# Topological $R^4$ inflation

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We consider the possibility that higher-curvature corrections could drive inflation after the compactification to four dimensions. Assuming that the low-energy limit of the fundamental theory is eleven-dimensional supergravity to the lowest order, including curvature corrections and taking the descent from eleven dimensions to four via an intermediate five-dimensional theory, as favored by recent considerations of unification at some scale around $\sim 10^{16}$ GeV, we may obtain a simple model of inflation in four dimensions. The effective degrees of freedom are two scalar fields and the metric. The scalars arise as the large five-dimensional modulus and the self-interacting conformal mode of the metric. The effective potential has a local maximum in addition to the more usual minimum. However, the potential is quite flat at the top, and admits topological inflation. We show that the model can resolve cosmological problems and provide a mechanism for structure formation with very little fine tuning. [S0556-2821(99)02808-8]

## I. INTRODUCTION

One of the central problems confronting inflation [1] is the identity of the inflaton, the field responsible for driving inflation, and the manner in which it fits in with unified field theories and/or string theory, notably *M* theory. The birth of the inflaton came with the demise of the notion that inflation is driven by an adjoint Higgs field in some grand unified theory (GUT) such as SU(5). While the production from quantum fluctuations of the cosmological perturbations necessary to generate structure in the universe is one of the great successes of inflation [2], the required smallness of the amplitude of these fluctuations undermined the possibility that a field with couplings of gauge strength could drive inflation. Rather, it is often assumed that the inflaton couples to matter only through gravitational interactions [3]. In this context, options such as inflation via higher-dimensional curvature terms, including as the $R^2$ inflation model proposed by Starobinsky [4], a hybrid inflationary model combining curvature and inflaton effects, as discussed by Kofman, Linde and Starobinsky [5], or string theory, which possesses many gauge-singlet fields such as the dilaton [6,7], may become quite interesting.

The most stringent constraints on inflation arise from the observations of the cosmic microwave background. The naive interpretation of the Cosmic Background Explorer (COBE) and other data on fluctuations in the microwave background radiation is that the density of vacuum energy during inflation is $V \sim (10^{16}$ GeV$)^4$, so that inflation is associated with an energy scale $V^{1/4} \sim 10^{16}$ GeV. One of the key points in the application of *M* theory to phenomenology

is the reconciliation of the bottom-up calculation of $M_{GUT} \sim 10^{16}$ GeV with the string unification scale, which is close to the four-dimensional Planck mass scale $M_4 \sim 10^{19}$ GeV. This is achieved by postulating a large fifth dimension $\mathcal{R}_5 \gg M_{GUT}^{-1}$, which is not felt by the gauge interactions, but causes the gravitational interactions to rise with energy much faster than in the conventional four dimensions. In this type of scenario, one could expect that inflation should be considered within a five-dimensional framework.

It is now known that ten-dimensional strongly coupled heterotic string theory is related through dualities to weakly coupled type I string theories, as well as to eleven-dimensional *M* theory [8]. In each case, the corresponding thresholds imply the presence of one large scale dimension below the unification point [8,9]. Within this general five-dimensional framework, two favored ranges for the magnitude of $\mathcal{R}_5$ can be distinguished. One is relatively close to $M_{GUT}^{-1}: \mathcal{R}_5^{-1} \sim 10^{12}$ to $10^{15}$ GeV, and the other could be as low as $\mathcal{R}_5^{-1} \sim 1$ TeV [10]. The latter is motivated in particular by Scherk-Schwarz models of supersymmetry breaking, in which the gravitino mass $M_{3/2} \sim \mathcal{R}_5^{-1}$. In this latter case, the large dimension is not necessarily the conventional fifth dimension of *M* theory. Indeed, in models studied in [10] the large dimension may be related to what is normally considered as one of the six ''small'' dimensions that is conventionally compactified in the manner of Calabi and Yau.

It has been known for quite some time that it is very difficult to incorporate conventional inflationary scenarios into the low-energy limit of string theory. The principal obstacle in this course has been the fact that the low-energy dynamics contains massless scalar fields with non-minimal couplings to gravity. Their coupling constants are precisely given by conformal symmetry and/or the dualities of string theory. In an expanding universe, these fields typically roll during the course of the expansion, consuming the available energy and hence decreasing the rate of expansion. One typi-

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cally finds solutions where the scale factor of the universe grows as a power of time, with the power determined by the scalar coupling constants. Once the numerical values of these constants dictated by string theory are taken into account, it has been found that the resulting power laws are too slow to give an inflationary universe [11]. Alternatively, if the scalars are endowed with masses which arise from some kind of non-perturbative supersymmetry breaking, the resulting models suffer from the graceful exit problem, as we discuss below. In string theory, higher-dimensional curvature terms are present in the action, appearing in the expansion in powers of the string tension. One might think that the inclusion of higher-derivative terms, which are low-energy signatures of the massive excitations of string theory, could produce several possibilities for curvature-driven inflation.

In this work, we consider a variant of the Starobinsky model based on $R^4$ curvature terms, as curvature-squared terms are not known to be present in the action of the full five- or eleven-dimensional theory. We assume throughout that the remaining six dimensions are fixed.[1] In a four-dimensional context, Maeda [14] derived the potentials for a general $R + R^n$ theory. For $n \neq 2$, the potential is not flat, but rather shows a peak and a well-defined minimum. We argue that it possible for this potential to inflate in a so-called topological manner [15]. As we discuss, a fully successful inflationary model of this sort would still require a potential for the dilaton. Whilst this model results in an inflationary stage with a guaranteed exit, the magnitude and spectral index of the resulting density fluctuations force us to consider some possible additional ingredients. Either curvature-squared terms that might appear at the level of the five-dimensional theory or quantum corrections to $T_{\mu\nu}{}^2$ would be sufficient to provide a self-consistent inflationary model.

Although such a solution is not directly derived from string or $M$ theory, it captures several elements that we expect to be present in the low-energy theory. As such, it represents a novel and motivated possible solution to the problem of inflation in string theory, which may hold some promise.

## II. CURVATURE-DRIVEN INFLATION

Among the first utilizations of higher-derivative curvature terms is the Starobinsky model [4], which is based on obtaining a self-consistent solution of Einstein's equations when they are modified to include one-loop quantum corrections to the stress-energy tensor $T_{\mu\nu}$. In its simplest form, the model is equivalent to a theory of gravity with an $R^2$ correction. When one considers the contributions of the back-reaction to the stress-energy, one finds a term which is equivalent to starting with an action of the form [16]

$$S = \frac{1}{2\kappa^2} \int d^4x \sqrt{g} (R + R^2/6M^2) \tag{1}$$

where $\kappa^2 = 8\pi G_N$. It is well known that this theory is conformally equivalent to a theory of Einstein gravity plus a scalar field [17]. By a field redefinition

$$\tilde{g}_{\mu\nu} = \left(1 + \frac{1}{3M^2}\phi\right)g_{\mu\nu}, \quad \phi' = \sqrt{\frac{3}{2}}\ln\left(1 + \frac{1}{3M^2}\phi\right), \tag{2}$$

the action can be simplified to

$$S = \frac{1}{2\kappa^2} \int d^4x \sqrt{\tilde{g}}\left(R - \partial^\mu\phi'\partial_\mu\phi' - \frac{3}{2}M^2(1 - e^{-\sqrt{2/3}\phi'})\right). \tag{3}$$

The potential is extremely flat for $\phi' \gg M_4$ and has a minimum at $\phi' = 0$ with $V(\phi' = 0) = 0$. For large initial values of $\phi'$, one can recognize this as an excellent model for chaotic inflation [18].

More generally, quantum corrections to the right-hand side of Einstein's equation in the absence of matter can be written as [19]

$$\langle T_{\mu\nu}\rangle = \left(\frac{k_2}{2880\pi^2}\right)\left(R^\rho_\mu R_{\nu\rho} - \frac{2}{3}RR_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R^{\rho\sigma}R_{\rho\sigma}\right.$$
$$+ \frac{1}{4}g_{\mu\nu}R^2\right) + \frac{1}{6}\left(\frac{k_3}{2880\pi^2}\right)\left(2R_{;\mu;\nu} - 2g_{\mu\nu}R^{;\rho}_{;\rho}\right.$$
$$\left.- 2RR_{\mu\nu} + \frac{1}{2}g_{\mu\nu}R^2\right) \tag{4}$$

where $k_2$ and $k_3$ are constants that appear in the process of regularization. We recall that $k_2$ is related to the number of light spin states, which can be very large in variants of string theories based on $M$ theory, as we will discuss below. On the other hand, the coefficient $k_3$ is independent of the number of light states. This term is none other but the variation of the $R^2$ term in the effective action. The theory admits a de Sitter solution which can be found from the 00 component of gravitational equation of motion [20]. Defining $H' = 2880\pi^2/k_2$ and $M^2 = 2880\pi^2/k_3$, and setting the spatial curvature $k=0$, one finds [21]

$$H^2(H^2 - H'^2) = (H'^2/M^2)(2\ddot{H}H + 2H^2\dot{H} - \dot{H}^2) \tag{5}$$

where $H = \dot{a}/a$ is the Hubble parameter. The de Sitter solution corresponds to $H = H'$ and of course $\dot{H} = \ddot{H} = 0$.

In order to avoid the overproduction of gravitons there is a *lower* limit on the parameters $k_{2,3}$ [22,21]: $k_2 \gtrsim 10^{10}$ implying the need for billions of spin degrees of freedom to be present. While this seems like an inordinately large number, it is possible to generate very large numbers of degrees of freedom in theories with extra dimensions, as we will now argue. Although this may not necessarily be the framework for $M$ phenomenology that is eventually adopted, we use the general form of the effective low-energy theory derived from

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[1]For recent discussions of fully eleven-dimensional cosmological solutions, see [12,13].

[2]The former can also be derived as a quantum correction to the energy momentum tensor.

$M$-theory compactification on a Calabi-Yau manifold to illustrate the discussion of the possibly large magnitude of $k_2$. The effective low-energy field theory has the form of a five-dimensional supergravity theory: as such, it contains a graviton supermultiplet, vector supermultiplets and scalar hypermultiplets. We recall that the graviton supermultiplet contains five graviton states, three graviphoton states and eight gravitino states. Each vector supermultiplet contains three vector states, one real scalar and four fermion states, and each scalar hypermultiplet contains four real scalars and four fermion states. The numbers of vector hypermultiplets $n_V$ and scalar hypermultiplets $n_H$ are related to the topological properties of the Calabi-Yau manifold:

$$n_V = n_{11} - 1, \quad n_H = n_{21} + 1. \tag{6}$$

Some of these states have even parity when the fifth dimension is compactified on $S_1/Z_2$, and some are odd, but this is not essential for our purpose. We are interested in the number of excited supermultiplets that appear below the effective inflationary scale, which we identify approximately with $10^{16}$ GeV $\sim M_{GUT}$. The number of such Kaluza-Klein excitations is given asymptotically by $n_{KK} \sim M_{GUT} \mathcal{R}_5$. Hence we estimate

$$k_2 \sim n_{KK} \times (16 + 8(n_{11} + n_{21})). \tag{7}$$

In realistic models, we expect that $n_{11} = 3$ and $n_{21} = n_{11} + 2\chi$, where the Euler characteristic $\chi = 3$. In this case, $k_2 \sim 88 n_{KK}$, which exceeds $10^{10}$ if $n_{KK} \sim M_{GUT} \mathcal{R}_5 \gtrsim 10^8$.[3]

Although large, such a value of $n_{KK}$ is perhaps not impossible as we shall see. In the Starobinsky model, the bound for $k_3$ is $k_3 \gtrsim 10^9$, corresponding to $M \lesssim 10^{14}$ GeV. This can be seen as follows. In order to produce sufficient expansion to solve cosmological problems, the effective cosmological constant, which by Eq. (1) is $\sim M^2 M_4^2$, would have to be $M^2 M_4^2 \lesssim 10^{64} (\text{GeV})^4$. From this, we would find $M \lesssim 10^{13}$ GeV. Alternatively, this is just the requirement that the Starobinsky model embodies chaotic inflation, and simultaneously satisfies the observational constraints. In our case, however, this requirement can be relaxed since the energy for inflation will not be supplied by the quadratic curvature term. As we will see below, our inflation mass scale will still satisfy a similar inequality, but without overconstraining $k_3$.

## III. ELEVEN-DIMENSIONAL SUPERGRAVITY AND HIGHER DERIVATIVE CURVATURE TERMS

As we have indicated above, the presence of the dilaton field and its universal coupling to other terms in the low-energy effective action hamper the embedding of standard inflationary models in string theory. One must remember that the derivative corrections to the string effective action are not uniquely defined. Their general form is fixed by requir-

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[3] There is also the possibility of additional matter and gauge fields associated with $D$ branes in the bulk, but we do not discuss them further here.

ing that the effective action reproduces at two loops the $\beta$ functions of the string world-sheet loop expansion [23]. However there arise divergences, and they must be renormalized. The results thus depend on the particular subtraction scheme adopted in the $\sigma$-model formulation. The renormalization-group transformations relate different schemes, which changes the form of the background $\sigma$-model couplings, while leaving the physics invariant. These renormalization group transformations viewed as maps of the target-space fields are called string field redefinitions. They change the form of the effective action while leaving the physics unaffected [24].

If we return to the issue of the dilaton evolution, we see that in general its equation of motion will be of the form $\nabla^2 \phi - (\nabla \phi)^2 + R/4 \sim \alpha' J$, where $J$ is the contribution of the higher derivative terms. It may contain higher derivatives of $\phi$, such as $\nabla^4 \phi$ in addition to curvature terms [23–25]. These terms could make the dilaton equation fourth-order in time derivatives. However, if we take the limit $\alpha' \to 0$ in the dilaton equation, for anything from $J$ to survive, it must diverge to cancel $\alpha'$. Solutions of this type cannot be perturbatively matched to the vacuum sector of the low-energy theory. Because of this one cannot be certain that they will remain unaltered by higher-order corrections. Also, the solutions are not uniquely determined at the given order of truncation because of the string field redefinitions discussed above. In terms of the dilaton field, this suggests that the only physically meaningful effect of the source $J$ at any given order of truncation is a perturbative correction in $\alpha'$. This could be enforced at the level of the effective theory by using field redefinitions to go to a unitary scheme, where higher than second derivatives of fields are automatically absent.

An example of such a unitary scheme is given by a four-dimensional string gravitational action [26] in the Einstein frame:

$$S = \frac{1}{2\kappa^2} \int d^4x \sqrt{g} \left\{ R - 2 \partial_\mu \phi \partial^\mu \phi + \frac{\alpha'}{8} e^{-2\phi} \widetilde{R}^2 \right\} \tag{8}$$

where $\widetilde{R}^2 = R_{abcd} R^{abcd} - 4 R_{ab} R^{ab} + R^2$ is the Gauss-Bonnet combination, and we have kept only the dilaton- and metric-dependent terms. In the absence of a potential for the dilaton, it has been shown that the combined dilaton and gravitational equations of motion do not admit de Sitter solutions [27,28]. In fact, this result remains true when terms of higher order in $\alpha'$ are considered [29]. As we show below, this result also does not depend on the fact that it is the Gauss-Bonnet combination. The coefficients of $R_{ab} R^{ab}$ and $R^2$ are arbitrary up to a field redefinition of the metric and dilaton [24]. For any choice of these coefficients, the only solution with a constant dilaton is Minkowski space.

When a potential for the dilaton due to, e.g., gaugino condensation [30] is included along with a cosmological term, due to, e.g., a central charge deficit, it is possible to generate an approximate de Sitter solution at the $\mathcal{O}(\alpha'^0)$ level [28,31] when $\alpha'$ terms are kept. However, in this case there is no exit from the inflationary period, and the dilaton is already sitting at its minimum and is constant. We are led therefore

to a particular difficulty with field-theoretic inflation in the gravitational sector of string theory.[4]

One can also choose to work in a non-unitary scheme, since all of the schemes are physically equivalent. The spurious degrees of freedom can be kept under control by expanding the source and solving the equations iteratively. In such situations, the dilaton would acquire $\mathcal{O}(\alpha')$ corrections as a response to the source. While this rolling would appear to be adverse for inflation, it is tempting to ask if it might only represent a disguise. For example, it might happen that, by a field redefinition, an apparently non-inflationary solution with a rolling dilaton to $\mathcal{O}(\alpha')$ is mapped onto an inflationary solution with a constant dilaton. This would require retaining some of the spurious degrees of freedom, because the constant dilaton ansatz would demand cancellations between $\mathcal{O}(1)$ and $\mathcal{O}(\alpha')$ terms. While perhaps unreliable, such solutions are still of interest. They might be a starting point for further study via more stringy methods.

Rather curiously, it turns out that imposing a constant dilaton in an *arbitrary* subtraction scheme requires that the space-time is exactly flat, and hence given by the Minkowski metric. To see this, let us consider the effective action to $\mathcal{O}(\alpha')$. In four dimensions, it can be written as

$$S = S_0 + S_1 \tag{9}$$

where

$$S_0 = \int d^4x \sqrt{g}\, e^{-2\phi} \{ R + 4(\nabla \phi)^2 \} \tag{10}$$

and

$$S_1 = \alpha' \lambda_0 \int d^4x \sqrt{g}\, e^{-2\phi} \Bigg\{ R_{\mu\nu\lambda\sigma}^2$$
$$+ 2(R + 4\nabla^2\phi - 4(\nabla\phi)^2)\delta\phi$$
$$+ \left( R^{\mu\nu} + 2\nabla^\mu\nabla^\nu - \frac{1}{2} g^{\mu\nu}(R + 4\nabla^2\phi - 4(\nabla\phi)^2) \right) \delta g_{\mu\nu} \Bigg\} \tag{11}$$

where we have chosen to work in the string frame, signified by the presence of $\exp(-2\phi)$. The parameter $\lambda$ varies between different string theories, being 1/4 in bosonic, 1/8 in heterotic and 0 in superstring theories. To $\mathcal{O}(\alpha')$, only the square of the Riemann tensor is unambiguous. The coefficients of all other terms are ambiguous, and can be set to zero by redefining the fields in $S_0$ by terms to order $\alpha'$. This is signified by the terms $\delta g_{\mu\nu}$ and $\delta\phi$, which are

$$\delta\phi = c_1 R + c_2(\nabla\phi)^2 + c_3\nabla^2\phi \tag{12}$$

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[4]We note in passing a model which attempts to use running moduli to solve cosmological problems: for the details of the model, see [32], and for the discussion of its viability, see [33].

$$\delta g_{\mu\nu} = b_1 R_{\mu\nu} + b_2 \nabla_\mu\phi\nabla_\nu\phi$$
$$+ g_{\mu\nu}(b_3 R + b_4(\nabla\phi)^2 + b_5\nabla^2\phi)$$

and are the most general expressions for the counterterms consistent with dimensional analysis and target-space general covariance. The coefficients $b_k$ and $c_k$ are completely arbitrary.

Let us now consider the case when the coefficients $b_k$ and $c_k$ are chosen such that the source for the dilaton vanishes in a cosmological Friedmann-Robertson-Walker (FRW) background. Because of the dilaton equation of motion, this would place a constraint on the curvature, which would select only those general metric solutions of the dilaton-less theory that are a consistent truncation of the $\mathcal{O}(\alpha')$ string effective action. First, we see that if the dilaton is a constant, all contributions to the action proportional to $(\nabla\phi)^2$ would produce terms at least $\sim \nabla\phi$ in the equations of motion, and so would vanish. Inserting Eq. (12) into Eq. (11), we see that the $\mathcal{O}(\alpha')$ action of interest to us is

$$S_1^{eff} = \alpha'\lambda \int d^4x \sqrt{g}\, e^{-2\phi}$$
$$\times \{ R_{\mu\nu\lambda\rho}^2 + b_1 R_{\mu\nu}^2 + \beta_1 R^2 + \beta_2\nabla^2\phi R$$
$$+ 2b_1 R_{\mu\nu}\nabla^\mu\nabla^\nu\phi \} \tag{13}$$

where, in terms of the original coefficients in Eq. (12), we have $\beta_1 = 2c_1 - b_3 - b_1/2$ and $\beta_2 = 8c_1 + 2c_3 - b_5 - 6b_3 - 2b_1$.

If we now vary the action $S = S_0 + S_1^{eff}$ with respect to $\phi$, and demand that $\phi = 0$ is a solution, we obtain the following constraint on the curvature:

$$2R + \alpha'\lambda\{ 2R_{\mu\nu\lambda\rho}^2 + 2b_1 R_{\mu\nu}^2 + 2\beta_1 R^2 - (b_1 + \beta_2)\nabla^2 R \} = 0. \tag{14}$$

So when we consider the action $S_0 + S_1^{eff}$, we can set $\phi = 0$ and impose the constraint (14) at the end. Further, all FRW solutions are conformally flat. This means that the Riemann curvature is given completely in terms of the Ricci tensor and Ricci scalar. Simple algebra then shows that

$$R_{\mu\nu\lambda\rho}^2 = 2R_{\mu\nu}^2 - \frac{1}{3}R^2. \tag{15}$$

The contributions of the Weyl tensor vanish because $C^\mu{}_{\nu\mu\lambda} = 0$, which is true for any geometry, and because on FRW backgrounds $C_{\mu\nu\lambda\rho} = 0$. Thus, even variations of the $C$-dependent terms would vanish on FRW backgrounds. Since in four dimensions the Gauss-Bonnet term is purely topological, i.e., is equal to a total divergence, we can also write

$$R_{\mu\nu\lambda\rho}^2 = 4R_{\mu\nu}^2 - R^2 + \nabla_\mu J^\mu \tag{16}$$

for some vector field $J^\mu$ which is irrelevant for our consideration. Combining the identities (15) and (16), we find that on FRW backgrounds

$$R^2_{\mu\nu} = \frac{1}{3}R^2 + \frac{1}{2}\nabla_\mu J^\mu. \qquad (17)$$

Since we are looking for solutions with $\phi = 0$, we can drop the boundary term, as its variation would always be proportional to $\nabla\phi$.

Using Eqs. (15) and (17), we find that the effective action on FRW backgrounds with a constant dilaton is precisely the action of the Starobinsky's model:

$$S^{eff} = \int d^4x \sqrt{g}\left\{R + \alpha'\lambda\left(\frac{b_1}{3} + \beta_1 + \frac{1}{3}\right)R^2\right\} \qquad (18)$$

which must, however, be supplemented with the constraint (14). If we vary this action with respect to the metric, and trace the result, we get

$$R = \alpha'\lambda(6\beta_1 + 2b_1 + 2)\nabla^2R \qquad (19)$$

which is the equation of motion for the conformal mode of the metric, that has become massive because of the derivative corrections. In order for this equation to be consistent with the constraint arising from requiring $\phi = 0$, we must ensure that the terms in the dilaton constrain proportional to $R^2_{\mu\nu}$ and $R^2$ cancel identically. Using Eq. (15), we find that this requires setting $\beta_1 = 1/3$ and $b_1 = -2$. This requirement is dictated by general covariance, and cannot be relaxed to order $\alpha'$ if we wish to have a constant dilaton. But when we insert this in the trace equation (19), we find that the derivative term drops out, and we get

$$R = 0 \qquad (20)$$

as a result. Worse yet, we see that the coefficient of the $R^2$ term in the action is zero, and so the vanishing of the dilaton requires that the metric is a solution of the flat space equations to order $\alpha'$ and not only to order 1. Hence we see that the only solution of the $\mathcal{O}(\alpha')$ action in any string theory with a constant dilaton is Minkowski space, regardless of the subtraction scheme.

However, while some ways of compactifying the eleven-dimensional theory to five dimensions are known [8], it is not yet clear if the compactification procedure is completely unique. Discussion so far has centered on Calabi-Yau compactification, whose features may depend on the mutual ratios of sizes of the interval $S^1/Z_2$ and 1-cycles on the Calabi-Yau spaces, as has already been pointed out in, for example, [10]. We recall that there are alternative compactifications of the weakly-coupled ten-dimensional heterotic string, and they may turn out to have elevations to the eleven-dimensional theory. In this article, we therefore consider a general approach in which we assume the internal manifold to be decoupled, with its size a massive field from the point of view of the five-dimensional theory. There have been several calculations of higher-order $R^n$ terms in ten and eleven dimensions [34]. Those in eleven dimensions are not known to yield $R^2$ terms, but may yield $R^4$ terms. On the other hand, the calculations by Hořava and Witten on the ten-dimensional boundary in their formulation of $M$ theory do yield $R^2$ terms, but only in the boundary effective actions.

We will not discuss their precise form, which in five dimensions may depend on the details of compactification of the eleven-dimensional theory. If we dimensionally reduce the five-dimensional theory to four dimensions, we will obtain an even more complicated-looking expression involving contractions of the four-dimensional Riemann tensor and terms with up to two derivatives of the size of the internal dimension, in addition to the four-dimensional version of the five-dimensional expression. Now, as long as the four-dimensional space-time is conformally flat, its Riemann tensor can be expressed completely in terms of the Ricci tensor and scalar, and thus any quartic curvature term could be written as a linear combination of terms like $R^4, R^2_{\mu\nu}$ and $R_{\mu\nu}R_{\lambda\sigma}R^{\mu\lambda}R^{\nu\sigma}$. Moreover, in four dimensions, the Gauss-Bonnet identity allows us to replace the square of the Ricci tensor by a square of the Ricci scalar, indicating that the scalar curvature modes play the most important role in cosmological dynamics. Here we repeat that the scalar modes must be endowed with mass in order that they decouple. This means that the terms proportional to its derivatives would all drop out, and hence we will ignore the scalar-tensor couplings which must depend on the derivatives of the scalar fields.

To model the possibility of curvature-driven inflation, we assume that the five-dimensional action contains a scalar $R^4$ contribution, and perform a dimensional reduction to four dimensions, ignoring the boundaries. In the context of the theory with walls, this merely means that we assume that the bulk can be foliated by identical and mutually non-interacting copies of the wall. Alternatively, it is clear that this dimensional reduction is identical to the standard Kaluza-Klein reduction on a circle. This approach produces a four-dimensional theory with two scalar fields, an inflaton and a dilaton. The inflaton potential has a maximum, which we will show to be sufficiently flat to support inflation, in a manner resembling the topological inflationary scenario. A problem with the scenario based only on the $R^4$ term, without the mass term for the compacton, is that it wants to decompactify the space time. This is obvious if we consider the $R^4$ term from the point of view of five dimensions. Since it behaves as an effective cosmological constant, according to the cosmological no-hair theorem, it forces the five-dimensional space to isotropize. Thus we must include a mass term for the compacton, which from the five-dimensional point of view will break the rotation symmetry, and allow the four-dimensional space to inflate while keeping the compacton fixed.

## IV. INFLATION IN FOUR DIMENSIONS

In this section, we will begin the investigation of the possibility of higher-curvature-driven inflation. We first give details of the dimensional reduction from five dimensions to four dimensions, in order to derive the four-dimensional effective theory. The pure gravity sector of the five-dimensional action, which we will consider is

$$S = \int d^5x \sqrt{G_5}\left\{\frac{M_5^3}{16\pi}R_5 + \alpha M_5^{-3}R_5^4\right\} \qquad (21)$$

where $M_5$ is the five-dimensional Planck mass. In the context of the eleven-dimensional theory, our five-dimensional Planck mass is related to the eleven-dimensional Planck mass by $M_5^3 = M_{11}^9 V_6$, where $V_6$ is the six-volume of the compactified space. We assume that $V_6^{-1/6} \sim M_{11} \sim M_{\text{GUT}}$ in general. The scale of the fifth dimension is then given by $\mathcal{R}_5^{-1} \sim (4/\alpha) M_{11}^3 M_4^{-2}$ [10]. We note that this effective action is not exactly what one finds after dimensional reduction of the eleven-dimensional supergravity with quartic corrections on an interval $S^1/Z_2$, which would contain couplings to two ten-dimensional boundaries with $R^2$ terms (see, e.g., [13]). Thus, our action (21) is not a direct descendant of a known reduction of $M$ theory on an interval. But such actions may nevertheless arise in some compactifications of the theory, and are simple enough to illustrate our main point.

Let us first outline the reduction procedure we will follow here. We first conformally transform the action (21), following [14], to represent it as a five-dimensional gravity with a minimally coupled self-interacting scalar field. The scalar arises because the conformal mode of the metric becomes dynamical thanks to the $R^4$ term [17]. Then we dimensionally reduce this action to four dimensions and apply another, four-dimensional, conformal transformation, to put the resulting four-dimensional action in the canonical form. This will produce another scalar field, the compacton (or dilaton) of the four-dimensional theory, which is related to the size of the fifth dimension.

We now give the formulas appropriate for this procedure. The conformal transformation which brings Eq. (21) to the Einstein form is

$$\bar{G}_{AB} = |1 + 64\pi\alpha M_5^{-6} R_5^3|^{2/3} G_{AB},  \qquad (22)$$

and the action is [14]

$$\bar{S}_5 = \int d^5x \sqrt{\bar{G}_5} \left\{ \frac{M_5^3}{16\pi} \bar{R}_5 - \frac{1}{2}\bar{G}^{AB}\partial_A\tilde{\chi}\partial_B\tilde{\chi} - U_5(\tilde{\chi}) \right\},  \qquad (23)$$

where the ''intermediate'' scalar field and its potential are

$$\tilde{\kappa}\tilde{\chi} \equiv \frac{2}{\sqrt{3}} \ln|1 + 64\pi\alpha M_5^{-6} R_5^3|,$$

$$U_5(\tilde{\chi}) = \frac{3M_5^5}{256\pi^{4/3}\alpha^{1/3}} e^{-(5\sqrt{3}/6)\tilde{\kappa}\tilde{\chi}} (e^{(\sqrt{3}/2)\tilde{\kappa}\tilde{\chi}} - 1)^{4/3},  \qquad (24)$$

with $\tilde{\kappa} \equiv \sqrt{(8\pi)/(M_5^3)}$. The standard Kaluza-Klein compactification ansatz, with the simultaneous conformal transformation of the four-dimensional metric to the canonical form, is

$$d\bar{s}_5^2 = e^{-\sqrt{2/3}\kappa\phi} g_{\mu\nu} dx^\mu dx^\nu + \frac{M_4^4}{M_5^6} e^{2\sqrt{2/3}\kappa\phi} dz^2,  \qquad (25)$$

where $g_{\mu\nu}$ is the four-dimensional Einstein-frame metric. The action (23) then reduces to the following four-dimensional action:
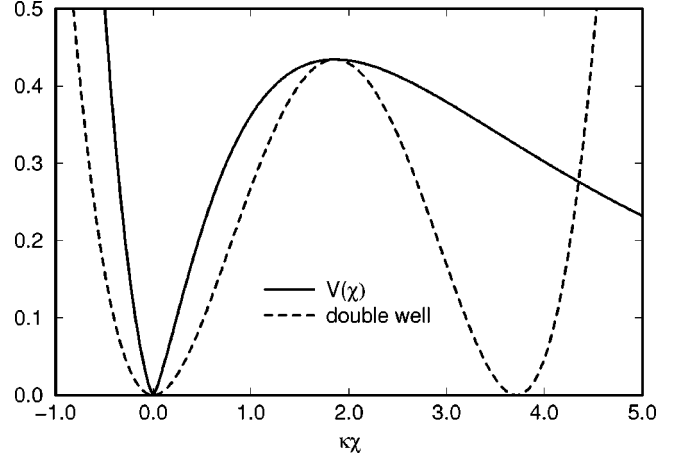


FIG. 1. Comparison of the potential $V(\chi)$ produced by a conformal rescaling of the $R^4$ term in the five-dimensional action and a simple double-well potential.

$$S_4 = \int d^4x \sqrt{g_4} \left\{ \frac{M_4^2}{16\pi} R_4 - \frac{1}{2} g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - \frac{1}{2} g^{\mu\nu}\partial_\mu\chi\partial_\nu\chi \right.$$
$$\left. - U(\varphi,\chi) \right\},  \qquad (26)$$

where $\chi = M_4 M_5^{-3/2}\tilde{\chi}$. In terms of these fields, the potential is given by

$$U(\varphi,\chi) = \frac{3M_4^2 M_5^2}{256\pi^{4/3}\alpha^{1/3}} e^{-\sqrt{2/3}\kappa\varphi} e^{-(5\sqrt{3}/6)\kappa\chi} (e^{(\sqrt{3}/2)\kappa\chi} - 1)^{4/3}$$

$$\equiv \frac{3M_4^2 M_5^2}{256\pi^{4/3}\alpha^{1/3}} e^{-\sqrt{2/3}\kappa\varphi} V(\chi).  \qquad (27)$$

Before we undertake a detailed investigation of Eq. (26), we note that the effective four-dimensional potential (27) is a product of an exponential and a function with a maximum. If we ignore the variation of $\chi$, we recall that in a universe dominated by a scalar field with an exponential potential $V(\phi) \propto e^{-\lambda\kappa\phi}$, and foliated by flat spatial hyperplanes, the expansion of the FRW scale factor $a(t)$ obeys a power law $a(t) \propto t^p$, with the power index given by $p = 2/\lambda^2$ [35].

Next, we ignore the dilaton factor $e^{-\sqrt{2/3}\kappa\varphi}$ in the potential and consider the dynamics of the $\chi$ field. The $\chi$-dependent factor in the potential, $V(\chi)$, is depicted in Fig. 1. It has a global minimum $V = 0$ at $\chi = 0$, a local maximum at $\chi = \chi_m = 2\ln 5/(\sqrt{3}\kappa) = 0.37 M_4$, and diverges as $V(\chi) \propto e^{-(5\sqrt{3}/6)\kappa\chi}$ for $\chi \to -\infty$. This is too steep for power-law inflation. For $\chi \to \infty$, $V(\chi)$ asymptotically approaches zero with $V(\chi) \propto e^{-(\sqrt{3}/6)\kappa\chi}$, which may be flat enough for inflation but it will lead to an unphysical universe with a runaway behavior towards the regime of extremely large curvature, in terms of the original five-dimensional description.

As a result, chaotic inflation [18] with a large initial value of $\chi$ is impossible here. Nevertheless there may remain the possibility to realize inflationary expansion in this model by using the potential energy around the local maximum, $V(\chi_m)$, as in the topological inflation scenario of Linde and Vilenkin [15]. In this scenario, if the scalar field $\chi(x)$ is randomly distributed initially with a large dispersion, some part of the universe will roll to $\chi = 0$, while in other parts it will run away to infinity. Between any two such regions there will appear domain walls, containing a large energy density, $\rho \sim V(\chi_m)$. If the wall is thicker than the Hubble radius of this energy density, there will exist a sufficiently large quasi-homogeneous region, filled with large potential energy, where inflationary expansion naturally sets in.

The condition for a domain wall to inflate has been investigated numerically in [36] for the case of a simple double-well potential, $V_{dw}(\phi) = (\lambda/4)(\phi^2 - \eta^2)^2$. There it has been found that a domain wall will undergo inflation if $\eta$ exceeds a critical value $\eta_{cr} = 0.33 M_4$, regardless of the value of $\lambda$. When $\eta = \eta_{cr}$, the ratio of the thickness of the wall — characterized by the curvature scale of the potential at the origin, $r_w \equiv (V''_{dw}(\phi = 0))^{-1/2}$, to the horizon $H^{-1} = (\kappa^2 V_{dw}(\phi = 0)/3)^{-1/2}$ is given by $r_w H = 0.48$, and is again independent of $\lambda$. An explicit check shows that, in our model, the distance between the potential minimum and the local maximum, $\chi_m = 0.37 M_4$, exceeds $\eta_{cr}$. Furthermore, the ratio of the characteristic thickness of the wall to the horizon scale is given by $r_w H = (\kappa^2 V(\chi_m)/3 V''(\chi_m))^{1/2} = 4/\sqrt{15} \simeq 1.0$, which is larger than the critical case of [36]. In Fig. 1 we have also depicted a double-well potential which has the same global minimum and the local maximum as $V(\chi)$. As is seen there, the latter is much flatter than the former around the local maximum.

Having seen that the potential $V(\chi)$ is flat enough around the local maximum, we now return to the exact form of the potential given in Eq. (27), and take the dilaton factor into account. This does not change $\chi_m$ nor $r_w H$. However, the rolling dilaton field hampers exponential inflation. Since $\chi$ moves slowly near $\chi = \chi_m$, as compared to the dilaton, we practically have an exponential potential

$$U(\varphi, \chi) = \frac{3 M_4^2 M_5^2}{256 \pi^{4/3} \alpha^{1/3}} e^{-\sqrt{2/3}\kappa\varphi} V(\chi_m) \equiv U_0 e^{-\sqrt{2/3}\kappa\varphi}.$$

$$(28)$$

The ensuing solution for the FRW scale factor obeys the power law $a(t) \propto t^3$ with $\kappa\varphi(t) = \kappa\varphi(t_i) + \sqrt{6}\ln(t/t_i)$, and is an asymptotic attractor [37] for the scale factor. Since the power is greater than unity, this solution still appears to be slowly inflating. However, when we look at the scalar field, we find $\exp(\sqrt{2/3}\kappa\varphi) = t^2 (\pi U_0)/(3 M_4^2)$. This evolution of the scalar field is too rapid, as we can easily verify by substituting the solutions back into the five-dimensional metric (25). Indeed, in the original frame the solution is $\bar{a}(\bar{t}) \propto \exp(\sqrt{(4\pi U_0)/(3 M_5^3)}\bar{t})$, and $\exp(\sqrt{2/3}\kappa\varphi) \propto (\sqrt{(4\pi U_0)/(3 M_5^3)}\bar{t})$. The physical interpretation of this behavior is simple. Substituting $\chi = \chi_m$ in the potential corresponds to adding a positive cosmological constant in five

dimensions, and, according to Wald's cosmological no-hair theorem [38], the anisotropic five-dimensional spacetime we are dealing with must approach the de Sitter space due to the effective cosmological constant.

Hence some mechanism is needed to stabilize the size of the fifth dimension. This requires breaking the residual five-dimensional gauge invariance, which equates the fifth direction with the other four in the case discussed above. The problem is linked to that of fixing the vacuum expectation value (VEV) of the dilaton, which presumably involves ill-understood non-perturbative phenomena such as supersymmetry breaking and perhaps gaugino condensation [30]. Scenarios for these have been proposed, and it seems quite possible that these may occur at an energy-momentum scale above that of the five-dimensional Kaluza-Klein excitations. In this case, the internal radius may be regarded as fixed, for our purposes here.

After the internal radius is stabilized, we can recover exponential inflation in three spatial dimensions in both conformal frames, as long as $\chi$ stays near the local maximum $\chi_m$. In this limit, the potential can be approximated as

$$U(0, \chi) = \frac{3 M_4^2 M_5^2}{256 \pi^{4/3} \alpha^{1/3}} V(\chi)$$

$$\sim U(0, \chi_m) - \frac{1}{2} m^2 (\chi - \chi_m)^2,$$

$$m^2 \equiv \frac{5}{16} \kappa^2 U(0, \chi_m), \qquad (29)$$

where we have used the relation $V''(\chi_m) = -\frac{5}{16}\kappa^2 V(\chi_m)$. Thus the standard slow-roll solution is

$$a(t) \propto e^{H_m t}, \quad H_m \equiv \sqrt{\frac{\kappa^2}{3} U(0, \chi_m)}, \qquad (30)$$

$$\chi(t) = \chi_m - (\chi_m - \chi_f)\exp\left(\frac{m^2}{3 H_m^2} H_m (t - t_f)\right)$$

$$= \chi_m - (\chi_m - \chi_f) e^{(5/16) H_m (t - t_f)},$$

where $t < t_f$, and where $t_f$ and $\chi_f$ stand for the time and the field amplitude at the end of inflation.

It is important to note that the model we are presently considering does not share the graceful exit problem of typical string-dilaton inflationary models. Generally, as we have described above, inflationary (or de Sitter) solutions with a dilaton coupled to gravity require that the dilaton be fixed. When the dilaton doubles as the inflaton as well, no means have been found to cancel the vacuum energy which drives inflation. This is different from the graceful exit problem in the pre-big-bang scenarios for which inflation is not driven by vacuum energy density, though these models also suffer from a graceful exit problem [39]. In our present case, these two issues are separated. Although the dilaton still needs to be fixed, inflation is driven by the $R^4$-induced potential of the conformal field $\chi$. Since the model we are

considering is of the "topological" type, we are sitting on the top of this potential and we are *guaranteed* that inflation will end as the field $\chi$ rolls to its minimum.

We can now check the magnitude and spectral index of the induced density fluctuations. The amplitude of a linear curvature fluctuation, $\Phi_H$, [40] on a comoving scale $l = 2\pi/k$ is given by

$$
\begin{aligned}
\Phi_H\left(l = \frac{2\pi}{k}\right) &= \frac{fH_m^2}{2\pi|\dot{\chi}(t_k)|} \\
&= \frac{3}{2\pi}\frac{fH_m^3}{m^2|\chi_m - \chi_f|}\exp\left(\frac{m^2}{3H_m^2}H_m(t_f - t_k)\right) \quad (31) \\
&= \frac{8fH_m}{5\pi|\chi_m - \chi_f|}e^{(5/16)H_m(t_f - t_k)},
\end{aligned}
$$

where $f = 3/5$ (2/3) in the matter- (radiation-) dominated era, and $t_k$ is the time when the $k$ mode left the Hubble radius during inflation. The spectral index, $n_s$, of density perturbation is given by

$$
n_s = 1 - \frac{2m^2}{3H_m^2}. \quad (32)
$$

We recall that, in the model we are discussing, $m^2/H_m^2 = 15/16$ so that $n_s = 3/8$, which is significantly different from the scale-invariant value $n_s = 1$ and is in disagreement with observations. Furthermore, the large-angle (an)isotropy of cosmic microwave background (CMB) radiation [41] requires $\delta T/T = -\Phi_H/3 = 10^{-5}$ on the comoving scale leaving the Hubble radius about 60 expansion times before the end of inflation, namely,

$$
\frac{\delta T}{T} = \frac{8H_m}{25\pi|\chi_m - \chi_f|}e^{(5/16)\times 60} \simeq 10^{-5}. \quad (33)
$$

Since we find $|\chi_m - \chi_f| \sim 0.1M_4$, the isotropy of CMB sets the scale of inflation as $H_m \sim 10^{-14}M_4$, which implies the Planck mass in five dimensions must be unacceptably small: $M_5 \sim 10^{-13}\alpha^{1/6}M_4$.

These results appear disconcerting. While our model showed some initial promise, it seems to fail the contact with the observations. A closer scrutiny of the dynamics shows that these problems arise because the potential which is generated by the $R^4$ term is a bit too steep to produce a satisfactory perturbation spectrum. In the next section, we discuss possible remedies.

## V. CURES FOR THE DENSITY FLUCTUATION PROBLEM

As we have seen in the previous section, although the relatively simple model described there, which is based on a compactified $R + R^4$ theory with a fixed dilaton, has all the ingredients necessary for inflation, we cannot obtain an acceptable magnitude for density fluctuations unless we choose the scale $M_5 \simeq 10^{-14}M_4$. But if on the other hand we have

$M_5 \sim M_{11} \sim M_{GUT}$, as occurs when $V_6^{-1/6} \sim M_{11}$ with $M_5^3 \sim M_{11}^9 V_6$, then by setting $M_5 \sim M_{GUT} \sim 10^{-3}M_4$ we would have greatly overproduced the magnitude of density fluctuations in the model. Moreover, we would still have the problem that the spectral index is equal to 3/8. We could foresee two possible solutions to this dilemma. The presence of an $R^2$ term in the five-dimensional action would flatten the potential further, possibly curing the problems with density fluctuations. In addition, the renormalization of the stress energy tensor, as in the original Starobinsky model, could also lower the effective value of $m^2/H_m^2$, which was at the root of the problems above.

The general classification of higher-order $R^n$ terms which may appear in the curvature expansion of the effective low-energy field-theory limit of $M$ theory is not available at present. It is however known that the supersymmetry of the theory rules out terms which are quadratic and cubic in curvature in the bulk, and that the lowest possible terms are quartic. Upon dimensional reduction of the eleven-dimensional theory on the interval $S^1/Z_2$ in the manner of Hořava and Witten, such terms could however produce terms which are quadratic in curvature in the effective action of the boundary theory. Our considerations here are different, since we do not consider theories with matter degrees of freedom confined only to the boundary. We should note that higher-order formulations of higher-dimensional gravity and super-gravity theories have been discussed in [42]. While the known types of quadratic corrections arising from $M$ theory are not explicitly of the type we need here, their form being restricted by supersymmetry, we recall that such constraints are relaxed in cases when supersymmetry is broken. Moreover, if we consider the effect of particle production and its back-reaction on the geometrical environment, we recall that this effect could be derived from effective counterterms in the action which are quadratic in curvature.

Therefore, we boldly consider the case where the five-dimensional action contains both quadratic and quartic terms in $R_5$:

$$
S_5 = \int d^5x\sqrt{G_5}\left\{\frac{M_5^3}{16\pi}R_5 + bM_5R_5^2 + cM_5^{-3}R_5^4\right\}. \quad (34)
$$

As we noted above, because of the absense of boundary terms, this action is not directly related to the Hořava-Witten reduction of M theory. We can again carry out the reduction to four dimensions along the same lines as discussed at the beginning of the previous section. Assuming $b$, $c > 0$, we apply the conformal transformation

$$
\bar{G}_{AB} = |1 + 32\pi bM_5^{-2}R_5 + 64\pi cM_5^{-6}R_5^3|^{2/3}G_{AB}, \quad (35)
$$

to obtain the equivalent action in the Einstein frame [14]:

$$
\bar{S}_5 = \int d^5x\sqrt{\bar{G}_5}\left\{\frac{M_5^3}{16\pi}\bar{R}_5 - \frac{1}{2}\bar{G}^{AB}\partial_A\hat{\chi}\partial_B\hat{\chi} - \hat{U}_5(\hat{\chi})\right\}, \quad (36)
$$

with

$$\tilde{\kappa}\hat{\chi} \equiv \frac{2}{\sqrt{3}}\ln|1 + 32\pi b M_5^{-2}R_5 + 64\pi c M_5^{-6}R_5^3|. \quad (37)$$

Since we are assuming $b$, $c > 0$, there is a one-to-one correspondence between $\hat{\chi}$ and $R_5$ for $1 + 32\pi b M_5^{-2}R_5 + 64\pi c M_5^{-6}R_5^3 > 0$, and from Eq. (37) we can solve for $\hat{\chi}$ to find the potential

$$\hat{U}_5(\hat{\chi}) = M_5^5 e^{-(5\sqrt{3}/6)\tilde{\kappa}\hat{\chi}}\left(-\frac{bR_5^2}{2M_5^4} + 2u(\hat{\chi})\frac{R_5}{M_5^2}\right),$$

$$u(\hat{\chi}) \equiv \frac{e^{(\sqrt{3}/2)\tilde{\kappa}\hat{\chi}} - 1}{128\pi c}, \quad (38)$$

with

$$R_5 = M_5^2\left\{u(\hat{\chi}) + \left[u^2(\hat{\chi}) + \left(\frac{b}{6c}\right)^3\right]^{1/2}\right\}^{1/3} - \left\{-u(\hat{\chi}) + \left[u^2(\hat{\chi})\right.\right.$$

$$\left.\left. + \left(\frac{b}{6c}\right)^3\right]^{1/2}\right\}^{1/3}. \quad (39)$$

We compactify to four dimensions and apply another conformal transformation, as before, to obtain the following four-dimensional Einstein action:

$$S_4 = \int d^4x \sqrt{g_4}\left\{\frac{M_4^2}{16\pi}R_4 - \frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - \frac{1}{2}g^{\mu\nu}\partial_\mu\chi\partial_\nu\chi \right.$$

$$\left. - \hat{U}(\varphi,\chi)\right\}, \quad (40)$$

where the potential is given by

$$\hat{U}(\varphi,\chi) = M_4^2 M_5^2 e^{-\sqrt{2/3}\kappa\varphi}e^{-(5\sqrt{3}/6)\kappa\chi}\left(-\frac{bR_5^2}{2M_5^4}\right.$$

$$\left. + \frac{e^{(\sqrt{3}/2)\kappa\chi} - 1}{64\pi c}\frac{R_5}{M_5^2}\right), \quad (41)$$

with $\kappa\chi = \tilde{\kappa}\hat{\chi}$ now.

If we had only the curvature-squared term in the original action (34), we would find the potential

$$\hat{U}(\varphi,\chi) = M_4^2 M_5^2 e^{-\sqrt{2/3}\kappa\varphi}e^{-(5\sqrt{3}/6)\kappa\chi}(e^{(\sqrt{3}/2)\kappa\chi} - 1)^2, \quad (42)$$

which has no local maxima in the $\chi$ direction. It diverges as $\hat{U}(\varphi,\chi) \propto e^{(\sqrt{3}/6)\kappa\chi}$ for $\chi \to \infty$ [43]. On the other hand, in the presence of both $R_5^2$ and $R_5^4$ terms in Eq. (34), the latter term eventually dominates the former, and the potential approaches zero asymptotically as in the pure quartic model discussed above. Thus the curvature-squared term is expected to increase $\chi_m$ and flatten the potential around the local maximum.

Assuming that the dilaton has been stabilized as before, we have numerically analyzed the potential $\hat{U}(0,\chi)$ for vari-
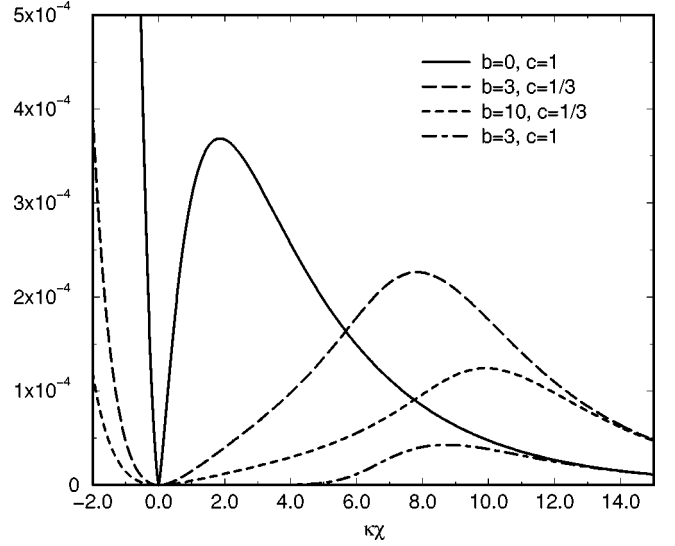


FIG. 2. The potential from Eq. (41) for different values of the parameters $b$ and $c$ controlling the relative strengths of the $R^2$ and $R^4$ terms in the five-dimensional action.

ous values of $b$ and $c$. As is seen in Fig. 2, the potential becomes flatter as we increase $b$. At the same time, however, we find that the height of the potential at the local maximum decreases. As a result we find the maximal possible value of the spectral index to be $n_s = 0.722$ which is realized for $b \gtrsim 10$ and $c \sim 1/3$. For example, taking $b = 3$ and $c = 1/3$, we find $\chi_m = 1.6M_4$, $\hat{U}(0,\chi_m) = 2.3 \times 10^{-4}M_4^2 M_5^2$, and $m^2/H_m^2 = 0.42$. The spectral index is given by $n_s = 0.72$. From $\delta T/T = 10^{-5}$ on the angular scale probed by COBE, the scale of inflation is determined as $H_m \simeq 0.04 M_5 = 6 \times 10^{11}$ GeV or $M_5 \simeq 2 \times 10^{13}$ GeV. This is somewhat on the small side, but we consider such an estimate relatively encouraging, given the crudity of our model.

The scale problem concerning the $R^4$ inflation could also be alleviated if consider the possible effects of the quantum correction terms to the stress energy tensor as described in Eq. (4) [4]. Here we will concentrate on the term proportional to $k_2$. Recall that the coefficient $k_2$ is proportional to the number of degrees of freedom and has been estimated to be $k_2 \sim O(100)\mathcal{R}_5 M_5$. If we ignore the $k_3$ term — which is actually just the variation of the $R^2$ term in the action, and has already been considered above — and include our potential (27), Eq. (5) becomes

$$H^2(1 - H^2/H'^2) = \frac{\kappa^2}{3}U(\phi,\chi) \quad (43)$$

where now $H'^2 = 360\pi M_4^2/k_2$. Near the maximum of the potential at $\chi = \chi_m$, the potential is $V(\chi_m) \simeq 10^{-3}M_4^2 M_5^2$. It is convenient to define $\epsilon = (1 - H^2/H'^2) \simeq m^2/H^2$ where $m^2 \sim 3 \times 10^{-3}M_5^2 M_4^2$ is the curvature of the potential at its maximum. Recall that our problem was related to the fact that $m^2/H^2 \sim 1$.

Using Eq. (43), we can now determine a consistent value for $\epsilon$ and $m^2$ and therefore $H'$. We can then check whether or not the resulting value of $k_2$ makes any sense with these

choices. To obtain the correct magnitude for density fluctuations over the last 60 e-foldings of inflation, we need $H \sim 10^{-5}M_4\epsilon e^{-20\epsilon}$. But recall that $H = m/\sqrt{\epsilon}$, so that we require $M_5 \simeq 2 \times 10^{-3}M_4\epsilon^{3/2}e^{-20\epsilon}$. We see that $M_5$ is maximized when $\epsilon = 0.075$. In this case, we have $M_5 \sim 10^{-6}M_4 \sim 10^{13}$ GeV. As commented above, such an estimate is rather too small, but we do not consider this too disastrous, given the present level of sophistication of this class of model.

We should next determine whether or not we can obtain a value of $k_2$ which is consistent with the desired values of $\epsilon$ and $M_5$. Substituting $k_2$ into $H'$, we find $H' \sim 3M_4/(M_5\mathcal{R}_5)^{1/2}$. For our value of $\epsilon$, we need $H'$ to be within 10% of this value,[5] and we can solve for $\mathcal{R}_5^{-1}$. The result is $\mathcal{R}_5^{-1} \sim 4 \times 10^{-15}M_5$, or about 40 GeV. We note that some estimates of $\mathcal{R}_5$, based on a version of the Scherk-Schwarz approach, could lead one to expect $\mathcal{R}_5^{-1} \sim 100M_5^3/M_4^2 \sim 1$ TeV. Therefore, we do not regard such a large value of $\mathcal{R}_5$ as necessarily unmotivated. Finally, we can trivially check that not only do we get sufficient inflation with an acceptable magnitude for density fluctuations, but also find the spectral index to be in agreement with the existing data. For $n_s$, using $\epsilon = 3/40$, we have that $n_s = 1 - (1/20) = 0.95$, which is a relatively encouraging result, well within the present experimental uncertainties.

## VI. CONCLUSION

Many of the problems associated with inflation in string theory can be traced to a rolling dilaton which precludes a (quasi) de Sitter expansion. In this paper we have presented a model based on higher-order curvature terms such as $R^4$ in the context of a five-dimensional theory. This has some features in common with what may arise in general compactifications of $M$ theory. In this context, the universe passes through an extended five-dimensional phase (down from its initial eleven-dimensional formulation) before it is seen as a four- dimensional space-time with one relatively large extra spatial dimension. We have made use of the conformal properties of the theory to analyze inflation in the familiar four-dimensional Einstein frame. This has enabled us to identify the relevant degrees of freedom, and consider their influence on cosmological dynamics.

We have found that a pure $R + R^4$ theory may provide a potential suitable for topological inflation. However, in this

theory the spectral index for density fluctuations is too small, and the magnitude of density fluctuations is too large, unless the unification point is taken at an absurdly low energy scale. We have considered several ways of repairing these problems. One approach may be to include $R^2$ corrections in the action. Such corrections are not known to arise in superstring theories, but may appear if the supersymmetry breaking scale and the unification scale are close to each other. They can soften the effective inflaton potential further. Alternatively, a similar flattening of the potential may arise as a consequence of a large number of particle degrees of freedom, expected as the Kaluza-Klein modes associated with an extended compact spatial dimension, which could be produced in the course of cosmological evolution. Particle production can likewise raise the spectral index while keeping the magnitude of fluctuations reasonable. This result calls for a rather low unification scale of $10^{13}$ GeV and a scale $\mathcal{R}_5^{-1} \sim 1$ TeV for the fifth dimension. Given the crudity of this model, these results must be seen as encouraging.

To conclude, we note that our arguments do not depend strongly on the specific form of higher-order corrections in the effective action. We only need some higher-order curvature corrections which produce a local extremum of the effective inflaton potential. It seems plausible that such terms may play a significant role in some region of a highly curved early universe. The usual fine tuning of the coefficients of these terms, which was detrimental for the Starobinsky model, could be avoided here because of the dilaton VEV. In turn, the quadratic corrections may be employed to flatten the potential in order to produce a satisfactory density perturbation spectrum, while not being overconstrained thanks to the fact that inflation is driven by higher corrections. Finally, the graceful exit problem is very easy to solve, due to the inherent instability of the solutions. The inflaton rolls down to the minimum, slowly at first but accelerating along the way. In light of this, we take this model to be a reasonable candidate for standard quasi-exponential inflation at scales close to the unification scale, which might offer some clues how to embed inflation in string theory.

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[5] This is hardly fine tuning by any standards.

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