(x)\alpha_s\right) R(x), \tag{3.27}$$

with

$$\mathcal{P}_i(x) := \frac{\xi_i(x)}{\phi_j(x) - \phi_k(x)} \quad (\text{cyclic permutation } i \neq j \neq k), \tag{3.28}$$

where the ξ_i are the $SO(3)$ left-invariant Killing vectors in terms of Euler angles $\chi_i = (\psi,\theta,\phi)$,

$$\xi_k(x) := \mathcal{M}(\theta,\psi)_{kl} p_{\chi_l}, \tag{3.29}$$

with the matrix

$$\mathcal{M}(\theta,\psi) := \begin{pmatrix} \sin\psi/\sin\theta, & \cos\psi, & -\sin\psi\cot\theta \\ -\cos\psi/\sin\theta, & \sin\psi, & \cos\psi\cot\theta \\ 0, & 0, & 1 \end{pmatrix}. \tag{3.30}$$

The ξ_i describe the Noetherian spin density part of formula (3.2), $S_i = \epsilon_{ijk}(PQ)_{jk}$, in the ''intrinsic frame,'' $S_i = R^T_{ij}\xi_j$. The antisymmetric part $\vec{E}$ of the electric field appearing in the unconstrained Hamiltonian (2.17) is given by the following expansion in a $1/g$ series, analogous to Eqs. (2.21)–(2.23):

$$E_i = R^T_{is}\sum_{n=0}^{\infty} \mathcal{E}_s^{(n)}, \tag{3.31}$$

with the zeroth order term

$$\mathcal{E}_i^{(0)} := -\frac{\xi_i}{\phi_j + \phi_k} \quad (\text{cycl. permut. } i \neq j \neq k), \tag{3.32}$$

the first order term given from $\mathcal{E}^{(0)}$ via

$$\mathcal{E}_i^{(1)} := -\frac{1}{g}\frac{1}{\phi_j+\phi_k}[((\nabla_{X_j}\vec{\mathcal{E}}^{(0)})_k - (\nabla_{X_k}\vec{\mathcal{E}}^{(0)})_j) - \Xi_i], \tag{3.33}$$

with cyclic permutations of $i \neq j \neq k$, and the higher order terms of the expansion determined via the recurrence relations

$$\mathcal{E}_i^{(n+1)} := -\frac{1}{g}\frac{1}{\phi_j+\phi_k}((\nabla_{X_j}\vec{\mathcal{E}}^{(n)})_k - (\nabla_{X_k}\vec{\mathcal{E}}^{(n)})_j). \tag{3.34}$$

Here the components of the covariant derivatives ∇_{X_k} in the direction of the vector fields $X_i(x) := R_{ik}\partial_k$,

$$(\nabla_{X_i}\vec{\mathcal{E}})_b := X_i\mathcal{E}_b + \Gamma^d_{ib}\mathcal{E}_d, \tag{3.35}$$

are determined by the connection depending only on the Euler angles

$$\Gamma^b{}_{ia} := (RX_iR^T)_{ab}. \tag{3.36}$$

Note that the connection $\Gamma^b{}_{ia}$ can be written in the form

$$\Gamma^b{}_{ia} = i(J^s)_{ab}(\mathcal{M}^{-1})_{ks}X_i\chi_k, \tag{3.37}$$

using the matrix $\mathcal{M}$ given in terms of the Euler angles $\chi_i = (\psi,\theta,\phi)$ in Eq. (3.30), which expresses the dual nature of the Killing vectors ξ_i in Eq. (3.30), and the Maurer-Cartan one-forms ω_i defined by

$$RdR^T =: \omega_i J_i, \quad \omega_i = i(\mathcal{M}^{-1})_{ki} d\chi_k. \tag{3.38}$$

The source terms Ξ_k in Eq. (3.33), finally, are given as

$$\Xi_1 = \Gamma^1{}_{22}(\pi_1 - \pi_2) + \tfrac{1}{2}X_1\pi_1 - \tfrac{1}{2}\Gamma^2_{23}\mathcal{P}_2 - \tfrac{1}{2}\Gamma^1_{23}\mathcal{P}_1 - \Gamma^1{}_{12}\mathcal{P}_3 + \tfrac{1}{2}X_2\mathcal{P}_3 + (2\to 3), \tag{3.39}$$

and its cyclic permutations Ξ_2 and Ξ_3.

The unconstrained Hamiltonian therefore takes the form

$$H = \frac{1}{2}\int d^3x \left(\sum_{i=1}^{3} \pi_i^2 + \frac{1}{2}\sum_{cycl.} \frac{\xi_i^2}{(\phi_j - \phi_k)^2} + \frac{1}{2}\vec{\mathcal{E}}^2 + V \right), \tag{3.40}$$

where the potential term V

$$V[\phi,\chi] = \sum_{i=1}^{3} V_i[\phi,\chi] \tag{3.41}$$

is the sum of

$$\begin{aligned} V_1[\phi,\chi] = &(\Gamma^1{}_{12}(\phi_2-\phi_1) - X_2\phi_1)^2 \\ &+ (\Gamma^1{}_{13}(\phi_3-\phi_1) - X_3\phi_1)^2 \\ &+ (\Gamma^1{}_{23}\phi_3 + \Gamma^1_{32}\phi_2 - g\phi_2\phi_3)^2, \end{aligned} \tag{3.42}$$

and its cyclic permutations. We see that through the main-axis transformation of the symmetric second rank tensor field Q the rotational degrees of freedom, the Euler angles χ and their canonical conjugate momenta p_χ, have been isolated from the scalars under spatial rotations and appear in the unconstrained Hamiltonian only via the three Killing vector fields ξ_k, the connections Γ, and the derivative vectors X_k.

IV. THE INFRARED LIMIT OF UNCONSTRAINED $SU(2)$ GLUODYNAMICS

A. The strong coupling limit of the theory

From the expression (3.40) for the unconstrained Hamiltonian one can analyze the classical system in the strong coupling limit up to order $O(1/g)$. Using the leading order (3.33) of the $\vec{\mathcal{E}}$ we obtain the Hamiltonian

$$H_S = \frac{1}{2}\int d^3x \left(\sum_{i=1}^{3} \pi_i^2 + \sum_{cycl.} \xi_i^2 \frac{\phi_j^2+\phi_k^2}{(\phi_j^2-\phi_k^2)^2} + V[\phi,\chi] \right). \tag{4.1}$$

For spatially constant fields the integrand of this expression reduces to the Hamiltonian of $SU(2)$ Yang-Mills mechanics considered in previous work [23]. For the further investigation of the low energy properties of $SU(2)$ field theory a thorough understanding of the properties of the leading order g^2 term in Eq. (3.41)

$$V_{\mathrm{hom}}[\phi_i] = g^2[\phi_1^2\phi_2^2 + \phi_2^2\phi_3^2 + \phi_3^2\phi_1^2], \tag{4.2}$$

containing no derivatives, is crucial. The stationary points of the potential term (4.2) are

$$\phi_1 = \phi_2 = 0, \quad \phi_3 - \text{arbitrary}, \tag{4.3}$$

and its cyclic permutations. Analyzing the second order derivatives of the potential at the stationary points one can conclude that they form a continous line of degenerate absolute minima at zero energy. In other words the potential has a ''valley'' of zero energy minima along the line $\phi_1 = \phi_2 = 0$. They are the unconstrained analogs of the toron solutions [35] representing constant Abelian field configurations with vanishing magnetic field in the strong coupling limit. The special point $\phi_1 = \phi_2 = \phi_3 = 0$ corresponds to the ordinary perturbative minimum.

In terms of the variables ρ, Φ and α the homogeneous potential (4.2) reads

$$V_{\mathrm{hom}} := \frac{g^2}{3}\left(\Phi^4 + \frac{3}{4}\rho^4 - \sqrt{2}\Phi\rho^3\cos 3\alpha \right), \tag{4.4}$$

showing that the α parametrizes the strength of the coupling between the spin-0 and spin-2 fields. The valley of minima is given by $\rho = \sqrt{2}\Phi$, $\alpha = 0$, Φ arbitrary, and the perturbative vacuum by $\rho = \Phi = \alpha = 0$.

For the investigation of configurations of higher energy it is necessary to include the part of the kinetic term in Eq. (4.1) containing the angular momentum variables ξ_i. Since the singular points of this term just correspond to the absolute minima of the potential there will a competition between an attractive and a repulsive force. At the balance point we shall have a local minimum corresponding to a classical configuration with higher energy.

B. Nonlinear sigma model type effective action as the infrared limit of the unconstrained system

We would like to find in this paragraph the effective classical field theory to which the unconstrained theory reduces in the limit of infinite coupling constant g, if we assume that the classical system spontaneously chooses one of the classical zero energy minima of the leading order g^2 part (4.2) of the potential (3.41). As discussed in the proceeding section these classical minima include apart from the perturbative vacuum, where all fields vanish, also field configurations with one scalar field attaining arbitrary values. Let us therefore put without loss of generality (explicitly breaking the cyclic symmetry)

$$\phi_1 = \phi_2 = 0, \tag{4.5}$$

such that the potential (4.2) vanishes. In this case the part of the potential (3.41) containing derivatives takes the form

$$V_{\text{inhom}}=\phi_3(x)^2[(\Gamma^2{}_{13}(x))^2+(\Gamma^2{}_{23}(x))^2+(\Gamma^2{}_{33}(x))^2+(\Gamma^3{}_{11}(x))^2+(\Gamma^3{}_{21}(x))^2+(\Gamma^3{}_{31}(x))^2]+[(X_1\phi_3)^2+(X_2\phi_3)^2]+2\phi_3(x)[\Gamma^3{}_{31}(x)X_1\phi_3+\Gamma^3{}_{32}(x)X_2\phi_3]. \tag{4.6}$$

Introducing the unit vector

$$n_i(\phi,\theta):=R_{3i}(\phi,\theta), \tag{4.7}$$

pointing along the 3-axis of the ''intrinsic frame,'' one can write

$$V_{\text{inhom}}=\phi_3(x)^2(\partial_i\vec{n})^2+(\partial_i\phi_3)^2-(n_i\partial_i\phi_3)^2-(n_i\partial_i n_j)\partial_j(\phi_3^2). \tag{4.8}$$

Concerning the contribution from the nonlocal term in this phase, we obtain for the leading part of the electric fields

$$\mathcal{E}_1^{(0)}=-\xi_1/\phi_3, \quad \mathcal{E}_2^{(0)}=-\xi_2/\phi_3. \tag{4.9}$$

Since the third component $\mathcal{E}_3^{(0)}$ and $\mathcal{P}_3$ are singular in the limit $\phi_1,\phi_2\to 0$, it is necessary to have $\xi_3\to 0$. The assumption of a definite value of ξ_3 is in accordance with the fact that the potential is symmetric around the 3-axis for small ϕ_1 and ϕ_2, such that the intrinsic angular momentum ξ_3 is conserved in the neighborhood of this configuration. Hence we obtain the following effective Hamiltonian up to order $O(1/g)$

$$H_{\text{eff}}=\frac{1}{2}\int d^3x\left[\pi_3^2+\frac{1}{\phi_3^2}(\xi_1^2+\xi_2^2)+(\partial_i\phi_3)^2+\phi_3^2(\partial_i\vec{n})^2-(n_i\partial_i\phi_3)^2-(n_i\partial_i n_j)\partial_j(\phi_3^2)\right]. \tag{4.10}$$

After the inverse Lagrangian transformation we obtain the corresponding nonlinear sigma model type effective Lagrangian for the unit vector $\vec{n}(t,\vec{x})$ coupled to the scalar field $\phi_3(t,\vec{x})$

$$L_{\text{eff}}[\phi_3,\vec{n}]=\frac{1}{2}\int d^3x[(\partial_\mu\phi_3^2)^2+\phi_3^2(\partial_\mu\vec{n})^2+(n_i\partial_i\phi_3)^2+n_i(\partial_i n_j)\partial_j(\phi_3^2)]. \tag{4.11}$$

In the limit of infinite coupling the unconstrained field theory in terms of six physical fields equivalent to the original $SU(2)$ Yang-Mills theory in terms of the gauge fields A_μ^a reduces therefore to an effective classical field theory involving only one of the three scalar fields and two of the three rotational fields summarized in the unit vector $\vec{n}$. Note that this nonlinear sigma model type Lagrangian admits singular hedgehog configurations of the unit vector field $\vec{n}$. Due to the absence of a scale at the classical level, however, these are unstable. Consider for example the case of one static monopole placed at the origin,

$$n_i:=x_i/r, \quad \phi_3=\phi_3(r), \quad r:=\sqrt{x_1^2+x_2^2+x_3^2}. \tag{4.12}$$

Minimizing its total energy E,

$$E[\phi_3]=4\pi\int dr\,\phi_3^2(r), \tag{4.13}$$

with respect to $\phi_3(r)$ we find the classical solution $\phi_3(r)\equiv 0$. There is no scale in the classical theory. Only in a quantum investigation a mass scale such as a nonvanishing value for the condensate $\langle 0|\hat{\phi}_3^2|0\rangle$ may appear, which might be related to the string tension of flux tubes directed along the unit-vector field $\vec{n}(t,\vec{x})$. The singular hedgehog configurations of such string-like directed flux tubes might then be associated with the glueballs. The pure quantum object $\langle 0|\hat{\phi}_3^2|0\rangle$ might be realized as a squeezed gluon condensate [31]. Note that for the case of a spatially constant condensate,

$$\langle 0|\hat{\phi}_3^2|0\rangle=:2m^2=\text{const.}, \tag{4.14}$$

the quantum effective action corresponding to Eq. (4.11) should reduce to the lowest order term of the effective soliton Lagangian discussed very recently by Faddeev and Niemi [24]

$$L_{\text{eff}}[\vec{n}]=m^2\int d^3x(\partial_\mu\vec{n})^2. \tag{4.15}$$

As discussed in [24], for the stability of these knots furthermore a higher order Skyrmion-like term in the derivative expansion of the unit-vector field $\vec{n}(t,\vec{x})$ is necessary. To obtain it from the corresponding higher order terms in the strong coupling expansion of the unconstrained Hamiltonian (3.40) is under present investigation.

First steps towards a quantum treatment of the unconstrained formulation obtained in the preceding paragraphs will be undertaken in the next section.

V. QUANTUM GROUND STATE WAVE FUNCTIONAL AND THE CLASSICAL CONFIGURATION OF LOWEST ENERGY

In this section we shall give the unconstrained analog of the well-known nonnormalizable ground-state wave functional which solves the Schrödinger equation with zero energy and analyze it in the strong coupling limit.

A. Exact ground state solution of the Schrödinger equation

For the original constrained system of $SU(2)$ gluodynamics in terms of the gauge fields $A_i^a(x)$ with the Hamiltonian

$$\mathcal{H}(A):=\frac{1}{2}\int d^3x\left(-\left(\frac{\delta}{\delta A_i^a(x)}\right)^2+B^2(x)\right) \tag{5.1}$$

and the Gauss law operators

$$\mathcal{G}^a(x) := (\partial_i \delta^a_b - g\epsilon^{abc} A^c_i(x)) \frac{\delta}{\delta A^b_i(x)} \tag{5.2}$$

in the Schrödinger functional formalism, a physical state has to satisfy both the functional Schrödinger equation and the Gauss law constraints

$$\mathcal{H}\Psi[A] = E\Psi[A], \tag{5.3}$$

$$\mathcal{G}^a(x)\Psi[A] = 0. \tag{5.4}$$

Remarkably, an exact solution for the ground state wave functional $\Psi[A]$ can be given [25]

$$\Psi[A] = \exp(-8\pi^2 W[A]) \tag{5.5}$$

in terms of the so called ''winding number functional'' $W[A]$ defined as the integral over 3-space

$$W[A] := \int d^3x K_0(x) \tag{5.6}$$

of the zero component of the Chern-Simons secondary characteristic class vector [32][7]

$$K^\mu(A) := -\frac{1}{16\pi^2}\epsilon^{\mu\nu\sigma\kappa}\mathrm{Tr}\left(F_{\nu\sigma}A_\kappa - \frac{2}{3}gA_\nu A_\sigma A_\kappa\right). \tag{5.8}$$

Since $W[A]$ obeys the functional differential equation

$$\frac{\delta}{\delta A^a_i(x)} W[A] = B^a_i(x) \tag{5.9}$$

the wave functional (5.5) satisfies the above Schrödinger equation. However this exact solution for the functional Schrödinger equation with the zero energy is known to be nonnormalizable and hence does not seem to have a physical meaning [33].

In the following we shall now analyze how such an exact solution arises in the unconstrained formalism.

[7]Note that whereas the topological invariant, the Pontryagin index

$$\nu[A] := \int d^4x \partial_\mu K^\mu(A) \tag{5.7}$$

and the corresponding Pontryagin density $\mathrm{Tr}({}^*F^{\mu\nu}F_{\nu\mu})$, are gauge invariant quantities, the Chern-Simons vector K^μ is not gauge invariant.

B. Exact ground state solution for the unconstrained Hamiltonian

Quantizing the variables Q and P of the unconstrained Hamiltonian (2.17) analogously to the A^a_i above[8] we have

$$\mathcal{H} = \frac{1}{2}\int d^3x\left(-\left(\frac{\delta}{\delta Q_{ij}(x)}\right)^2 + B^2(x) + \frac{1}{2}\vec{E}^2\left(Q, \frac{\delta}{\delta Q}\right)\right), \tag{5.10}$$

and hence the functional Schrödinger equation

$$\mathcal{H}\Psi[Q] = E\Psi[Q]. \tag{5.11}$$

The Gauss law has already been implemented by the reduction to the physical variables.

A corresponding exact zero energy solution can indeed be found for the reduced Schrödinger equation (5.11). For this we note the following two important properties of the potential terms present in the Schrödinger equation (5.11). First, the reduced magnetic field $B_{ij}(Q)$ can be written as the functional derivative of the functional $\mathcal{W}[Q]$

$$\mathcal{W}[Q] := \frac{1}{32\pi^2}\int d^3x\left[\mathrm{Tr}(BQ) - \frac{1}{12}g\big(\mathrm{Tr}(Q^3) + \mathrm{Tr}^3(Q) - 2\,\mathrm{Tr}(Q)\mathrm{Tr}(Q^2)\big)\right] \tag{5.12}$$

such that

$$\frac{\delta}{\delta Q_{ij}(x)}\mathcal{W}[Q] = B_{ij}(x). \tag{5.13}$$

Furthermore, the nonlocal term in the Schrödinger equation (5.11) annihilates $\mathcal{W}[Q]$

$$\vec{E}^2\left[Q, \frac{\delta}{\delta Q_{ij}(x)}\right]\mathcal{W}[Q] = 0. \tag{5.14}$$

The last equation can easily be found to hold if one takes into account that the magnetic field $B_i = {}^*F_{0i}$ satisfies the Bianchi identity $D_i {}^*F_{0i} = 0$.

Thus the corresponding ground state wave functional solution for the unconstrained Hamiltonian is

$$\Psi[Q] = \exp(-8\pi^2\mathcal{W}[Q]). \tag{5.15}$$

[8]Note that due to the positive definiteness of the elements of the matrix field Q we have to solve the Schrödinger equation in a restricted domain of functional space. Special boundary conditions have to be imposed on the wave functional such that all operators are well defined (e.g. Hermiticity of the Hamiltonian). A discussion on this subject in gauge theories can be found e.g. in the review [34].

In order to investigate the relation of the $\mathcal{W}[Q]$ to the above winding number functional $W[A]$ we write the zero component of the the Chern-Simons secondary characteristic class vector K^μ, given in Eq. (5.8), in terms of the new variables Q and q_i

$$K^0(Q,q)=\mathcal{K}^0(Q)-\frac{1}{24\pi^2}\epsilon_{ijk}\left[\frac{2}{3}g\mathrm{Tr}(\Omega_i\Omega_j\Omega_k)-\partial_i\mathrm{Tr}(Q_j\Omega_k)\right]. \tag{5.16}$$

The first term

$$\mathcal{K}^0(Q):=-\frac{1}{16\pi^2}\epsilon_{ijk}\mathrm{Tr}\left(F_{ij}Q_k-\frac{2}{3}gQ_iQ_jQ_k\right) \tag{5.17}$$

is a functional only of the physical Q of a form similiar to that of the original Chern-Simons secondary characteristic class vector. Here we have introduced the $SU(2)$ matrices $Q_l:=Q_{li}\tau_i$, with the Pauli matrices τ_i, and

$$\Omega_i(q):=\frac{1}{g}U^{-1}(q)\partial_iU(q)=\frac{1}{g}\Omega_{ls}(q)\frac{\tau_s}{2}\left(\frac{\partial q_l}{\partial x_i}\right), \tag{5.18}$$

with the $SU(2)$ matrices $U(q)$ related to the 3×3 orthogonal matrix $O(q)$ via $O_{ab}(q)=\frac{1}{2}\mathrm{Tr}(U^{-1}(q)\tau_aU(q)\tau_b)$ and the 3×3 matrix Ω_{ij} defined in Eq. (2.10).

We observe that the space integral over the first term coincides with the above functional $\mathcal{W}[Q]$ of Eq. (5.12)

$$\int d^3x\mathcal{K}^0(Q)=\mathcal{W}[Q]. \tag{5.19}$$

Using the usual boundary condition[9]

$$U(q)\to\pm I, \tag{5.20}$$

we see that the space integral over the second term is proportional to the natural number n representing the winding of the mapping of compactified three space into $SU(2)$

$$\frac{g^3}{24\pi^2}\int d^3x\epsilon_{ijk}\mathrm{Tr}(\Omega_i\Omega_j\Omega_k)=n. \tag{5.21}$$

Assuming here the vanishing of the physical field Q at spatial infinity there is no contribution from the third term. Hence we obtain the relation

$$\Psi[A]=\exp\left[-\frac{8\pi^2}{g^2}n\right]\Psi[Q] \tag{5.22}$$

between the ground state wave functional (5.5) of the extended quantization scheme and the reduced (5.15). We find that the winding number of the original gauge field A only appears as an unphysical normalization prefactor originating from the second term in Eq. (5.16), which depends only on the unphysical q_i. Furthermore, we note that the power $(8\pi^2/g^2)n$ is the classical Euclidean action of $SU(2)$ Yang-Mills theory of self-dual fields [36] with winding number n.

The physical part of the wave function, $\Psi[Q]$, on the other hand however, has the same unpleasant property as Eq. (5.5) that it is nonnormalizable. In order to shed some light on the reason for its nonnormalizability, it is useful to limit to the homogeneous case and to analyze the properties of $\Psi[Q]$ in the neighborhood of the classical minima of the potential.

C. Analysis of the exact ground state wave functional in the strong coupling limit

In the strong coupling limit the ground state wave functional (5.15) reduces to the very simple form

$$\Psi[\phi_1,\phi_2,\phi_3]=\exp[-g\phi_1\phi_2\phi_3]. \tag{5.23}$$

This wave functional is obviously nonnormalizable. In difference to the Abelian case, however, where an analogous nonnormalizable exact zero energy solution exists,[10] the exponent of Eq. (5.23) is free of any sign ambiguities due to the positivity of the symmetric matrix Q in the polar representation (2.6), and hence the positivity of the diagonal fields ϕ_i, see Eq. (3.23). In order to investigate the reason for the nonnormalizability of Eq. (5.23) we analyze it near the classical zero-energy minima, that is, without loss of generality, in the neighborhood of the line $\phi_1=\phi_2=0$ of minima of the classical potential (4.2). It is useful to pass from the variables ϕ_1 and ϕ_2 transverse to the valley to the new variables $\phi_\perp$ and γ via

$$\phi_1=\phi_\perp\cos\gamma,\quad \phi_2=\phi_\perp\sin\gamma\quad \left(\phi_\perp\geqslant0,\quad 0\leqslant\gamma\leqslant\frac{\pi}{2}\right). \tag{5.24}$$

The classical potential then reads

$$V(\phi_3,\phi_\perp,\gamma)=g^2\left(\phi_3^2\phi_\perp^2+\frac{1}{4}\phi_\perp^4\sin^2(2\gamma)\right), \tag{5.25}$$

and the ground state wave function (5.23) becomes

$$\Phi[\phi_3,\phi_\perp,\gamma]=\exp\left[-\frac{g}{2}\phi_3\phi_\perp^2\sin(2\gamma)\right]. \tag{5.26}$$

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[9] Note that we have no information about the behavior of the unphysical variables q_i. For example the requirement of the finiteness of the action usually used to fix the behavior of the physical fields does not apply to the unphysical field q_i.

[10] In (Abelian) electrodynamics the unconstrained form of the corresponding exact zero-energy ground state wave functional is an exponential of $\int dkka_1(k)a_2(k)$, where a_1,a_2 are the (momentum space) polarization modes of A. This false ground state is nonnormalizable due to the sign indefiniteness of the exponent.

We see that close to the bottom of the valley, for small $\phi_\perp$, the potential is that of a harmonic oscillator and the wave functional correspondingly a Gaussian with a maximum at the classical minimum line $\phi_\perp = 0$. The height of the maximum is constant along the valley. The nonnormalizability of the ground state wave function in the infrared region is therefore due to the outflow of the wave function with constant values along the valley to arbitrarily large values of the field ϕ_3. This may result in the formation of condensates with macroscopically large fluctuations of the field amplitude. To establish the connection between this phenomenon and the model of the squeezed gluon condensate [31] will be an interesting task for further investigation.

VI. CONCLUDING REMARKS

Following the Dirac formalism for constrained Hamiltonian systems we have formulated several representations for the classical $SU(2)$ Yang-Mills gauge theory entirely in terms of unconstrained gauge invariant local fields. All transformations which have been used, canonical transformations and the Abelianization of the constraints, maintain the canonical structures of the generalized Hamiltonian dynamics. We identify the unconstrained field with a symmetric positive definite second rank tensor field under spatial rotations. Its decomposition into irreducible representations under spatial rotations leads to the introduction of two fields, a five-dimensional vector field $\mathbf{Y}(x)$ and a scalar field $\Phi(x)$. Their dynamics is governed by an explicitly rotational invariant non-local Hamiltonian. It is different from the local Hamiltonian obtained by Goldstone and Jackiw [1] as well as by Izergin *et al.* [3]. They used the so-called electric field representation with vanishing antisymmetric part of the electric field. A representation for the Hamiltonian with a nonlocal interaction of the unconstrained variables similar to ours has been derived in the work of Simonov [8] based on another separation of scalar and rotational degrees of freedom. Our separation of the unconstrained fields into scalars under spatial rotations and into rotational degrees of freedom, however, leads to a simpler form of the Hamiltonian, which in particular is free of operator ordering ambiguities in the strong coupling limit. Our unconstrained representation of the Hamiltonian furthermore allows us to derive an effective low energy Lagrangian for the rotational degrees of freedom coupled to one of the scalar fields suggested by the form of the classical potential in the strong coupling limit. The dynamics of the rotational variables in this limit is summarized by the unit vector describing the orientation of the intrinsic frame. Due to the absence of a scale in the classical theory the singular hedgehog configurations of the unit vector field are found to be unstable classically. In order to obtain a nonvanishing value for the vacuum expectation value for one of the three scalar field operators, which would set a scale, a quantum treatment at least to one loop order is necessary and is under present investigation. For the case of a spatially constant scalar quantum condensate we expect to obtain the first term of a derivative expansion proposed recently by Faddeev and Niemi [24]. As shown in their work such a soliton Lagragian allows for stable massive knotlike configurations which might be related to glueballs. For the stability of the knots higher order terms in the derivative expansion, such as the Skyrme type fourth order term in [24], are necessary. Their derivation in the framework of the unconstrained theory, proposed in this paper, is under investigation. First steps towards a complete quantum description have been done in this paper. In particular, we have investigated the famous ground state wave functional, which solves the Schrödinger equation with zero energy eigenvalue. In conclusion we would like to emphasize that our investigation of low energy aspects of non-Abelian gauge theories directly in terms of the physical unconstrained fields offers an alternative to the variational calculations using approximate projection onto gauge invariant states [37,38].

The reason for trying to construct the physical variables entirely in internal terms without the use of any gauge fixing is the aspiration to maintain all local and global properties of the initial gauge theory. Several questions in connection with the global aspects of the reduction procedure are arising at this point. In the paper we describe how to project SU(2) Yang-Mills theory onto the constraint shell defined by the Gauss law. It is well known that the exponentiation of infinitisimal transformations generated by the Gauss law operator can lead only to homotopically trivial gauge transformations, continiously deformable to unity. However, the initial classical action is invariant under all gauge transformations, including the homotopically nontrivial ones. What trace does the existence of large gauge transformations leave on the unconstrained system? First steps towards a clarification of these important issues have been undertaken in this paper, a more complete analysis is under present investigation.

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APPENDIX A: NOTATIONS AND SOME FORMULAS

1. Spin 1 matrices and eigenvectors

For generators of spin-1 obeying the algebra $[J_i, J_j] = i\epsilon_{ijk}J_k$ we use the following matrix realizations:

$$J_1 = i\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}, \quad J_2 = i\begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix},$$

$$J_3 = i\begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

Furthermore the representation of rotations $R(\chi)$ in terms of Euler angles $\chi=(\theta,\psi,\phi)$ is used

$$R(\psi,\theta,\phi) = e^{-i\psi J_3} e^{-i\theta J_1} e^{-i\phi J_3}. \tag{A1}$$

The eigenfunctions of J^2 and J_3 are

$$\vec{e}_{+1} = \frac{1}{\sqrt{2}}\begin{pmatrix} -1 \\ -i \\ 0 \end{pmatrix}, \quad \vec{e}_0 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \quad \vec{e}_{-1} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -i \\ 0 \end{pmatrix}$$

which are orthogonal with respect to the metric $\eta_{\alpha\beta} := (-1)^\alpha \delta_{\alpha,-\beta}$

$$(\vec{e}_\alpha \cdot \vec{e}_\beta) = \eta_{\alpha\beta} \tag{A2}$$

and satisfy the completeness condition

$$e_i^\alpha e_j^\beta \eta_{\alpha\beta} = \delta_{ij}. \tag{A3}$$

2. Spin-0, spin-1 and spin-2 tensors basis

To obtain a matrix representation for spin-0, spin-1 and spin-2 basis matrices we use the Clebsh-Gordon decomposition for the direct product of spin-1 eigenvectors e_i^α into the irreducible components $3\otimes 3 = 0 \oplus 1 \oplus 2$. To distinguish the matrices corresponding to the different spins we use boldface notation for spin 2.

For spin-0 they read explicitly

$$I_0 := \frac{1}{\sqrt{3}}(\vec{e}_0\otimes\vec{e}_0 - \vec{e}_{+1}\otimes\vec{e}_{-1} - \vec{e}_{-1}\otimes\vec{e}_{+1})$$
$$= \frac{1}{\sqrt{3}}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

for spin-1

$$J_{+1} := (\vec{e}_0\otimes\vec{e}_{+1} - \vec{e}_{+1}\otimes\vec{e}_0) = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & i \\ -1 & -i & 0 \end{pmatrix},$$

$$J_{-1} := (\vec{e}_{-1}\otimes\vec{e}_0 - \vec{e}_0\otimes\vec{e}_{-1}) = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & -i \\ -1 & i & 0 \end{pmatrix},$$

$$J_0 := (\vec{e}_{-1}\otimes\vec{e}_{+1} - \vec{e}_{+1}\otimes\vec{e}_{-1}) = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

for spin-2

$$\mathbf{T}_{+2} = \sqrt{2}(\vec{e}_{+1}\otimes\vec{e}_{+1}) = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & i & 0 \\ i & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

$$\mathbf{T}_{-2} = \sqrt{2}(\vec{e}_{-1}\otimes\vec{e}_{-1}) = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -i & 0 \\ -i & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

$$\mathbf{T}_{+1} := (\vec{e}_{+1}\otimes\vec{e}_0 + \vec{e}_0\otimes\vec{e}_{+1}) = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 0 & -1 \\ 0 & 0 & -i \\ -1 & -i & 0 \end{pmatrix},$$

$$\mathbf{T}_{-1} := (\vec{e}_{-1}\otimes\vec{e}_0 + \vec{e}_0\otimes\vec{e}_{-1}) = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & -i \\ 1 & -i & 0 \end{pmatrix},$$

$$\mathbf{T}_0 := \frac{1}{\sqrt{3}}(\vec{e}_{+1}\otimes\vec{e}_{-1} + 2\vec{e}_0\otimes\vec{e}_0 + \vec{e}_{-1}\otimes\vec{e}_{+1})$$
$$= \frac{1}{\sqrt{3}}\begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 2 \end{pmatrix}.$$

They obey the following orthonormality relations:

$$\mathrm{Tr}(\mathbf{T}_A\mathbf{T}_B) = 2\eta_{AB}, \quad \mathrm{Tr}(\mathbf{T}_A J_\alpha) = 0, \quad \mathrm{Tr}(J_\alpha J_\beta) = 2\eta_{\alpha\beta}, \tag{A4}$$

the completeness condition

$$\frac{1}{10}\sum_A (\mathbf{T}_A)_{il}(\mathbf{T}_A)_{km} + (I_0)_{il}(I_0)_{km} = \frac{1}{4}(\delta_{im}\delta_{lk} + \delta_{il}\delta_{mk}), \tag{A5}$$

and the following commutation and anticommutation relations:

$$[\mathbf{T}_A, \mathbf{T}_B]_+ = \frac{4}{\sqrt{3}}\eta_{AB} I_0 + \frac{2}{\sqrt{3}} d^{(2)}_{ABC}\mathbf{T}^C, \tag{A6}$$

$$[\mathbf{T}_A, \mathbf{T}_B]_- = c^{(2)}_{AB\gamma} J^\gamma; \tag{A7}$$

$$[J_\alpha, J_\beta]_+ = \frac{4}{\sqrt{3}}\eta_{\alpha\beta} I_0 + d^{(1)}_{\alpha\beta C}\mathbf{T}^C, \tag{A8}$$

$$[J_\alpha, J_\beta]_- = c^{(1)}_{\alpha\beta\gamma} J^\gamma; \tag{A9}$$

$$[J_\alpha, \mathbf{T}_B]_+ = d^{(1)}_{\alpha\gamma B} J^\gamma, \tag{A10}$$

TABLE I. The coefficients $d^{(1)}_{\alpha\beta C}$, $d^{(2)}_{ABC}$ and $c^{(2)}_{AB\gamma}$.

A	B	C	$d^{(2)}_{ABC}$	A	B	γ	$c^{(2)}_{AB\gamma}$	α	β	C	$d^{(1)}_{\alpha\beta C}$
-2	2	0	-1	-2	2	0	-2	-1	1	0	$-1/\sqrt{3}$
-2	1	1	$\sqrt{3/2}$	-2	1	1	$-\sqrt{2}$	-1	0	1	1
-2	0	2	-1					-1	-1	2	$-\sqrt{2}$
-1	2	-1	$\sqrt{3/2}$	-1	2	-1	$-\sqrt{2}$	0	1	-1	1
-1	1	0	$-1/2$	-1	1	0	1	0	0	0	$-2/\sqrt{3}$
-1	0	1	$-1/2$	-1	0	1	$-\sqrt{3}$	0	-1	1	1
-1	-1	2	$\sqrt{3/2}$								
0	2	-2	-1	0	1	-1	$-\sqrt{3}$	1	1	-2	$-\sqrt{2}$
0	1	-1	$-1/2$	0	0	0	0	1	0	-1	1
0	0	0	1	0	-1	1	$\sqrt{3}$	1	-1	0	$-1/\sqrt{3}$
0	-1	1	$-1/2$								
0	-2	2	-1								
1	1	-2	$\sqrt{3/2}$	1	0	-1	$\sqrt{3}$				
1	0	-1	$-1/2$	1	-1	0	-1				
1	-1	0	$-1/2$	1	-2	1	$\sqrt{2}$				
1	-2	1	$\sqrt{3/2}$								
2	0	-2	-1	2	-1	-1	$\sqrt{2}$				
2	-1	-1	$\sqrt{3/2}$	2	-2	0	2				
2	-2	0	-1								

$$[J_\alpha, \mathbf{T}_B]_- = c^{(2)}_{BD\alpha}\mathbf{T}^C. \tag{A11}$$

The coefficients $c^{(1)}_{\alpha\beta\gamma}$ are totally antisymmetric with $c^{(1)}_{-1,+1,0}=1$ and $(J_\gamma)_{ij} = -c^{(1)}_{\alpha\beta\gamma}e^\alpha_i e^\beta_j$. The coefficients $d^{(1)}_{\alpha\beta C}$, $d^{(2)}_{ABC}$ and $c^{(2)}_{AB\gamma}$ are given in Table I. Note that

$$(\mathbf{T}_A)_{ij} = -d^{(1)}_{\alpha\beta A}e^\alpha_i e^\beta_j. \tag{A12}$$

3. Generators for the D functions

Define the five-dimensional spin matrices

$$(\mathbf{J}_\gamma)_A{}^B := -\eta^{BC}c^{(2)}_{AC\gamma} \tag{A13}$$

such that

$$\mathbf{J}_{+1} = \begin{pmatrix} 0 & -\sqrt{2} & 0 & 0 & 0 \\ 0 & 0 & \sqrt{3} & 0 & 0 \\ 0 & 0 & 0 & \sqrt{3} & 0 \\ 0 & 0 & 0 & 0 & -\sqrt{2} \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix},$$

$$\mathbf{J}_{-1} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ \sqrt{2} & 0 & 0 & 0 & 0 \\ 0 & -\sqrt{3} & 0 & 0 & 0 \\ 0 & 0 & -\sqrt{3} & 0 & 0 \\ 0 & 0 & 0 & \sqrt{2} & 0 \end{pmatrix},$$

$$\mathbf{J}_0 = \begin{pmatrix} 2 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & -2 \end{pmatrix}.$$

The corresponding Cartesian components $(\mathbf{J}_i)_A{}^B := e^\alpha_i (J_\alpha)_A{}^B$ are

$$\mathbf{J}_1 = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & -\sqrt{3/2} & 0 & 0 \\ 0 & -\sqrt{3/2} & 0 & -\sqrt{3/2} & 0 \\ 0 & 0 & -\sqrt{3/2} & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix},$$

$$\mathbf{J}_2 = i\begin{pmatrix} 0 & -1 & 0 & 0 & 0 \\ 1 & 0 & \sqrt{3/2} & 0 & 0 \\ 0 & -\sqrt{3/2} & 0 & \sqrt{3/2} & 0 \\ 0 & 0 & -\sqrt{3/2} & 0 & -1 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix},$$

$$\mathbf{J}_3 = \mathbf{J}_0$$

satisfying the $so(3)$ algebra

$$[\mathbf{J}_a, \mathbf{J}_b] = i\epsilon_{abc}\mathbf{J}_c. \tag{A14}$$

We use the D-functions as representation of rotations in 3-space defined in terms of Euler angles $\chi=(\theta,\psi,\phi)$

$$D(\psi,\theta,\phi)=e^{-i\psi \mathbf{J}_3}e^{-i\theta \mathbf{J}_1}e^{-i\phi \mathbf{J}_3}. \tag{A15}$$

They can be obtained from the corresponding 3-dimensional representation [28] via the formula

$$D(\chi)_{AB}=\frac{1}{2}\mathrm{Tr}(R(\chi)\mathbf{T}_A R^T(\chi)\mathbf{T}_B). \tag{A16}$$

4. Basis for symmetric matrices

We use the orthogonal basis $(\overline{\alpha_i},\alpha_i)$ for symmetric matrices. They read explicitly

$$\overline{\alpha_1}=\begin{pmatrix}1&0&0\\0&0&0\\0&0&0\end{pmatrix},\quad \overline{\alpha_2}=\begin{pmatrix}0&0&0\\0&1&0\\0&0&0\end{pmatrix},$$

$$\overline{\alpha_3}=\begin{pmatrix}0&0&0\\0&0&0\\0&0&1\end{pmatrix},$$

$$\alpha_1=\begin{pmatrix}0&0&0\\0&0&1\\0&1&0\end{pmatrix},\quad \alpha_2=\begin{pmatrix}0&0&1\\0&0&0\\1&0&0\end{pmatrix},$$

$$\alpha_3=\begin{pmatrix}0&1&0\\1&0&0\\0&0&0\end{pmatrix}.$$

They obey the following orthonormality relations:

$$\mathrm{tr}(\overline{\alpha_i}\overline{\alpha_j})=\delta_{ij},\quad \mathrm{tr}(\alpha_i\alpha_j)=2\delta_{ij},\quad \mathrm{tr}(\overline{\alpha_i}\alpha_j)=0. \tag{A17}$$

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