Dynamics in the conformal window in QCD-like theories

V. A. Miransky
Bogolyubov Institute for Theoretical Physics, 252143, Kiev, Ukraine
and Department of Physics, Nagoya University, Nagoya 464-8602, Japan
(Received 14 December 1998; published 26 March 1999)

The dynamics with an infrared stable fixed point in the conformal window in QCD-like theories (with a relatively large number of fermion flavors) is studied. The dependence of masses of colorless bound states on a bare fermion mass is described. In particular it is shown that in such dynamics glueballs are much lighter than bound states composed of fermions, if the value of the infrared fixed point is not too large. This yields a clear signature for the conformal window, which in particular can be useful for lattice computer simulations of these gauge theories. [S0556-2821(99)04408-2]

PACS number(s): 11.30.Rd, 11.30.Qc, 12.38.Aw

Recently, there has been considerable interest in the existence of a nontrivial conformal dynamics in $(3+1)$-dimensional nonsupersymmetric vectorlike gauge theories, with a relatively large number of fermion flavors N_f [1–6]. The roots of this problem go back to a work by Banks and Zaks [7] who were the first to discuss the consequences of the existence of an infrared-stable fixed point $\alpha=\alpha^*$ for $N_f>N_f^*$ in vectorlike gauge theories. The value N_f^* depends on the gauge group: in the case of the SU(3) gauge group, $N_f^*=8$ in the two-loop approximation. New insight in this problem [1,2] has been, on the one hand, connected with using the results of the analysis of the Schwinger-Dyson (SD) equations describing chiral symmetry breaking in QCD (for a review, see Refs. [8,9]) and, on the other hand, with the discovery of the conformal window in $N=1$ supersymmetric QCD [10].

In particular, Appelquist, Terning, and Wijewardhana [1] suggested that, in the case of the gauge group SU(N_c), the critical value $N_f^{\rm cr}\simeq 4N_c$ separates a phase with no confinement and chiral symmetry breaking ($N_f>N_f^{\rm cr}$) and a phase with confinement and with chiral symmetry breaking ($N_f<N_f^{\rm cr}$). The basic point for this suggestion was the observation that at $N_f>N_f^{\rm cr}$ the value of the infrared fixed point α^* is smaller than a critical value

$$\alpha_{\rm cr}\simeq \frac{2N_c}{N_c^2-1}\,\frac{\pi}{3},$$

presumably needed to generate the chiral condensate [8,9].

The authors of Ref. [1] considered only the case when the running coupling constant $\alpha(\mu)$ is less than the fixed point α^*. In this case the dynamics is asymptotically free (at short distances) both at $N_f<N_f^{\rm cr}$ and $N_f^{\rm cr}<N_f<N_f^{**}\equiv 11N_c/2$.

Yamawaki and the author [2] analyzed the dynamics in the whole (α,N_f) plane and suggested the (α,N_f)-phase diagram of the SU(N_c) theory (see Fig. 1 below).[1] In particular, it was pointed out that one can get an interesting nonasymptotically free dynamics when the bare coupling constant $\alpha^{(0)}$ is *larger* than α^*, though not very large.

The dynamics with $\alpha^{(0)}>\alpha^*$ admits a continuum limit and is interesting in itself. Also, its better understanding can be important for establishing the conformal window in lattice computer simulations of the SU(N_c) theory with such large values of N_f. In order to illustrate this, let us consider the following example. For $N_c=3$ and $N_f=16$, the value of the infrared fixed point α^* is small: $\alpha^*\simeq 0.04$ (see below). To reach the asymptotically free phase, one needs to take the bare coupling $\alpha^{(0)}$ less than this value of α^*. However, because of large finite size effects, the lattice computer simulations of the SU(3) theory with such a small $\alpha^{(0)}$ would be unreliable. Therefore, in this case, it is necessary to consider the dynamics with $\alpha(\mu)>\alpha^*$.

One of the goals of this paper is establishing a clear signature of the existence of the infrared fixed point α^*, which would be useful for lattice computer simulations. The signature we will suggest is the spectrum of low-energy excitations in the presence of a bare fermion mass. In particular, we will show that in this case, unlike the familiar QCD with a small N_f ($N_f=2$ or 3), glueballs are much lighter than bound states composed of fermions, if the value of the infrared fixed point is not too large. Another characteristic point is a strong (and simple) dependence of the masses of all the colorless bound states on the bare fermion mass, even if the latter is tiny.

We begin by recalling the basic facts concerning the two-loop β function in an SU(N_c) theory. The β function is

$$\beta=-b\alpha^2-c\alpha^3 \tag{1}$$

with [11]

$$b=\frac{1}{6\pi}(11N_c-2N_f), \tag{2a}$$

$$c=\frac{1}{24\pi^2}\left(34N_c^2-10N_cN_f-3\frac{N_c^2-1}{N_c}N_f\right). \tag{2b}$$

[1]This phase diagram is essentially different from the original Banks-Zaks diagram [7]. For details, see Sec. VII in Ref. [2].

While these two coefficients are invariant under change of a renormalization scheme, the higher-order coefficients are scheme dependent. Actually, there is a renormalization scheme in which the two-loop β function is (perturbatively) exact [12]. We will use such a renormalization scheme.

If $b>0$ ($N_f<N_f^{**}\equiv 11N_c/2$) and $c<0$, the β function has a zero, corresponding to a infrared-stable fixed point, at

$$\alpha=\alpha^*=-\frac{b}{c}. \tag{3}$$

When N_f is close to N_f^{**}, the value of α^* is small. For example, from Eqs. (2a), (2b), and (3), one gets $\alpha^*\simeq 0.04$, 0.14, 0.28, and 0.47 for $N_c=3$ and $N_f=16$, 15, 14, and 13, respectively.

The value of α^* becomes equal to

$$\alpha_{\rm cr}=\frac{2N_c}{N_c^2-1}\,\frac{\pi}{3}$$

at N_f close to $N_f\simeq 4N_c$ and the fixed point disappears at the value $N_f=N_f^*$, when the coefficient c becomes positive (N_f^* is $N_f^*\simeq 8.05$ for $N_c=3$). The β function (1) leads to the following solution for the running coupling:

$$b\,\log\left(\frac{q}{\mu}\right)=\frac{1}{\alpha(q)}-\frac{1}{\alpha(\mu)}-\frac{1}{\alpha^*}\log\left(\frac{\alpha(q)[\alpha(\mu)-\alpha^*]}{\alpha(\mu)[\alpha(q)-\alpha^*]}\right). \tag{4}$$

We emphasize that this solution is valid both for $\alpha(\mu)<\alpha^*$ and $\alpha(\mu)>\alpha^*$.

Let us first consider the case with $\alpha(\mu)<\alpha^*$. It is convenient to introduce the parameter [1]

$$\Lambda\equiv\mu\,\exp\left[-\frac{1}{b\alpha^*}\log\left(\frac{\alpha^*-\alpha(\mu)}{\alpha(\mu)}\right)-\frac{1}{b\alpha(\mu)}\right]. \tag{5}$$

Then, Eqs. (4) and (5) imply that

$$\frac{1}{\alpha(q)}=b\,\log\left(\frac{q}{\Lambda}\right)+\frac{1}{\alpha^*}\log\left(\frac{\alpha(q)}{\alpha^*-\alpha(q)}\right). \tag{6}$$

Taking $q=\Lambda$, we find that

$$\frac{\alpha^*}{1+e^{-1}}\simeq 0.73\alpha^*<\alpha(\Lambda)<\alpha^*. \tag{7}$$

One may think that Λ plays here the same role as $\Lambda_{\rm QCD}$ in the confinement phase. However, as we will see, its physical meaning is somewhat different.

Equation (6) implies that

$$\alpha(q)\simeq\frac{1}{b\,\log(q/\Lambda)} \tag{8}$$

for $q\gg\Lambda$ (the usual behavior in asymptotically free theories), and

$$\alpha(q)\simeq\frac{\alpha^*}{1+e^{-1}(q/\Lambda)^{b\alpha^*}} \tag{9}$$

for $q\ll\Lambda$, governed by the infrared fixed point α^*. Let us turn to a less familiar case with $\alpha(\mu)>\alpha^*$. One still can use Eq. (4). Introduce now the parameter $\tilde\Lambda$ as

$$\tilde\Lambda\equiv\mu\,\exp\left[-\frac{1}{b\alpha^*}\log\left(\frac{\alpha(\mu)-\alpha^*}{\alpha(q)}\right)-\frac{1}{b\alpha(\mu)}\right] \tag{10}$$

[compare with Eq. (5)]. Then, Eqs. (4) and (10) imply

$$\frac{1}{\alpha(q)}=b\,\log\frac{q}{\tilde\Lambda}+\frac{1}{\alpha^*}\log\left(\frac{\alpha(q)}{\alpha(q)-\alpha^*}\right). \tag{11}$$

What is the meaning of $\tilde\Lambda$? It is a Landau pole at which $\alpha(q)|_{q=\tilde\Lambda}=\infty$. Indeed, taking $q=\tilde\Lambda$ in Eq. (11), one gets

$$\frac{1}{\alpha(\tilde\Lambda)}=\frac{1}{\alpha^*}\log\frac{\alpha(\tilde\Lambda)}{\alpha(\tilde\Lambda)-\alpha^*}. \tag{12}$$

The only solution of this equation is $\alpha(\tilde\Lambda)=\infty$.

The presence of the Landau pole implies that the dynamics is not asymptotically free. To get more insight into this dynamics, let us introduce an ultraviolet cutoff M with the bare coupling constant $\alpha^{(0)}\equiv\alpha(q)|_{q=M}$. Now all momenta q satisfy $q\leqslant M$.

Equation (11) implies that at finite $\alpha^{(0)}=\alpha(M)$, the cutoff M is less than $\tilde\Lambda$, with $\alpha(\tilde\Lambda)=\infty$. Therefore the Landau pole is unreachable in the theory with cutoff M and with $\alpha^{(0)}<\infty$. Still one can of course use $\tilde\Lambda$ (10) for a convenient parametrization of the running coupling $\alpha(q)$ [see Eq. (11)]. However, one should remember that momenta q satisfy $q\leqslant M<\tilde\Lambda$.

Equation (11) implies that

$$\alpha^2(q)\simeq\frac{\alpha^*}{2b\,\log(\tilde\Lambda/q)} \tag{13}$$

for $\alpha(q)\gg\alpha^*$, and

$$\alpha(q)\simeq\frac{\alpha^*}{1-e^{-1}(q/\tilde\Lambda)^{b\alpha^*}} \tag{14}$$

when $\alpha(q)$ is close to α^*, i.e., when $\alpha(q)-\alpha^*\ll\alpha^*$. Thus, now $\alpha(q)$ approaches the fixed point α^* from above [compare with Eq. (9)]. And, in general, Eq. (11) implies that $\alpha(q)$ monotonically decrease with q, from $\alpha(q)=\alpha^{(0)}$ at $q=M$ to $\alpha(q)=\alpha^*$ at $q=0$.

Does a meaningful continuum limit exist in this case? The answer is of course ''yes.'' As it follows from Eq. (11), when M (and therefore $\tilde\Lambda$) goes to infinity, and the bare coupling $\alpha^{(0)}>\alpha^*$ is arbitrary but fixed, $\alpha(q)$ is equal to the fixed value, $\alpha(q)=\alpha^*$, for all $q<\infty$. Therefore it is a nontrivial conformal field theory.

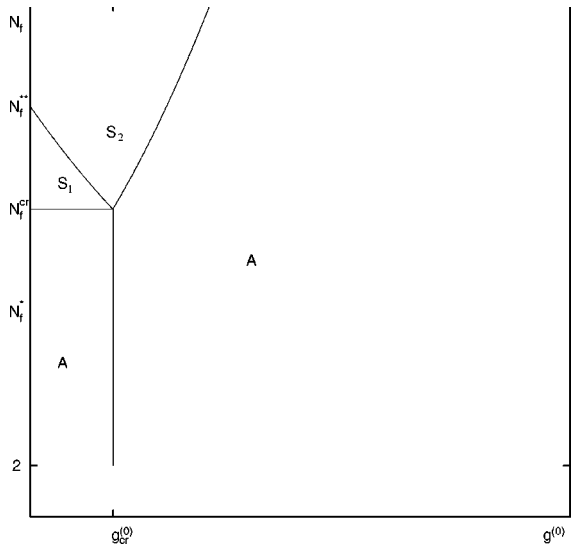

FIG. 1. The phase diagram in an SU(N_c) gauge model. The coupling constant $g^{(0)}=\sqrt{4\pi\alpha^{(0)}}$ and S and A denote symmetric and asymmetric phases, respectively.

So far we considered the solution for $\alpha(q)$ connected with the perturbative (and perturbatively exact in the 't Hooft renormalization scheme [12]) β function (1). However, unlike ultraviolet stable fixed points, defining dynamics at high momenta, infrared-stable fixed points (defining dynamics at low momenta) are very sensitive to nonperturbative dynamics leading to the generation of particle masses. For example, if fermions acquire a dynamical mass, they decouple from the infrared dynamics, and therefore the perturbative infrared fixed point (3) will disappear.

The phase diagram in the $(\alpha^{(0)},N_f)$ plane in this theory was suggested in Ref. [2]. It is shown in Fig. 1. The left-hand portion of the curve in this figure coincides with the line of the infrared-stable fixed points $\alpha^*(N_f)$ in Eq. (3). It separates two symmetric phases, S_1 and S_2, with $\alpha^{(0)}<\alpha^*$ and $\alpha^{(0)}>\alpha^*$, respectively. Its lower end is $N_f=N_f^{\rm cr}$ (with $N_f^{\rm cr}\simeq 4N_c$ if $\alpha_{\rm cr}\simeq[2N_c/(N_c^2-1)](\pi/3)$): at $N_f^*<N_f<N_f^{\rm cr}$ the infrared fixed point is washed out by generating a dynamical fermion mass.

The horizontal, $N_f=N_f^{\rm cr}$, line describes a phase transition between the symmetric phase S_1 and the phase with confinement and chiral symmetry breaking. As it was suggested in Refs. [1,2], based on a similarity of this phase transition with that in quenched four-dimensional QED ($\rm QED_4$) [8,9,13] and in $\rm QED_3$ [14], there is the following scaling law for $m_{\rm dyn}^2$:

$$m_{\rm dyn}^2\sim\Lambda_{\rm cr}^2\exp\left(-\frac{C}{\sqrt{\alpha^*(N_f)/\alpha_{\rm cr}-1}}\right), \qquad (15)$$

where the constant C is of order one and $\Lambda_{\rm cr}$ is a scale at which the running coupling is of order $\alpha_{\rm cr}$.

It is a continuous phase transition with an essential singularity at $N_f=N_f^{\rm cr}$. The characteristic point of this phase transition is that the critical line $N_f=N_f^{\rm cr}$ separates phases with essentially different spectra of low-energy excitations [1,2] and the different structure of the equation for the divergence of the dilatation current (i.e., with essentially different realizations of the conformal symmetry) [2]. It was called the conformal phase transition in Ref. [2].

At present it is still unclear whether the phase transition on the line $N_f=N_f^{\rm cr}$ is indeed a continuous phase transition with an essential singularity or it is a first order phase transition [3,6]. However, anyway, the two properties (the abrupt change of the spectrum of excitations and the different structure of the equation for the divergence of the dilatation current in those two phases) have to take place.

At last, the right-hand portion of the curve on the diagram occurs because at large enough values of the bare coupling, spontaneous chiral symmetry breaking takes place for any number N_f of fermion flavors. This portion describes a phase transition called a bulk phase transition in the literature, and it is presumably a first order phase transition.[2] The vertical line ends above $N_f=0$ since in pure gluodynamics there is apparently no phase transition between weak-coupling and strong-coupling phases.

Up to now we have considered the case of a chiral invariant action. But how will the dynamics change if a bare fermion mass term is added in the action? This question is in particular relevant for lattice computer simulations: for studying a chiral phase transition on a finite lattice, it is necessary to introduce a bare fermion mass. We will show that adding even an arbitrary small bare fermion mass results in a dramatic changing the dynamics both in the S_1 and S_2 phases.

Recall that in the case of confinement SU(N_c) theories, with a small, $N_f<N_f^{\rm cr}$, number of fermion flavors, the role of a bare fermion mass $m^{(0)}$ is minor if $m^{(0)}\ll\Lambda_{\rm QCD}$ (where $\Lambda_{\rm QCD}$ is a confinement scale). The only relevant consequence is that massless Nambu-Goldstone pseudoscalars get a small mass [the PCAC (partial conservation of axial vector current) dynamics].

The reason for that is the fact that the scale $\Lambda_{\rm QCD}$, connected with a scale anomaly, describes the breakdown of the conformal symmetry connected *both* with perturbative and nonperturbative dynamics: the running coupling and the formation of bound state. Certainly, a small bare mass $m^{(0)}\ll\Lambda_{\rm QCD}$ is irrelevant for the dynamics of those bound states.

Now let us turn to the phase S_1 and S_2, with $N_f>N_f^{\rm cr}$. At finite Λ in S_1 and $\tilde\Lambda$ in S_2, there is still conformal anomaly: because of the running of the effective coupling constant, the conformal symmetry is broken. It is restored only if $\Lambda\to 0$ in S_1 and $\tilde\Lambda>M\to\infty$ in S_2. However, the essential difference with respect to confinement theories is that both Λ and $\tilde\Lambda$ have nothing with the dynamics forming bound states: since at $N_f>N_f^{\rm cr}$ the effective coupling is relatively weak, it is

[2]The fact that spontaneous chiral symmetry breaking takes place for any number of fermion flavors, if $\alpha^{(0)}$ is large enough, is valid at least for lattice theories with Kogut-Susskind fermions. Notice, however, that since the bulk phase transition is a lattice artifact, the form of this portion of the curve can depend on the type of fermions used in simulations (for details, see Ref. [2]).

impossible to form bound states from *massless* fermions and gluons (recall that the S_1 and S_2 phases are chiral invariant).

Therefore the absence of a mass for fermions and gluons is a key point for *not* creating bound states in those phases. The situation changes dramatically if a bare fermion mass is introduced: indeed, even weak gauge, Coulomb-like, interactions can easily produce bound states composed of massive constituents, as it happens, for example, in QED, where electron-positron (positronium) bound states are present.

To be concrete, let us first consider the case when all fermions have the same bare mass $m^{(0)}$. It leads to a mass function $m(q^2) \equiv B(q^2)/A(q^2)$ in the fermion propagator $G(q) = [\hat{q}A(q^2) - B(q^2)]^{-1}$. The current fermion mass m is given by the relation

$$m(q^2)|_{q^2=m^2} = m. \tag{16}$$

For the clearest exposition, let us consider a particular theory with a finite cutoff M and the bare coupling constant $\alpha^{(0)} = \alpha(q)|_{q=M}$ being not far away from the fixed point α^*. Then, the mass function is changing in the ''walking'' regime [15] with $\alpha(q^2) \simeq \alpha^*$. It is

$$m(q^2) \simeq m^{(0)} \left(\frac{M}{q} \right)^{\gamma_m}, \tag{17}$$

where the anomalous dimension $\gamma_m \simeq 1 - (1 - \alpha^*/\alpha_{\rm cr})^{1/2}$ [8,9]. Equations (16) and (17) imply that

$$m \simeq m^{(0)} \left(\frac{M}{m^{(0)}} \right)^{\gamma_m/(1+\gamma_m)}. \tag{18}$$

There are two main consequences of the presence of the bare mass.

(a) Bound states, composed of fermions, occur in the spectrum of the theory. The mass of a n-body bound state is $M^{(n)} \simeq nm$.

(b) At momenta $q < m$, fermions and their bound states decouple. There is a pure SU(N_c) Yang-Mills theory with confinement. Its spectrum contains glueballs.

To estimate glueball masses, notice that at momenta $q < m$, the running of the coupling is defined by the parameter $\overline{b}$ of the Yang-Mills theory:

$$\overline{b} = \frac{11}{6\pi} N_c. \tag{19}$$

Therefore the glueball masses $M_{\rm gl}$ are of order

$$\Lambda_{\rm YM} \simeq m \exp\left(-\frac{1}{\overline{b}\alpha^*} \right). \tag{20}$$

For $N_c = 3$, we find from Eqs. (2a), (2b), and (19) that $\exp(-1/\overline{b}\alpha^*)$ is 6×10^{-7}, 2×10^{-2}, 10^{-1}, and 3×10^{-1} for $N_f = 16$, 15, 14, and 13, respectively. Therefore at $N_f = 16$, 15, and 14, the glueball masses are essentially lighter than the masses of the bound states composed of fermions. The situation is similar to that in confinement QCD with heavy quarks, $m \gg \Lambda_{\rm QCD}$. However, there is now a new important point: in the conformal window, *any* value of $m^{(0)}$ (and therefore m) is ''heavy'': the fermion mass m sets a new scale in the theory, and the confinement scale $\Lambda_{\rm YM}$ (20) is less, and rather often much less, than this scale m. This leads to a spectacular ''experimental'' signature of the conformal window in lattice computer simulations: glueball masses rapidly, as $(m^{(0)})^{1/(1+\gamma_m)}$, decrease with the bare fermion mass $m^{(0)}$ for *all* values of $m^{(0)}$ less than cutoff M.

A few comments are in order.

(1) The phases S_1 and S_2 have essentially the same long distance dynamics. They are distinguished only by their dynamics at short distances: while the dynamics of the phase S_1 is asymptotically free, that of the phase S_2 is not. In particular, when all fermions are massive (with the current mass m), the continuum limit $M \to \infty$ of the S_2 theory is a nonasymptotically free confinement theory. Its spectrum includes colorless bound states composed of fermions and gluons. For $q < m$ the running coupling $\alpha(q)$ is the same as in pure SU(N_c) Yang-Mills theory, and for all $q > m$ $\alpha(q)$ is very close to α^* (''walking,'' actually, ''standing'' dynamics). For those values N_f for which α^* is small (as $N_f = 16$, 15, and 14 at $N_c = 3$), glueballs are much lighter than the bound states composed of fermions. Notice that, unlike the case with $m = 0$, there exists an S matrix in this theory.

(2) In order to get the clearest exposition, we assumed such estimates as $N_f^{\rm cr} \simeq 4N_c$ for $N_f^{\rm cr}$ and $\gamma_m = 1 - \sqrt{1 - \alpha^*/\alpha_{\rm cr}}$ for the anomalous dimension γ_m. While the latter should be reasonable for $\alpha^* < \alpha_{\rm cr}$ (and especially for $\alpha^* \ll \alpha_{\rm cr}$) [8,9], the former is based on the assumption that

$$\alpha_{\rm cr} \simeq \frac{2N_c}{N_c^2 - 1} \frac{\pi}{3}$$

which, though it seems reasonable, might be crude for some values of N_c. It is clear, however, that the dynamical picture presented in this paper is essentially independent of those assumptions.

(3) So far we have considered the case when all fermions have the same bare mass $m^{(0)}$. The generalization to the case when different fermions may have different bare masses is evident.

(4) Lattice computer simulations of the SU(3) theory with a relatively large number of N_f [16,17] indeed indicate the existence of a symmetric phase. However, the value of the critical number $N_f^{\rm cr}$ is different in different simulations: it varies from $N_f^{\rm cr} = 6$ [17] through $N_f^{\rm cr} = 12$ [16].

We hope that the signature of the conformal window suggested in this paper can be useful to settle this important issue.

I thank M. Creutz, D. Sinclair, and K. Yamawaki for useful discussions. I wish to acknowledge the JSPS (Japan Society for the Promotion of Science) for its support during my stay at Nagoya University. My special thanks to Koichi Yamawaki for his warm hospitality.

[1] T. Appelquist, J. Terning, and L. C. R. Wijewardhana, Phys. Rev. Lett. **77**, 1214 (1996); T. Appelquist, A. Ratnaweera, J. Terning, and L. C. R. Wijewardhana, Phys. Rev. D **58**, 105017 (1998).

[2] V. A. Miransky and K. Yamawaki, Phys. Rev. D **55**, 5051 (1997); **56**, 3768(E) (1997).

[3] R. S. Chivukula, Phys. Rev. D **55**, 5238 (1997).

[4] M. Velkovsky and E. Shuryak, Phys. Lett. B **437**, 398 (1998).

[5] E. Gardi and M. Karliner, Nucl. Phys. **B529**, 383 (1998); E. Gardi, G. Grunberg, and M. Karliner, J. High Energy Phys. **07**, 007 (1998).

[6] E. Gardi and G. Grunberg, hep-th/9810192.

[7] T. Banks and A. Zaks, Nucl. Phys. **B196**, 189 (1982).

[8] P. I. Fomin, V. P. Gusynin, V. A. Miransky, and Yu. A. Sitenko, Riv. Nuovo Cimento **6**, 1 (1983); C. D. Roberts and A. G. Williams, Prog. Part. Nucl. Phys. **33**, 477 (1994).

[9] V. A. Miransky, *Dynamical Symmetry Breaking in Quantum Field Theories* (World Scientific, Singapore, 1993).

[10] N. Seiberg, Phys. Rev. D **49**, 6857 (1994); K. Intriligator and N. Seiberg, Nucl. Phys. B (Proc. Suppl.) **45B and C**, 1 (1996).

[11] D. R. T. Jones, Nucl. Phys. **B75**, 531 (1974); W. E. Caswell, Phys. Rev. Lett. **33**, 244 (1974).

[12] G. 't Hooft, in *Recent Developments in Gauge Theories*, Vol. 59 of NATO Advanced Study Institute Series B: Physics, edited by G.'t Hooft *et al.* (Plenum, New York, 1980).

[13] P. I. Fomin, V. P. Gusynin, and V. A. Miransky, Phys. Lett. **78B**, 136 (1978); V. A. Miransky, Nuovo Cimento A **90**, 149 (1985); Int. J. Mod. Phys. A **8**, 135 (1993).

[14] T. Appelquist, D. Nash, and L. C. R. Wijewardhana, Phys. Rev. Lett. **60**, 2575 (1988); D. Nash, *ibid.* **62**, 3024 (1989); T. Appelquist, J. Terning, and L. C. R. Wijewardhana, *ibid.* **75**, 2081 (1995).

[15] B. Holdom, Phys. Rev. D **24**, 1441 (1981); Phys. Lett. **150B**, 301 (1985); K. Yamawaki, M. Bando, and K. Matumoto, Phys. Rev. Lett. **56**, 1335 (1986); T. Akiba and T. Yanagida, Phys. Lett. **169B**, 308 (1986); T. Appelquist, D. Karabali, and L. C. R. Wijewardhana, Phys. Rev. Lett. **57**, 957 (1986).

[16] J. B. Kogut and D. R. Sinclair, Nucl. Phys. **B295**, 465 (1998); F. Brown *et al.*, Phys. Rev. D **46**, 5655 (1992).

[17] Y. Iwasaki, K. Kanaya, S. Kaya, S. Sakai, and T. Yoshie, Nucl. Phys. B (Proc. Suppl.) **53**, 449 (1997); D. Chen and R. D. Mawhinney, *ibid.* **53**, 216 (1997); R. D. Mawhinney, hep-lat/9705030.