Anyonic physical observables and spin phase transition

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The quantization of a charged matter system coupled to Chern-Simons gauge fields is analyzed in covariant gauge fixing and gauge invariant, physical anyon operators satisfying fractional statistics are constructed in a symmetric phase, based on Dirac's recipe performed on QED. This method provides us a definite way of identifying physical spectrums free from gauge ambiguity and constructing physical anyon operators under a covariant gauge fixing and we analyze the statistical spin phase transition in a symmetry-broken phase predicted by Wen and Zee. The Higgs mechanism transmutes an anyon satisfying fractional statistics into a canonical boson, a spin 0 Higgs boson, or a topologically massive photon which is a Chern-Simons gauge field absorbed would-be Goldstone boson. [S0556-2821(99)01206-0]

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I. INTRODUCTION

In (2+1)-dimensional field theories such as Chern-Simons gauge theory with a charged matter field [1], the nonlinear σ model with a Hopf term [2], and the CP^1 model with a Chern-Simons term [3], it has been known that there exist excitations with exotic statistics, anyons [4,5], which continuously interpolate between bosons and fermions. The fundamental role of fractional statistics in condensed matter physics has been proposed in order to describe a fractional quantum Hall effect [6] and the behavior of two-dimensional materials such as vortices in superfluid helium films [7] and the Cu-O planes of the copper-oxide superconductors [8].

However, in (2+1)-dimensional quantum field theory, the explicit construction of anyon operators exhibiting fractional statistics has led to much controversy and debate [9]. Although the physical results must be independent of a gauge fixing condition, different results with different gauge choices had been reported [10]. Because of the difficulty of identifying physical degrees of freedom, doubt was cast on the results thus obtained and the gauge independent analysis was attempted [11,12].

Since the representation of a *physical* operator [13] can be varied with each gauge fixing, the construction of a physical operator under a gauge fixing condition must be treated carefully. One of the reasons for consideration of *physical* field variables is that the formulation of dynamics in terms of a Lagrangian (or Hamiltonian) and the equations of motion make use of a larger field algebra which includes nonobservable fields [14]. It is important to notice that gauge invariance of an operator does not necessarily imply it to be *physical*. Additional care must be paid to identify the physical spectrums with correct quantum numbers such as spin and charge.

There are also different opinions on the anyonic properties including the existence of statistical spin phase transition in a symmetry-broken phase [15,16], which is an interesting problem that may be relevant to high-T_c superconductivity [17]. In Ref. [16], Boyanovsky argued that the excitations relative to ground state being not rotational invariant are spin 0 bosons. But this result seems to contradict the result of Deser and Yang [18], observing that Higgs mechanism transmutes a nondynamical Chern-Simons term into topologically massive, parity-violating, spin 1 theory [19].

In this paper, we will perform a careful analysis on the charged matter fields coupled to Chern-Simons gauge fields in a covariant gauge along Dirac's method performed on QED [13]. In Sec. II, we will analyze the Maxwell theory of a symmetric and a symmetry-broken phase in a covariant gauge in order to illustrate the definite way of identifying a physical spectrum free from gauge ambiguity arising from gauge fixing and to manifest the importance of consideration of *physical* fields. In Sec. III, the quantization of the Chern-Simons matter system will be presented and physical anyon operators will be constructed in a covariant gauge based on the approach in Sec. II. We will also analyze the quantization of the Chern-Simons matter system in the symmetry-broken phase and show that the Higgs mechanism transmutes an anyon satisfying fractional statistics into a spin 0 Higgs boson and a topologically massive photon which is a Chern-Simons gauge field absorbed would-be Goldstone boson. Thus the Higgs effect influences the spin phase of the anyon and interestingly induces the statistical spin phase transition, predicted by Wen and Zee [15]. As we will see, the result is consistent with the observation of Deser and Yang [18]. Section IV contains our conclusion. In the Appendix, we will analyze the Poincaré algebra of massive vector fields—Proca and massive Chern-Simons theories—and extract the spin content of massive vector fields with no ambiguity.

II. MAXWELL THEORY IN A COVARIANT GAUGE

A. Symmetric phase

In this subsection, we will briefly review the analysis of the Maxwell theory subject to a covariant gauge fixing in a symmetric phase. The Lagrangian is given by

$$\mathcal{L}=-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}+|D_\mu\phi|^2+\mathcal{L}_{GF},$$

$$\mathcal{L}_{GF}=-\frac{1}{2}(\partial_\mu A^\mu)^2, \tag{2.1}$$

where $D_\mu = \partial_\mu + ieA_\mu$ is a covariant derivative with metric $g_{\mu\nu} = (+1,-1,-1)$. The equal-time commutators are as follows (a time argument of operators is suppressed):

$$[A_\mu(\mathbf{x}), A_\nu(\mathbf{y})] = [E_\mu(\mathbf{x}), E_\nu(\mathbf{y})] = 0,$$

$$[E_\mu(\mathbf{x}), A_\nu(\mathbf{y})] = i g_{\mu\nu} \delta^2(\mathbf{x}-\mathbf{y}),$$

$$[\phi(\mathbf{x}), \pi(\mathbf{y})] = [\phi^*(\mathbf{x}), \pi^*(\mathbf{y})] = i\delta^2(\mathbf{x}-\mathbf{y}), \qquad (2.2)$$

where we put

$$E_\mu(\mathbf{x}) = \frac{\partial A_\mu}{\partial x_0}, \quad \pi(\mathbf{x}) = (D_0\phi)^*(\mathbf{x}), \quad \pi^*(\mathbf{x}) = (D_0\phi)(\mathbf{x}).$$

In order to get consistent quantum theory in the covariant gauge, we must impose a supplementary condition for any state $|P\rangle$ on an indefinite state space [13],

$$\partial_\mu A^\mu(x)|P\rangle = 0 \Rightarrow \partial_\mu A^\mu \approx 0. \qquad (2.3)$$

We also require that this condition is satisfied afterwards:

$$\partial_0 \partial_\mu A^\mu \approx 0. \qquad (2.4)$$

According to the arguments of Dirac [13], one can easily see Eqs. (2.3) and (2.4) are the only independent supplementary conditions affecting dynamical variables at one instant of time. Then the state $|P\rangle$ satisfying the conditions (2.3) and (2.4) is defined as a *physical* state. The condition for a dynamical variable Φ to be *physical* is that

$$[F, \Phi] \approx 0 \qquad (2.5)$$

for each supplementary condition $F|P\rangle = 0$.

To find the physical fields, let us decompose $A_i(\mathbf{x})$ and $E_i(\mathbf{x})$ into transverse and longitudinal parts,

$$A_i(\mathbf{x}) = \mathcal{A}_i(\mathbf{x}) + \frac{\partial V}{\partial x^i} \qquad (2.6)$$

with $\partial \mathcal{A}^i/\partial x^i = 0$ and

$$E_i(\mathbf{x}) = \mathcal{E}_i(\mathbf{x}) + \frac{\partial U}{\partial x^i} \qquad (2.7)$$

with $\partial \mathcal{E}^i/\partial x^i = 0$ and $U = \partial V/\partial x^0$. Observe that, for massless theory, the transverse-longitudinal decompositions, Eqs. (2.6) and (2.7), are free from the ambiguity existing in zero-momentum limit for massive theory [20] and the Poincaré algebra in terms of the physical variables is well defined. Indeed, this situation corresponds to the vanishing of topological Chern-Simons term, $\mu = 0$, in Ref. [20], which shows that its Poincaré algebra is naturally free from zero-momentum anomaly. The conditions (2.3) and (2.4) can then be rewritten as

$$F \equiv \partial_\mu A^\mu = E_0 - \nabla^2 V \approx 0, \qquad (2.8)$$

$$G \equiv \partial_i F^{io} - J_0 = \nabla^2(U - A_0) - J_0 \approx 0, \qquad (2.9)$$

where charge density $J_0(\mathbf{x})$ is given by

$$J_0(\mathbf{x}) = ie\{\pi^*(\mathbf{x})\phi^*(\mathbf{x}) - \pi(\mathbf{x})\phi(\mathbf{x})\}$$

and G is a Gauss-law constraint.

From the commutation relation (2.2), one can obtain a useful relation [13]

$$[U(\mathbf{x}), V(\mathbf{y})] = iG(\mathbf{x}-\mathbf{y}), \qquad (2.10)$$

where $G(\mathbf{x}-\mathbf{y})$ is a two-dimensional Green's function

$$\nabla^2 G(\mathbf{x}-\mathbf{y}) = \delta^2(\mathbf{x}-\mathbf{y}), \quad G(\mathbf{x}-\mathbf{y}) = \frac{1}{2\pi}\ln|\mathbf{x}-\mathbf{y}|.$$

Then one can see that the transverse field variables $\mathcal{A}^i$ and $\mathcal{E}^i$ evidently commute with the supplementary conditions (2.8) and (2.9) and so are physical, while the operators ϕ and ϕ^* are unphysical. Define

$$\tilde{\phi}(x) = e^{ieV(x)}\phi(x). \qquad (2.11)$$

$\tilde{\phi}(x)$ now commutes with F and G and hence is physical. Similarly, $\tilde{\phi}^*(x)$ is physical. In the covariant gauge, the physical operators $\tilde{\phi}$ and $\tilde{\phi}^*$ describe charged particles surrounded by their Coulomb fields. Accordingly, the composite nonlocal operators $\tilde{\phi}$ and $\tilde{\phi}^*$ just correspond to the physical processes of creation and annihilation of charged particles, since these processes must always be accompanied by an appropriate Coulomb change in an electric field around the point where the particle is created or annihilated [13]. The nonlocality of the physical field variables $\tilde{\phi}$ and $\tilde{\phi}^*$ are not surprising since a field carrying a nonzero charge whose current obeys a local field equation, $\partial_\nu F^{\nu\mu} = J^\mu$, cannot be local [14].

The covariant derivatives of ϕ and ϕ^* can then be represented by the physical fields $\tilde{\phi}$ and $\tilde{\phi}^*$:

$$D_i\phi(x) = e^{-ieV(x)}(\partial_i + ie\mathcal{A}_i)\tilde{\phi}(x) \equiv e^{-ieV(x)}\tilde{D}_i\tilde{\phi}(x),$$

$$\pi(x) = e^{ieV(x)}\{\partial_0 - ie(A_0 - U)\}\tilde{\phi}^*(x) \equiv e^{ieV(x)}\tilde{\pi}(x),$$

$$(D_i\phi(x))^* = e^{ieV(x)}(\partial_i - ie\mathcal{A}_i)\tilde{\phi}^*(x) \equiv e^{ieV(x)}(\tilde{D}_i\tilde{\phi}(x))^*,$$

$$\pi^*(x) = e^{-ieV(x)}\{\partial_0 + ie(A_0 - U)\}\tilde{\phi}(x) \equiv e^{-ieV(x)}\tilde{\pi}^*(x). \qquad (2.12)$$

The variables $\mathcal{A}^i, \mathcal{E}^i, \tilde{\phi}, \tilde{\pi}, \tilde{\phi}^*$, and $\tilde{\pi}^*$ are the only independent physical variables, apart from the quantities (2.8) and

(2.9). Now we can find the relationships between Coulomb gauge and covariant gauge in the state space constructed by only physical variables. The field variables in covariant gauge $\mathcal{A}_i = A_i - \partial_i V, A_0 - U, \tilde{\phi}$, and $\tilde{\phi}^*$ correspond to A_i, A_0, ϕ, and ϕ^* in Coulomb gauge, respectively. Indeed, we can reexpress the commutation relations (2.2) as those of the physical field variables using the relation (2.10):

$$
\begin{aligned}
&[\mathcal{A}_i(\mathbf{x}), \mathcal{A}_j(\mathbf{y})] = [\mathcal{E}_i(\mathbf{x}), \mathcal{E}_j(\mathbf{y})] = 0,\\
&[\mathcal{E}_i(\mathbf{x}), \mathcal{A}_j(\mathbf{x}')] = -i\,\delta_{ij}^{tr}(\mathbf{x}-\mathbf{y}),\\
&[\tilde{\phi}(\mathbf{x}), \tilde{\pi}(\mathbf{y})] = [\tilde{\phi}^*(\mathbf{x}), \tilde{\pi}^*(\mathbf{y})] = i\,\delta^2(\mathbf{x}-\mathbf{y}),\\
&[\tilde{\phi}(\mathbf{x}), \mathcal{A}_i(\mathbf{y})] = [\tilde{\phi}(\mathbf{x}), \mathcal{E}_i(\mathbf{y})] = 0,
\end{aligned}
\tag{2.13}
$$

where $\delta_{ij}^{tr}(\mathbf{x}-\mathbf{y})$ is given by

$$
\delta_{ij}^{tr}(\mathbf{x}-\mathbf{y}) = \int \frac{d^2k}{(2\pi)^2} e^{i\mathbf{k}\cdot(\mathbf{x}-\mathbf{y})}\left(\delta_{ij} - \frac{k_i k_j}{\mathbf{k}^2}\right).
$$

Also using the relation (2.10), we obtain

$$
[A_0(\mathbf{x}) - U(\mathbf{x}), \tilde{\phi}(\mathbf{y})] = eG(\mathbf{x}-\mathbf{y})\tilde{\phi}(\mathbf{y}).
$$

These are exactly the same as the commutation relations of Coulomb gauge in the presence of interaction [21].

For simplicity, we can drop the gauge fixing term in the Langrangian (2.1) without loss of generality since we impose the supplementary conditions (2.8) and (2.9) for physical states and all the dynamical and physical variables commute with these conditions. If we keep the gauge fixing term in the Lagrangian (2.1), we will only obtain the *weakly* identical results after working out all the commutator algebra.

Let us define an angular momentum operator constructed from the gauge invariant, symmetric energy-momentum tensor defined as

$$
\delta S = \frac{1}{2}\int_{\mathcal{M}} \sqrt{g}\,\delta g^{\mu\nu} T_{\mu\nu}. \tag{2.14}
$$

The energy-momentum tensor $T_{\mu\nu}$ for the Langrangian (2.1) is then given by

$$
\begin{aligned}
T_{\mu\nu} = & -F_{\mu\lambda}F^{\lambda}_{\nu} + (D_\mu\phi)^* D_\nu\phi + (D_\nu\phi)^* D_\mu\phi\\
& - g_{\mu\nu}\left\{-\frac{1}{4}F_{\lambda\rho}F^{\lambda\rho} + |D_\lambda\phi|^2\right\}.
\end{aligned}
\tag{2.15}
$$

Our interest is the rotational property of physical fields. In order to calculate the angular momentum operator L, we begin with the symmetric energy-momentum tensor in Eq. (2.15):

$$
\begin{aligned}
L = & \int d^2y\,\epsilon^{ij} y_i T_{0j} = \int d^2y\,\epsilon^{ij} y_i \{\tilde{\pi}(\mathbf{y})\partial_j\tilde{\phi}(\mathbf{y})\\
& + \tilde{\pi}^*(\mathbf{y})\partial_j\tilde{\phi}^*(\mathbf{y})\} - \int d^2y\,\epsilon^{ij} y_i \mathcal{A}_j(\mathbf{y}) J_0(\mathbf{y})\\
& + \int d^2y\,\epsilon^{ij} y_i \{E^k(\mathbf{y}) - \partial^k A_0(\mathbf{y})\}\{\partial_k\mathcal{A}_j(\mathbf{y}) - \partial_j\mathcal{A}_k(\mathbf{y})\}\\
= & \int d^2y\,\epsilon^{ij} y_i \{\tilde{\pi}(\mathbf{y})\partial_j\tilde{\phi}(\mathbf{y}) + \tilde{\pi}^*(\mathbf{y})\partial_j\tilde{\phi}^*(\mathbf{y})\}\\
& + \int d^2y\,\epsilon^{ij} y_i \dot{\mathcal{A}}_k(\mathbf{y})\partial_j\mathcal{A}_k(\mathbf{y}) - \int d^2y\,\epsilon^{ij}\dot{\mathcal{A}}_i(\mathbf{y})\mathcal{A}_j(\mathbf{y}).
\end{aligned}
\tag{2.16}
$$

In order to find the final expression (2.16), we used the Gauss-law constraint (2.9). Because of a particular feature in three dimensions, the last term in Eq. (2.16), would-be-spin term, identically vanishes since the massless vector theory has only one degree of freedom so that there is no additional degree of freedom available to form nonzero spin states (if we take $\mathcal{A}_i = \epsilon^{ij}\partial_j\xi, \epsilon^{ij}\dot{\mathcal{A}}_i\mathcal{A}_j$ is then the total derivative). Although the second term in Eq. (2.16) does not verify the commutation relation characteristic of angular momentum [remember $[\mathcal{A}_i(\mathbf{x}), \dot{\mathcal{A}}_j(\mathbf{y})] = i\,\delta_{ij}^{tr}(\mathbf{x}-\mathbf{y})$], it can be shown that this term is independent of the possible polarizations of photons, so purely ''orbital'' in terms of an appropriate choice of the polarization vectors satisfying transversality condition. Thus a massless vector theory in three dimensions is spinless, confirming the result of Binegar [22].

Note that one can make use of supplementary conditions only after we have worked out all the commutators [24]. Following this rule, we get

$$
\begin{aligned}
& [L, \tilde{\phi}(\mathbf{x})]\\
& = \int d^2y\,\epsilon^{ij} y_i [\tilde{\pi}(\mathbf{y})\partial_j\tilde{\phi}(\mathbf{y}) + \tilde{\pi}^*(\mathbf{y})\partial_j\tilde{\phi}^*(\mathbf{y}), \tilde{\phi}(\mathbf{x})]\\
& \quad - \int d^2y\,\epsilon^{ij} y_i \mathcal{A}_j(\mathbf{y})[J_0(\mathbf{y}), \tilde{\phi}(\mathbf{x})]\\
& \quad + \int d^2y\,\epsilon^{ij} y_i [E^k(\mathbf{y}), \tilde{\phi}(\mathbf{x})]\{\partial_k\mathcal{A}_j(\mathbf{y}) - \partial_j\mathcal{A}_k(\mathbf{y})\}\\
& = -i\,\epsilon^{ij} x_i \partial_j \tilde{\phi}(\mathbf{x}).
\end{aligned}
\tag{2.17}
$$

Since the angular momentum operator is Hermitian, we also obtain

$$
[L, \tilde{\phi}^*(\mathbf{x})] = -i\,\epsilon^{ij} x_i \partial_j \tilde{\phi}^*(\mathbf{x}). \tag{2.18}
$$

These are natural results since there is no reason for an anomalous spin in the Maxwell theory.

However, if we do not use the physical fields commuting with supplementary conditions, we will have extra terms which depend on gauge fields. They will make the physical interpretation about rotational property of fields obscure. It suggests that before turning to dynamics, one should first solve constraints (2.8) and (2.9), i.e., one has to find all physical objects. This is one of the reasons we consider the physical variables.

B. Symmetry-broken phase

For the Maxwell theory of symmetric phase, our prescription of physical variables on gauge fields has based on transverse-longitudinal decomposition of the fields. However, in the case of symmetry-broken phase, we will be faced with a problem, zero-momentum ambiguity in the transverse-longitudinal decomposition of gauge fields. This ambiguity interrupts us from defining the Poincaré algebra in terms of physical variables and thus extracting the spin content of gauge fields [20]. Thus we will use two alternative and complementary prescriptions in order to identify physical spectrums.

To begin with, we introduce a symmetry breaking potential $V(\phi)$ in the Lagrangian (2.1). And, for definite physical spectrums, consider the following parametrization of a charged scalar field $\phi(x)$:

$$\phi(x)=\frac{1}{\sqrt{2}}\{v+\varphi(x)\}e^{i\chi(x)/v}, \tag{2.19}$$

where vacuum expectation value v of ϕ is nonzero. Now the Lagrangian (2.1) with the potential $V(\phi)$ becomes in terms of the Higgs field $\varphi(x)$ and the would-be-Goldstone boson $\chi(x)$,

$$\mathcal{L}=-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}+\frac{1}{2}(\partial_\mu\varphi)^2+\frac{1}{2}e^2(v+\varphi)^2\overline{A}_\mu^2-V(v,\varphi)+\mathcal{L}_{GF}, \tag{2.20}$$

where $\overline{A}_\mu$ is defined by

$$\overline{A}_\mu\equiv A_\mu+\frac{1}{m}\partial_\mu\chi, \quad m=ev. \tag{2.21}$$

The Lagrangian (2.20) has no constraint-nonsingular theory, so the canonical quantization is straightforward. From the Lagrangian (2.20), one can obtain the conjugate momenta π_φ and π_χ of φ and χ,

$$\pi_\varphi=\frac{\partial\mathcal{L}}{\partial\dot{\varphi}}=\dot{\varphi}, \quad \pi_\chi=\frac{\partial\mathcal{L}}{\partial\dot{\chi}}=\frac{e}{v}(v+\varphi)^2\overline{A}_0,$$

and their equal-time commutation relations are

$$\begin{aligned}
&[\varphi(\mathbf{x}),\pi_\varphi(\mathbf{y})]=[\chi(\mathbf{x}),\pi_\chi(\mathbf{y})]=i\delta^2(\mathbf{x}-\mathbf{y}),\\
&[\varphi(\mathbf{x}),\pi_\chi(\mathbf{y})]=[\chi(\mathbf{x}),\pi_\varphi(\mathbf{y})]=0,\\
&[\varphi(\mathbf{x}),\chi(\mathbf{x}')]=[\pi_\varphi(\mathbf{x}),\pi_\chi(\mathbf{y})]=0.
\end{aligned} \tag{2.22}$$

The commutation relations of gauge fields are identical with those in the symmetric phase.

Note that the gauge symmetry still remains in the symmetry-broken phase as long as we keep the would-be Goldstone boson χ in the Lagrangian (2.20) except the gauge fixing term. The supplementary conditions in the symmetry-broken phase correspond to this gauge symmetry. They are also equal to Eqs. (2.8) and (2.9), except that $J_0(\mathbf{x})$ is given by

$$J_0(\mathbf{x})=-m\pi_\chi(\mathbf{x}). \tag{2.23}$$

According to these supplementary conditions, one can find that the variables $\mathcal{A}^i,\mathcal{E}^i,\varphi,\pi_\varphi,A_i^L\equiv\partial_i(V+\chi/m)$, and $\pi_{A_i^L}\equiv m\partial_i\pi_\chi/(-\nabla^2+m^2)+m^2\partial_i(U-A_0)/(-\nabla^2+m^2)$ are the only independent physical variables, i.e., commute with F and G.

Consider the symmetric energy-momentum tensor defined by Eq. (2.14):

$$\begin{aligned}
T_{\mu\nu}=&-F_{\mu\lambda}F^\lambda_{\ \nu}+\partial_\mu\varphi\partial_\nu\varphi+e^2(v+\varphi)^2\overline{A}_\mu\overline{A}_\nu\\
&-g_{\mu\nu}\left\{-\frac{1}{4}F_{\lambda\rho}F^{\lambda\rho}+\frac{1}{2}(\partial_\lambda\varphi)^2\right.\\
&\left.+\frac{1}{2}e^2(v+\varphi)^2\overline{A}_\lambda^2-V(v,\varphi)\right\}.
\end{aligned} \tag{2.24}$$

As in the symmetric phase, we have dropped the gauge fixing term $\mathcal{L}_{GF}$ from the Lagrangian (2.20) since we will deal with only physical variables commuting with the gauge fixing term. According to Eq. (2.24), for a small excitation φ, the Hamiltonian is

$$\begin{aligned}
H\approx&\frac{1}{2}\int d^2x\{\mathcal{E}_i^2+\mathcal{B}^2+\pi_\varphi^2\\
&+(\nabla\varphi)^2+\pi_\chi^2+m^2(A_i^L)^2+m^2\mathcal{A}_i^2\}\\
&-\frac{1}{2}\int d^2xd^2yJ_0(\mathbf{x})G(\mathbf{x}-\mathbf{y})J_0(\mathbf{y})+H_{int}+V(v,\varphi),\\
=&\frac{1}{2}\int d^2x\left\{\mathcal{E}_i^2+\mathcal{B}^2+\pi_\varphi^2+(\nabla\varphi)^2+\pi_{A_i^L}\right.\\
&\left.\times\left(1-\frac{\nabla^2}{m^2}\right)\pi_{A_i^L}+m^2(A_i^L)^2+m^2\mathcal{A}_i^2\right\}+H_{int}+V(v,\varphi),
\end{aligned} \tag{2.25}$$

where $\mathcal{B}\equiv\epsilon^{ij}\partial_i\mathcal{A}^j$. The Coulomb-like energy appears as the result of applying the supplementary condition (2.9) with Eq. (2.23). After a canonical transformation, which is a Bogoliubov transformation with respect to A_i^L defined by $\tilde{\pi}_{A_i^L} = \sqrt{1-(\nabla^2/m^2)}\pi_{A_i^L}$ and $\tilde{A}_i^L = A_i^L/\sqrt{1-\nabla^2/m^2}, H$ finally becomes

$$H=\frac{1}{2}\int d^2x\{\mathcal{E}_i^2+\mathcal{B}^2+\pi_\varphi^2+(\nabla\varphi)^2+\tilde{\pi}_{A_i^L}{}^2+(\nabla\tilde{A}_i^L)^2 + m^2\mathcal{A}_i^2+m^2(\tilde{A}_i^L)^2\}+H_{int}+V(v,\varphi). \tag{2.26}$$

This result confirms that the vector fields are excitations with mass m.

However, note that the following expression about the angular momentum L does not show the result of Binegar [22] that a massive vector field is spin 1:

$$\begin{aligned} L &= \int d^2y\,\epsilon^{ij}y_iT_{0j} \\ &= \int d^2y\,\epsilon^{ij}y_i\{\pi_\varphi(\mathbf{y})\partial_j\varphi(\mathbf{y}) \\ &\quad + \dot{\mathcal{A}}_k(\mathbf{y})\partial_j\mathcal{A}_k(\mathbf{y})+\pi_{A_k^L}(\mathbf{y})\partial_jA_k^L(\mathbf{y})\}, \end{aligned} \tag{2.27}$$

where we have dropped would-be-spin terms on vector fields since they are total derivatives and vanishes at spatial infinity. Notice that, for massive theory, the transverse-longitudinal decompositions, Eqs. (2.6) and (2.7), are ambiguous in zero-momemtum limit [20]. Since the spin of one-particle states can be characterized by the value of the angualar momentum of the particle about an arbitrary axis when the particle is at rest, it is not unfortunately a good prescription for the spin contents of massive vector fields to deal with the transverse and longitudinal components separately. Of course, as considered in Ref. [20], after the removal of an infrared singularity of boost generators, one can fix the spin of vector excitations. In Ref. [23], the determination of the spin of massive vector fields was already performed on along this line, where it was shown that the massive vectors carry spin 1. This result implies a spin phase transition in the three-dimensional Maxwell-Higgs theory.

If now the would-be Goldstone boson χ is gauged away, there is no supplementary condition because there is no gauge symmetry. And the canonical variables A_0 and π_0 are removed through Dirac brackets since the constraints, $\pi_0 \approx 0$ and $\dot{\pi}_0 \approx 0$, are second-class [24,25]. Thus, there is no need to explicitly specify physical variables as the field algebra includes only observable fields, A_1 and A_2, as already confirmed. In addition, if we would not take the transverse-longitudinal decomposition of the massive vector fields which is observer-dependent, the two degrees of freedom will be available to form parity doubled, nonzero spin states. Then, it can be shown with no wonder the Poincaré algebra is well-defined and so the spin contents of vector fields can be extracted from it with no ambiguity. We will perform this in the Appendix.

III. ANYONIC PHYSICAL OBSERVABLES AND SPIN PHASE TRANSITION

A. Symmetric phase

In $(2+1)$-dimensional Maxwell theory, we showed that the fields have no rotational anomaly in any phase, so that there is no exotic statistics. But this story is dramatically changed in Chern-Simons theory. The model we wish to analyze is Chern-Simons gauge theory coupled to complex scalar fields. The Lagrangian density in a symmetric phase is given by

$$\begin{aligned} \mathcal{L} &= \frac{\kappa}{4}\epsilon^{\mu\nu\lambda}A_\mu F_{\nu\lambda}+|D_\mu\phi|^2+\mathcal{L}_{GF}, \\ \mathcal{L}_{GF} &= -\frac{1}{2}(\partial_\mu A^\mu)^2. \end{aligned} \tag{3.1}$$

In spite of gauge fixing term, there are primary constraints given by

$$\pi^i-\frac{\kappa}{2}\epsilon^{ij}A_j\approx 0, \quad i,j=1,2, \tag{3.2}$$

which are second-class constraints which no longer lead to secondary constraints. We shall follow the Dirac procedure to eliminate these constraints. We now proceed to quantize the theory canonically by introducing Dirac brackets in the standard manner [24,25]. The nonvanishing set of equal-time commutation relations is

$$\begin{aligned} &[A_0(\mathbf{x}),\pi_0(\mathbf{y})]=i\delta^2(\mathbf{x}-\mathbf{y}), \\ &[A_i(\mathbf{x}),A_j(\mathbf{y})]=\frac{i}{\kappa}\epsilon_{ij}\delta^2(\mathbf{x}-\mathbf{y}), \\ &[\phi(\mathbf{x}),\pi(\mathbf{y})]=[\phi^*(\mathbf{x}),\pi^*(\mathbf{y})]=i\delta^2(\mathbf{x}-\mathbf{y}). \end{aligned} \tag{3.3}$$

As in the Maxwell theory, let us decompose the gauge field $A_i(\mathbf{x})$ into transverse and longitudinal parts,

$$A_i(\mathbf{x})=\mathcal{A}_i(\mathbf{x})+\frac{\partial V}{\partial x^i}=\epsilon_{ij}\frac{\partial U}{\partial x^j}+\frac{\partial V}{\partial x^i}.$$

Then one can show that the fields $U(\mathbf{x})$ and $V(\mathbf{y})$ satisfy the following commutation relation using the second relation in Eq. (3.3):

$$[U(\mathbf{x}),V(\mathbf{y})]=-\frac{i}{\kappa}G(\mathbf{x}-\mathbf{y}). \tag{3.4}$$

As disscussed in Maxwell theory, we must impose the supplementary conditions on any state $|P\rangle$ in order to return to the consistent original theory from the modified Lagrangian (3.1). All the supplementary conditions affecting dynamical variables at one instant of time are

$$F\equiv\partial_\mu A^\mu=\partial_0A_0-\nabla^2V\approx 0, \tag{3.5}$$

$$G\equiv-\kappa\epsilon^{ij}\partial_iA_j+J_0=\kappa\nabla^2U+J_0\approx 0, \tag{3.6}$$

where charge density $J_0(\mathbf{x})$ is given by

$$J_0(\mathbf{x}) = ie\{\pi^*(\mathbf{x})\phi^*(\mathbf{x}) - \pi(\mathbf{x})\phi(\mathbf{x})\}.$$

Of course, F and G commute with each other. Then any components of Chern-Simons gauge field A_μ or any combinations of them are not physical, i.e., do not commute with F and G. Even any component of the field tensor $F_{\mu\nu}$ is not physical, although it is gauge invariant. As a well-known fact [1], there exists no real photon in pure Chern-Simons theory of symmetric phase.

The variable ϕ is not physical since it commutes with F but not with G. However, if we define [13]

$$\tilde{\phi}(x) = e^{ieV(x)}\phi(x), \tag{3.7}$$

$\tilde{\phi}(x)$ commutes with F and G and thus physical. Likely, $\tilde{\phi}^*(x)$ is physical. Then, the variables $\tilde{\phi}(x), \tilde{\pi}(x), \tilde{\phi}^*(x)$, and $\tilde{\pi}^*(x)$ are the only independent physical variables, apart from the quantities (3.5) and (3.6). From the commutation relations (3.3) and (3.4), we observe that these physical field operators $\tilde{\phi}(\mathbf{x})$ and $\tilde{\phi}^*(\mathbf{x})$ create a flux quantum as well as a $U(1)$ charge,

$$Q\{\tilde{\phi}(\mathbf{x})|P\rangle\} = (q-e)\tilde{\phi}(\mathbf{x})|P\rangle,$$

$$\Phi\{\tilde{\phi}(\mathbf{x})|P\rangle\} = \left(b + \frac{e}{\kappa}\right)\tilde{\phi}(\mathbf{x})|P\rangle,$$

$$Q\{\tilde{\phi}^*(\mathbf{x})|P\rangle\} = (q+e)\tilde{\phi}^*(\mathbf{x})|P\rangle,$$

$$\Phi\{\tilde{\phi}^*(\mathbf{x})|P\rangle\} = \left(b - \frac{e}{\kappa}\right)\tilde{\phi}^*(\mathbf{x})|P\rangle, \tag{3.8}$$

where $Q = \int d^2x J_0(\mathbf{x})$, $\Phi = \int d^2x B(\mathbf{x})$ and a state $|P\rangle$ is assumed to be a simultaneous eigenstate of Q and Φ with eigenvalues q and b, respectively. These properties show a manifest evidence that Chern-Simons gauge fields attach a flux quantum proportional to $U(1)$ charge to complex scalar field, which is a dual picture that a physical electron in Maxwell theory carries Coulomb fields surrounding the charge. In the presence of Chern-Simons gauge fields, the complex scalar field, $\tilde{\phi}(\mathbf{x})$ or $\tilde{\phi}^*(\mathbf{x})$, becomes a boson plus flux composite and, as we will see, this composite dynamically will be an anyon by the Aharonov-Bohm effect [26]. That is, if we interchange the two field quanta with charge q, the statistical phase by the Aharonov-Bohm effect is equal to $q^2/2\kappa$. Indeed, we will construct the composite anyon operators satisfying fractional statistics [4] under a covariant gauge fixing.

One can solve the Gauss-law constraint in Fock space represented by the physical variables which satisfy supplementary conditions (3.5) and (3.6):

$$U(\mathbf{x}) = -\frac{1}{\kappa}\int d^2y G(\mathbf{x}-\mathbf{y})J_0(\mathbf{y}), \tag{3.9}$$

$$V(\mathbf{x}) = \int d^2y G(\mathbf{x}-\mathbf{y})\partial_0 A_0(\mathbf{y}). \tag{3.10}$$

The gauge field $A_i(\mathbf{x})$ can be thus expressed as the following combination:

$$A_i(\mathbf{x}) = -\frac{1}{\kappa}\epsilon_{ij}\partial_j\int d^2y G(\mathbf{x}-\mathbf{y})J_0(\mathbf{y}) + \partial_i\int d^2y G(\mathbf{x}-\mathbf{y})\partial_0 A_0(\mathbf{y}).$$

The angular momentum operator obtained from symmetric energy-momentum tensor given by

$$T_{\mu\nu} = (D_\mu\phi)^* D_\nu\phi + (D_\nu\phi)^* D_\mu\phi - g_{\mu\nu}|D_\lambda\phi|^2$$

is represented as follows:

$$\begin{aligned} L &= \int d^2y \epsilon^{ij} y_i T_{0j} \\ &= \int d^2y \epsilon^{ij} y_i \{\tilde{\pi}(\mathbf{y})\partial_j\tilde{\phi}(\mathbf{y}) + \tilde{\pi}^*(\mathbf{y})\partial_j\tilde{\phi}^*(\mathbf{y})\} \\ &\quad - \int d^2y \epsilon^{ij} y_i \mathcal{A}_j(\mathbf{y})J_0(\mathbf{y}) \\ &= \int d^2y \epsilon^{ij} y_i \{\tilde{\pi}(\mathbf{y})\partial_j\tilde{\phi}(\mathbf{y}) + \tilde{\pi}^*(\mathbf{y})\partial_j\tilde{\phi}^*(\mathbf{y})\} + \frac{Q^2}{4\pi\kappa}, \end{aligned} \tag{3.11}$$

where we used Eq. (3.9) in the final step. For the same reason as in Sec. II, we have safely dropped the gauge fixing term. First, note that physical and gauge invariant scalar fields $\tilde{\phi}(x)$ and $\tilde{\phi}^*(x)$ have a anomalous spin. That is,

$$\begin{aligned} [L, \tilde{\phi}(\mathbf{x})] &= -i\epsilon^{ij}x_i\partial_j\tilde{\phi}(\mathbf{x}) \\ &\quad - \int d^2y \epsilon^{ij} y_i [\mathcal{A}_j(\mathbf{y})J_0(\mathbf{y}), \tilde{\phi}(\mathbf{x})] \\ &= -i\epsilon^{ij}x_i\partial_j\tilde{\phi}(\mathbf{x}) \\ &\quad + \frac{e}{\kappa}\int d^2y (x_i\partial_i^x + y_i\partial_i^y)G(\mathbf{x}-\mathbf{y})J_0(\mathbf{y})\tilde{\phi}(\mathbf{x}) \\ &= -i\epsilon^{ij}x_i\partial_j\tilde{\phi}(\mathbf{x}) - \frac{eQ}{2\pi\kappa}\tilde{\phi}(\mathbf{x}), \end{aligned} \tag{3.12}$$

where we have computed the commutator using the relations (3.3) and (3.4) and used the expression (3.9) after this computation. Similarly,

$$[L, \tilde{\phi}^*(\mathbf{x})] = -i\epsilon^{ij}x_i\partial_j\tilde{\phi}^*(\mathbf{x}) + \frac{eQ}{2\pi\kappa}\tilde{\phi}^*(\mathbf{x}). \tag{3.13}$$

These results agree with the previous ones [1] under Coulomb gauge.

In symmetric phase, the Chern-Simons gauge fields are by themselves nondynamical as the result of ‘‘too much symmetry''—diffeomorphism invariance. As a result of this ''too much symmetry,'' these gauge fields remain confined, but change a boundary condition of coupled fields in the same way that the unphysical fields of Maxwell theory, i.e., scalar and longitudinal photons, result in an infrared dressing of static Coulomb field to charged matter fields [13]. So there can be two points of view describing the charged matter system coupled to Chern-Simons gauge fields. One is that a part of dynamical informations (boundary condition of the coupled fields) assigns to the Chern-Simons gauge fields through an interaction. The other is that all the dynamical informations are assigned to the charged matter fields by removing the Chern-Simons gauge fields through a singular gauge transformation initiated by Semenoff [4]. However, in quantum field theory described by *smooth* fields, the removing of a topological term with diffeomorphism invariance—Chern-Simons term—through singular gauge transformations is in general impossible and instead remains a remnant [27]. This phenomenon seems to be a quite general feature of bosonization in a continuum field theory in higher dimension $D \geqslant 3$.

The presence of nondynamical Chern-Simons gauge fields leads a *scalar field* to anomalous spin term. In order to incorporate the relation between spin and statistics, we shall construct anyon operators satisfying graded commutation relations showing the fractional statistics [4]. This is based on the fact that the choice *à la* Semenoff with respect to the physical variables about complex fields ϕ and ϕ^* is also true:

$$\hat{\phi}(x) = e^{\{i(e/2\pi\kappa)\int d^2y \Theta(\mathbf{x}-\mathbf{y}) J_0(\mathbf{y}) + ieV(\mathbf{x})\}} \phi(x) \equiv S(x)\phi(x), \tag{3.14}$$

$$\hat{\phi}^*(x) = e^{\{-i(e/2\pi\kappa)\int d^2y \Theta(\mathbf{x}-\mathbf{y}) J_0(\mathbf{y}) - ieV(\mathbf{x})\}} \phi^*(x) \equiv S^{-1}(x)\phi^*(x). \tag{3.15}$$

We introduced the multivalued function $\Theta(\mathbf{x})$ satisfying ''Cauchy-Riemann'' equations

$$\partial_i G(\mathbf{x}) = \frac{1}{2\pi} \epsilon_{ij} \partial_j \Theta(\mathbf{x}) \tag{3.16}$$

and the resulting function $\Theta(\mathbf{x})$ satisfies the following properties:

$$\tan \Theta(\mathbf{x}) = \frac{x_2}{x_1}, \quad \partial_i \Theta(\mathbf{x}) = -\epsilon_{ij} \frac{x^j}{\mathbf{x}^2},$$
$$\nabla \times \nabla \Theta(\mathbf{x}) = 2\pi \delta^2(\mathbf{x}), \quad \nabla^2 \Theta(\mathbf{x}) = 0. \tag{3.17}$$

Now we want to see the properties of the *hat* fields through careful analysis. First, let us express $\pi(x)$ in terms of the hat fields:

$$\begin{aligned} \pi(x) &= (D_0\phi(x))^* \\ &= \{(\partial_0 + ieA_0) S^{-1}(x) \hat{\phi}(x)\}^* \\ &= S(x)\left[\left\{\partial_0 - ie\int d^2y G(\mathbf{x}-\mathbf{y}) \Box A_0(\mathbf{y}) \right.\right. \\ &\quad \left.\left. - i\frac{e}{2\pi\kappa}\int d^2y \Theta(\mathbf{x}-\mathbf{y}) \partial_i J_i(\mathbf{y})\right\}\hat{\phi}(\mathbf{x})\right]^*, \end{aligned}$$

where we used Eq. (3.10) and the current conservation law. After using the equation of motion about A_0 and integration by part, we arrive at the following result:

$$\begin{aligned} & S(x)\left[\left\{\partial_0 - i\frac{e}{\kappa}\int d^2y\, \epsilon_{ij}\partial_i^y G(\mathbf{x}-\mathbf{y}) J_j(\mathbf{y}) \right.\right. \\ &\quad \left.\left. - i\frac{e}{2\pi\kappa}\int d^2y \Theta(\mathbf{x}-\mathbf{y})\partial_i^y J_i(\mathbf{y})\right\}\hat{\phi}(\mathbf{x})\right]^* \\ &= S(x)\left[\left\{\partial_0 - i\frac{e}{2\pi\kappa}\int d^2y\, \partial_i^y(\Theta(\mathbf{x}-\mathbf{y}) J_i(\mathbf{y}))\right\}\hat{\phi}(\mathbf{x})\right]^* \\ &= S(x)\{\partial_0\hat{\phi}(x)\}^* \equiv S(x)\hat{\pi}(x), \end{aligned} \tag{3.18}$$

where we used ''Cauchy-Riemann'' equation (3.16) and assumed that current density rapidly decreases at large r. In the same way, we can show that

$$\pi^*(x) = S^{-1}(x)\{\partial_0\hat{\phi}(x)\} \equiv S^{-1}(x)\hat{\pi}^*(x). \tag{3.19}$$

Second, express $D_i\phi(x)$ in terms of the hat fields:

$$\begin{aligned} D_i\phi(x) &= (\partial_i + ieA_i)\{S^{-1}(x)\hat{\phi}(x)\} \\ &= S^{-1}(x)\partial_i\hat{\phi}(x) + ieS^{-1}(x) \\ &\quad \times\left\{\mathcal{A}_i(x) - \frac{1}{2\pi\kappa}\partial_i\int d^2y \Theta(\mathbf{x}-\mathbf{y}) J_0(\mathbf{y})\right\}\hat{\phi}(x). \end{aligned} \tag{3.20}$$

As a consequence of Eqs. (3.9) and (3.16), $\mathcal{A}_i(x)$ can be rewritten as

$$\mathcal{A}_i(x) = \frac{1}{2\pi\kappa}\int d^2y\, \partial_i^x \Theta(\mathbf{x}-\mathbf{y}) J_0(\mathbf{y}). \tag{3.21}$$

However, unless the charge density $J_0(\mathbf{y})$ is sufficiently well localized, the interchange of integral and derivative in Eq. (3.21) and thus displaying $\mathcal{A}_i(x)$ as a pure gauge is in general not correct [27]. When $J_0(\mathbf{y})$ is smoothly distributed over an extended region, the correct expression for $\mathcal{A}_i$ is

$$\begin{aligned} \mathcal{A}_i(\mathbf{x}) &= \frac{1}{2\pi\kappa}\partial_i\int d^2y \Theta(\mathbf{x}-\mathbf{y}) J_0(\mathbf{y}) \\ &\quad - \frac{1}{2\pi\kappa}\epsilon_{ij}\int d\theta\, d\mathbf{z}_\theta^j J_0(\mathbf{x};\mathbf{z}), \end{aligned} \tag{3.22}$$

where the line integral of J_0 is along the cut line introduced to integrate the multivalued function Θ and θ is the polar angle of the cut line. In order to remove an arbitrary directional dependence of minimal coupling arising from the choice of cut-line, we considered the averaging process for all possible choices of cut-line. This expression is still consistent with Gauss-law constraint $B=\nabla\times\mathbf{A}=-(1/\kappa)J_0$. For the sufficiently well localized charge density such as nonrelativistic quantum field theory [27] or theory on the lattice [28], the second term in Eq. (3.22) can be dropped with impunity. Anyway, if we can *neglect* the contribution of the remnant in Eq. (3.22) for some distribution J_0, we finally arrive at the following result:

$$D_i\phi(x)=S^{-1}(x)\partial_i\hat{\phi}(x),\quad (D_i\phi(x))^*=S(x)\partial_i\hat{\phi}^*(x).$$

The Hamiltonian of hat fields then takes the form

$$H=\int d^2x\left\{\frac{1}{2}\hat{\pi}^2+\frac{1}{2}(\nabla\hat{\phi})^2\right\},\tag{3.23}$$

and their angular momentum operator is given by

$$L=\int d^2x\epsilon^{ij}x_i\{\hat{\pi}\partial_j\hat{\phi}+\hat{\pi}^*\partial_j\hat{\phi}^*\}.\tag{3.24}$$

To study the statistics of hat fields, we use the identities by the Baker-Campbell-Hausdorff formula,

$$\begin{aligned}S(x)\phi(z)S^{-1}(x)&=e^{-i(e^2/2\pi\kappa)\Theta(\mathbf{x}-\mathbf{z})}\phi(z),\\ S(x)\pi(z)S^{-1}(x)&=e^{i(e^2/2\pi\kappa)\Theta(\mathbf{x}-\mathbf{z})}\pi(z).\end{aligned}\tag{3.25}$$

The commutation relations of hat fields now obey the graded commutation relations [4],

$$\begin{aligned}\hat{\phi}(\mathbf{x})\hat{\phi}(\mathbf{y})&=e^{-i(e^2\Delta/2\pi\kappa)}\hat{\phi}(\mathbf{y})\hat{\phi}(\mathbf{x}),\\ \hat{\phi}(\mathbf{x})\hat{\phi}^*(\mathbf{y})&=e^{i(e^2\Delta/2\pi\kappa)}\hat{\phi}^*(\mathbf{y})\hat{\phi}(\mathbf{x}),\\ \hat{\phi}(\mathbf{x})\hat{\pi}(\mathbf{y})&=i\delta^2(\mathbf{x}-\mathbf{y})+e^{i(e^2\Delta/2\pi\kappa)}\hat{\pi}(\mathbf{y})\hat{\phi}(\mathbf{x}),\\ \hat{\phi}(\mathbf{x})\hat{\pi}^*(\mathbf{y})&=e^{-i(e^2\Delta/2\pi\kappa)}\hat{\pi}^*(\mathbf{y})\hat{\phi}(\mathbf{x}),\\ \hat{\pi}(\mathbf{x})\hat{\pi}(\mathbf{y})&=e^{-i(e^2\Delta/2\pi\kappa)}\hat{\pi}(\mathbf{y})\hat{\pi}(\mathbf{x}),\\ \hat{\pi}(\mathbf{x})\hat{\pi}^*(\mathbf{y})&=e^{i(e^2\Delta/2\pi\kappa)}\hat{\pi}^*(\mathbf{y})\hat{\pi}(\mathbf{x}),\end{aligned}\tag{3.26}$$

with multivalued phase

$$\Delta=\Theta(\mathbf{x}-\mathbf{y})-\Theta(\mathbf{y}-\mathbf{x})=\pi \text{ mod } 2\pi n.$$

These multivalued operators carry fractional statistics and may be regarded as anyon operators since they create a state with arbitray spin when acting on a physical state. The statistical phases in the graded commutation relations (3.26) are exactly equal to the Aharonov-Bohm phase for the field quanta satisfying Eq. (3.8). Consequently, we see that the Aharonov-Bohm effect is the origin of anyon statistics.

Note that the representation of a physical variable depends on the gauge fixing and one needs to first find all physical variables before turning to dynamics in order to obtain correct results. This is the lesson learned from Dirac [13].

B. Symmetry-broken phase

We shall now consider when spontaneous symmetry breaking occurs [15]. In the same way as the Maxwell theory, we introduce the symmetry breaking potential $V(\phi)$ and consider the same parametrization of a complex scalar field ϕ. Then the Lagrangian density in the symmetry-broken phase is given by

$$\begin{aligned}\mathcal{L}=&\frac{\kappa}{4}\epsilon^{\mu\nu\lambda}A_\mu F_{\nu\lambda}+\frac{1}{2}(\partial_\mu\varphi)^2\\ &+\frac{1}{2}e^2(v+\varphi)^2(\overline{A_\mu})^2-V(v,\varphi)+\mathcal{L}_{GF},\end{aligned}\tag{3.27}$$

where $\overline{A_\mu}$ is defined by

$$\overline{A_\mu}\equiv A_\mu+\frac{1}{ev}\partial_\mu\chi.\tag{3.28}$$

The effective vector action obtained from Eq. (3.27) is

$$\mathcal{L}=\frac{\kappa}{4}\epsilon^{\mu\nu\lambda}A_\mu F_{\nu\lambda}+\frac{1}{2}e^2v^2A_\mu^2,\tag{3.29}$$

after a gauge transformation $A_\mu\to A_\mu-(1/ev)\partial_\mu\chi$. The ''self-dual'' first order system (3.29) has been shown [19] to be equivalent to topologically massive spin 1 theory [20] even in the presence of interaction. Thus one expects that the Chern-Simons gauge field absorbed a would-be Goldstone boson is transmuted into topologically massive helicity 1 excitation in the Higgs' phase. This is the observation of Deser and Yang [18]. But there also exist arguments [16] that the excitation relative to ground state being not rotational invariant is spin 0 boson. We shall resolve this inconsistency based on the same analysis as in Sec. II.

The equal-time commutation relations of Chern-Simons gauge fields are equal to Eq. (3.3) and matter parts are equal to Eq. (2.22). The supplementary conditions are also identical to Eqs. (3.5) and (3.6), except that $J_0(\mathbf{x})$ is given by

$$J_0(\mathbf{x})=-ev\,\pi_\chi(\mathbf{x}).\tag{3.30}$$

According to these supplementary conditions, we can find that the variables $\varphi(x),\pi_\varphi$, $A_i^L\equiv\partial_i(V+\chi/ev)$, and $\pi_{A_i^L}\equiv ev\,\partial_i\pi_\chi/(-\nabla^2+m^2)+\kappa m^2\partial_i U/(-\nabla^2+m^2)$ $(m=e^2v^2/\kappa)$ are the only independent physical variables, apart from the quantities (3.5) and (3.6). Note that the longitudinal Chern-Simons gauge field absorbed would-be Goldstone boson can be a dynamical variable and restore the vector field's dynamics [18] in the Higgs' phase. That is, the Chern-Simons-Higgs theory will have one massive spin 1 and one massive spin 0 (Higgs field) propagating modes. In the case of Maxwell-Chern-Simons-Higgs theory, we again have two parity-violating massive spin 1 modes [23,29]. Although the scalar form $V+\chi/ev$ identified by Boyanovsky is *physical* in the Dirac's sense, the more appropriate choice will be vector form as seen from the experience obtained by the analysis of the Maxwell-Higgs theory in Sec. II. In addition, Boyanovsky's choice will encounter an infrared singularity since $V+\chi/ev=\partial_i A_i^L/\nabla^2$ and this will bring about a superfluous infrared singularity in Poincaré generators.

In order to examine the physical spectrum of the involved fields, consider the symmetric energy-momentum tensor defined by Eq. (2.14):

$$T_{\mu\nu}=\partial_\mu\varphi\partial_\nu\varphi+e^2(v+\varphi)^2\overline{A}_\mu\overline{A}_\nu -g_{\mu\nu}\left\{\frac{1}{2}(\partial_\lambda\varphi)^2+\frac{1}{2}e^2(v+\varphi)^2\overline{A}_\lambda^2-V(v,\varphi)\right\}. \tag{3.31}$$

Then the Hamiltonian becomes

$$\begin{aligned}H&=\int d^2x\left\{\frac{1}{2}(\dot{\varphi}^2+(\nabla\varphi)^2+e^2(v+\varphi)^2\overline{A}_0\overline{A}_0 +e^2(v+\varphi)^2\overline{A}_i\overline{A}_i)+V(v,\varphi)\right\}\\&=\frac{1}{2}\int d^2x\left\{\pi_\varphi^2+(\nabla\varphi)^2+\pi_{A_i^L}\left(\frac{e^2v^2}{\kappa^2}-\frac{\nabla^2}{e^2v^2}\right)\pi_{A_i^L} +e^2v^2(A_i^L)^2\right\}+H_{int}+V(v,\varphi),\end{aligned} \tag{3.32}$$

where Coulomb-like energy appears as the result of applying the supplementary condition (3.6). As in Sec. II, after a Bogoliubov transformation with respect to A_i^L defined by $\tilde{\pi}_{A_i^L}=\sqrt{(e^2v^2/\kappa^2)-(\nabla^2/e^2v^2)}\pi_{A_i^L}$ and $\tilde{A}_i^L=A_i^L/\sqrt{(e^2v^2/\kappa^2)-(\nabla^2/e^2v^2)}$, H becomes

$$H=\frac{1}{2}\int d^2x\left\{\pi_\varphi^2+(\nabla\varphi)^2+\tilde{\pi}_{A_i^L}{}^2+(\nabla\tilde{A}_i^L)^2 +\frac{e^4v^4}{\kappa^2}(\tilde{A}_i^L)^2\right\}+H_{int}+V(v,\varphi). \tag{3.33}$$

This result confirms that the vector field is an excitation with mass $m=e^2v^2/\kappa$.

The angular momentum operator obtained from the symmetric energy-momentum tensor (3.31) is represented by using the supplementary condition (3.6) as follows:

$$\begin{aligned}L&=\int d^2y\,\epsilon^{ij}y_iT_{0j}\\&=\int d^2y\,\epsilon^{ij}y_i\{\pi_\varphi(\mathbf{y})\partial_j\varphi(\mathbf{y})+ev\,\pi_\chi(\mathbf{y})A_j^L(\mathbf{x})\}-\int d^2y\,\epsilon^{ij}y_i\mathcal{A}_j(\mathbf{y})J_0(\mathbf{y})\\&=\int d^2y\,\epsilon^{ij}y_i\{\pi_\varphi(\mathbf{y})\partial_j\varphi(\mathbf{y})+\pi_{A_k^L}(\mathbf{y})\partial_jA_k^L(\mathbf{y})\}+\frac{Q^2}{4\pi\kappa},\end{aligned} \tag{3.34}$$

where we have again dropped a would-be-spin term on vector field since it is total derivative and vanishes at spatial infinity. The angular momentum operator L in Eq. (3.34) seems to have the anomalous term as in Ref. [16] but it is misleading since the last term in Eq. (3.34) induces no effect on fields, i.e.,

$$\left[\int d^2y\,\epsilon^{ij}y_i\mathcal{A}_j(\mathbf{y})J_0(\mathbf{y}),A_i^L(x)\right]=\left[\int d^2y\,\epsilon^{ij}y_i\mathcal{A}_j(\mathbf{y})J_0(\mathbf{y}),\varphi(x)\right]=0. \tag{3.35}$$

Moreover, it is expected that, in the massive vector theory, we will encounter the zero-momentum ambiguity on the transverse-longitudinal decomposition of vector fields, which brings about the zero-momentum anomaly in Poincaré algebra as in Sec. II. Thus the correct spin content of vector fields should be determined by the removing of zero-momentum singularity of boost generator or prescription free from zero-momentum ambiguity which abandons the transverse-longitudinal decomposition of vector fields. In the Appendix, we will obtain the result that the Chern-Simons gauge field A_i^L is a spin 1 excitation, by checking the Poincaré algebra in terms of the prescription free from zero-momentum ambiguity which does not take the transverse-longitudinal decomposition of vector fields.

Deser and Jackiw found [19] that massive Chern-Simons theory is equivalent (by a Legendre transformation) to topologically massive gauge theory [20] which has a parity-violating spin 1 excitation, where the spin of the vector excitation was fixed by the removal of zero-momentum singularity of Poincaré algebra. In Ref. [18], Deser and Yang observed that Higgs mechanism transmutes a non-dynamical Chern-Simons term into topologically massive, parity-violating, spin 1 theory. According to the results, we can conclude that the Chern-Simons gauge field in Higgs' phase is a spin 1 excitation. Consequently, the excitations in Higgs' phase have no anomalous spin and the Chern-Simons gauge field absorbed a would-be Goldstone boson is transmuted into topologically massive helicity 1 excitation. Therefore, the Higgs mechanism transmutes an anyon satisfying fractional statistics into a spin 0 or a spin 1 boson and so there exists interestingly a statistical spin-phase transition as the observation of Wen and Zee [15].

IV. CONCLUSION

We have presented the quantization of a charged matter system coupled to a Chern-Simons gauge field in a covariant gauge fixing. Our approach is based on Dirac's method performed on QED [13] which provides us a definite way of identifying physical spectrums free from gauge ambiguity arising from the gauge fixing and illustrates the importance of consideration of physical field variables. The important point is that the formulation of dynamics in terms of a Lagrangian (or Hamiltonian) and the equations of motion make use of a larger field algebra which includes nonobservable fields and thus one must find all physical variables before turning to dynamics in order to obtain correct results. Then it can be shown quite generally that the physical charged fields are described by nonlocal fields carrying static field, for example, Coulomb fields for Maxwell theory and magnetic flux for Chern-Simons theory. In the case of Chern-Simons theory, we have shown that the static field, i.e., the magnetic flux, attached to charged matter fields is the origin of fractional statistics.

We have also presented the quantization of the Chern-Simons matter system in a symmetry-broken phase and ensured that the Higgs mechanism transmutes a nondynamical Chern-Simons term into topologically massive, parity-violating, spin 1 theory. Thus the Higgs effect transmutes an anyon satisfying fractional statistics into a canonical boson, a spin 0 Higgs boson or a topologically massive photon which is a Chern-Simons gauge field absorbed would-be Goldstone boson. In order to identify correct spectrums, we have used two alternative and complementary prescriptions and found the consistent result with Deser and Jackiw on the spin of massive vector fields and thus removed an inconsistency between Boyanovsky and Deser and Yang. Consequently, it implies that the Higgs effect induces the statistical spin phase transition predicted by Wen and Zee.

We think that the same approach performed in this paper will be applied to Maxwell-Chern-Simons theory as well. For the Maxwell-Chern-Simons theory, there also exist different opinions [10,30] on anyon statistics in symmetric phase. In Higgs' phase, this theory has two parity-violating massive spin 1 photons [23,29] and one spin 0 Higgs field. Thus the problem on the existence of anyon statistics in this model will be involved with the statistical spin phase transition. Study on these issues will be also interesting.

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APPENDIX A: POINCARÉ ALGEBRA OF MASSIVE VECTOR FIELDS IN 2+1 DIMENSIONS

1. Proca theory

In this appendix, we will analyze the Poincaré algebra of Proca theory in 2+1 dimensions and show that the massive vector field in the Proca theory is canonically spin 1.

Consider the following Proca Lagrangian:

$$\mathcal{L}_P = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m^2 A_\mu^2 . \tag{A1}$$

After the canonical quantization of the theory by introducing a Dirac bracket, we obtain the following commutation relations:

$$[A_0(\mathbf{x}), \pi_0(\mathbf{y})] = 0,$$
$$[A_i(\mathbf{x}), \pi^j(\mathbf{y})] = i\delta_{ij}\delta^2(\mathbf{x}-\mathbf{y}), \tag{A2}$$

where conjugate momenta $\pi^j \equiv \partial\mathcal{L}_P/\partial\dot{A}_j = F^{j0}$. Then the energy-momentum tensor $T_{\mu\nu}$ for the Proca theory (A1) is given as

$$T_{\mu\nu} = -F_{\mu\lambda}F^\lambda_{\ \nu} + m^2 A_\mu A_\nu - g_{\mu\nu}\mathcal{L}_P . \tag{A3}$$

With the energy-momentum tensor, the Poincaré generators can be expressed as the following forms:

$$P_i = \int d^2x T_{0i} = \int d^2x \pi^k(x)\partial_i A_k(x),$$

$$H = \int d^2x T_{00} = \int d^2x \left\{ \frac{1}{2}\pi^i(x)\pi^i(x) + \frac{1}{4}F^{ij}(x)F_{ij}(x) + \frac{1}{2m^2}(\partial_i\pi^i(x))^2 + \frac{m^2}{2}A^i(x)A^i(x) \right\},$$

$$L = \int d^2x \epsilon^{ij}x_i T_{0j} = \int d^2x \epsilon^{ij}x_i \pi^k(x)\partial_j A_k(x) - \int d^2x \epsilon^{ij}\pi^i(x)A_j(x), \tag{A4}$$

$$M_{i0} = \int d^2x x^i \left\{ \frac{1}{2}\pi^j(x)\pi^j(x) + \frac{1}{4}F^{jk}(x)F_{jk}(x) + \frac{1}{2m^2}(\partial_j\pi^j(x))^2 + \frac{m^2}{2}A^j(x)A^j(x) \right\} - tP_i ,$$

using the equation of motion, $A_0 = -(1/m^2)\partial_i F^{i0} = -(1/m^2)\partial_i\pi^i$. After a straightforward calculation with the commutation relations (A2), one can check that the Poincaré algebra for the Proca theory is well defined, especially,

$$[M_{i0}, P_j] = -i\delta_{ij}H,$$
$$[M_{i0}, M_{j0}] = -i\epsilon_{ij}L. \tag{A5}$$

The Poincaré algebra is free from zero-momentum anomaly and the angular momentum operator L in Eq. (A4) has a canonical expression for spin 1 theory contrary to that of Eq. (2.27). Here, we confirm the result of Binegar [22] that a massive vector field is spin 1.

2. Massive Chern-Simons theory

In this Appendix, we will analyze the Poincaré algebra of massive Chern-Simons theory in 2+1 dimensions which has only one propagating mode and show that this massive mode in the Chern-Simons theory is canonically spin 1.

Consider the following massive Chern-Simons Lagrangian [19]

$$\mathcal{L}_{CS}=\frac{\kappa}{4}\epsilon^{\mu\nu\lambda}A_\mu F_{\nu\lambda}+\frac{\mu}{2}A_\mu^2. \tag{A6}$$

After the canonical quantization of the theory by introducing a Dirac bracket, we obtain the following commutation relations:

$$[A_0(\mathbf{x}),\pi_0(\mathbf{y})]=0,$$
$$[A_i(\mathbf{x}),A_j(\mathbf{y})]=\frac{i}{\kappa}\epsilon_{ij}\delta^2(\mathbf{x}-\mathbf{y}). \tag{A7}$$

Equation (A7) shows that the Chern-Simons gauge fields A_1 and A_2 are not independent due to the symplectic structure of $\mathcal{L}_{CS}$.

The energy-momentum tensor $T_{\mu\nu}$ for the massive Chern-Simons theory (A6) is given by

$$T_{\mu\nu}=\mu A_\mu A_\nu-\frac{\mu}{2}g_{\mu\nu}A_\lambda A^\lambda. \tag{A8}$$

With the energy-momentum tensor, the Poincaré generators can be expressed as the following forms:

$$\begin{aligned}
P_i&=\int d^2x T_{0i}=\int d^2x\,\kappa B(x)A_i(x)\\
&=\int d^2x\,\pi^k(x)\partial_i A_k(x),\\
H&=\int d^2x T_{00}=\int d^2x\left\{\frac{\kappa^2}{2\mu}(B(x))^2+\frac{\mu}{2}A^i(x)A^i(x)\right\},\\
L&=\int d^2x\,\epsilon^{ij}x_i T_{0j}=\kappa\int d^2x\,\epsilon^{ij}x_i B(x)A_j(x)\\
&=\int d^2x\,\epsilon^{ij}x_i\pi^k(x)\partial_j A_k(x)-\int d^2x\,\epsilon^{ij}\pi^i(x)A_j(x),\\
M_{i0}&=\int d^2x\,x^i\left\{\frac{\kappa^2}{2\mu}(B(x))^2+\frac{\mu}{2}A^j(x)A^j(x)\right\}-tP_i,
\end{aligned} \tag{A9}$$

where $B(x)=\epsilon^{ij}\partial_i A^j(x)$, $\pi^i(x)=(\kappa/2)\epsilon^{ij}A_j(x)$, and $A_0=(\kappa/\mu)B(x)$. After a straightforward calculation using the commutation relations (A7) and $[B(x),B(y)]=0$, one can also check that the Poincaré algebra for the massive Chern-Simons theory is well defined, especially,

$$[M_{i0},P_j]=-i\delta_{ij}H,$$
$$[M_{i0},M_{j0}]=-i\epsilon_{ij}L. \tag{A10}$$

The Poincaré algebra is also free from zero-momentum anomaly and the angular momentum operator L in Eq. (A9) has a canonical expression for spin 1 theory. Thus, we confirm the result of Deser and Jackiw [19] that the excitation of the massive Chern-Simons theory is spin 1.

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