

Some remarks on theories with large compact dimensions and TeV-scale quantum gravity

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We comment on some implications of theories with large compactification radii and TeV-scale quantum gravity. These include the behavior of high-energy gravitational scattering cross sections and consequences for ultra-high-energy cosmic rays and neutrino scattering, the question of how to generate naturally light neutrino masses, the issue of quark-lepton unification, the equivalence principle, and the cosmological constant. [S0556-2821(99)03108-2]

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I. INTRODUCTION

The intriguing idea that fundamental interactions can be understood as operating in a spacetime of dimension higher than $d=4$ dates back at least to the work of Kaluza and Klein (KK) [1]. A number of studies were carried out subsequently of higher-dimensional field theories, which we shall generically refer to as Kaluza-Klein theories [2]. In a modern context, Kaluza-Klein theories arise naturally from (super)string theories in the limit where relevant energies E are much less than the string mass scale $M_s \sim (\alpha')^{-1/2}$, where α' is the slope parameter. In both generic Kaluza-Klein and string theories, there has thus always been the question of what dynamical mechanism is responsible for compactification and at what scale(s) $\{R\}$ the extra n spacetime dimensions are compactified, leaving the observed four spacetime dimensions. A conventional view has been that the corresponding compactification mass scale(s) $\{R^{-1}\}$ would be high, $\approx M_s$, with M_s being given (in a perturbative analysis) by $M_s \sim g_s M_{Pl} / \sqrt{8\pi}$, where $M_{Pl} = (\hbar c / G_N)^{1/2} = 1.2 \times 10^{19}$ GeV is the Planck mass, G_N is Newton's constant, and $g_s^2 / (4\pi) \sim 0.04$. However, recently, there has been considerable interest in the very different and provocative possibility that some inverse compactification mass scale(s), r_c^{-1} , is (are) much less than the Planck scale [3–16]. A related feature of this theoretical development is a profound change in the role of the Newton constant and Planck mass; rather than being fundamental constants of nature, these become derived quantities, reflecting the change in spacetime dimensionality, from $d=4$ at large distances, to a higher dimensionality at distances $r < r_c$ (where for simplicity, we assume throughout this paper that there is single compactification radius relevant for gravity) and the resultant change of the gravitational force from

$$F = \frac{G_N m_1 m_2}{r^2} = \frac{m_1 m_2}{M_{Pl}^2 r^2} \quad \text{for } r \gg r_c \quad (1.1)$$

to

$$F = \frac{G_{N,4+n} m_1 m_2}{r^{2+n}} = \frac{m_1 m_2}{\hat{M}_{4+n}^{n+2} S_{3+n} r^{2+n}} \quad \text{for } r \ll r_c \quad (1.2)$$

where

$$S_d = \frac{2\pi^{d/2}}{\Gamma(d/2)} \quad (1.3)$$

is the area of the unit sphere in \mathbb{R}^d . Since r_c depends on n , it will be denoted as r_n . Setting $M_{4+n}^{n+2} = (2\pi)^n \hat{M}_{4+n}^{n+2}$, as in Ref. [7] [motivated by toroidal compactification, in which the volume of the compactified space is $V_n = (2\pi r_n)^n$] and applying Gauss's law at $r \ll r_n$ and $r \gg r_n$, one finds that

$$M_{Pl}^2 = r_n^n M_{4+n}^{2+n} \quad (1.4)$$

i.e.,

$$\begin{aligned} r_n &= M_{4+n}^{-1} \left(\frac{M_{Pl}}{M_{4+n}} \right)^{2/n} \\ &= (2.0 \times 10^{-17} \text{ cm}) \left(\frac{1 \text{ TeV}}{M_{4+n}} \right) \left(\frac{M_{Pl}}{M_{4+n}} \right)^{2/n}. \end{aligned} \quad (1.5)$$

Assuming that the higher-dimensional theory at short distances is a string theory, one expects that the fundamental string scale M_s and Planck mass M_{4+n} are not too different (a perturbative expectation is that $M_s \sim g_s M_{4+n}$). Thus, a compactification radius such that $r_n^{-1} \ll M_{Pl}$ corresponds to a short-distance Planck scale and string mass M_s which are also $\ll M_{Pl}$.

It is a striking fact that there is an vast extrapolation of 33 orders of magnitude between the smallest scale of O(1) cm to which Newton's law has been tested [5,17,18] and the scale that has conventionally been regarded as being characteristic

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of quantum gravity, namely the Planck length, $L_{Pl} = \hbar/(M_{Pl}c) \sim 10^{-33}$ cm, and it is not at all implausible that new phenomena could occur in these 33 decades that would significantly modify the nature of gravity. It is therefore instructive to explore how drastically one can change the conventional scenario in which both gauge and gravitational interactions occur in four-dimensional spacetime up to energies comparable to the Planck mass. Of course if one had a truly fundamental theory of everything, it would predict and explain the scale(s) of compactification and the changes in dimensionality that occur. Here we shall take a phenomenological attitude of considering various values of n . From Eq. (1.4) it follows that for fixed M_{4+n}, r_n is a monotonically decreasing function of n . [From Eq. (1.5), one sees that in the formal limit $n \rightarrow \infty, r_n$ approaches M_{4+n}^{-1} from above; in a string theory context, the values of n up to 6 are of interest since this corresponds to spacetime dimensions up to 10 at short distances.] Consequently, the strongest challenge to the conventional paradigm is obtained for the smallest values of M_{4+n} and n . From this point of view, one is therefore motivated to consider values of M_{4+n} as low as the 1–10 TeV region. For such values, the case $n=1$ would yield a compactification radius larger than the solar system, and hence is clearly excluded by existing measurements of gravity and tests of Newton's law. For $M_{4+n} \sim 30$ TeV (which in fact is a lower bound [12]), the case $n=2$ yields a compactification radius

$$r_{n=2} \equiv r_2 \approx 2.7 \text{ } \mu\text{m, i.e., } r_2^{-1} \sim 0.07 \text{ eV}$$

for

$$M_{4+n} = M_6 = 30 \text{ TeV.} \quad (1.6)$$

As r decreases below this scale, the gravitational force changes from a $1/r^2$ to a $1/r^4$ behavior. Currently planned experiments plan to probe gravity somewhat below the present limit of O(1) cm [17]. We shall concentrate on the case $n=2$ because, among the allowed values $n \geq 2$, it yields (for a given M_{4+n}) the largest value of the compactification radius and hence the strongest contrast to the conventional paradigm.

Another reason for considering theories with very low string scales not too far above the electroweak scale, $M_{ew} = 2^{-1/4} G_F^{-1/2} = 250$ GeV, is that this essentially removes the old hierarchy problem, i.e., the problem of preventing the Higgs boson mass from getting large radiative corrections that would naturally raise it to the grand unified theory (GUT) or conventional string (\sim Planck) scale. One must acknowledge that a new hierarchy appears, namely the large ratio between the compactification mass r_n^{-1} and the string scale. For $n=2$, with $M_s \sim 1$ TeV, and r_2^{-1} as given in Eq. (1.6), this ratio is $\sim (1 \text{ TeV})/(10^{-1}) \text{ eV} = 10^{13}$, which is almost as large as the old hierarchy M_{GUT}/M_{ew} or M_{Pl}/M_{ew} . Obviously, supersymmetry cannot be used to stabilize this new hierarchy since it is broken at a scale of at least the electroweak level; some ideas for how this stabilization might occur have been discussed recently [15,16]. Note that if, indeed, the string scale is as low as ~ 1 TeV, so

that the conventional hierarchy is absent, one motivation for supersymmetry would be removed, although its original motivation — to avoid tachyons in string theory — would still be present, given that the quantum theory of gravity is assumed to be a string theory.

In a theory with extra dimensions compactified at a scale r_n , it would naively seem that for distances much less than the compactification scale, all of the fields would depend on the coordinates of the higher-dimensional space. Of course, if this were true, then low values of n including $n=2$ would clearly be ruled out since, among other things, QCD and electroweak interactions have been well measured up to energies of order 10^2 GeV (lengths down to 10^{-16} cm), and the data shows that these interactions take place in a spacetime of dimension 4. Hence, to avoid the danger of a contradiction with experiment, one is led to require that the known fermions and gauge fields be confined to four-dimensional spacetime at least down to distances of about $(1 \text{ TeV})^{-1} \sim 10^{-17}$ cm. Several possible mechanisms for this dimensional confinement of standard model fields have been suggested [5,7,9]. A particularly appealing mechanism is present in modern string theories with Dirichlet p -branes D_p (commonly denoted D-branes [19]); see, e.g., [5,9,10,13,20] and references therein. Calculations of scattering processes involving D-branes suggest that when probed at high energy, these exhibit a thickness [21]

$$\delta \sim M_s^{-1}. \quad (1.7)$$

Specifically, as r decreases past r_n , gravity would feel the extra n dimensions, but the usual gauge and matter fields would be confined to a $p=3$ D_p -brane sweeping out the usual Minkowski 4-dimensional spacetime. The fact that the gravitons do propagate in all $4+n$ dimensions is responsible for the change in the gravitational force law from $1/r^2$ to $1/r^{2+n}$ at distance scales below the compactification scale r_n . There are several specific scenarios of this type. One example features a type-I string theory with 5-branes and 9-branes sweeping out 6-dimensional and 10-dimensional spacetime volumes, respectively, and each having noncompact 4-dimensional Minkowski submanifolds [8,10]. As r decreases below r_c , gravitons (the closed-strings) change from propagating in four dimensions to propagating in six dimensions, so that $n=2$ and $r_c = r_2$, but gauge and matter fields (corresponding to open string states) continue to reside on 3-branes in the 5-branes. As r decreases through an additional compactification scale slightly above M_s^{-1} , the gauge and matter fields extend to a 9-brane sweeping out the full 10-dimensional spacetime. In this region the gauge couplings run rapidly, since they have dimensions; studies of how these couplings might unify at the TeV scale (using several different specific models) include Refs. [9,10]. If, as in the standard model and supersymmetric generalizations thereof, the fermions gain their masses from Yukawa couplings, then these also run rapidly for the same reason.

Given the provocative new features of these proposed models with large compact dimensions and TeV scale strings, there is strong motivation for immediate phenomenological studies to assess their experimental viability, and

these have been initiated in a number of works, e.g., Refs. [7–13]. Important issues that have been studied include the above-mentioned constraints due to experimental gravity tests, and also proton decay, possible contributions to flavor-changing neutral currents and precision electroweak observables, effects mimicking compositeness and changes in scattering processes measured in current e^+e^- and $\bar{p}p$ collider experiments, rare decays, and astrophysical and cosmological effects. One serious concern is that the contributions of Kaluza-Klein modes to the mean mass energy would overclose the universe; however, it has been argued that the theory can evade this problem [12]. Another severe constraint arises from the effects of KK-graviton emission on cooling of supernovae. This has been used to infer the lower bound [12]

$$M_{4+n} \gtrsim 10^{(15-4.5n)/(n+2)} \text{ TeV} \quad (1.8)$$

i.e., for the case of main interest here, $n=2$,

$$M_6 \gtrsim 30 \text{ TeV}. \quad (1.9)$$

It has been argued that this may still be consistent with a fundamental string scale M_s of $O(1)$ TeV [12]. Implications for the cosmological constant have also been discussed (e.g., Refs. [12,13,15,16]). Effects on dispersion of light travelling over cosmological distances may also yield serious constraints [22]. The problem of stabilizing the new hierarchy $M_s r_c \gg 1$ has been addressed in several papers, including Refs. [15,16].

In this paper we shall remark on some other phenomenological implications of these theories with large compact dimensions and TeV-scale strings. In Sec. II we review how the exchange of KK modes of gravitons can produce relatively large effects. We then give some estimates of their effects on high-energy scattering cross sections. In Sec. III we address the problem of obtaining light neutrino masses in the absence of the conventional methods (seesaw mechanism and higher-dimension operators). Sections IV and V contain some discussion of the equivalence principle and the cosmological constant.

II. GRAVITON+KK EXCHANGES AND HIGH-ENERGY BEHAVIOR OF CROSS SECTIONS

In the theories under consideration here, there are several relevant ranges for the center-of-mass energy \sqrt{s} of a given process: (i) the extreme low-energy region, $\sqrt{s} < r_2^{-1}$; (ii) the large range $r_2^{-1} < \sqrt{s} < M_s$, which includes energies up to the TeV scale at its upper end; and (iii) the range of energies above the string scale, $\sqrt{s} > M_s \sim 1$ TeV. In this section, by “high-energy” behavior of cross sections, we shall mean intervals (ii) and (iii). Let us denote the momentum of a graviton as $k = k_L = (\{k_\lambda\}, k_1, \dots, k_n)$, where the usual spacetime Lorentz index $\lambda = 0, 1, 2, 3$. Because of the compactification, the extra n components of the graviton momenta are quantized. With the simplifying assumption that the compactification radii of all of the extra n dimensions are the

same, one has, for toroidal compactification, with the circumferences

$$L_i = 2\pi r_n \equiv L_n, \quad i = 1, \dots, n \quad (2.1)$$

the quantization of KK momenta

$$k_i = \frac{2\pi l_i}{L_n} = \frac{l_i}{r_n}, \quad i = 1, \dots, n. \quad (2.2)$$

To an observer in the usual four-dimensional spacetime, the above graviton would thus appear to be a massive Kaluza-Klein (KK) state with mass

$$\mu_{l_1, \dots, l_n} = \left(\sum_{i=1}^n l_i^2 \right)^{1/2} r_n^{-1}. \quad (2.3)$$

All of these KK states have the same Lorentz structure as the graviton as regards their couplings to other particles. Since the gravitons propagate in the full $(4+n)$ -dimensional spacetime, their self-interactions must conserve not only the ordinary 4-momenta, but also the KK momentum components. That is, if one envisions a scattering process involving N gravitons with momenta $k_L^{(1)}, \dots, k_L^{(N)}$ (directed into the vertex, say), then $\sum_{j=1}^N k_L^{(j)} = 0$, so that $\sum_{j=1}^N k_\lambda^{(j)} = 0$ for the usual spacetime components $\lambda = 0, 1, 2, 3$, and also $\sum_{j=1}^N k_i^{(j)} = 0$ for $i = 1, \dots, n$, whence $\sum_{j=1}^N l_i^{(j)} = 0$ for $i = 1, \dots, n$. However, since other particles are assumed to be confined to the thin membrane of thickness $\delta \sim M_s^{-1}$, which breaks translational invariance in the extra n dimensions, they do not have well-defined KK momenta in these extra dimensions. Therefore the interactions of gravitons with such particles do not, in general, conserve the KK momentum components, $\{l_i/r_n\}, i = 1, \dots, n$ — at least so long as these are smaller than M_s , the ultimate cutoff scale in the field theory, at which it goes over into a string theory, i.e., so long as

$$\frac{l_i}{r_n} \lesssim M_s. \quad (2.4)$$

Defining the graviton field as

$$g_{MN} = \eta_{MN} + \frac{h_{MN}}{(M_{4+n}^{2+n})^{1/2}} \quad (2.5)$$

[where $\eta_{\mu\nu}$ is the usual flat-space metric tensor, with signature which we take as $(+, -, -, -)$, and the details of η_{MN} in the extra dimensions depend on the nature of the compactified manifold], the resulting interactions of the gravitons with the usual gauge and matter fields on the 3-brane are given by $T_{\mu\nu}(x)h_{\mu\nu}(x)/(M_{4+n}^{2+n})^{1/2}$ for x restricted to lie on this 3-brane. Equivalently, one can treat the graviton-KK emission in a four-dimensional framework, where the coupling is $1/M_{Pl}$ in an amplitude; the rate is then proportional to $1/M_{Pl}^2$ times a factor reflecting the multiplicity of KK-graviton emission. Since this factor is $\sim (s^{1/2}r_n)^n$, where $s^{1/2}$ is the center-of-mass energy available for graviton-KK emission, when one substitutes the expression for r_n from Eq.

(1.4), the factor of $1/M_{Pl}^2$ is exactly cancelled, and the final product is $s^{n/2}/M_{4+n}^{n+2}$, as one would obtain directly from Eq. (2.5) [7]. Thus, from a four-dimensional viewpoint, although the KK-gravitons are coupled extremely weakly, this is compensated by their very large multiplicity, so that their net effect involves in the denominator a mass scale in the range of 10 TeV instead of M_{Pl} .

Let us study the implications of this further. In considering the exchange of gravitons, and in particular, their KK momentum components, in some process, one should formally consider all $l_i \in \mathbb{Z}$ for each $i=1, \dots, n$ and sum over all of these exchanges. In the theories of interest here, as \sqrt{s} becomes comparable to the string scale, M_s , one changes over from a field theory (with effects of D-branes included) to a fully stringlike picture, so that M_s serves as an upper cutoff to what is really the low-energy effective field theory with which we work. Accordingly, we shall impose an upper cutoff

$$l_i < l_{max} = M_s r_n \quad (2.6)$$

on the sums over KK modes, which thus run over the range

$$l_i = 0, \pm 1, \dots, \pm l_{max} \quad \text{for } i=1, \dots, n. \quad (2.7)$$

The value of $l_{max} = M_s r_n$ is very large; for example, for the case of primary interest here, $n=2$, for $M_s \sim 1$ TeV, one has $l_{max} = M_s r_2 \sim 10^{16}$ while for $n=3,4, l_{max} = M_s r_n \sim 10^{11}$ and 10^8 , respectively. When one is interested in the effect of the exchange of these KK components of gravitons on the static gravitational potential generated between two test masses at a distance r , the resultant contribution of the higher KK modes is suppressed by a Yukawa-type factor:

$$\frac{V}{m_1 m_2} = \frac{G_N}{r} \sum_{l_1, \dots, l_n} e^{-\mu r} = \frac{G_N}{r} \sum_{l_1, \dots, l_n} \exp \left[- \left(\sum_{j=1}^n l_j^2 \right)^{1/2} r \right]. \quad (2.8)$$

Formally, the summation is over each $l_i \in \mathbb{Z}$, but the actual contribution depends on the value of r relative to r_n since only the number

$$\nu_{eff} \sim \left(\frac{r_n}{r} \right)^n \quad (2.9)$$

of KK modes which do not suffer exponential suppression in Eq. (2.8) contribute significantly. Thus, when $r \gg r_n$, only the term with $(l_1, \dots, l_n) = (0, \dots, 0)$ contributes and one recovers the usual Newton law $V \propto 1/r$, but as r decreases through r_n , more KK modes contribute, and finally, for $r \ll r_n$, all of the KK modes up to $M_s r$ contribute, giving rise to the crossover in the behavior of the gravitational interaction $V \rightarrow 1/r^{1+n}$.

The consideration of the exchange or production of these KK components of gravitons in various processes, while not exhausting the full implications of the new theory, can serve as a general guideline for studying the possible novel phenomenological implications. Note also that unlike other possible signatures such as missing energy, apparent compositeness, etc. which could originate from other modifications/

extensions of the standard model, the multi-KK exchange and some of its consequences are unique to the present class of models.

Already at present, cosmic ray interactions have provided us with pp collisions having lab and center of mass energies up to $\sim 3 \times 10^8$ TeV and $\sim 10^3$ TeV, respectively [23], far above the assumed string scale M_s , the ultimate cutoff of the field theory. It has been heuristically argued that since the bulk of the hadronic collisions are soft, the new TeV region physics will not be strongly manifest in these collisions [12]. One of our purposes in this paper will be to examine this issue and the more general question of the impact of graviton+KK exchange(s) in quasi-diffractive processes which are defined here to be $2 \rightarrow 2$ scattering processes with cm mass energy squared $s \gg M_s^2$ but $Q^2 < M_s^2$ (or even $Q^2 \ll M_s^2$). Using the strong lower bound (1.8), which forces M_{4+n} to be substantially above the electroweak scale, we obtain results that confirm the arguments of Refs. [12].

Before proceeding, a comment is in order: in a generic Kaluza-Klein theory, there would also be KK excitations of usual standard-model gauge, fermion, and Higgs fields. However, in the type of theory that we consider here, since the compactification radius r_2 is so large, and hence, as noted above, the standard-model fields must be confined to usual 4-dimensional spacetime even for $r < r_c$, down to distances $r \sim M_s^{-1}$ where a fully stringlike picture emerges, it follows that standard-model fields do not have KK excitations in the energy range $\sqrt{s} \lesssim M_s$.

Let us consider the scattering of two usual standard-model particles, indicated by their momenta, $p_1 + p_2 \rightarrow p'_1 + p'_2$. These could be leptons and/or the quark/gluon constituents of energetic protons. As stated above, we concentrate on the case of $n=2$ extra dimensions whose compactification radius is given by (1.6). We define $P = p_1 + p_2 = p'_1 + p'_2$ and $q = p_1 - p'_1 = p'_2 - p_2$. The center of mass energy squared is then given by

$$s = P^2 = \sum_{M,N} P_M P_N g^{MN} = \sum_{\mu, \nu=0,1,2,3} P_\mu P_\nu g^{\mu\nu} \quad (2.10)$$

and the momentum transfer squared $t = q^2 = -Q^2$ by

$$t = q^2 = \sum_{M,N} q_M q_N g^{MN} = \sum_{\mu, \nu=0,1,2,3} q_\mu q_\nu g^{\mu\nu} \quad (2.11)$$

where we have used the fact that the usual standard-model particles are confined to four-dimensional spacetime. The third kinematic invariant is $u = (p_1 - p'_2)^2$ satisfying the relation that $s+t+u$ is equal to the sums of the squared masses of the colliding particles. A vector gauge boson V ($= \gamma, Z^0, W$, or gluon) exchanged between the particles yields a t-channel Born amplitude which, for

$$s \gg m_V^2, \quad s \gg t, \quad \text{and } s \gg m_i^2 \quad (2.12)$$

(whence $u \sim -s$) is of the form

$$\begin{aligned}
A_{1v} &\simeq \frac{g_1 g_2 (p_1 + p'_1)_\mu (p_2 + p'_2)_\nu g^{\mu\nu}}{t - m_V^2} \simeq \frac{g_1 g_2 (s - u)}{t - m_V^2} \\
&= \frac{2g_1 g_2 s}{t - m_V^2} \quad (2.13)
\end{aligned}$$

where g_1, g_2 are appropriate gauge coupling constants, the $(p_j + p'_j)_\mu$ factors arise from the gauge-fermion couplings, and m_V is the mass of the exchanged vector boson. We neglect the contributions from the $q_\mu q_\nu / m_W^2$ term in the W propagator numerator which give negligible contributions, and the respective $q_\mu q_\nu / m_Z^2$ and (gauge-dependent) $q_\mu q_\nu / q^2$ term in the Z and γ propagator numerators, which give zero contributions because the Z and γ have diagonal, current-conserving, couplings.

We next consider the graviton exchange amplitude and begin with the usual four-dimensional massless graviton exchange. It has long been recognized that the (perturbatively calculated) scattering amplitudes involving the exchange of a particle with spin J in the t channel grow like s^J ; indeed, this was one of the motivations for the development of Regge theory, in which the spins of the exchanged particles were effectively made into a variable $\alpha(t)$ and in which the resultant energy-dependence of the amplitude was much softer (e.g., [24]). Since gravity couples to the (conserved, traceless, symmetric) energy-momentum tensor, the amplitude for the exchange of one graviton between the same particles, with the same kinematic condition (2.12) yields an amplitude

$$\begin{aligned}
A_{1g} &\simeq G_N [(p_1 + p'_1)_\mu (p_1 + p'_1)_\nu - (p_1 + p'_1)^2 g_{\mu\nu}] \\
&\quad \times [(p_2 + p'_2)_\rho (p_2 + p'_2)_\sigma \\
&\quad - (p_2 + p'_2)^2 g_{\rho\sigma}] \frac{(g^{\mu\rho} g^{\nu\sigma} + \text{perms})}{t} \propto \frac{G_N s^2}{t} \quad (2.14)
\end{aligned}$$

where the conserved nature of the graviton-fermion couplings means that terms involving momenta q in the numerator of the graviton propagator do not contribute. The s^2 behavior in Eq. (2.14) conforms to the expected s^J behavior for spin- J exchange. Comparing with Eq. (2.13), we see that

$$\frac{A_{1g}}{A_{1v}} \simeq \frac{s/M_{Pl}^2}{g_1 g_2} \quad (2.15)$$

This ratio becomes of order unity for $\sqrt{s} \simeq M_{Pl}$, i.e., ordinary four-dimensional quantum gravity becomes strong only at superplanckian energies.

However, the contribution of the many KK components of gravitons changes this picture significantly. Reverting to the case of general n , we observe that, from Eqs. (2.4) and (2.7), this contribution has the effect of replacing the propagator $1/Q^2$ by

$$\frac{1}{Q^2} \rightarrow \sum_{l_1} \cdots \sum_{l_n} \frac{1}{-Q_0^2 + |\mathbf{q}|^2 + \sum_{i=1}^n (l_i/r_n)^2} \quad (2.16)$$

where the range of the indices $l_i, i=1, \dots, n$ in the summation is given by Eq. (2.7). To obtain an approximate value for this expression, we note that there is a large range of each KK momentum squared, $(l_i/r_n)^2$, from 0 to M_s^2 , and over much of this range, it is $\gg Q^2$ for typical Q^2 values of $Q^2 \sim \Lambda_{QCD}^2 \sim (150 \text{ MeV})^2$ that give the dominant contribute to hadronic cross sections. Similar comments can be made about cross sections for other types of reactions. Therefore, we consider the approximation of keeping only the KK graviton modes in Eq. (2.16) satisfying

$$\sum_{i=0}^n (l_i/r_n)^2 \gg Q^2. \quad (2.17)$$

For $n \geq 3$, we then drop the Q^2 term in the denominator of Eq. (2.16). Further, recalling that l_{max} is very large, it is reasonable to approximate the summations by integrations, yielding

$$\frac{1}{Q^2} \rightarrow r_n^2 \int_{-l_{max}}^{l_{max}} \cdots \int_{-l_{max}}^{l_{max}} \frac{d^n l}{l^2} = \frac{r_n^2 S_n (M_s r_n)^{n-2}}{n-2} \quad \text{for } n \geq 2 \quad (2.18)$$

where S_n was given in Eq. (1.3). For $n=2$, we must retain the Q^2 term in the denominator of Eq. (2.16), and, again approximating the summations by integrations, we obtain

$$\frac{1}{Q^2} \rightarrow \pi r_2^2 \ln \left(1 + \frac{M_s^2}{Q^2} \right) \quad \text{for } n=2. \quad (2.19)$$

Next, substituting Eq. (2.16) in Eq. (2.14), using the above approximations, and inserting Eq. (1.5) for r_n , we find for $n=2$

$$\tilde{A}_{grav.} = \sum_{l_1, \dots, l_n} A_{1g+KK} = \frac{\pi s^2}{M_6^4} \ln \left(1 + \frac{M_s^2}{Q^2} \right) \quad (2.20)$$

and for $n \geq 3$,

$$\tilde{A}_{grav.} = \sum_{l_1, \dots, l_n} A_{1g+KK} = \frac{S_n s^2}{M_{4+n}^4} \left(\frac{M_s}{M_{4+n}} \right)^{n-2}. \quad (2.21)$$

As one might expect, the amplitude $\tilde{A}_{grav.}$ becomes large once s significantly exceeds M_{4+n}^2 . In particular, we find, in this new Born approximation of one graviton + KK exchange, a differential cross section which, for $n=2$, is

$$\frac{d\sigma}{dt} = \frac{1}{16\pi s^2} |\tilde{A}_{grav.}|^2 \simeq \frac{\pi s^2}{16M_6^8} \ln^2 \left(1 + \frac{M_s^2}{Q^2} \right) \quad (2.22)$$

and, for $n \geq 3$, is

$$\frac{d\sigma}{dt} \simeq \frac{S_n^2 s^2}{16\pi M_{4+n}^8} \left(\frac{M_s}{M_{4+n}} \right)^{2(n-2)}. \quad (2.23)$$

In the following we shall restrict ourselves to momentum transfers that are soft on the scale of M_s , i.e., to

$$Q^2 \lesssim \xi M_s^2 \quad (2.24)$$

where we take $\xi \lesssim 10^{-2}$. We also apply a similar softness cutoff to the allowed KK graviton exchanges by having the sums in Eq. (2.16) extend not over the range $0 \leq |l_i| \leq M_s r_n$ but instead only over the range $0 \leq |l_i| \leq \xi^{1/2} M_s r_n$. This will simply introduce an extra $\xi^{(n-2)/2}$ factor in Eq. (2.18) and, for $n=2$ will remove the logarithmic enhancement factors in Eqs. (2.19) and (2.22). Integrating over the region $0 \leq -t \leq \xi M_s^2$, we find the total cross sections, for $n=2$,

$$\begin{aligned} \sigma &= \int \frac{d\sigma}{dt} dt \geq \int_{-\xi M_s^2}^0 \frac{d\sigma}{dt} dt \simeq \frac{\pi \xi s^2 M_s^2}{16 M_6^8} \\ &\simeq \xi \left(\frac{s}{1 \text{ TeV}^2} \right)^2 \left(\frac{1 \text{ TeV}}{M_6} \right)^6 \left(\frac{M_s}{M_6} \right)^2 \times 10^{-34} \text{ cm}^2 \end{aligned} \quad (2.25)$$

and, for $n \geq 3$,

$$\begin{aligned} \sigma &\simeq \frac{\xi S_n^2 s^2 M_s^2}{16\pi M_{4+n}^8} \left(\frac{M_s}{M_{4+n}} \right)^{2(n-2)} \\ &\simeq \xi S_n^2 \left(\frac{s}{1 \text{ TeV}^2} \right)^2 \left(\frac{1 \text{ TeV}}{M_{4+n}} \right)^8 \left(\frac{M_s}{M_{4+n}} \right)^{2(n-2)} \times 10^{-35} \text{ cm}^2. \end{aligned} \quad (2.26)$$

As noted before, this $\sigma \propto s^2$ growth is characteristic of a process involving the exchange of a spin-2 particle (see also Refs. [25,26]). One can, in addition, consider the contributions of higher-order graviton+KK exchanges, but it is a challenging problem to calculate these (see further below).

We next consider a different approach which yields, by construction, a unitarized amplitude and cross section. As elsewhere in this paper, we focus on the case $n=2$. The unitarization procedure starts by performing a partial wave decomposition of the original, non-unitarized Born amplitude. The resulting partial waves are (in general, away from possible resonances) monotonically decreasing with l . For our estimate, let us assume that for $l \leq l_0(s)$, all of the partial waves exceed the unitarity bound $a_l(s) \leq 1$ and for all $l > l_0(s)$ they obey this bound. The method is then to set all $a_l(s)$ for $l < l_0(s)$ equal to unity. In the present context of small momentum transfer, $s \gg -t$, and hence small center-of-mass scattering angle $\theta \equiv \theta_{cm}$ (recall $\theta_{cm}^2 \sim -4t/s$ in this limit), we can make use of the eikonal or related peripheral approximation (for a review, see, e.g., [24]). We introduce the notation $\mathbf{k} \equiv \mathbf{p}_{1cm}$ and $\mathbf{k}' \equiv \mathbf{p}'_1$, and note that $k = |\mathbf{k}| = |\mathbf{k}'| \simeq \sqrt{s}/2$ in the limit that we are considering. The impact parameter \mathbf{b} satisfies $\mathbf{b} \perp \hat{\mathbf{k}}$, so $\mathbf{q} \cdot \mathbf{b} = -\mathbf{k}' \cdot \mathbf{b} = kb \theta \cos \phi$,

where ϕ is the azimuthal angle relative to $\hat{\mathbf{k}}$. Denoting $|\mathbf{b}| = b$, we recall that $l = |\mathbf{q}|b$. We can thus write an integral transform defining the eikonal amplitude $\tilde{a}(s, b)$:

$$A(s, t) = s \int \frac{d^2 \mathbf{b}}{2\pi} e^{i\mathbf{q} \cdot \mathbf{b}} \tilde{a}(s, b) \quad (2.27)$$

$$= s \int_0^\infty b db \int_0^{2\pi} d\phi e^{-ikb\theta \cos \phi} \tilde{a}(s, b) \quad (2.28)$$

$$= s \int_0^\infty b db J_0(kb\theta) \tilde{a}(s, b). \quad (2.29)$$

The inverse relation is then, formally,

$$\tilde{a}(s, b) = \frac{1}{s} \int \frac{d^2 \mathbf{q}}{2\pi} e^{i\mathbf{q} \cdot \mathbf{b}} A(s, t) \quad (2.30)$$

where it is understood that $Q^2 \simeq |\mathbf{q}|^2 < \xi M_s^2 \ll M_s^2$ because of the quasi-diffractive kinematic conditions assumed for our estimate. In accordance with our unitarization, we set the eikonal function $\tilde{a}(s, b)$ equal to unity for $b \leq b_0$, where $b_0 = l_0/|\mathbf{q}|$. A rough estimate of the total cross section is then

$$\sigma \simeq \pi b_0^2. \quad (2.31)$$

Substituting Eq. (2.20) into the inverse transform (2.30), we obtain the dimensional estimate

$$\tilde{a}(s, b) \sim \text{const} \times \frac{s}{M_6^4 b^2}. \quad (2.32)$$

This function $\tilde{a}(s, b)$ decreases monotonically as a function of b , and

$$\tilde{a}(s, b) = 1 \quad \text{for } b_0^2 \sim \text{const} \times \frac{s}{M_6^4}. \quad (2.33)$$

Hence, from Eq. (2.31) we have the unitarized estimate

$$\sigma \propto \frac{s}{M_6^4}. \quad (2.34)$$

Evidently, the behavior $\sigma \propto s$ is a less rapid growth than that suggested by the analysis of 1-graviton exchange above, which yielded the behavior $\sigma \propto s^2$. We consider the eikonal method to be more reliable because, by construction, it produces a unitarized amplitude, whereas the 1-graviton+KK exchange calculation (and the additions of multiple graviton+KK exchange) does not automatically do this.

Another way of seeing this result is to consider two particles of energy $W/2$, each moving in opposite directions in the center of mass frame, so that the compound system has energy $W (= \sqrt{s})$. Now recall that the Schwarzschild radius of a black hole of mass m is $R_{Sch.} = 2G_N m = 2m/M_{Pl}^2$. Therefore, if the impact parameter (size) of the system is

smaller than $\sim 2W/M_{Pl}^2$, we might expect it to collapse to a black hole. The critical impact parameter $b_0 \approx 2W/M_{Pl}^2$ may thus serve as an effective interaction range. Note that this argument applies for both elastic scattering and inelastic scattering involving multiparticle production. This then leads to the estimate for the total cross section $\sigma \approx \pi b_0^2 \approx (4\pi s)/M_{Pl}^4$. Making the replacement $M_{Pl} \rightarrow M_{4+n}$ appropriate for the models considered here (provided that $b_0 < r_n$), we obtain for the total cross section

$$\sigma \approx \frac{4\pi s}{M_{4+n}^4}. \quad (2.35)$$

In particular, for the case $n=2$ of main interest here,

$$\sigma \approx \frac{4\pi s}{M_6^4} \approx 5 \left(\frac{s}{1 \text{ TeV}^2} \right) \left(\frac{1 \text{ TeV}}{M_6} \right)^4 \times 10^{-33} \text{ cm}^2 \quad (2.36)$$

which exhibits a $\sigma \propto s$ behavior, in agreement with Eq. (2.34). Given the lower bound $M_6 \gtrsim 30 \text{ TeV}$ in Eq. (1.9), we have

$$\sigma \leq 0.6 \left(\frac{s}{1 \text{ TeV}^2} \right) \times 10^{-38} \text{ cm}^2. \quad (2.37)$$

We shall discuss the phenomenological implications of this below.

In passing, we note that the behavior $\sigma \approx s$ of Eqs. (2.34) and (2.36) may be viewed as the analogue of the Froissart bound [27]

$$\sigma \leq \frac{4\pi}{\mu^2} \ln^2 \left(\frac{s}{s_0} \right) \quad (2.38)$$

in ordinary hadronic physics. Since the quantity μ^2 that appears in Eq. (2.38) is the squared mass of the lightest particle exchanged in the t channel, which for the present case is the massless graviton, the original Froissart bound [27] is clearly not applicable here. In the following we will adopt the conservative estimate (2.36).

We have seen that the behavior $\sigma \propto s$ does not conflict with any unitarity bounds; in fact, we have incorporated these bounds for our estimate of the total cross section. Our arguments above based on the eikonal model and black-hole considerations are rather general and do not depend on calculating specific field-theory diagrams. This is important, since in the energy region $s > M_s^2$, there are stringy effects. Although it is not necessary for our arguments, we briefly comment on these. Once $W > M_s$, the massive string states consisting of a tower $M^2 = rM_s^2$, where $r=1,2,\dots$ can be excited. These have exponentially rising degeneracy $\rho_r \sim \exp(\sqrt{r})$. A description in terms of s -channel string state production would be challenging, especially given the necessity of carrying out fully the multi-loop string unitarization program to all orders in order to avoid poles on the real s axis and obtain physical behavior involving finite-wide reso-

nances in $A(s,t)$ [28,29]. However, precisely because of the basic feature of string models (which actually predates strings), namely the Horn-Schmid duality [30], we need not compute $A_{2 \rightarrow 2}(s,t)$ by summing the contributions of s -channel resonances. This specific duality implies that we can equally well represent $A_{2 \rightarrow 2}(s,t)$ by summing over t -channel contributions. In particular, for the region of interest here, namely $s \gg M_s^2$, $-t \ll M_s^2$, the second, asymptotic series description is much easier to compute, as it consists of only the contribution from the leading Regge trajectories. Hence,

$$A(s,t) \approx \beta(t) s^{\alpha(t)} \quad \text{for } s \rightarrow \infty \quad \text{and} \quad \frac{-t}{s} \ll 1. \quad (2.39)$$

In the present case we know what the leading Regge trajectory is, namely the graviton, with intercept $\alpha(t=0) = J=2$ and slope α' :

$$\alpha_G(t) = 2 - \alpha' t \quad \text{with} \quad \alpha' = \frac{1}{4\pi T} = \frac{1}{4\pi M_s^2} \quad (2.40)$$

where T is the string tension. The estimate that we performed of the $2 \rightarrow 2$ scattering amplitude is equivalent to this t -channel summation, with two modifications: first, in approximating $s^{\alpha_G(t)} \approx s^2$, we neglected the shift $\alpha' t$; however, since $-t < \xi M_s^2$ with $\xi \ll 10^{-2}$, the corrections due to this shift is $O(10^{-3})$ and therefore negligible. The second modification is that together with the graviton, we summed also over the t -channel KK excitations. But this is, indeed, a basic feature of these models, as we recall from the fact that it is actually responsible for the crossover of the gravitational force from $1/r^2$ to $1/r^{2+n}$ as r decreases below r_n . Note further that whatever the intermediate s -channel states are, our general arguments above show that they cannot conspire to remove the s growth of the cross section.

Finally, we would like to address the issue of compositeness of the colliding particles, since hadrons are certainly composites and even quarks and leptons might exhibit compositeness as length scales smaller than the current limits of order $(\text{few} \times \text{TeV})^{-1}$. It is well known that because of the universality of the static gravitational interaction, it is independent of the compositeness of the objects involved, and only depends on their total masses, as $F = G_N m_1 m_2 / r^2$. It is interesting to observe that the asymptotic behavior of the cross section, $\sigma \propto s / M_6^4$ is also independent of the possible (partonic or preonic) structure of the colliding particles. To show this, we let the colliding particles labelled 1 and 2 consist, respectively, of N_1 and N_2 partons (preons). We first note that for particles with masses satisfying $m_i^2 \ll s$ and hence negligible for the present purposes, one can approximate the kinematics by setting $m_i = 0$, so that $s = 2p_1 \cdot p_2 = 4E_1 E_2$, where the energies E_1 and E_2 are specified in the center-of-mass frame or any Lorentz frame obtained from it via a boost along the collision axis $\hat{\mathbf{k}}$. Writing the energy of each particle as the sum of the energies of its constituents,

$$E_1 = \sum_{i=1}^{N_1} E_{1i}, \quad E_2 = \sum_{i=1}^{N_2} E_{2i} \quad (2.41)$$

and substituting, we obtain, for the total cross section

$$\sigma_{12} = \kappa \frac{s}{M_6^4} = \frac{4\kappa E_1 E_2}{M_6^4} = \frac{\kappa}{M_6^4} \sum_{i=1}^{N_1} \sum_{j=1}^{N_2} E_{1i} E_{2j} = \sum_{i=1}^{N_1} \sum_{j=1}^{N_2} \sigma_{ij} \quad (2.42)$$

where $\kappa = 4\pi$ in Eq. (2.36) but its precise value is not important here. That is, we write the particle-1 particle-2 cross section σ_{12} as the incoherent sum of the $N_1 N_2$ parton-parton (preon-preon) cross sections and find that, to this order, σ_{12} is invariant under composition. This is to be contrasted with a $\sigma_{12} \propto s^2$ cross section, which would not have this invariance. In passing, we also note that because of the specific limitation $-t < \xi M_s^2 \ll M_s^2$ that we used, and the fact that the string scale is M_s^2 , we do not expect here any form-factor effects on the graviton+KK couplings. Thus, the cross section behavior $\sigma \propto s$ in Eq. (2.36) can be interpreted in a certain sense, e.g., in the context of composite models, as preserving the counting of degrees of freedom. (The criterion of preserving degrees of freedom and associated information content has also been used to infer a holographic principle in quantum gravity [31,32].)

One may wonder whether the growth of the cross section over many decades of accessible energies [33,34] does not constitute a fundamental difficulty and potential fatal flaw, in principle, of the model. If the truly naive estimate $\sigma \propto s^2 N$ from the consideration of the N -graviton exchange amplitude were valid, then it would appear that this would, indeed, be a serious problem. If, however, the cross sections grow only like s , as suggested by our discussion above, then there may not be any fatal difficulty. To see this point, let us assume that the forward dispersion relation for some $2 \rightarrow 2$ scattering amplitude, such as $\nu + N \rightarrow \nu + N$ holds (where $N = \text{nucleon}$). Then, with the optical theorem that the imaginary part of the forward scattering amplitude $A(s, t=0)$ satisfies $\text{Im}(A(s, t=0)) \propto s \sigma(s) \propto s^2$, it follows that in these models, this forward dispersion relation would need one more subtraction than in the usual case where $\sigma \leq \text{const} \times \ln^2 s$. The subtraction term is likely to involve the factor $\alpha' s \sim s/M_s^2$, which is also what the field-theory limit, via the Taylor-series expansion in $\alpha' s$, of string theory scattering amplitudes, would suggest.

III. EFFECTS OF HIGH-ENERGY GRAVITATIONAL SCATTERING CROSS SECTION

A. Ultra-high energy cosmic rays

In recent years, there has been considerable progress in the measurement of ultra-high-energy UHE cosmic rays. These have been observed with lab energies up to $E \sim 3 \times 10^{20}$ eV [23]. The cosmic rays consist of protons together with some nuclei having charge $Z > 1$ (the relative fractions depending on the energy). The highest-energy cosmic rays are of considerable current interest because their energies are above the Greisen-Zatsepin-Kuzmin (GZK) upper bound

[35] of $E_{GZK} \approx 5 \times 10^{19}$ eV, which was based on the fact that above this energy, the cosmic rays would lose energy by scattering off the cosmic microwave background photons and producing pions, via the reaction

$$p + \gamma_{CMB} \rightarrow N + \pi \quad (3.1)$$

which can proceed resonantly through the $\Delta(1232)$. A number of mechanisms have been proposed to try to account for this [23]. For the illustrative energy $E = 10^{19}$ eV, i.e., $s \approx 2 \times 10^4$ TeV², the cross section estimate (2.36) for the new graviton+KK contribution to pp scattering, together with the lower bound (1.9), gives $\sigma_{grav+KK} \lesssim 10^{-34}$ cm². This is quite small compared to the usual hadronic pp cross section, which, at this energy, is $\sigma_{pp} \approx 150$ mb = 1.5×10^{-25} cm². Thus, using our unitarized estimate (2.36), we find that the phenomenological impact of models with two large compact dimensions on UHE pp scattering is innocuous.

B. Ultra-high energy neutrino scattering

A striking feature of the new contribution to high-energy scattering cross sections due to graviton+KK exchange, Eq. (2.36), is the fact that it is projectile-independent, and thus the same for the scattering of $pp, \gamma p, \nu p$, etc. We focus here on ultra-high energy neutrinos; one of the reasons why these are of current interest is that they occur as the decay products of the pions produced by the reaction (3.1) and analogous reactions in which the protons scatter off of the radiation field of a source such as a gamma-ray burster (GRB) [36,37]. Calculations of UHE νp and $\bar{\nu} p$ weak charged and neutral current cross sections find that these grow with energy approximately as $E^{0.36}$ [38]. Since this is a lower power than the linear growth in s , and hence E , exhibited by our estimate (2.36) of the cross section due to graviton+KK exchange, the latter would eventually exceed the neutrino cross sections based on W and Z exchange. Using the lower bound (1.9) with our estimate (2.36), yields, for the contribution of gravitational scattering to νp (or $\bar{\nu} p$) scattering, the result $\sigma_{grav+KK} \sim 10^{-44} (E_\nu/\text{GeV})$ cm², so that at $E_\nu \sim 10^{21}$ eV, this gravitational contribution is $\sigma_{grav+KK} \sim 10^{-32}$ cm², roughly 10 % of the sum of the conventional charged and neutral current νp (or $\bar{\nu} p$) scattering cross sections, as calculated in Ref. [38]. Clearly our estimate (2.36) and the lower bounds (1.8), (1.9) are only approximate, so the implication that we draw from the present discussion is that in theories with large compactification radii and TeV scale quantum gravity, it is possible for gravitational scattering to make a non-negligible contribution to ultra-high energy νp and $\bar{\nu} p$ total scattering cross sections. If, indeed, this new gravitational contribution to νp and $\bar{\nu} p$ scattering is significant, this will be important for large detectors like such as the Antarctic Muon and Neutrino Detector Array (AMANDA) [36] and the Auger Cosmic Ray Observatory [39] which will measure these UHE neutrinos; if one just used the conventional CC + NC cross sections, one would infer a somewhat larger (anti)neutrino flux than would actually be present. Since at energies $E \geq 10^{15}$ eV, the interaction length for (anti)neutrinos is smaller than the diameter of the

earth, and for $E \geq 10^{18}$ eV, the earth is opaque to (anti)neutrinos [38], it follows that this possible enhancement of the observed event rate would only occur for downward going (anti)neutrinos. It should be emphasized, however, that our discussion concerning the gravitational cross section is subject to the uncertainty regarding the characteristics of the dominant final states in gravitational scattering of UHE νp and $\bar{\nu} p$ collisions. In particular, it could be that these collisions involve considerable multi-graviton-KK production, in which case it would be interesting also to study the resultant propagation and interaction of these KK modes.

IV. LIGHT NEUTRINOS

For a number of years there has been accumulating evidence for neutrino masses and associated lepton mixing; at present, it is generally considered that the strongest evidence arises from the deficiency of the solar neutrino flux measured by the Homestake, Kamiokande, SuperKamiokande, GALLEX, and SAGE detectors [41] and the atmospheric neutrino anomaly first observed by the Kamioka and IMB detectors, confirmed by the Soudan-2 detector, and recently measured with high statistics by the SuperKamiokande detector [42]. An appealing explanation of this data is neutrino oscillations and associated neutrino masses and mixing. As is well known, fits to the solar neutrino data yield values of $\Delta m_{ij}^2 = |m(\nu_i)^2 - m(\nu_j)^2|$ of order 10^{-5} eV² for $\nu_e \rightarrow \nu_j$ (although ‘‘just-so’’ oscillations feature $\Delta_{1j} \sim 10^{-10}$ eV²), and the SuperKamiokande experiment fits its atmospheric neutrino anomaly as being due to the neutrino oscillations $\nu_\mu \rightarrow \nu_\tau$ (or $\nu_\mu \rightarrow \nu_s$, where ν_s is a light electroweak-singlet neutrino) with central value $\Delta_{23} \sim 2 \times 10^{-3}$ eV² [42,43]. This data, then, suggests neutrino masses in the range ≤ 0.1 eV for the mass eigenstates $\nu_i, i=1,2,3$ whose linear combinations comprise the neutrinos ν_e, ν_μ , and ν_τ . Many details require further experimental clarification, but the most striking aspect of this is the smallness of these masses relative to the masses of the quarks and charged leptons. Indeed, independently of these suggestions for nonzero neutrino masses, direct mass limits yield upper bounds on the masses of the mass eigenstates comprising $\nu_l, l=e, \mu, \tau$ are much smaller than the masses of the respective charged leptons.

A fundamental theory should therefore give an explanation of why neutrinos are so light. The well-known seesaw mechanism does this [44]. However, we are not aware of any mechanism of comparable simplicity and elegance in theories in which the highest fundamental scale is of order 10 TeV. To see the problem, we recall how the seesaw mechanism works. This is based on the assumption that there exist some number of electroweak-singlet neutrinos, $\{N_R\}$. In SO(10) grand unified theory, this is not an *ad hoc* hypothesis but an intrinsic feature; the fermions of each generation transform according to the 16-dimensional spinor representation, $16_L = 10_L + 5_L^c + 1_L$, where the SU(5) representations are indicated, and the SU(5)-singlet state is the N_L^c (equivalent to N_R). This also fixes the number of electroweak-singlet neutrinos as equal to the number of the usual fermion generations, $N_{gen} = 3$. These electroweak-singlet neutrinos lead to Dirac mass terms of the form

$$\sum_{i,j} \bar{\nu}_{j,L} M_{D,ij} N_{i,R} + \text{H.c.} \quad (4.1)$$

and Majorana mass terms of the form $\sum_{i,j} N_{i,R}^T M_{R,ij} N_{j,R} + \text{H.c.}$, where i, j are generation indices. Because the Majorana terms are electroweak-singlet operators, the coefficients in the matrix M_R should not be related to the electroweak symmetry breaking scale, but instead, taking a top-down view, would naturally be of order the largest mass scale in the theory. The specific SO(10) GUT provides an explicit realization of this expectation, with the coefficients in M_R naturally of order the GUT scale. If M_R has maximal rank, then the diagonalization of the full Dirac + Majorana neutrino mass matrix yields a set of heavy, mainly weak-singlet Majorana neutrinos with masses of order M_{GUT} and the three light mass Majorana eigenstates, which are mainly linear combinations of the observed isodoublet weak neutrino eigenstates. The masses of the observed left-handed neutrinos are determined, to leading order in M_{ew}/M_R , as the eigenvalues of the matrix

$$M_\nu = -M_D M_R^{-1} M_D^T. \quad (4.2)$$

For a generic matrix M_R (with maximal rank), this provides a natural explanation of why these masses are much smaller than the scale of the quarks and charged leptons. At a more phenomenological level, even without any explicit consideration of electroweak-singlet neutrinos, one can also get naturally small neutrino masses by dimension-5 operators of the form $(1/M_X) \mathcal{L}_L^T C \mathcal{L}_L \phi \phi$, where $\mathcal{L} = (\nu_l, l)$ is the l 'th lepton doublet and ϕ is the $Y=1$ Higgs boson in the standard model or its supersymmetric extension (for an explicit expression, see, e.g., Ref. [45]). Such higher-dimension operators occur naturally if one regards the standard model as an effective low-energy field theory; however, the success of this model requires that one make $M_X \gg M_{ew}$. The resulting neutrino masses are $\sim M_{ew}^2/M_X$ and hence are naturally small.

In a theory in which there are no fundamental mass scales $\gg M_{ew}$, this type of mechanism is not available to explain small neutrino masses. It is true that rapid running of gauge and Yukawa couplings g_a and Y_{ij} will occur once the energy gets to the point where the standard model fields feel the extra dimensions, since $\dim(g) = \dim(Y) = (4-d)/2$ and hence are dimensional [12]. However, the corresponding powers of E in the effective couplings divide out in ratios, so it is not clear how this mechanism would specifically pick out the neutrinos to be so light.

Of course, one notices that for the case $n=2$, with M_6 near its lower bound (1.9), the theory exhibits a mass scale $r_2^{-1} \sim 0.07$ eV, as in Eq. (1.6), and the square of this scale, $r_2^{-2} \sim 5 \times 10^{-3}$, is comparable to the value of Δ_{23} inferred by the SuperKamiokande collaboration to fit their atmospheric anomaly. But it is not clear to us how to relate the compactification mass scale r_2^{-1} to masses of fermions, and in particular, neutrinos. It is also not clear how to get a generational (family) structure, and corresponding spectrum of mass values, out of a single r_2 , although one should remem-

ber that the assumption of a single compactification radius was only made as a simplification, and, in the absence of some symmetry to guarantee equal compactification radii for the different extra dimensions, the generic expectation is that these radii would be at least somewhat different.

V. QUARK-LEPTON UNIFICATION

One of the deepest goals of a physical theory is to provide a unified understanding of a wide range of different quantities and phenomena. Thus, one of the greatest appeals of grand unified theories is that they provide (i) a basic unification of the QCD and electroweak gauge interactions, embedding the associated $SU(3)$ and $SU(2) \times U(1)_Y$ gauge groups in a simple group; and (ii) correspondingly, a unification of quarks and leptons, since these are transformed into each other (or their conjugates) by the GUT gauge transformations. Proton and bound neutron decay is a natural consequence of this unification, and the slow, logarithmic running of the gauge couplings is nicely concordant with the high mass scale M_{GUT} which is necessary to suppress nucleon decay adequately to agree with experimental limits. Consequently, a fundamental concern with theories whose largest fundamental scale is of order 10 TeV, say, is that this scale may be so close to the electroweak scale, where the standard model gauge group is definitely not unified in a larger group, that it may be unnatural for such a unification to occur. Even if the rapid power-law running of gauge couplings were to lead to some sort of unification of these couplings, one would obviously not want this to be accompanied by the onset of a simple grand unified group with GUT scale of order 10 TeV, since this would generically lead to prohibitively rapid proton decay. This is not to say that one cannot contrive special mechanisms to suppress proton decay in such models (see, e.g., [12,9,10]), but true quark-lepton unification generically leads to proton decay at the scale of unification, so if one attempts such unification at the TeV scale, one must resort to special suppression mechanisms.

VI. EQUIVALENCE PRINCIPLE

A basic experimental property of gravity is the equivalence principle, that inertial and gravitational mass are equal, or, in Einstein's formulation, the statement that the effects of a uniform gravitational field are equivalent to the effects of a uniform acceleration of the coordinate frame and, related to this, that gravity couples only to mass, independent of other quantum numbers of the particles involved. This has been tested in laboratory and solar system measurements down to the level of about 10^{-12} [18,46,47]. This principle can be derived from the properties of the coupling of the massless spin-2 graviton required to yield Lorentz-invariant amplitudes [48,46]. Thus, modifications to this formalism will, in general, lead to violations of the equivalence principle. For example, it was recognized that such violations would result from a massless or sufficiently light ($m \lesssim 10^{-3}$ eV) dilaton in string theories of quantum gravity [49]. Moreover, one might question how natural it is to propose that the fundamental scale of gravity is close to the scale of an interaction, namely

the electroweak interaction, which does not couple to particles in a manner dictated by the principle of equivalence but instead according to their gauge group representations. We obtain a rough estimate of the violation of the equivalence principle in the present class of theories as follows. As discussed before, the KK modes of the gravitons couple like the graviton, but, from a four-dimensional point of view, have a mass given by Eq. (2.3). The minimal value of this mass is just $\mu = r_n^{-1}$ and, in the $n=2$ case where r_n is largest, this is listed in Eq. (1.6) for $M_6 = 30$ TeV as $\mu \sim 0.07$ eV. The long-distance effects of this lightest KK particle are suppressed by a usual exponential, $e^{-\mu r}$. For values of r in the macroscopic range, say $r \sim 10$ cm, this suppression factor is far below the 10^{-12} level.

VII. COSMOLOGICAL CONSTANT

The issue of the cosmological constant Λ and the associated vacuum energy density

$$\rho_\Lambda = \frac{\Lambda}{8\pi G_N} \quad (7.1)$$

has been a longstanding and vexing one in cosmology. As is well known, vacuum fluctuations of quantum fields naturally yield contributions to ρ_Λ that are many orders of magnitude larger than the upper limit $\Omega_\Lambda \lesssim 2$ derived from the requirement not to overclose the universe, where

$$\Omega_\Lambda = \frac{\rho_\Lambda}{\rho_c} = \frac{\Lambda^2}{3H_0^2} \quad (7.2)$$

and the critical mean mass density, ρ_c , is given by

$$\begin{aligned} \rho_c &= \frac{3H_0^2}{8\pi G_N} = 1.05 \times 10^{-5} h_0^2 \text{ GeV/cm}^3 \\ &= 3.97 \times 10^{-11} \left(\frac{h_0}{0.7} \right)^2 \text{ eV}^4 \end{aligned} \quad (7.3)$$

with H_0 the current value of the Hubble constant H defined as $H = R^{-1} dR/dt$, where $R(t)$ is the scale factor in the standard Friedmann-Robertson-Walker metric. In terms of $h_0 = H_0/(100 \text{ km/sec/Mpc})$, the current value of the Hubble constant is $0.5 < h_0 < 0.85$ (e.g. Ref. [40]). Specifically, in quantum gravity without supersymmetry one would generate a contribution to ρ_Λ of order M_{Pl}^4 , a factor of 10^{125} times larger than the upper limit. Even if this is avoided by supersymmetry, the electroweak symmetry breaking contributes a term of order M_{ew}^4 , which is 10^{65} times larger than ρ_c , and this cannot be avoided by supersymmetry, since supersymmetry must be broken at energies of at least the scale M_{ew} . From a field-theory point of view, one would argue that since no symmetry prevents a nonzero Λ , it should be present [50]. Indeed, recent data on type Ia supernovae [51,52] and other observational data [53] suggests that Ω_Λ is nonzero and of order unity (see, e.g., Figs. 6 and 7 of Ref. [52]).

It is natural that whenever a new type of theory is proposed, one tries to explore its implications for the cosmologi-

cal constant. In any theory with compactified dimensions, one is led to investigate the effects of compactification on vacuum fluctuations and resultant contributions to the cosmological constant [54]. One observation is that if one simply extracts mass and length scales phenomenologically from ρ_Λ by using the value $\Omega_\Lambda \sim 1$ suggested by the supernovae data [52,51], setting

$$\rho_\Lambda \equiv r_\Lambda^{-4} \quad (7.4)$$

then one obtains

$$r_\Lambda^{-1} \sim 2.5 \times 10^{-3} \text{ eV}, \text{ i.e., } r_\Lambda \sim 0.8 \times 10^{-2} \text{ cm.} \quad (7.5)$$

This is intriguingly close to the value of the compactification radius $r_2 \sim O(1)$ millimeter implied by Eq. (1.5) if one were to use the original estimate $M_{4+n} = M_6 \sim 1$ TeV for the $n = 2$ case. The supernova cooling constraint of $M_6 \gtrsim 30$ TeV in Eq. (1.9) reduces r_2 to \sim a micron rather than \sim a millimeter, thereby removing a close similarity to r_Λ . Nevertheless, in view of the disparity of 65 orders of magnitude between M_{ew}^4 and the upper bound on ρ_Λ , there is perhaps a hint of some interesting physics relating r_n and r_Λ . In the theoretical context of both KK theories and D-branes, one is also led to consider Casimir-type contributions to the vacuum energy density [55], which are of the form $U_{Cas.} \propto r^{-4}$, where r is a relevant distance scale, such as the compactification scale. In this context, there is, indeed, a natural connection between ρ_Λ and r_n^{-4} (see also Refs. [10,13,15,56]).

VIII. CONCLUSIONS

Theories with large compactification radii and a very low string scale represent a radical departure from previous fun-

damental particle theory including grand unification and conventional string theory. They serve to emphasize that a vast 33 orders of magnitude separate the length scale of about a centimeter at which gravity has been experimentally measured from the Planck length of 10^{-33} cm, and it is quite possible that new phenomena occur in these 33 decades of energy or length which significantly modify gravity from one's naive extrapolations. We believe that it is healthy to formulate and study such radical challenges to the orthodox paradigm in order to assess how experimentally viable they are. Indeed, an intense effort to analyze the phenomenology of these types of theories is currently underway. In these theories the Planck mass becomes a secondary rather than basic quantity, and is expressed via Eq. (1.5) as a function of the short-distance string scale and the compactification radius. In this paper we have contributed a few results to this effort. We have made an estimate of high-energy gravitational scattering cross sections and have discussed some of the implications for ultra-high-energy cosmic ray and neutrino scattering. We have also commented on some other topics, including naturally light neutrino masses, quark-lepton unification, the equivalence principle, and the cosmological constant. Many more interesting questions deserve study.

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