

## Upper bounds on all $R$ -parity-violating $\lambda\lambda''$ combinations from proton stability

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In an  $R$ -parity-violating supersymmetric theory, we derive upper bounds on all the  $\lambda''_{ijk}\lambda'_{i'j'k'}$ -type combinations from the consideration of proton stability, where  $\lambda''_{ijk}$  are baryon-number-violating couplings involving three baryonic fields and  $\lambda'_{i'j'k'}$  are lepton-number-violating couplings involving three leptonic fields. [S0556-2821(99)01809-3]

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In the minimally supersymmetrized standard model (MSSM), the superpotential contains the following  $R$ -parity conserving terms:

$$W_0 = f_e^{ij} L_i H_d E_j^c + f_d^{ij} Q_i H_d D_j^c + f_u^{ij} Q_i H_u U_j^c + \mu H_d H_u. \quad (1)$$

Here,  $L_i$  and  $Q_i$  are  $SU(2)$ -doublet lepton and quark superfields;  $E_i^c, U_i^c, D_i^c$  are  $SU(2)$ -singlet charged lepton, up- and down-quark superfields;  $H_d$  and  $H_u$  are Higgs superfields responsible for the down- and up-type masses respectively. The generation indices are assumed to be summed over.

$R$ -parity is a discrete symmetry which is defined as  $R = (-1)^{(3B+L+2S)}$ , where  $B$  is the baryon number,  $L$  is the lepton number and  $S$  is the spin of the particle.  $R$  is  $+1$  for all standard model particles and  $-1$  for their superpartners. If one allows  $R$ -parity violation [1], the most general superpotential includes the following  $L$ - and  $B$ -violating terms:

$$W' = \frac{1}{2} \lambda_{ijk} L_i L_j E_k^c + \lambda'_{ijk} L_i Q_j D_k^c + \frac{1}{2} \lambda''_{ijk} U_i^c D_j^c D_k^c + \mu_i L_i H_u. \quad (2)$$

Here  $\lambda''_{ijk}$  are  $B$ -violating while  $\lambda_{ijk}, \lambda'_{ijk}$  and  $\mu_i$  are all  $L$ -violating couplings. Considering the antisymmetry in the first (last) two flavor indices in  $\lambda(\lambda'')$ , namely

$$\lambda_{ijk} = -\lambda_{jik}, \quad \lambda''_{ijk} = -\lambda''_{ikj}, \quad (3)$$

there are 48 additional parameters. These are constrained from various experimental searches [2].

The simultaneous presence of  $B$ - and  $L$ -violating couplings drives proton decay. Therefore, in an  $R$ -parity-violating ( $\mathcal{R}$ ) theory, what can be derived from proton decay are, for example, bounds on  $\lambda''$  correlated with any of the  $L$ -violating couplings. As mentioned before,  $L$ -violating sources are of 3 types. The couplings  $\lambda'_{ijk}$  constitute one type of source, and the correlated bounds in this case exist in the literature [3,4]. At the tree level the bounds apply only to a select set of the couplings, and one obtains [3], assuming superpartner masses around 1 TeV,

$$\lambda'_{11k} \lambda''_{11k} \lesssim 10^{-24}, \quad (4)$$

where  $k=2,3$ . At the one-loop level, one can always find at least one diagram in which any  $\lambda'_{ijk}$  in conjunction with any  $\lambda''_{lmn}$  contributes to proton decay [4]. It follows that, for superparticle masses of order 1 TeV,

$$\lambda'_{ijk} \lambda''_{lmn} \lesssim 10^{-9}. \quad (5)$$

If one admits tree level flavor-changing squark mixing, the bounds are strengthened by two orders of magnitude.

Recently, contributions to proton decay originating from the  $L$ -violating parameters  $\mu_i$  in conjunction with the  $\lambda''$ 's have been investigated [5]. Here, the diagram at the tree level produces a constraint, for an exchanged scalar mass of 1 TeV,

$$\lambda''_{112} \epsilon_l < 10^{-21}, \quad (6)$$

where  $\epsilon_l = \mu_l / \mu$ , with  $\mu$  assumed to be of order 1 TeV. Constraints on the other  $\lambda''_{ijk} \epsilon_l$ -type combinations originate from loop diagrams and hence are weaker. They are typically of order  $10^{-10}$ – $10^{-14}$ , always assuming superparticle masses of order 1 TeV.

The aim of the present paper is to examine the other source of lepton number violation, viz.  $\lambda_{ijk}$ , and derive bounds on  $\lambda''_{ijk} \lambda'_{i'j'k'}$  products with any choices of flavor indices.

The fact that  $\lambda$  and  $\lambda''$  together can drive proton decay has been noted before [6] in the context of an extended gauge model. The idea however applies to any general framework of  $R$ -parity violation, for example, as the one in the present paper. We will note as we proceed further that any one of the nine  $\lambda''$ -couplings in association with any one of the nine  $\lambda$ -couplings can contribute to proton decay at one- or two-loop order if not at the tree level. Figure 1 shows a generic diagram involving two blobs. The  $\lambda''_{ijk}$ -blob on one side represents either a tree or a one-loop diagram containing

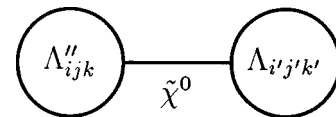


FIG. 1. Generic structure of diagrams involving  $\lambda''$  and  $\lambda$  couplings that lead to proton decay.

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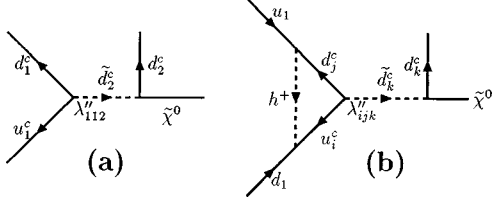


FIG. 2. The  $B$ -violating blob  $\Lambda''_{ijk}$  for various combinations of the indices.  $\tilde{\chi}^0$  is the neutralino coupled to the blob.

a  $\lambda''_{ijk}$ -vertex with external quark lines. The  $\Lambda_{i'j'k'}$ -blob on the other side contains a  $\lambda_{i'j'k'}$ -vertex with external lepton lines through a tree or a one-loop diagram. The two blobs are connected by virtual neutralinos. The amplitude of this generic diagram can be written as

$$G_R \approx \frac{\Lambda''_{ijk} \Lambda_{i'j'k'}}{m_{\tilde{\chi}^0}}, \quad (7)$$

where  $m_{\tilde{\chi}^0}$  is the mass of the exchanged neutralino. The maximum contribution is expected to come from the exchange of the lightest neutralino, which we assume to be predominantly a gaugino (the Higgsino exchanged graphs will be suppressed by light masses). The next task is to decipher the explicit structures of the two blobs  $\Lambda''$  and  $\Lambda$ , which involve all possible combinations of flavor indices associated with  $\lambda''$  and  $\lambda$ , in a case by case basis.

We start with the evaluation of  $\Lambda''_{ijk}$  for various combinations of the indices. Note that, due to the antisymmetry of the indices mentioned in Eq. (3), we can always take  $k \neq 3$ . Among the independent couplings now, we can distinguish two cases. This part is very similar to the discussion appearing in our earlier work [5].

Case (a). For  $\Lambda''_{112}$ , we obtain a tree graph, which is shown in Fig. 2(a). The strength of the blob is given by

$$\Lambda''_{112}^{(a)} \approx \frac{g \lambda''_{112}}{m_{\tilde{\chi}^0}}. \quad (8)$$

Case (b). The other  $\lambda''_{ijk}$ 's cannot appear in tree diagrams because they would involve at least one heavy quark ( $c$ ,  $b$  or  $t$ ) in the outer legs. However, they couple through loop diagrams involving the exchange of charged scalars. As a result of the  $\mu$ -term, the physical charged Higgs  $h^+$  is a combination of  $H_u^+$  and  $H_d^+$ . The diagram involving  $h^+$  is shown in Fig. 2(b) where flavor violation occurs via Cabibbo-Kobayashi-Maskawa (CKM) projections. This diagram is infinite. In the supersymmetric limit, it cancels with a similar diagram containing the longitudinal  $W$ -boson in place of the charged Higgs boson. Since supersymmetry is broken, the sum of the two diagrams gives a finite quantity which depends logarithmically on the masses of these two scalars. For a charged Higgs boson mass of order 1 TeV that we assume in this paper, we can safely omit this logarithm for an order-of-magnitude estimate, yielding

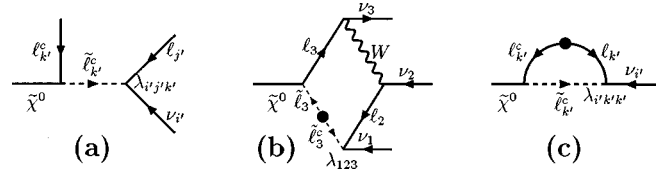


FIG. 3. The  $L$ -violating blob  $\Lambda_{i'j'k'}$  for various combinations of the indices.  $\tilde{\chi}^0$  is the neutralino coupled to the blob.

$$\Lambda''_{ijk} \approx \frac{f_u^i f_d^j}{16\pi^2} (V_{i1}^* V_{1j}) \frac{g \lambda''_{ijk}}{m_{\tilde{d}_k^c}}. \quad (9)$$

We now discuss the lepton-number violating blob  $\Lambda_{i'j'k'}$ . Here, there are three cases which should be distinguished.

Case (a). Six of the nine different  $\lambda_{i'j'k'}$ 's, characterized by  $k' \neq 3$ , contribute to the blob at the tree level as shown in Fig. 3(a). The strength of this blob can be written as

$$\Lambda_{i'j'k'}^{(a)} \approx \frac{g \lambda_{i'j'k'}}{m_{\tilde{\tau}_{k'}}} \quad (k' \neq 3). \quad (10)$$

Case (b). For  $\Lambda_{123}$ , the dominant contribution comes through the box shown in Fig. 3(b). Again, an order of magnitude estimate for the strength of this blob yields

$$\Lambda_{123}^{(b)} \approx \frac{g^3 \lambda_{123}}{16\pi^2 M_W^2} \frac{m_\tau}{m_{\tilde{\tau}}}. \quad (11)$$

Here we have parametrized the left-right slepton mixing blob as  $m_\tau m_{\tilde{\tau}}$ , where  $m_{\tilde{\tau}}$  is some kind of an average of the masses of  $\tilde{\tau}_L$  and  $\tilde{\tau}_R$ .

Case (c). There is another type of contribution to the blob  $\Lambda_{i'j'k'}$  for  $j' = k'$  [or equivalently  $i' = k'$  on account of the antisymmetry shown in Eq. (3)]. These are self-energy type diagrams shown in Fig. 3(c). These include some of the couplings described in case (a), viz., those with  $j' = k' = 1$  and  $j' = k' = 2$ . For the remaining case  $j' = k' = 3$ , we do not have the tree diagram described in case (a) because that would have implied a  $\tau$ -lepton in the final state. The strength of this blob is given by

$$\Lambda_{i'k'k'}^{(c)} \approx \frac{g \lambda_{i'k'k'}}{16\pi^2} m_{l_{k'}}. \quad (12)$$

Since these blobs lead to only one particle carrying lepton number in the final state, the dimension of  $\Lambda_{i'k'k'}^{(c)}$  is not the same as that of  $\Lambda_{i'j'k'}^{(a)}$  or  $\Lambda_{123}^{(b)}$ . Since such self-energy diagrams involve lower dimensional operators, the constraints on  $\lambda_{i'11}$  and  $\lambda_{i'22}$  derived from these diagrams, despite suffering loop suppressions, happen to be stronger than those derived from tree diagrams in case (a).

We can now put the various combinations of  $\Lambda''_{ijk}$  and  $\Lambda_{i'j'k'}$  into Eq. (7) to obtain the strength of the baryon and lepton number violating couplings which are responsible for

TABLE I. Decay modes for the proton and bounds derived on the couplings for all possible combinations of the baryon violating and the lepton violating vertices. The bounds are on the products  $\lambda''_{ijk}\lambda''_{i'j'k'}$ . All superpartner masses are assumed to be 1 TeV. The ranges in the last column indicate the variation due to different CKM projections.

$\lambda''_{i'j'k'}$	$\lambda''_{112}$		all other $\lambda''_{ijk}$	
	Final states	Bounds	Final states	Bounds
$i' \neq j' \neq k', k' \neq 3$	$K^+ e^\pm \mu^\mp \bar{\nu}$	$10^{-16}$	$\pi^+(K^+) e^\pm \mu^\mp \bar{\nu}$	$10^{-5}-10^{-7}$
$i' \neq j' \neq k', k' = 3$	$K^+ + 3\nu$	$10^{-14}$	$\pi^+(K^+) + 3\nu$	$10^{-3}-10^{-5}$
$j' = k' = 1$ (or $i' = k' = 1$ )	$K^+ \bar{\nu}$	$10^{-17}$	$\pi^+(K^+) \bar{\nu}$	$10^{-6}-10^{-8}$
$j' = k' = 2$ (or $i' = k' = 2$ )	$K^+ \bar{\nu}$	$10^{-20}$	$\pi^+(K^+) \bar{\nu}$	$10^{-9}-10^{-11}$
$j' = k' = 3$ (or $i' = k' = 3$ )	$K^+ \bar{\nu}$	$10^{-21}$	$\pi^+(K^+) \bar{\nu}$	$10^{-10}-10^{-12}$

proton decay. For lepton number violation through case (c), the effective coupling  $G_R$  has mass dimension  $-2$ , and proton lifetime is given by

$$\tau_p = (m_p^5 G_R^2)^{-1}. \quad (13)$$

In other cases, the final states will have three particles carrying lepton number, the effective coupling  $G_R$  will have mass dimension  $-5$ , and the proton lifetime will be given by

$$\tau_p = (m_p^{11} G_R^2)^{-1}. \quad (14)$$

We present the bounds on the combinations  $\lambda''_{ijk}\lambda''_{i'j'k'}$  in Table I for superparticle masses of the order of 1 TeV. In deriving the bounds, we have taken  $\tau_p$  to be  $10^{32}$  yr [7], except for final states with charged leptons for which a

benchmark value of  $10^{31}$  yr has been assumed. Moreover, for our order of magnitude estimates, we have neglected all final state particle masses and phase space factors, as in all earlier estimates [2–5]. Consideration of 4-body phase space can relax the bounds in the first two rows of Table I by about two orders of magnitude.

To conclude, we have derived new constraints on all products of the form  $\lambda''_{ijk}\lambda''_{i'j'k'}$  from proton stability. These bounds are complementary to other  $L$  and  $B$  violating products [4,5] that contribute to proton decay. In most cases, our bounds are orders of magnitude stronger than the products of upper bounds on individual couplings [2,8–10].

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- [1] G. Farrar and P. Fayet, Phys. Lett. **76B**, 575 (1978); S. Weinberg, Phys. Rev. D **26**, 287 (1982); N. Sakai and T. Yanagida, Nucl. Phys. **B197**, 533 (1982); C. Aulakh and R. N. Mohapatra, Phys. Lett. **119B**, 136 (1982); L. Hall and M. Suzuki, Nucl. Phys. **B231**, 419 (1984); J. Ellis *et al.*, Phys. Lett. **150B**, 142 (1985); G. Ross and J. W. F. Valle, *ibid.* **151B**, 375 (1985); S. Dawson, Nucl. Phys. **B261**, 297 (1985); R. Barbieri and A. Masiero, *ibid.* **B267**, 679 (1986).
- [2] For reviews, see G. Bhattacharyya, hep-ph/9709395, Nucl. Phys. B (Proc. Suppl.) **52A**, 83 (1997); H. Dreiner, in *Perspectives on Supersymmetry*, edited by G. Kane (World Scientific, Singapore, 1998), hep-ph/9707435.

- [3] I. Hinchliffe and T. Kaeding, Phys. Rev. D **47**, 279 (1993).
- [4] A. Y. Smirnov and F. Vissani, Phys. Lett. B **380**, 317 (1996). See also C. Carlson, P. Roy, and M. Sher, *ibid.* **357**, 99 (1995).
- [5] G. Bhattacharyya and P. B. Pal, Phys. Lett. B **439**, 81 (1998).
- [6] H. N. Long and P. B. Pal, Mod. Phys. Lett. A **13**, 2355 (1998).
- [7] Particle Data Group, C. Case *et al.*, Eur. Phys. J. C **3**, 1 (1998).
- [8] F. Zwirner, Phys. Lett. **132B**, 103 (1983); J. L. Goity and M. Sher, Phys. Lett. B **346**, 69 (1995); **385**, 500(E) (1996).
- [9] G. Bhattacharyya, D. Choudhury, and K. Sridhar, Phys. Lett. B **355**, 193 (1995).
- [10] V. Barger, G. Giudice, and T. Han, Phys. Rev. D **40**, 2987 (1989).