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Inflation and large internal dimensions

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We consider some aspects of inflation in models with large internal dimensions. If inflation occurs on a 3D wall after the stabilization of internal dimensions in the models with low unification scale ($M \sim 1$ TeV), the inflaton field must be extremely light. This problem may disappear in models with an intermediate ($M \sim 10^{11}$ GeV) to high ($M \sim 10^{16}$ GeV) unification scale. However, in all of these cases the wall inflation does not provide a complete solution to the horizon and flatness problems. To solve them, there must be a stage of inflation in the bulk before the compactification of internal dimensions. [S0556-2821(99)50110-0]

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The hierarchy problem has been one of the long-standing challenges to theoretical physics. It is a puzzle concerning masses of scalar fields, which are all quadratically divergent in the loop expansion of a generic quantum field theory. Since the natural cutoff of any quantum field theory is the Planck mass M_p, the renormalization effects should drive all scalar masses up to the Planck scale. As a result, all interacting scalar fields should be very heavy. On the other hand, in the standard model the Higgs field must have a mass $m_H \sim 1$ TeV in order for the model to be consistent. This is the hierarchy problem: the light scalars are needed in the theory, but ensuring that they are light requires not only that their masses are very small classically, but also that there is some mechanism which will keep the masses small after radiative corrections.

However, if one starts with a fundamental theory which is higher-dimensional, and identifies the higher-dimensional fundamental Planck scale with the gauge unification scale M, it may be possible to recover the very large Planck scale of the four-dimensional (4D) world if the higher-dimensional theory is compactified to 4D on a large internal space [1–8]. There have also been previous considerations of models of unification scales below the 4D Planck scale in string theory [9–12]. If the size of the internal dimensions is r_0, using Newton's law in $D+4$ dimensions, at distances much larger than the size r_0 of the internal space, one finds [1]

$$M_p^2 \sim r_0^D M^{2+D}. \tag{1}$$

Thus if $r_0 \gg M^{-1}$, the reduced Planck mass may appear to be many orders of magnitude larger than the fundamental Planck mass. The hierarchy problem then becomes the problem of choosing the radius of stabilization of the internal space, which should be large compared to the fundamental scale. In all of these models, it has been assumed that the 4D world is a three-dimensional analogue of a domain wall, or a 3-brane in the modern M-theory parlance, which is embedded in a higher-dimensional theory. The proposal that the world may be a hypersurface in a higher-dimensional spacetime goes back to [13] (see also [14]), but has been reinvigorated by recent developments in string and M-theory, which may provide the mechanism to explain why matter degrees of freedom are stuck to the wall. In this paper we will give some comments concerning the possibility to have inflation in such a scenario.

Generically, inflation may begin within a small island of $D+4$ dimensional space of Planck size M^{-1}. Then it may proceed differently in three uncompactified directions and in the remaining D dimensions, which grow from M^{-1} to r_0 and then stabilize. Unfortunately, it is very difficult to study this possibility since many aspects of compactification and stabilization of D dimensions in this theory still remain rather speculative. Therefore prior to the investigation of this generic but complicated regime, one may try to analyze a simpler possibility, assuming that inflation (or at least its latest stages) occur only in three uncompactified directions after the stabilization of internal dimensions.

In [7,15] it has been argued that having all of inflation occur after compactification may require an extremely light wall inflaton, as compared to the unification scale. Indeed, the effective potential along the wall in this scenario cannot be greater than M^4. This follows from the assumption that the thickness of the wall cannot be much greater than M^{-1}, and the Planck density in $D+4$ is M^{D+4}. Then the Hubble constant during inflation on the wall is given by

$$H \sim \sqrt{\frac{8\pi V(\phi)}{3M_p^2}} \lesssim \frac{M^2}{M_p}. \tag{2}$$

Inflation occurs only if the inflaton mass m is smaller than H, which implies the constraint

$$m \lesssim H \lesssim \frac{M^2}{M_p}. \tag{3}$$

This bound is completely independent of the number of internal dimensions. In the particular case $D=2$, this constraint shows that the Compton wavelength of the inflaton field should be greater than the size of internal dimensions.

If one takes $M \sim 1$ TeV, as proposed in [1], one gets an extremely strong constraint on the inflaton mass, $m \lesssim 10^{-4}$ eV [7,15]. In principle, supersymmetry may provide some flat directions with an extremely small curvature $V'' = m^2 < (10^{-4}\ \text{eV})^2$, but this forces one to make a step back from the original motivation for the models of this type. Even if this is allowed, one still encounters severe problems in constructing inflationary models of such type.

For example, if one considers a simplest version of chaotic inflation with $V(\phi) = (m^2/2)\phi^2$ [16], one finds unacceptably density perturbations

$$\frac{\delta\rho}{\rho}\sim 50\frac{m}{M_p}\lesssim 10^{-31}. \tag{4}$$

The situation becomes slightly better for hybrid inflation driven by the potential $V(\phi,\sigma)=(1/4V)(M^2-\lambda\sigma^2)^2+(m^2/2)\phi^2+(g^2/2)\phi^2\sigma^2$ [17,18]. In this case

$$\frac{\delta\rho}{\rho}=\frac{2\sqrt{6\pi}g}{5\lambda^{3/2}}\frac{M^5}{M_p^3 m^2}. \tag{5}$$

Let us take $M\sim 1$ TeV and $\lambda, g\sim O(1)$, as should be if the inflaton is to be a particle from the spectrum of the wall gauge theory. Compared to the Cosmic Background Explorer (COBE) data, which give $\delta\rho/\rho\sim 5\times 10^{-5}$ at redshifts corresponding to the last 60 *e*-foldings of inflation, we find the desirable value of m:

$$m\sim 10^{-10}\ \text{eV}. \tag{6}$$

This is six orders of magnitude worse than the constraint $m\lesssim 10^{-4}$ eV obtained in [7,15] from the condition of existence of the inflationary regime (3). A different possibility was discussed in [15], but it requires the existence of a small coupling constant $\lambda\sim 10^{-8}$.

Note, that in the hybrid inflation scenario discussed above, the mass of the inflaton field ϕ *after* inflation is equal to $gM/\sqrt{\lambda}\gg m$. Therefore one can have efficient reheating and baryogenesis in this model. Thus, it is possible to have a consistent inflationary scenario of this type, if one finds a mechanism which maintains the extreme flatness of the effective potential during inflation. Supersymmetry may help here, but typically supersymmetry induces the inflaton mass $m=O(H)$. There exist several mechanisms which may help to avoid this complication [19]. However, according to Eq. (6), in the model described above the mass of the inflaton field during inflation must be six orders of magnitude smaller than H, which may be rather difficult to achieve.

The constraint on the inflaton mass can be relaxed by assuming that the scale M is much larger than 1 TeV. For example, if we take $M\sim 10^{11}$ GeV, as suggested in [6,12], the constraint on the inflaton mass is

$$m<M^2/M_p\sim 1\ \text{TeV}. \tag{7}$$

It fits perfectly in the hybrid inflation scenario. Indeed, in the original version of the hybrid inflation model [17], it has been proposed to take the parameters $M=10^{11}$ GeV, $m=10^2$ GeV, $g^2=\lambda=0.1$, which satisfy the constraint $m<M^2/M_p$ and give the proper amplitude of density perturbations.

Another interesting possibility is if the unification scale is $M=10^{16}-10^{17}$ GeV [9]. This would lead to the constraint

$$m\lesssim 10^{13}-10^{15}\ \text{GeV}. \tag{8}$$

This condition is satisfied in the simplest version of chaotic inflation scenario with $V(\phi)=m^2/2\phi^2$ and $m\sim 10^{13}$ GeV [20]. Hybrid inflation works in this case as well, for a smaller value of m [19].

The discussion above shows that having inflation on the wall when the unification scale is low, ~ 1 TeV, requires incredibly small masses and couplings. On the other hand, it is rather easy to fit some of the mainstream inflationary models on the wall in theories with large internal dimensions which have stabilized, if the unification scale is medium to high.

However we must point out that having all of inflation after the internal dimensions are stabilized cannot be the whole picture. For very low unification scales, the compactification scale r_0^{-1} and the unification scale M differ by many orders of magnitude. In such models, inflation after stabilization of the internal dimensions requires extreme fine tuning of the initial conditions in the early universe.

Indeed, the only natural time scale for the beginning of inflation in this model is given by the higher-dimensional Planck time M^{-1}, when the density of the universe was of the order M^{D+4}. The last condition is consistent with the requirement that the 4D density of the wall is $O(M^4)$. However, as we already mentioned, the Hubble parameter H at this time is smaller than $M^2/M_p\ll M$, which implies that inflation occurs on a time scale much greater than M^{-1}. Thus, the universe must be sufficiently large and homogeneous from the very beginning to survive and not to loose the homogeneity during the long period of time from $t\sim M^{-1}$ to $t\sim H^{-1}$. (If $M\sim 1$ TeV and $H\sim 10^{-4}$ eV, these two time scales differ by 16 orders of magnitude.) On the other hand, one cannot simply assume that the universe must be homogeneous at all times, because at the beginning of inflation it must be strongly inhomogeneous: its density at the wall must be many orders of magnitude greater than the density in the bulk. Indeed, suppose for definiteness that the initial value of H during inflation is equal to M^2/M_p (in the case $H\ll M^2/M_p$, the problem will be even more pronounced). In this case the initial energy density on the wall will be close to its higher-dimensional Planck value, M^{D+4}. If one wants to neglect the influence of the bulk on the expansion of the universe, one should require that the total energy concentrated there should be smaller than the energy at the wall. Using Eq. (1), one can show that this condition implies that the density of matter in the bulk at that time must be smaller than the Planck density M^{D+4} by the factor of $M^2/M_p^2\ll 1$. This means that the density of matter in the bulk must be nearly empty as compared with the density at the wall, and this emptiness must be preserved on a scale $r_0=M^{-1}(M_p/M)^{2/D}\gg M^{-1}$. In addition, inflation on the wall requires the distribution of matter on the wall to be homogeneous on scale $\sim H^{-1}\gg M_p/M^2\gg M^{-1}$. Since M^{-1} is the only natural scale for homogeneity, one can hardly explain from first principles how this specific structure could be formed unless there was a previous stage of inflation, simultaneously in the bulk and on the wall, which could extend the Planck scale M^{-1} to the scale M_p/M^2.

Since this subject is rather complicated, we will consider here, for purely illustrative purposes, a toy model of the wall inflation. It is different from the model of Ref. [1], but has some obvious similarities.

It is well known that the domain walls in the theories with spontaneous symmetry breaking in the thin wall approxima-

tion can be described as objects with the energy-momentum tensor $T^{\mu}_{\nu}=\sigma\delta(x)\ \ \mathrm{diag}(1,1,1,0)$, where x is the direction orthogonal to the domain wall, and σ is its surface tension. The metric corresponding to this distribution of matter describes an inflating 2D domain wall in a 4D spacetime [22].[1]

We have found a similar solution describing a 3D inflating domain wall in a 5D space-time. It is produced by scalar field on the 3D domain wall, which in the thin wall approximation has the energy-momentum tensor [24] $T^{\mu}_{\nu}=-\sigma\delta(w)\ \ \mathrm{diag}(1,1,1,1,0)$. Here w runs along the 5th direction, orthogonal to the 3D wall, and σ is the surface tension of the 3D wall in the 5D space-time. We could solve Einstein's equations with this source to determine the 3-brane metric. It is

$$ds_5^2=(1-H|w|)^2(-dt^2+e^{2Ht}(dx^2+dy^2+dz^2))+dw^2 \quad (9)$$

where H is the Hubble constant along the 3-brane, given by $H=4\pi\sigma/3M^3$ [24]. This metric is perfectly regular at the event horizon $Hw=1$, and singular at the domain wall, where $w=0$. This is the place where the δ-function source T^{μ}_{ν} must reside.

The solution (9) can be interpreted as an inflating 3-brane, with the event horizon on the brane given by H^{-1}. In the direction transversal to the brane, the metric resembles the Rindler metric. The gravitational field is repulsive, and it pushes the perturbations towards the Rindler horizon, located at $w=H^{-1}$. Any inflating point on the brane is completely surrounded by an event horizon, at a distance H^{-1} from it. The no-hair theorem [21] for this metric would show that if one perturbs this metric on a scale greater than $O(H^{-1})$, this perturbation is rapidly stretched in all directions by the expansion of the universe, just like in the usual inflationary universe scenario. For example, if instead of the domain wall at the plane $\omega=0$ one considers a spherical domain wall positioned at $x^2+y^2+z^2+\omega^2=r^2$ with $r\gg H^{-1}$ inflation will blow up this bubble, stretch its walls, and metric near the wall will be again described by our solution (9). However, if $r\ll H^{-1}$, then the surface tension of the domain wall will shrink its size to zero within the time $t\ll H^{-1}$, and inflation will never happen. This means, as we expected, that in order to find out whether inflation on the wall will happen, we need to adjust properly the initial conditions on a scale H^{-1}.

For the scenario the wall inflation in the theory proposed in Ref. [1] this would imply that the wall must be homogeneous on the scale $H^{-1}\sim M^3/\sigma\gg M^{-1}$. This is extremely difficult to achieve, especially with $M\sim 1$ TeV, when the size of the initial homogeneous domain H^{-1} must be 16 orders of magnitude greater than the Planck length M^{-1}.

In conclusion, we have found that inflation on the wall can be achieved in several versions of inflationary scenario. The simplest way to do so is to use the hybrid inflation scenario. It can be done even for $M\sim 1$ TeV, but it is much easier to do for larger M. However, to obtain a complete cosmological scenario, one will probably need to consider not only inflation on the wall, but also inflation in the bulk. In such a scenario, since the natural size of homogeneous islands in the early universe is given by the unification length, M^{-1}, it would be necessary to take the internal dimensions to be initially small, $\sim M^{-1}$, and allow them to expand until they reach the compactification scale r_0. During this expansion, the ratio M/M_p may change by many orders of magnitude, and may be much closer to unity in the beginning. This may relax the constraints on the mass of the inflaton [7,15] quite considerably. The possibility of constructing bulk inflation could thus play a crucial role in establishing feasibility of models with large internal dimensions. However, concrete details of this stage of early inflation may depend on the specifics of the model used to describe it, which lies beyond the scope of the present article. We will defer this discussion for the future.

Another possible resolution of the outlined problems is related to the eternal inflation scenario. Indeed, it is known that the inflationary universe in simplest versions of chaotic inflation scenario enters regime of self-reproduction [25]. This means that once inflation begins, it produces infinite amount of homogeneous space, whereas noninflationary parts of the universe produce only a finite amount of inhomogeneous space. This fact may make the problem of initial conditions irrelevant [26]. Note that the regime of self-reproduction occurs not only in chaotic inflation, but in new inflation as well [27]. On the other hand, this regime does not occur in the pre-big bang inflation, which makes the problems of initial conditions in this theory more difficult to resolve [28]. If inflation of the wall is eternal, then we may not necessarily need to have the preceding stage of inflation in the bulk.

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[18] To avoid terminological misunderstanding, we should emphasize that hybrid inflation is a particular version of chaotic inflation. As it was emphasized in [16], the borderline between chaotic inflation and the earlier generation of inflationary models (old inflation and new inflation) is not in the choice of the potentials. The main idea of chaotic inflation was that one does not need to assume that the initial position of the inflaton field is fixed by thermal effects; its initial distribution may well be chaotic. Hybrid inflation scenario [17] is based on the same idea.
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[24] Our solution (9) can be obtained by directly solving Einstein's equations in 5D with the 3 brane stress-energy $T^{\mu}_{\nu}=-\sigma\delta(w)$ diag(1,1,1,1,0), where σ is the generalized surface tension. The $(-)$ sign is due to our metric signature convention $(-,+,+,+,+)$. The general form of the metric (9) is an obvious generalization of the metric obtained in [22]. A somewhat nontrivial step is to find a relation between H and σ. To do so, we use the equations of motion, and find the Ricci tensor on the brane: $R^{\alpha}_{\beta}=(8\pi/3M^3)\sigma\delta(w)\delta^{\alpha}_{\beta}$, where $\alpha,\beta\in\{0,\ldots,3\}$. Since the curvature is singular, by symmetry all one needs to check is the divergence of R^0_0. The only singularity arises from $R^0_0(singular)=-\partial_w\Gamma^0_{w0}=2H\delta(w)$. Hence $H=(4\pi/3M^3)\sigma$, in contrast to [22], where $D=4$ and $H=(2\pi/M_p^2)\sigma$. This completes the derivation of our solution (9). The solution for a general case of a $D-2$ brane in D dimensions is very similar, with $H=[4\pi/(D-2)M^{D-2}]\sigma$. One may also consider a p brane in D dimensions. The stress energy tensor for it is $T^{\mu}_{\nu}=-\sigma\delta(w)(1_1,\ldots,1_{p+1},0_{p+2},\ldots,0_D)$, and the Ricci tensor on the brane is $R^{\alpha}_{\beta}=[8\pi(p+3-D)/(D-2)M^{D-2}]\sigma\delta(w)\delta^{\alpha}_{\beta}$. For the 3 brane in 6D, this gives $R^{\alpha}_{\beta}=0$. In general, for $p+3\leqslant$D one has $R\leqslant 0$. This suggests that the branes in a space with more than one uncompactified dimension would not inflate. A particular example is provided by cosmic strings in 4D, for which $p=1$ and $D=4$. Unlike domain walls, cosmic strings do not inflate. This result suggests that in order to have inflation on the wall, we need to consider models where at most one dimension remains uncompactified. The class of models proposed in [1] satisfies this condition.
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