Can the cosmological constant support a scalar field?

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(Received 6 July 1998; published 26 March 1999)

Static spherically symmetric gravitational equilibria of the real scalar field are discussed in the presence of the cosmological constant. We find nontrivial solutions if we take the self-interaction of the field into account while there is no such equilibria in the noninteracting case. The system has critical parameters beyond which new solutions disappear. They are determined by the ratio of the cosmological horizon to the ''Compton radius'' of the scalar field. We also discuss the stability of the solutions by means of a linear perturbation method and find that the number of unstable modes depends on the node number of the scalar field. [S0556-2821(99)03006-4]

PACS number(s): 04.20.Jb, 04.40.$-$b, 95.30.Sf

I. INTRODUCTION

In 1968 a new type of star solution called a boson star, which consists of a massive free complex scalar field, was discovered [1–3]. Such an object is a macroscopic quantum state that is only prevented from collapsing gravitationally by the Heisenberg uncertainty principle. Since the mass of the boson star is estimated as $M_{bs}\sim m_{\rm Pl}^2/m_\phi$, where $m_{\rm Pl}$ and m_ϕ are the Planck mass and the mass of the scalar field, respectively, m_ϕ must be extremely small in order to form a boson star with a solar mass. However, a dramatic change occurs if we take the self-interaction of the scalar field into account. The mass of the boson star is expected to be $M_{bs}\sim m_{\rm Pl}^3/m_\phi^2$ [4,5], which is of the order of neutron stars, and the boson star has a solar mass with m_ϕ comparable with, e.g., the axion mass. Thereafter, many altered solutions were derived, for example, soliton stars, which have a larger mass $M_{ss}\sim m_{\rm Pl}^4/m_\phi^3$ [6] than the boson star and Q stars [7]. These self-gravitating solutions with a nonzero finite mass and no singularity are called nontopological soliton stars. Cosmological effects of these structures may be significant because they can be one of the candidates for the dark matter of the universe, although we have to clarify the mass and the interaction of the particles of dark matter.

Several attempts have been made to construct a boson star solution consisting of a real scalar field. Since equilibrium soliton star solutions are possible due to the existence of conserved Noether currents [6,8] associated with a global U(1) symmetry, a system of self-gravitating real massive scalar field does not admit regular static solutions [9–12]. Hence the real scalar field must collapse to a black hole, escape to infinity, or oscillate periodically. Surprisingly the latter solution exists as Seidel and Suen pointed out [13]. This nonstatic solution is called the oscillating boson star.

In this paper we investigate the self-gravitating real scalar field in the presence of the cosmological constant and discuss whether the cosmological constant can support the real scalar field in the static spherically symmetric spacetime, motivated by the following reasons. The nonexistence of a regular static solution is partly due to the boundary condition at the origin or at infinity. Hence we expect that the previous result may be changed by the cosmological constant, which produces a cosmological horizon. Furthermore observational results support the existence of the cosmological constant [14]. In particular recent observation of the type Ia supernova at $z=0.83$ supports a universe with the cosmological constant [15]. Furthermore the existence of the static solution may produce new aspects to the dynamical evolution of the real scalar field in de Sitter spacetime, as we will discuss in Sec. V, and have an effect on physics in the inflationary or reheating era of the universe.

This paper is organized as follows. In the next section, we explain the model and derive the basic equations. We show nontrivial solutions obtained by numerical calculation, and discuss why there are critical parameters, beyond which no nontrivial solutions exist, in Sec. III. In Sec. IV we investigate the stability of the new solutions by using a perturbation analysis. We give our conclusions and remarks in the final section.

II. MODEL AND BASIC EQUATIONS

We start with the action

$$S=\int d^4x\sqrt{-g}\left[\frac{1}{16\pi G}(R-2\Lambda)-\frac{1}{2}\partial_\mu\phi\,\partial^\mu\phi-V(\phi)\right], \tag{1}$$

where ϕ is the real scalar field and $V(\phi)$ is its potential. We shall assume a spherically symmetric spacetime and adopt the following metric:

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$$ds^2 = -\left(1 - \frac{2Gm(t,r)}{r} - \frac{\Lambda}{3}r^2\right)e^{-2\delta(t,r)}dt^2 + \left(1 - \frac{2Gm(t,r)}{r} - \frac{\Lambda}{3}r^2\right)^{-1}dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2). \quad (2)$$

$m(t,r)$ is the mass function, which is the quasilocal mass defined by Ref. [16], and $\delta(t,r)$ is the lapse function.

The field equations derived from Eq. (1) with the metric (2) are

$$\tilde{m}' = 4\pi\tilde{r}^2\left[\frac{1}{2}B^{-1}e^{2\delta}\dot{\tilde{\phi}}^2 + \frac{1}{2}B\tilde{\phi}'^2 + \tilde{V}(\tilde{\phi})\right], \quad (3)$$

$$\delta' = -4\pi\tilde{r}[B^{-2}e^{2\delta}\dot{\tilde{\phi}}^2 + \tilde{\phi}'^2], \quad (4)$$

$$\dot{\tilde{m}} = 4\pi\tilde{r}^2 B\dot{\tilde{\phi}}\tilde{\phi}', \quad (5)$$

$$-[e^{\delta}B^{-1}\dot{\tilde{\phi}}]^{\cdot} + \frac{1}{\tilde{r}^2}[\tilde{r}^2 e^{-\delta}B\tilde{\phi}']' = e^{-\delta}\frac{d\tilde{V}(\tilde{\phi})}{d\tilde{\phi}}, \quad (6)$$

$$B \equiv 1 - \frac{2\tilde{m}}{F} - \frac{1}{3}\tilde{r}^2,$$

where we have used dimensionless variables normalized by Λ and G as $\tilde{t} \equiv \sqrt{\Lambda}t$, $\tilde{r} \equiv \sqrt{\Lambda}r$, $\tilde{m} \equiv \sqrt{\Lambda}Gm$, $\tilde{\phi} \equiv \sqrt{G}\phi$ and $\tilde{V} \equiv GV/\Lambda$. A dot and a prime denote derivatives with respect to $\tilde{t}$ and $\tilde{r}$, respectively.

In order to find a static solution, we have to discuss in advance the boundary conditions at the origin and on the cosmological horizon $(r = r_c)$. First, expanding the field functions around the origin and substituting them into static field equations, we find

$$\tilde{m} = \frac{8\pi}{6}\tilde{V}(\tilde{\phi}_0)\tilde{r}^3 + \cdots, \quad (7)$$

$$\delta = \delta_0 - \frac{4\pi}{9}\left(\frac{d\tilde{V}}{d\tilde{\phi}}\right)^2\Bigg|_{\tilde{\phi}=\tilde{\phi}_0}\tilde{r}^3 + \cdots, \quad (8)$$

$$\tilde{\phi} = \tilde{\phi}_0 + \frac{1}{6}\frac{d\tilde{V}}{d\tilde{\phi}}\Bigg|_{\tilde{\phi}=\tilde{\phi}_0}\tilde{r}^2 + \cdots. \quad (9)$$

We integrate the static field equations from the origin with shooting parameters $\tilde{\phi}_0$ and δ_0, which are chosen to satisfy the boundary conditions on the cosmological horizon. On the horizon we assume that curvature, field variables, and their derivatives are finite. In this paper we put $\delta(r_c) = 0$. If we are interested in a different boundary condition such as $\delta \to \delta^* \neq 0$, we can always have such a boundary condition without further calculation because the constant difference is absorbed in the time coordinate by rescaling. That is, introducing $\bar{\delta} \equiv \delta - \delta^*$, and rescaling the time coordinate as $\bar{t} = e^{-\delta^*}\tilde{t}$, we recover our boundary condition.

Now we discuss concrete forms of the potential of the scalar field. First we consider the convex potential $\tilde{V}(\tilde{\phi}) = m_\phi^2\tilde{\phi}^2$. We can put $\tilde{\phi}_0 \geqslant 0$ without loss of generality. Since $d\tilde{V}/d\tilde{\phi}|_{r=0} > 0$, $\tilde{\phi}''|_{r=0}$ is positive by Eq. (9). This means that the scalar field $\tilde{\phi}$ begins to increase near the origin. Eq. (6) is rewritten as

$$B\tilde{\phi}'' + \left[\left(\frac{2}{\tilde{r}} + 4\pi\tilde{r}\tilde{\phi}'^2\right)B^2 + B'\right]\tilde{\phi}' = \frac{d\tilde{V}(\tilde{\phi})}{d\tilde{\phi}}. \quad (10)$$

Here we assume that $\tilde{\phi}$ has the extremum, i.e., $\tilde{\phi}' = 0$ between the origin and the cosmological horizon. At the extremum of $\tilde{\phi}$ Eq. (10) becomes

$$B\tilde{\phi}'' = \frac{d\tilde{V}(\tilde{\phi})}{d\tilde{\phi}}. \quad (11)$$

When $\tilde{\phi} > 0$, i.e., $dV/d\tilde{\phi} > 0$, $\tilde{\phi}''$ is always positive. This means that the extremum is not the maximum but the minimum. Hence $\tilde{\phi}$ increases monotonically to the cosmological horizon. On the cosmological horizon Eq. (10) becomes

$$B'\tilde{\phi}' = \frac{d\tilde{V}(\tilde{\phi})}{d\tilde{\phi}}. \quad (12)$$

The left-hand side is negative because B' is always negative around the cosmological horizon, while the right-hand side is positive. This is a contradiction. As a result, there is no regular static solution with the convex potential except for the trivial solution $\tilde{\phi} \equiv 0$. This result is different from the usual boson star case [1–3], where the time dependence of a phase of the complex scalar field plays an important role and gives nontrivial configurations.

Next we investigate a double well potential $\tilde{V}(\tilde{\phi}) = \tilde{\lambda}(\tilde{\phi}^2 - \tilde{v}^2)^2/4$, where $\tilde{\lambda} = \lambda/G\Lambda$ and $\tilde{v} = \sqrt{G}v$ are the rescaled self-coupling constant and vacuum expectation value, respectively. Before we proceed to the nontrivial case, we mention the trivial solutions. There are two analytic solutions, one of which is (a) $\tilde{\phi} \equiv \tilde{v}$, $\tilde{m} \equiv 0$, $\delta \equiv 0$. This is the usual de Sitter solution. The other one is (b) $\tilde{\phi} \equiv 0$, $\tilde{m} = \pi\tilde{\lambda}\tilde{v}^2\tilde{r}^3/3$, $\delta \equiv 0$. This is another de Sitter solution. However, the latter includes not only the cosmological constant Λ but also the "effective cosmological constant" defined by $\Lambda_{\text{eff}} = 2\pi\lambda\tilde{v}^4$, because the scalar field sits on the top of the potential. Obviously, the latter is the excited solution. In the nontrivial case we can also restrict $\tilde{\phi}_0 > 0$ because the scalar field has reflection symmetry. If we put $\tilde{\phi}_0 > \tilde{v}$, the same discussion as in the convex potential case holds and there is no regular solution. If we put $0 < \tilde{\phi}_0 < \tilde{v}$, $d\tilde{V}/d\tilde{\phi}$ is negative and then $\tilde{\phi}''$ becomes negative by Eq. (9) and hence $\tilde{\phi}$ decreases near the origin. In the intermediate region Eq. (11) indicates that there is no local maximum when $-\tilde{v} < \tilde{\phi} < 0$

or $\tilde{\phi}>\tilde{v}$ and that there is no local minimum when $\tilde{\phi}<-\tilde{v}$ or $0<\tilde{\phi}<\tilde{v}$. On the cosmological horizon Eq. (12) says that $\tilde{\phi}'$ has the opposite sign of $d\tilde{V}/d\tilde{\phi}$, which means $\tilde{\phi}'>0$ if $\tilde{\phi}<-\tilde{v}$ or $0<\tilde{\phi}<\tilde{v}$, and $\tilde{\phi}'<0$ if $-\tilde{v}<\tilde{\phi}<0$ or $\tilde{\phi}>\tilde{v}$. Consequently the only consistent behavior for the scalar field is the following case: (c) $\tilde{\phi}$ starts with the value $0<\tilde{\phi}_0<\tilde{v}$ from the origin and goes over the potential barrier (passing $\tilde{\phi}=0$), and then it decreases monotonically to $-\tilde{v}<\tilde{\phi}(r_c)<0$ on the cosmological horizon or oscillates once or several times in the region of $-\tilde{v}<\tilde{\phi}<\tilde{v}$. In each oscillation $\tilde{\phi}$ must go over the top of the potential. Finally $\tilde{\phi}$ takes $-\tilde{v}<\tilde{\phi}<0$ with $\tilde{\phi}'<0$ or $0<\tilde{\phi}<\tilde{v}$ with $\tilde{\phi}'>0$ on the cosmological horizon.

III. STATIC SOLUTIONS

Now we turn to the numerical analysis. By adjusting the shooting parameters, we found the nontrivial solutions when $\tilde{\lambda}$ and $\tilde{v}$ satisfy a certain condition which we will discuss later. These solutions are classified into several families by the node number n of the scalar field. Figure 1 shows configurations of the scalar field with $n=1$ for $\tilde{v}=0.1$ and various $\tilde{\lambda}$. The structure spreads to the cosmological scale. For large $\tilde{\lambda}$, which means that the cosmological constant is small, the scalar field continues to stay $\tilde{\phi}\approx\tilde{v}$ even for quite large r. If $\tilde{\phi}$ is much different from its vacuum value near the origin, $\tilde{\phi}$ changes rapidly by Eq. (11) and has large kinetic energy. However, as the cosmological constant is not large enough to support the gravitational force produced by such energy, the scalar field will collapse. Hence $\tilde{\phi}$ must take almost vacuum value around the origin.

As $\tilde{\lambda}$ decreases, the amplitude of $\tilde{\phi}$ becomes small and finally the solution coincides with the excited de Sitter solution (b) ($\tilde{\phi}\equiv 0$) at some critical value of $\tilde{\lambda}=\tilde{\lambda}_{\rm cr}\cong 423$. We show the critical parameters in the $\tilde{\lambda}$-$\tilde{v}$ plane in Fig. 2. In the right-hand side of each line $(n=1,2,3)$, there exist nontrivial solutions with $n=1,2,3$. On the critical lines the solution becomes the excited de Sitter solution (b).

What is the physical meaning of existence of the critical parameters? In order to answer this question we examine nontrivial solutions near the critical lines. When we shift the parameters

$$\tilde{\lambda}_{\rm cr}\to\tilde{\lambda}=\tilde{\lambda}_{\rm cr}+\tilde{\lambda}_1\epsilon, \tag{13}$$

$$\tilde{v}_{\rm cr}\to\tilde{v}=\tilde{v}_{\rm cr}+\tilde{v}_1\epsilon, \tag{14}$$

where ϵ is an infinitesimal parameter, the field functions shift as

$$\tilde{\phi}_{\rm dS}\to\tilde{\phi}=\tilde{\phi}_{\rm dS}+\tilde{\phi}_1\epsilon+\frac{1}{2}\tilde{\phi}_2\epsilon^2+\cdots, \tag{15}$$

$$\tilde{m}_{\rm dS}\to\tilde{m}=\tilde{m}_{\rm dS}+\tilde{m}_1\epsilon+\frac{1}{2}\tilde{m}_2\epsilon^2+\cdots, \tag{16}$$

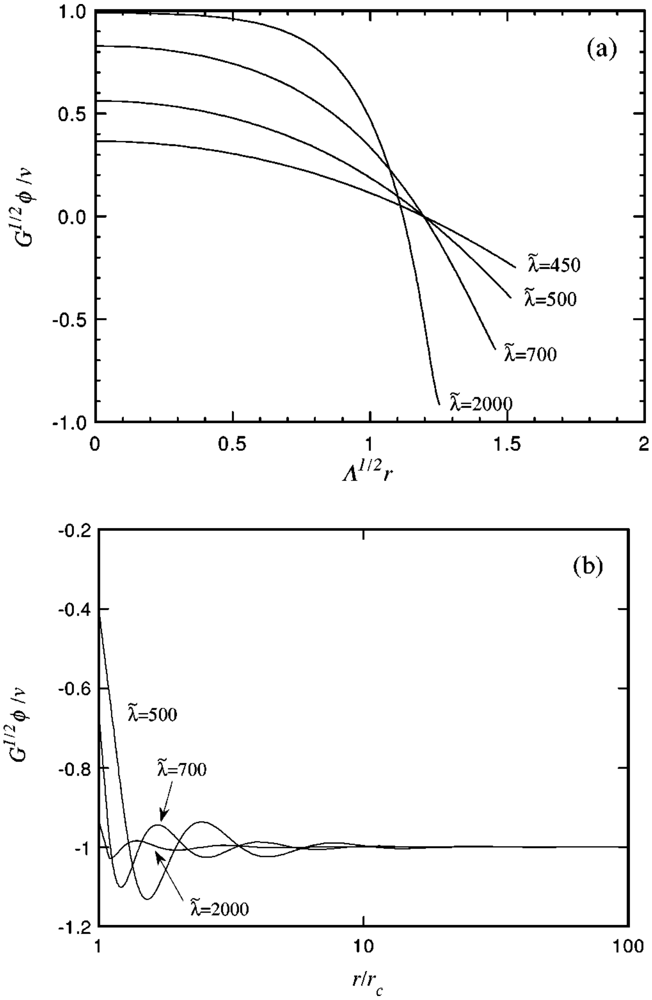

FIG. 1. The configurations of the scalar field $\tilde{\phi}=\sqrt{G}\phi$ with one node $(n=1)$. We set $\tilde{v}=\sqrt{G}v=0.1$ and show the solutions for $\tilde{\lambda}=\lambda/G\Lambda=450$, 500, 700, 2000. (a) shows between the origin and the cosmological horizon. As $\tilde{\lambda}$ becomes small, $\tilde{\phi}_0$ approaches zero and the solution coincides with the excited de Sitter solution in the $\tilde{\lambda}\to\tilde{\lambda}_{\rm cr}$ limit. (b) shows beyond the cosmological horizon. The radial coordinate is normalized by the radius of the cosmological horizons, which are $\tilde{r}_c=\sqrt{\Lambda}r_c=1.5124$, 1.4563, 1.2531 for $\tilde{\lambda}=\lambda/G\Lambda=500$, 700, 2000, respectively. The scalar field oscillates around its vacuum value $-\tilde{v}$ with damping. We can see that the spacetime approaches de Sitter spacetime asymptotically.

$$\delta_{\rm dS}\to\delta=\delta_{\rm dS}+\delta_1\epsilon+\frac{1}{2}\delta_2\epsilon^2+\cdots, \tag{17}$$

where $\tilde{\phi}_{\rm dS}=0$, $\delta_{\rm dS}=0$, and $\tilde{m}_{\rm dS}=\pi\tilde{\lambda}_{\rm cr}\tilde{v}_{\rm cr}^4\tilde{r}^3$. Substituting Eqs. (13)–(17) into Eq. (6) and taking the first order of ϵ, we obtain

$$\frac{1}{\tilde{r}^2}\left[\tilde{r}^2\left(1-\frac{2\tilde{m}_{\rm dS}}{\tilde{r}}-\frac{1}{3}\tilde{r}^2\right)\tilde{\phi}_1'\right]'=-\tilde{\lambda}_{\rm cr}v_{\rm cr}^2\tilde{\phi}_1. \tag{18}$$

Introducing new variables $\tilde{\psi}_1=\tilde{r}\tilde{\phi}_1$, $A=1+2\pi\tilde{\lambda}_{\rm cr}\tilde{v}_{\rm cr}^4$, and $\tilde{z}=\sqrt{A/3}\,\tilde{r}$, Eq. (18) becomes

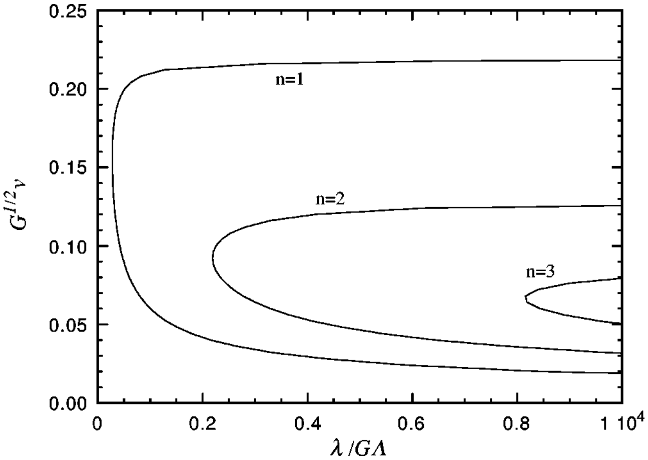

FIG. 2. The critical parameters of the nontrivial solutions with node $n=1$, 2, 3. In the right-hand side of each line the nontrivial solution exists. The critical lines are expressed as $\tilde{\lambda}_{\rm cr}^{(n)}=2n(2n+3)/[3\tilde{v}_{\rm cr}^{(n)2}-2n(2n+3)\pi\tilde{v}_{\rm cr}^{(n)4}]$ and approach $\tilde{v}_{\rm cr}^{(n)}=0$ and $\tilde{v}_{\rm cr}^{(n)}=\sqrt{3/4\pi n(2n+3)}$ in the large $\tilde{\lambda}_{\rm cr}^{(n)}$ limit. On the critical lines the nontrivial solutions coincide with the excited de Sitter solution.

$$\frac{d}{d\tilde{z}}\left[(1-\tilde{z}^2)\frac{d\tilde{\psi}_1}{d\tilde{z}}\right]+\left(2+\frac{3}{A}\tilde{\lambda}_{\rm cr}\tilde{v}_{\rm cr}^2\right)\tilde{\psi}_1=0, \tag{19}$$

This has the same form as the well known Legendre differential equation

$$\frac{d}{dz}\left[(1-z^2)\frac{du}{dz}\right]+\nu(\nu+1)u=0. \tag{20}$$

In the present case, the boundary conditions at the origin ($\tilde{r}=0$ and $\tilde{z}=0$) are $\tilde{\phi}_1=C_1(=\text{const})$ and $\tilde{\phi}_1'=0$, i.e., $\tilde{\psi}_1=0$ and $\tilde{\psi}'=C_2(=\text{const})$. On the other hand, the behavior of the Legendre function P_ν around the origin is

$$P_\nu(0)=-\frac{\sin\nu\pi}{2\sqrt{\pi^3}}\Gamma\left(\frac{\nu+1}{2}\right)\Gamma\left(-\frac{\nu}{2}\right). \tag{21}$$

When $\nu\notin\boldsymbol{Z}(\boldsymbol{Z}$: integer), $P_\nu(0)\neq 0$ from the properties of the Γ function. Hence it is enough to check its behavior when ν is a natural number n and zero because the Legendre function satisfies $P_\nu(0)=P_{-\nu-1}(0)$. When $\nu=2n-1$ is positive odd, $\sin(\nu\pi)=0$ and $0<\Gamma(\cdot)\Gamma(\cdot)<\infty$, then $P_\nu(0)=0$. On the other hand, when $\nu=2n$ is positive even or zero, $\sin(\nu\pi)=0$ and $\Gamma(\cdot)\Gamma(\cdot)=\infty$, then $P_\nu(0)$ is indefinite. By using properties of the Γ function Eq. (21) is rewritten as

$$P_\nu(0)=-\frac{1}{\sqrt{\pi^3}}\cos\left(\frac{\nu}{2}\pi\right)\sin\left(\frac{\nu}{2}\pi\right)\Gamma\left(\frac{\nu+1}{2}\right)\Gamma\left(-\frac{\nu}{2}\right)$$

$$=\frac{1}{\sqrt{\pi}}\cos\left(\frac{\nu}{2}\pi\right)\Gamma\left(\frac{\nu+1}{2}\right)\Bigg/\Gamma\left(\frac{\nu+2}{2}\right). \tag{22}$$

If we assume $\nu=2n$ or 0, $\cos(\nu\pi/2)=\pm 1$, and $0<\Gamma(\cdot)/\Gamma(\cdot)<\infty$, then $P_\nu(0)\neq 0$. As a result, when ν is positive odd or negative even, $P_\nu(0)=0$, and the solutions of the Legendre equation satisfy the boundary condition at the origin; moreover are also regular on the cosmological horizon ($r=r_c$ or $z=1$). Comparing Eq. (19) with Eq. (20), we obtain

$$\tilde{\lambda}_{\rm cr}=\frac{(\nu+2)(\nu-1)}{3\tilde{v}_{\rm cr}^2-2\pi\tilde{v}_{\rm cr}^4(\nu+2)(\nu-1)}. \tag{23}$$

When $\nu=1$ or -2, $\tilde{\lambda}_{\rm cr}$ becomes zero and the solutions correspond to the usual de Sitter solution (a). When $\nu=3$ or -4, $\tilde{\lambda}_{\rm cr}^{(1)}=10/(3\tilde{v}_{\rm cr}^{(1)2}-20\pi\tilde{v}_{\rm cr}^{(1)4})$, and when $\nu=5$ or -6, $\tilde{\lambda}_{\rm cr}^{(2)}=28/(3\tilde{v}_{\rm cr}^{(2)2}-56\pi\tilde{v}_{\rm cr}^{(2)4})$. These equations correspond to the critical lines with $n=1$ and $n=2$ in Fig. 2, respectively. Generally the critical line with node n is expressed by

$$\tilde{\lambda}_{\rm cr}^{(n)}=\frac{2n(2n+3)}{3\tilde{v}_{\rm cr}^{(n)2}-4\pi n(2n+3)\tilde{v}_{\rm cr}^{(n)4}}. \tag{24}$$

We can see that the critical lines approach $\tilde{v}_{\rm cr}^{(n)}=0$ and $\tilde{v}_{\rm cr}^{(n)}=\sqrt{3/4\pi n(2n+3)}$ in the large $\tilde{\lambda}_{\rm cr}^{(n)}$ limit.

Equation (24) is expressed as

$$\left(\frac{\tilde{R}_{\rm cos}}{\tilde{\lambda}_{\rm Comp}}\right)^2=n(2n+3), \tag{25}$$

where $\tilde{R}_{\rm cos}=\sqrt{3/(1+\tilde{\lambda}_{\rm eff})}=\sqrt{3/(1+2\pi\tilde{\lambda}\tilde{v}^4)}$ is the typical scale of the cosmological horizon and $\tilde{\lambda}_{\rm Comp}=\sqrt{2/\tilde{\lambda}\tilde{v}^2}$ is the "Compton radius" of the scalar field. Note that usually the Compton radius indicates the size of the region around the origin where the energy of the field concentrates. In the present case, however, the scalar field takes almost vacuum value around the origin and at the cosmological horizon. Our "Compton radius" indicates the size of the intermediate region where the scalar field is around the top of the potential barrier. Equation (25) shows that ratio of $\tilde{R}_{\rm cos}$ and $\tilde{\lambda}_{\rm Comp}$ are bounded from below by constant $n(2n+3)$ for each node number. This means that as $\tilde{\lambda}_{\rm Comp}$ becomes small ($\tilde{\lambda}\to\tilde{\lambda}_{\rm cr}$) the structure becomes large and it cannot be packed into the radius $\tilde{R}_{\rm cos}$. Finally nontrivial solutions cannot exist for $\tilde{\lambda}<\tilde{\lambda}_{\rm cr}$.

Since nontrivial solutions are smooth on the cosmological horizon, they can be extended beyond the cosmological horizon to null infinity $\mathcal{I}^+$. As the sign of B is changed on the cosmological horizon, the behavior of the scalar fields is quite different from that within the cosmological horizon. Let us examine them around the vacuum expectation value $\tilde{v}$. Note that Eq. (11) shows the scalar field can have the maximum (minimum) when $\tilde{\phi}>\tilde{v}$ ($\tilde{\phi}<\tilde{v}$). Introducing a new variable $\tilde{\varphi}\equiv\tilde{\phi}-\tilde{v}$ and neglecting the second and higher orders of $\tilde{\varphi}$, we find that Eq. (10) is rewritten as

$$\tilde{\varphi}''+\mu\tilde{\varphi}'=-\omega^2\tilde{\varphi}, \tag{26}$$

where

$$\mu = \left[\frac{2}{\tilde{r}} + B^{-1} B' \right], \tag{27}$$

$$\omega^2 = -2 \tilde{\lambda} \tilde{v}^2 B^{-1}. \tag{28}$$

As μ is always positive beyond the cosmological horizon, the second term of Eq. (26) works as a friction term. Hence the field $\tilde{\varphi}$ behaves as a damped oscillator around the vacuum value $\tilde{v}$ with the frequency ω. The frequency of the oscillation becomes smaller as the spacetime approaches infinity because of the B^{-1} effect as well as of the friction force. Around the other vacuum value $-\tilde{v}$, the situation is the same as above. Figure 1(b) shows the configurations of the scalar field of the nontrivial solutions with $n=1$ and $\tilde{v}=0.1$. We can find that they oscillate with damping around $\tilde{\phi}=-\tilde{v}$. For the $n=2$ solution, the scalar fields damp to $\tilde{\phi}=\tilde{v}$. As a result, the spacetime approaches the de Sitter solution asymptotically and has the same global structure as de Sitter spacetime.

IV. STABILITY ANALYSIS

In this section we investigate the stability of solutions obtained in the previous section. First we expand the field functions as

$$\tilde{\phi}(\tilde{t},\tilde{r}) = \tilde{\phi}_0(\tilde{t}) + \frac{\tilde{\phi}_1(\tilde{t},\tilde{r})}{\tilde{r}} \epsilon, \tag{29}$$

$$\tilde{m}(\tilde{t},\tilde{r}) = \tilde{m}_0(\tilde{t}) + \tilde{m}_1(\tilde{t},\tilde{r}) \epsilon, \tag{30}$$

$$\delta(\tilde{t},\tilde{r}) = \delta_0(\tilde{t}) + \delta_1(\tilde{t},\tilde{r}) \epsilon, \tag{31}$$

around the static solution $\tilde{\phi}_0$, $\tilde{m}_0$, and δ_0. Substituting them into Eqs. (3)–(6), we obtain

$$\dot{\tilde{m}}_1 = 4\pi \tilde{r}^2 B_0 \overline{\tilde{\phi}_0'} \dot{\tilde{\phi}}_1, \tag{32}$$

$$\begin{aligned} &-e^{\delta_0} B_0^{-1} \ddot{\tilde{\phi}}_1 + [e^{-\delta_0} B_0 \tilde{\phi}_1']' \\ &\quad - \left[\frac{1}{\tilde{r}} (e^{-\delta_0} B_0)' + 8\pi \tilde{r} e^{-\delta_0} \lambda (\tilde{\phi}_0^2 - \tilde{v}^2) \tilde{\phi}_0 \tilde{\phi}_0' \right. \\ &\quad \left. + \lambda e^{-\delta_0} (3\tilde{\phi}_0^2 - \tilde{v}^2) \right] \tilde{\phi}_1 \\ &\quad - \left[\frac{2}{\tilde{r}} (\tilde{r} e^{-\delta_0} \tilde{\phi}_0')' - 8\pi \tilde{r} e^{-\delta_0} \tilde{\phi}_0'^3 \right] \tilde{m}_1 = 0, \end{aligned} \tag{33}$$

where $B_0 = 1 - 2\tilde{m}_0/\tilde{r} - F^2/3$. Next $\tilde{\phi}_1$ and $\tilde{m}_1$ are approximated by harmonic functions as

$$\tilde{\phi}_1 = \xi(\tilde{r}) e^{i\tilde{\sigma}\tilde{t}}, \tag{34}$$

$$\tilde{m}_1 = \eta(\tilde{r}) e^{i\tilde{\sigma}\tilde{t}}. \tag{35}$$

From Eq. (32) the relation between ξ and η is

$$\eta = 4\pi \tilde{r} B_0 \tilde{\phi}_0' \xi. \tag{36}$$

With Eqs. (33) and (36), the perturbation equation of the scalar field becomes

$$-\frac{d^2 \xi}{d\tilde{r}^{*2}} + U(\tilde{r}) \xi = \tilde{\sigma}^2 \xi, \tag{37}$$

where we employ the tortoise coordinate $\tilde{r}^*$ defined by

$$\frac{d\tilde{r}^*}{d\tilde{r}} = e^{\delta_0} B_0^{-1}, \tag{38}$$

and the potential function is

$$\begin{aligned} U(r) = e^{-\delta_0} B_0 &\left[\frac{1}{\tilde{r}} (e^{-\delta_0} B_0)' + 8\pi \tilde{r} e^{-\delta_0} \lambda (\tilde{\phi}_0^2 - \tilde{v}^2) \tilde{\phi}_0 \tilde{\phi}_0' \right. \\ &+ \lambda e^{-\delta_0} (3\tilde{\phi}_0^2 - \tilde{v}^2) \\ &\left. + 4\pi \tilde{r} B_0 \tilde{\phi}_0' \left\{ \frac{2}{\tilde{r}} (\tilde{r} e^{-\delta_0} \tilde{\phi}_0')' - 8\pi \tilde{r} e^{-\delta_0} \tilde{\phi}_0'^3 \right\} \right]. \end{aligned} \tag{39}$$

If there exists even one bound state with negative eigenvalue $\tilde{\sigma}^2$, the perturbed variables $\tilde{\phi}_1$ and $\tilde{m}_1$ diverge exponentially and the solution becomes unstable.

By numerical calculation, we find one or two negative eigenmodes in the $n=1$ case depending on the parameters $\tilde{\lambda}$ and $\tilde{v}$. The boundary which divides the number of negative eigenmodes takes a value close to the critical parameters $\tilde{\lambda}_{\mathrm{cr}}$ and $\tilde{v}_{\mathrm{cr}}$. For example, there are two negative modes below $\tilde{\lambda} \sim 500$ for fixed $\tilde{v}=0.1$. Figure 3 shows the zero-node eigenfunction of the solutions with $n=1$, $\tilde{v}=0.1$, and several values of $\tilde{\lambda}$. We confirmed that the solutions with larger n have also n or $n+1$ negative eigenmodes. As a result, all of the nontrivial solutions are found to be unstable.

V. CONCLUSION

We investigated the massive real scalar field in the static spherically symmetric spacetime with the cosmological constant. We showed that there is no nontrivial solution if the field has only a mass term. However, taking the self-interaction of the field into account, we found nontrivial solutions. These structures are interesting because no static singularity free solution exists without the cosmological constant. The distribution of the structure spreads out to the cosmological scale and is far from the boson star picture whose energy is concentrated in a small region described by

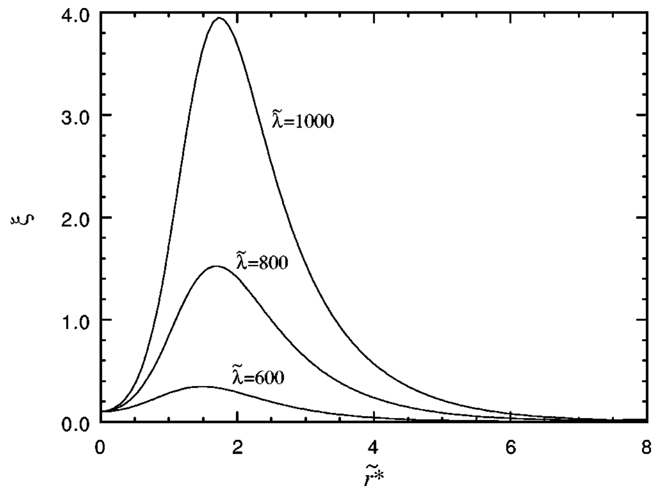

FIG. 3. The configurations of eigenfunction of the perturbation equation of the scalar field. We set $\tilde{v}=\sqrt{G}v=0.1$, $n=1$ and show the solutions for $\tilde{\lambda}=\lambda/G\Lambda=600$, 800, 1000. The eigenvalues for each eigenfunction are $\tilde{\sigma}^2=-0.911$, -0.594, -0.484.

its Compton radius. There are critical parameters $\tilde{\lambda}_{\rm cr}$ or $\tilde{v}_{\rm cr}$ beyond which there are no nontrivial solutions. The critical parameters are determined by the ratio of the cosmological horizon to the ''Compton radius'' of the scalar field. When the structure becomes too large, it cannot be packed into the cosmological horizon. New solutions are extendable beyond the cosmological horizon and the scalar field oscillates around its vacuum expectation values. The spacetime approaches the de Sitter solution asymptotically and has the same global structure as the de Sitter spacetime.

Linear perturbation analysis shows that new solutions are unstable even for radial perturbations. The reason of their instability is interpreted as follows. The scalar field is not fixed to the vacuum expectation value at the two boundaries (the origin and the cosmological horizon) but sits on the potential barrier. Hence when the scalar field is perturbed, it easily slips down. If enough friction is added in Eq. (6) by considering another potential or by adopting another matter field, this kind of structure may be stabilized.

Although we focused only on the static solutions, they may affect the dynamic evolution of the scalar field in de Sitter spacetime, in particular, the critical behavior of the scalar field in gravitational collapse discovered by Choptuik [17]. He showed that one parameter families of interpolating solutions have a critical value p^*, beyond which the black hole forms at infinitesimal mass. Since the black hole mass changes continuously as the parameter changes, this transition is called a type II transition. Interestingly the black hole mass has a simple power-law form $M_{\rm bh}\propto|p-p^*|^{\gamma}$, where γ is the critical exponent. Near the critical value the field asymptotically approaches the discretely self-similar critical solution. Recently a different type of transition has been discovered for the Einstein-Yang-Mills system [18] and for the Einstein-Klein-Gordon system [19]. In these systems the black hole formation can turn on at finite mass (type I transition) as well as at infinitesimal mass (type II transition). In the type I transition the critical solution is the Bartnik-McKinnon solution [20], which is a static singularity free solution of the Einstein-Yang-Mills system, and the oscillating soliton star solution in the unstable branch [13], which is a periodically oscillating singularity free solution in the Einstein-Klein-Gordon system, respectively. The necessary condition of the critical solution is that it has at least one unstable mode [21]. Hence we expect that our solutions with $n=1$ could be the critical solution of the type I transition of the gravitational collapse in de Sitter spacetime.

From a different point of view, our solution provides an example of cosmic no-hair conjecture. The scalar field may escape over the cosmological horizon and the spacetime approaches the de Sitter solution. Otherwise the scalar field may collapse to a black hole solution and the spacetime may develop into the Schwarzschild–de Sitter solution. Furthermore the present system has a black hole solution [22]. This is also an important issue, which is related to the fundamental problem of black hole no-hair conjecture.

ACKNOWLEDGMENTS

We would like to thank Dmitri V. Gal'tsov, Akio Hosoya, Hideki Ishihara, and Kei-ichi Maeda for useful discussions and Christian Baraldo and Wade Naylor for their critical reading of our paper. This work was supported partially by the Grant-in-Aid for Scientific Research Fund of the Ministry of Education, Science and Culture (T.T. and K.M.) and by the Grant-in-Aid for JSPS [No. 199704162 (T.T.) and 199605200 (K.M.)].

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