Role of resonances in nonleptonic hyperon decays

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We examine the importance of resonances for the nonleptonic hyperon decays in the framework of chiral perturbation theory. Lower lying resonances are included in the effective theory. Integrating out the heavy degrees of freedom in the resonance saturation scheme generates higher order counterterms in the effective Lagrangian, providing an estimate of the pertinent coupling constants. A fit to the eight independent decay amplitudes that are not related by isospin symmetry is performed and reasonable agreement for *both s* and *p* waves is achieved. [S0556-2821(99)08609-9]

I. INTRODUCTION

The matrix elements of nonleptonic hyperon decays can be described in terms of just two amplitudes—the parity-violating *s* wave and the parity-conserving *p* wave. Chiral perturbation theory provides a framework whereby these amplitudes can be expanded in terms of small four-momenta and the current masses m_q of the light quarks, $q = u, d, s$. At lowest order in this expansion the amplitudes are expressed in terms of two unknown coupling constants, the so-called low energy constants (LECs). However, there is no consensus for the determination of these two weak parameters. If one employs values which provide a good fit for the *s* waves, one obtains a poor description of the *p* waves. On the other hand, a good *p*-wave representation yields a poor *s*-wave fit [1]. In order to overcome this problem, one must go beyond leading order. In the paper of Bijnens *et al.* [2], a first attempt was made in calculating the leading chiral corrections to such decays. The authors worked in the limit $m_u = m_d = 0$ and kept only the nonanalytic logarithms from the Goldstone boson loops—no local counterterms were considered. However, the resulting *s*-wave predictions no longer agreed with the data, and the corrections for the *p* waves were even larger.

Jenkins reinvestigated this topic within the framework of heavy baryon chiral perturbation theory, explicitly including the spin-3/2 decuplet in the effective theory [3]. As in [2], no counterterms were included—only leading nonanalytic pieces from the meson loops were retained and $m_u = m_d = 0$ was assumed. Working in the $SU(6)$ limit (by neglecting the octet-decuplet mass splitting), she found significant cancellations between the octet and decuplet components in the loops. For the *s* waves good agreement between theory and experiment was restored, although in the case of the *p* waves, the chiral corrections did not provide a good description of the data. Thus, the inability to fit *s* and *p* waves simultaneously remains even after including the lowest non-analytic contributions.

In our recent paper [4] a calculation was performed which included *all* terms at one-loop order. This work suffers from the fact, however, that at this order too many new unknown LECs enter the calculation so that the theory lacks predictive power. In order to proceed these parameters were estimated by means of spin-3/2 decuplet resonance exchange. The results for the *p* waves were still in disagreement with the data and, therefore, additional counterterms that were not saturated by the decuplet had to be taken into account. An exact fit to the data was possible, but the question remains whether the LECs are estimated correctly.

Another intriguing possibility was examined by Le Yaouanc *et al.*, who assert that a reasonable fit for both *s* and *p* waves can be provided by appending pole contributions from $SU(6)(70,1^-)$ states to the *s* waves [5]. Their calculations were performed in a simple constituent quark model and appear to be able to provide a resolution of the *s*- and *p*-wave dilemma.

The purpose of the present work is to consider the validity of this approach within the framework of chiral perturbation theory. This would provide also an estimate of the counterterms involved in such a calculation, which have been neglected completely in [2] and [3]. Furthermore, we include the octet of spin-parity $1/2^+$ Roper-like states, which generalizes the considerations of [5]. We do not intend to provide a definitive solution of the problem of hyperon decay but rather to study the relevance of resonance saturation estimation of counterterms. We will show that one is able to successfully identify counterterms in chiral perturbation theory with the contributions found in the quark model. The calculations are performed at the tree level. Of course, for a more quantitative statement one has to include loop effects. However, this is beyond the scope of the present work.

In the following section then, we introduce the effective weak and strong Lagrangians including resonant states. By integrating out the heavy degrees of freedom from the theory the effects of the resonances are included in the counterterms and expressions for the decay amplitudes in terms of these constants are given. A least-squares fit to experiment is performed in Sec. III, while in Sec. IV we conclude with a brief summary.

II. RESONANCES IN HYPERON DECAYS

There exist seven experimentally accessible nonleptonic hyperon decays: $\Sigma^+ \to n\pi^+$, $\Sigma^+ \to p\pi^0$, $\Sigma^- \to n\pi^-$, $\Lambda \to p\pi^-$, $\Lambda \to n\pi^0$, $\Xi^- \to \Lambda\pi^-$ and $\Xi^0 \to \Lambda\pi^0$, and the matrix elements of these decays can each be expressed in terms

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of a parity-violating s-wave amplitude $A_{ij}^{(S)}$ and a parity-conserving p-wave amplitude $A_{ij}^{(P)}$:

$$\mathcal{A}(B_i \rightarrow B_j \pi) = \bar{u}_{B_j}\{A_{ij}^{(S)} + A_{ij}^{(P)}\gamma_5\}u_{B_i}. \quad (1)$$

The underlying strangeness-changing Hamiltonian transforms under $SU(3) \times SU(3)$ as $(8_L, 1_R) \oplus (27_L, 1_R)$ and, experimentally, the octet piece dominates over the 27-plet by a factor of twenty or so. Consequently, we will neglect the 27-plet in what follows. Isospin symmetry of the strong interactions implies then the relations

$$\mathcal{A}(\Lambda \rightarrow p\pi^-) + \sqrt{2}\mathcal{A}(\Lambda \rightarrow n\pi^0) = 0$$

$$\mathcal{A}(\Xi^- \rightarrow \Lambda\pi^-) + \sqrt{2}\mathcal{A}(\Xi^0 \rightarrow \Lambda\pi^0) = 0$$

$$\sqrt{2}\mathcal{A}(\Sigma^+ \rightarrow p\pi^0) + \mathcal{A}(\Sigma^- \rightarrow n\pi^-) - \mathcal{A}(\Sigma^+ \rightarrow n\pi^+) = 0, \quad (2)$$

which hold for both s and p waves. We choose $\Sigma^+ \rightarrow n\pi^+$, $\Sigma^- \rightarrow n\pi^-$, $\Lambda \rightarrow p\pi^-$ and $\Xi^- \rightarrow \Lambda\pi^-$ as the four independent decay amplitudes which are not related by isospin.

The purpose of this work is to study the role of resonances in hyperon decays. To this end, it is sufficient in this preliminary study to work at the tree level. We consider first the Lagrangian without resonances, which will be included in the following section. Our starting point is the relativistic effective chiral strong interaction Lagrangian for the pseudoscalar bosons coupled to the lowest-lying $1/2^+$ baryon octet

$$\mathcal{L}_{\phi B}^{(1)} = i\,\text{tr}(\bar{B}\gamma_\mu[D^\mu, B]) - \overset{\circ}{M}\,\text{tr}(\bar{B}B)$$
$$+ \frac{1}{2}D\,\text{tr}(\bar{B}\gamma_\mu\gamma_5\{u^\mu, B\}) + \frac{1}{2}F\,\text{tr}(\bar{B}\gamma_\mu\gamma_5[u^\mu, B]),$$
$$(3)$$

where the superscript denotes the chiral order and $\overset{\circ}{M}$ is the octet baryon mass in the chiral limit. We set $D = 3/4$ and $F = 1/2$, which are the $SU(6)$ values. The pseudoscalar Goldstone fields ($\phi = \pi, K, \eta$) are collected in the 3×3 unimodular, unitary matrix $U(x)$,

$$U(\phi) = u^2(\phi) = \exp\{2i\phi/\overset{\circ}{F}\}, \quad u_\mu = iu^\dagger\nabla_\mu Uu^\dagger, \quad (4)$$

with $\overset{\circ}{F}$ being the pseudoscalar decay constant in the chiral limit, and

$$\phi = \frac{1}{\sqrt{2}}\begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta \end{pmatrix}$$
$$(5)$$

represents the contraction of the pseudoscalar fields with the Gell-Mann matrices. We replace $\overset{\circ}{F}$ by the pion decay constant $F_\pi \simeq 92.4$ MeV which is consistent to the order we are working. B is the standard $SU(3)$ matrix representation of the low-lying spin-1/2 baryons (N, Λ, Σ, Ξ) and we work in the isospin limit $m_u = m_d = \hat{m}$.

The purely mesonic component of the Lagrangian can be decomposed into a strong and a weak interacting part

$$\mathcal{L}_\phi = \mathcal{L}_\phi^S + \mathcal{L}_\phi^W, \quad (6)$$

where $\mathcal{L}_\phi^S = \mathcal{L}_\phi^{S\,(2)}$ is the usual (strong and electromagnetic) mesonic Lagrangian at lowest order—cf., e.g., [6]. From the weak mesonic Lagrangian only the term

$$\mathcal{L}_\phi^W = \frac{F_\pi^2}{4}h_\pi\,\text{tr}(h_+ u_\mu u^\mu) \quad (7)$$

contributes to the order we are working. Here, we have defined

$$h_+ = u^\dagger h u + u^\dagger h^\dagger u, \quad (8)$$

with $h_b^a = \delta_2^a\delta_b^3$ being the weak transition matrix. Note that h_+ transforms as a matter field and the weak coupling h_π is well-determined from weak nonleptonic kaon decays—$h_\pi = 3.2 \times 10^{-7}$.

Turning to the weak component of the meson-baryon Lagrangian, the form of the lowest order Lagrangian is

$$\mathcal{L}_{\phi B}^{W\,(0)} = d\,\text{tr}(\bar{B}\{h_+, B\}) + f\,\text{tr}(\bar{B}[h_+, B]), \quad (9)$$

and d, f are the *only* weak counterterms considered in most previous calculations [1–3]. As discussed in the Introduction, use of this Lagrangian does not provide a simultaneously satisfactory fit to s and p waves, even after the inclusion of meson loops. In order to improve the agreement with experiment, one must account for additional weak counterterms, but, performing the calculation with the complete Lagrangian including counterterms from higher orders, one has no predictive power [4]. Indeed there exist only eight experimental numbers: the s- and p-wave amplitudes for the four independent hyperon decays, while on the other side, the theoretical predictions contain considerably more than eight low energy constants. We are not able to fix all the low-energy constants appearing in $\mathcal{L}_{\phi B}^W$ from data, even if we resort to large N_c arguments. We will therefore use the principle of resonance saturation in order to estimate the importance of these constants, as outlined in the following sections.

A. Inclusion of resonances

In order to include resonances one begins by writing down the most general Lagrangian at lowest order which exhibits the same symmetries as the underlying theory, i.e. Lorentz invariance and chiral symmetry. For the strong part we require invariance under C and P transformations separately, while the weak piece is invariant under CPS transformations, where the transformation S interchanges down and

strange quarks in the Lagrangian. We will work in the CP limit so that all LECs are real. (Of course, C and P invariance are not required separately for the weak interacting Lagrangian.)

We begin with the inclusion of the lowest, $1/2^-$, negative parity level in $(70,1^-)$. In [5] it was shown that such states dominate the contributions from $(70,1^-)$ and considerably improve agreement with experiment for the hyperon decays. Among the well established states one has $N(1535)$ and $\Lambda(1405)$. As for the remaining $1/2^-$ octet components there are a number of not so well-established states in the same mass range—cf. [5] and references therein. We denote the $1/2^-$ octet by R. Under CP transformations the fields behave as

$$B \rightarrow \gamma_0 C \bar{B}^T, \quad \bar{B} \rightarrow B^T C \gamma_0, \quad u^\mu \rightarrow -u_\mu^T,$$

$$h_+ \rightarrow h_+^T, \quad D^\mu \rightarrow -D_\mu^T,$$

$$R \rightarrow -\gamma_0 C \bar{R}^T, \quad \bar{R} \rightarrow -R^T C \gamma_0, \tag{10}$$

where C is the usual charge conjugation matrix. The kinetic term is straightforward

$$\mathcal{L}_R^{kin} = i \operatorname{tr}(\bar{R} \gamma_\mu [D^\mu, R]) - M_R \operatorname{tr}(\bar{R}R) \tag{11}$$

with M_R being the mass of the resonance octet in the chiral limit. The resonances R couple strongly to the $1/2^+$ baryon octet B via the Lagrangian

$$\mathcal{L}_{RB}^{(1)} = i s_d [\operatorname{tr}(\bar{R} \gamma_\mu \{u^\mu, B\}) - \operatorname{tr}(\bar{B} \gamma_\mu \{u^\mu, R\})]$$

$$+ i s_f [\operatorname{tr}(\bar{R} \gamma_\mu [u^\mu, B]) - \operatorname{tr}(\bar{B} \gamma_\mu [u^\mu, R])] \tag{12}$$

and the two coupling constants s_d and s_f can be determined from the strong decays of the resonances (cf. the Appendix), yielding the central values

$$s_d = 0.17, \quad s_f = -0.12. \tag{13}$$

A few remarks about the Lagrangian $\mathcal{L}_{RB}^{(1)}$ are in order. In principle, terms of the form $\bar{B} u^\mu D_\mu R$ are allowed by symmetry considerations. But, by use of the equation of motion for the resonance fields

$$i \gamma_\mu [D^\mu, R] - M_R R = 0 \tag{14}$$

one is able to reduce it to the terms already included in $\mathcal{L}_{RB}^{(1)}$. The interaction term $i \bar{R} \gamma_5 B$ also satisfies the symmetry constraints, but can be transformed away by a unitary transformation. The proof of this is as follows. Consider a Lagrangian of the form

$$\mathcal{L} = i \bar{B} \gamma_\mu D^\mu B - M_B \bar{B} B + i \bar{R} \gamma_\mu D^\mu R - M_R \bar{R} R + i \alpha \bar{R} \gamma_5 B$$

$$+ i \alpha \bar{B} \gamma_5 R \tag{15}$$

where we have supressed flavor indices and α is the off diagonal coupling. By introducing a doublet notation we can rewrite the Lagrangian

$$\mathcal{L} = i \bar{Q} \gamma_\mu D^\mu Q + \bar{Q}(A + i \gamma_5 C) Q \tag{16}$$

with

$$Q = \begin{pmatrix} B \\ R \end{pmatrix}, \quad A = \begin{pmatrix} -M_B & 0 \\ 0 & -M_R \end{pmatrix}, \quad C = \begin{pmatrix} 0 & \alpha \\ \alpha & 0 \end{pmatrix}. \tag{17}$$

Note, that A and C are Hermitian. Decomposing the doublet field Q as follows:

$$Q_R = \frac{1}{2}(1 + \gamma_5)Q, \quad Q_L = \frac{1}{2}(1 - \gamma_5)Q \tag{18}$$

the Lagrangian reads

$$\mathcal{L} = i \bar{Q}_R \gamma_\mu D^\mu Q_R + i \bar{Q}_L \gamma_\mu D^\mu Q_L + \bar{Q}_L M Q_R + \bar{Q}_R M^\dagger Q_L \tag{19}$$

with $M = A + iC$. Then by applying a bi-unitary transformation

$$Q_R \rightarrow R Q_R, \quad Q_L \rightarrow L Q_L \tag{20}$$

with unitary matrices R and L one can diagonalize the matrix M

$$L^\dagger M R = M_d \tag{21}$$

where M_d is diagonal with positive elements. The first two terms in Eq. (19) remain unchanged by this transformation. Expressing the Lagrangian in terms of Q we obtain

$$\mathcal{L} = i \bar{Q} \gamma_\mu D^\mu Q + \bar{Q} M_d Q \tag{22}$$

which is the desired result. Including the interaction terms from $\mathcal{L}_{RB}^{(1)}$ does not alter the proof. The interaction term of the form $i \bar{R} \gamma_5 B$ can therefore be neglected, which leads to significant simplifications of the weak Lagrangian between the resonances and the low-lying baryon octet.

We can then turn to the lowest order weak Lagrangian which reads

$$\mathcal{L}_{RB}^{W(1)} = i w_d [\operatorname{tr}(\bar{R}\{h_+, B\}) - \operatorname{tr}(\bar{B}\{h_+, R\})]$$

$$+ i w_f [\operatorname{tr}(\bar{R}[h_+, B]) - \operatorname{tr}(\bar{B}[h_+, R])] \tag{23}$$

with two unknown weak couplings w_d and w_f, which will be determined from a fit to the hyperon decays. (Again, a term of the form $\bar{R} \gamma_5 h_+ B$ is allowed by symmetry considerations, but a proof analogous to the one above shows that such terms can be transformed away.) Furthermore, terms with the structure $i \bar{R} \gamma_\mu h_+ u^\mu B$ and $\bar{R} \gamma_\mu \gamma_5 (D^\mu h_+) B$ are possible. But, after contraction with the vertices from $\mathcal{L}_{RB}^{(1)}$ in the resonance saturation scheme, they deliver contact terms of chiral order two and involve at least two outgoing mesons, which is clearly beyond our tree level considerations.

We will not include any additional resonances from the $(70,1^-)$ multiplet, which were the only states considered in [5]. But in many other applications the spin-$3/2^+$ decuplet

and the spin-$1/2^+$ Roper octet play an important role, cf. e.g. [7]. The decuplet is only 231 MeV higher in average than the ground state octet and the Roper octet masses are comparable to the $1/2^-$ states R. One should therefore presumably account for these resonances.

We first consider the decuplet. Due to angular momentum conservation the spin-3/2 decuplet states can couple to the spin-1/2 baryon octet only accompanied by Goldstone bosons—i.e. decuplet states contribute only through loop diagrams to nonleptonic hyperon decays. An explicit calculation shows that such diagrams saturate contact terms of the same chiral order, $O(p^2)$, as the loop corrections with the baryon octet [4]. Since we restrict ourselves to $O(p^0)$ and $O(p^1)$ we can disregard such decuplet contributions. In addition, the calculation of relativistic loop diagrams in the resonance saturation scheme leads to some complications. The integrals are in general divergent and have to be renormalized, which introduces new unknown parameters. The absence of a strict chiral counting scheme in the relativistic formulation leads to contributions from higher loop diagrams which are usually neglected in such calculations, cf. [7].

The lowest multiplet of excited states contributing to the chiral order $O(p^1)$ is the octet of even-parity Roper-like spin-1/2 fields. While it was argued in [8] that these play no role, a more recent study seems to indicate that one cannot neglect contributions from these states to, e.g., the decuplet magnetic moments [9]. It is thus important to investigate the possible contribution of these baryon resonances to the LECs. The octet consists of the $N^*(1440)$, $\Sigma^*(1660)$, $\Lambda^*(1600)$ and $\Xi^*(1620?)$. We denote the spin-$1/2^+$ resonance octet by B^*. The transformation properties of B^* under CP are the same as for the ground state baryons B, and the effective Lagrangian of the B^* octet coupled to the ground state baryons takes the form

$$\mathcal{L}_{B^*B} = \mathcal{L}_{B^*}^{kin} + \mathcal{L}_{B^*B}^{S} + \mathcal{L}_{B^*B}^{W} \qquad (24)$$

with the kinetic term

$$\mathcal{L}_{B^*}^{kin} = i \, \text{tr}(\bar{B}^* \gamma_\mu [D^\mu, B^*]) - M_{B^*} \, \text{tr}(\bar{B}^* B^*), \qquad (25)$$

a strong interaction part [7]

$$\mathcal{L}_{B^*B}^{S} = \frac{1}{4} D^* [\text{tr}(\bar{B}^* \gamma_\mu \gamma_5 \{u^\mu, B\}) + \text{tr}(\bar{B} \gamma_\mu \gamma_5 \{u^\mu, B^*\})]$$

$$+ \frac{1}{4} F^* [\text{tr}(\bar{B}^* \gamma_\mu \gamma_5 [u^\mu, B]) + \text{tr}(\bar{B} \gamma_\mu \gamma_5 [u^\mu, B^*])]$$

$$(26)$$

FIG. 1. Diagrams including resonances that contribute to s and p waves. Solid and dashed lines denote ground state baryons and Goldstone bosons, respectively. The double line represents a resonance. Solid squares and circles are vertices of the weak and strong interactions, respectively.

and a weak piece

$$\mathcal{L}_{B^*B}^{W} = d^* [\text{tr}(\bar{B}^* \{h_+, B\}) + \text{tr}(\bar{B} \{h_+, B^*\})]$$

$$+ f^* [\text{tr}(\bar{B}^* [h_+, B]) + \text{tr}(\bar{B} [h_+, B^*])]. \qquad (27)$$

The couplings D^* and F^* have already been determined from the strong decays of these resonances [7], with central values

$$D^* = 0.60, \quad F^* = 0.11, \qquad (28)$$

while the weak parameters d^* and f^* can be determined from a fit to the nonleptonic hyperon decays—cf. Sec. III.

Having written down the relevant Lagrangian for the resonances coupled to the ground state baryons we can proceed to integrate out the heavy degrees of freedom from the effective theory.

B. Resonance saturation

In this section we calculate the tree level diagrams involving resonances which contribute to nonleptonic hyperon decay. Allowing the resonance masses to become large with fixed ratios of coupling constant to mass, higher order terms in the effective meson-baryon Lagrangian are generated, the coefficients of which can be expressed in terms of a few resonance parameters.

Using the vertices from the Lagrangians developed in the preceeding section we calculate the diagrams in Fig. 1. Then, performing the limit $M_R, M_{B^*} \to \infty$ and using the Cayley-Hamilton identity for the two traceless 3×3 matrices u_μ and h_+,

$$\frac{3}{2} \text{tr}(\bar{B} \Gamma_\mu \{h_+, \{u^\mu, B\}\}) + \frac{3}{2} \text{tr}(\bar{B} \Gamma_\mu \{u^\mu, \{h_+, B\}\}) + \frac{1}{2} \text{tr}(\bar{B} \Gamma_\mu [h_+, [u^\mu, B]]) + \frac{1}{2} \text{tr}(\bar{B} \Gamma_\mu [u^\mu, [h_+, B]])$$

$$= 2 \, \text{tr}(\bar{B} \Gamma_\mu B) \text{tr}(h_+ u^\mu) + 2 \, \text{tr}(\bar{B} h_+) \Gamma_\mu \, \text{tr}(u^\mu B) + 2 \, \text{tr}(\bar{B} u^\mu) \Gamma_\mu \, \text{tr}(h_+ B), \qquad (29)$$

with $\Gamma_\mu = \gamma_\mu \gamma_5, \gamma_\mu$, one generates the effective Lagrangian

$$\mathcal{L}_{\phi B}^{W\,(1)} = g_1\{\mathrm{tr}(\bar{B}\gamma_\mu[h_+,\{u^\mu,B\}]) + \mathrm{tr}(\bar{B}\gamma_\mu\{u^\mu,[h_+,B]\})\} + g_2\{\mathrm{tr}(\bar{B}\gamma_\mu\{h_+,[u^\mu,B]\}) + \mathrm{tr}(\bar{B}\gamma_\mu[u^\mu,\{h_+,B\}])\}$$

$$+ g_3\{\mathrm{tr}(\bar{B}\gamma_\mu[h_+,[u^\mu,B]]) + \mathrm{tr}(\bar{B}\gamma_\mu[u^\mu,[h_+,B]])\} + g_4\mathrm{tr}(\bar{B}\gamma_\mu B)\mathrm{tr}(u^\mu h_+) + g_6\{\mathrm{tr}(\bar{B}\gamma_\mu\gamma_5[h_+,\{u^\mu,B\}])$$

$$+ \mathrm{tr}(\bar{B}\gamma_\mu\gamma_5\{u^\mu,[h_+,B]\})\} + g_7\{\mathrm{tr}(\bar{B}\gamma_\mu\gamma_5\{h_+,[u^\mu,B]\}) + \mathrm{tr}(\bar{B}\gamma_\mu\gamma_5[u^\mu,\{h_+,B\}])\} + g_8\{\mathrm{tr}(\bar{B}\gamma_\mu\gamma_5[h_+,[u^\mu,B]])$$

$$+ \mathrm{tr}(\bar{B}\gamma_\mu\gamma_5[u^\mu,[h_+,B]])\} + g_9\mathrm{tr}(\bar{B}\gamma_\mu\gamma_5 B)\mathrm{tr}(u^\mu h_+). \tag{30}$$

The couplings read, in terms of the resonance parameters,

$$g_1 = \frac{s_d w_f}{M_R}, \quad g_2 = \frac{s_f w_d}{M_R}, \quad g_3 = \frac{s_f w_f}{M_R} - \frac{s_d w_d}{3M_R},$$

$$g_4 = \frac{4 s_d w_d}{3 M_R}, \quad g_6 = \frac{D^* f^*}{4 M_{B^*}}, \quad g_7 = \frac{F^* d^*}{4 M_{B^*}},$$

$$g_8 = \frac{F^* f^*}{4 M_{B^*}} - \frac{D^* d^*}{12 M_{B^*}}, \quad g_9 = \frac{D^* d^*}{3 M_{B^*}}. \tag{31}$$

The Lagrangian $\mathcal{L}_{\phi B}^{W\,(1)}$ forms, together with the weak Lagrangians $\mathcal{L}_{\phi B}^{W\,(0)}$ at lowest order and $\mathcal{L}_\phi^W$ from Eq. (7), the strangeness changing Lagrangian which we employ for the calculation of the decay amplitudes. [Note that the most general Lagrangian $\mathcal{L}_{\phi B}^{W\,(1)}$ contains two additional terms

$$g_5\,\mathrm{tr}(\bar{B}h_+)\gamma_\mu\mathrm{tr}(u^\mu B) + g_{10}\,\mathrm{tr}(\bar{B}h_+)\gamma_\mu\gamma_5\mathrm{tr}(u^\mu B) + (\mathrm{H.c.}) \tag{32}$$

that are not generated by the resonances considered here.]

C. Heavy baryon limit

We evaluate the decay amplitudes in the extreme non-relativistic limit wherein the baryons are characterized by a four velocity v_μ [10]. A consistent chiral counting scheme emerges, i.e. a one-to-one correspondence between the Goldstone boson loops and the expansion in small momenta and quark masses. In the heavy baryon formulation the p wave must be modified, since γ_5 connects the light with the heavy degrees of freedom which are integrated out in this scheme. One therefore introduces the modified heavy baryon p-wave amplitude $\mathcal{A}_{ij}^{(P)}$ by

$$A_{ij}^{(P)} = -\frac{1}{2}(E_j + M_j)\mathcal{A}_{ij}^{(P)}, \tag{33}$$

where E_j and M_j are the energy and mass of the outgoing baryon, respectively. In the rest frame of the heavy baryon, $v_\mu = (1,0,0,0)$, the decay amplitude reduces to the non-relativistic form

$$\mathcal{A}(B_i \to B_j\,\pi) = \bar{\chi}_{B_j}\left\{\mathcal{A}_{ij}^{(S)} + \frac{1}{2}\mathbf{k}\cdot\sigma\,\mathcal{A}_{ij}^{(P)}\right\}\chi_{B_i}$$

$$= \bar{\chi}_{B_j}\{\mathcal{A}_{ij}^{(S)} + S\cdot k\,\mathcal{A}_{ij}^{(P)}\}\chi_{B_i}, \tag{34}$$

where k is the outgoing momentum of the pion and $2S_\mu = i\gamma_5\sigma_{\mu\nu}v^\nu$ is the Pauli-Lubanski spin vector, which in the rest frame is given by $S_\mu^{\mathbf{v}=0} = (0,\frac{1}{2}\sigma)$. The structure of the Lagrangian remains almost unchanged with γ_μ and $\gamma_\mu\gamma_5$ replaced by v_μ and $2S_\mu$, respectively. The only additional terms that contribute are relativistic corrections to the Dirac term and are of the form [4]

$$\mathcal{L}_{\phi B}^{(2)} = \mathcal{L}_{\phi B}^{(2,rc)} = -\frac{1}{2\overset{\circ}{M}}\mathrm{tr}(\bar{B}[D_\mu,[D^\mu,B]])$$

$$+ \frac{1}{2\overset{\circ}{M}}\mathrm{tr}(\bar{B}[v\cdot D,[v\cdot D,B]]). \tag{35}$$

We utilize the same notation for the baryon fields as in the relativistic case. These terms produce a finite shift to the bare masses of the baryons. Since we work with the physical masses of the baryons the effects of $\mathcal{L}_{\phi B}^{(2)}$ are already included in our expressions for the decay amplitudes and we can neglect Eq. (35).

The general structure of the s-wave decay amplitudes is

$$\mathcal{A}_{ij}^{(s)} = \frac{1}{\sqrt{2}\,F_\pi}\{\alpha_{ij}^{(s)} + v\cdot k\,\beta_{ij}^{(s)}\} \tag{36}$$

with

$$\alpha_{\Sigma^+ n}^{(s)} = 0, \quad \beta_{\Sigma^+ n}^{(s)} = -4g_1 + 4g_2 - 4g_3$$

$$\alpha_{\Sigma^- n}^{(s)} = d - f, \quad \beta_{\Sigma^- n}^{(s)} = -2g_1 - 2g_2 + 2g_3$$

$$\alpha_{\Lambda p}^{(s)} = -\frac{1}{\sqrt{6}}(d + 3f), \quad \beta_{\Lambda p}^{(s)} = -\frac{1}{\sqrt{6}}(10g_1 + 2g_2 + 6g_3)$$

$$\alpha_{\Xi^- \Lambda}^{(s)} = -\frac{1}{\sqrt{6}}(d - 3f), \quad \beta_{\Xi^- \Lambda}^{(s)} = \frac{1}{\sqrt{6}}(10g_1 + 2g_2 - 6g_3). \tag{37}$$

In the rest frame of the decaying baryon the energy of the meson may be written as

$$v\cdot k = \frac{1}{2M_i}(M_i^2 - M_j^2 + m_\pi^2). \tag{38}$$

We obtain very similar expressions for the resonance contributions to those found in the constituent quark model [5]. There the contributions to the s waves were found to be
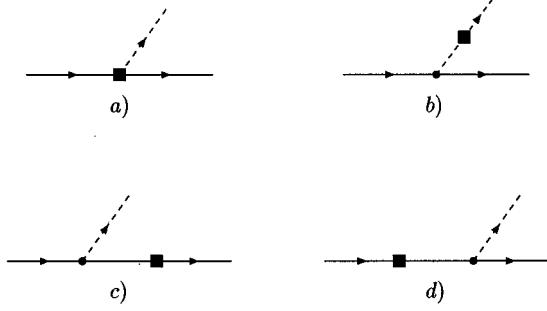
FIG. 2. Diagrams that contribute to s and p waves. (a) contributes both to s and p waves, whereas (b),(c),(d) contribute only to the p waves. Solid and dashed lines denote ground state baryons and Goldstone bosons, respectively. Solid squares and circles are vertices of the weak and strong interactions, respectively.

proportional to the mass difference of the decaying and the light baryon which differs from $v \cdot k$ only by terms quadratic in the meson masses. To the order we are working then we have agreement with the quark model calculation. The contact diagram that contributes to the s waves is shown in Fig. 2a.

For the p waves one finds the form

$$\mathcal{A}_{ij}^{(p)} = \frac{1}{\sqrt{2}\,F_\pi}\left\{ \alpha_{ij}^{(p)} + \beta_{ij}^{(p)} + \frac{1}{2}h_\pi \frac{m_\pi^2}{m_\pi^2 - m_K^2}\,\phi_{ij}^{(p)} \right\} \quad (39)$$

where $\alpha_{ij}^{(p)}$ denotes the baryon pole terms, $\beta_{ij}^{(p)}$ the contact terms in $\mathcal{L}_{\phi B}^{W(1)}$ and $\phi_{ij}^{(p)}$ the contribution from the weak decay of the meson. The diagrams which contribute to the p waves are depicted in Fig. 2, and yield

$$\alpha_{\Sigma^+ n}^{(p)} = -\frac{1}{M_\Sigma - M_N}2\,D\,(d-f) - \frac{1}{M_\Lambda - M_N}\frac{2}{3}D\,(d+3f)$$

$$\beta_{\Sigma^+ n}^{(p)} = -8g_6 + 8g_7 - 8g_8\,, \quad \phi_{\Sigma^+ n}^{(p)} = 0$$

$$\alpha_{\Sigma^- n}^{(p)} = -\frac{1}{M_\Sigma - M_N}2\,F\,(d-f) - \frac{1}{M_\Lambda - M_N}\frac{2}{3}D\,(d+3f)$$

$$\beta_{\Sigma^- n}^{(p)} = -4g_6 - 4g_7 + 4g_8\,, \quad \phi_{\Sigma^- n}^{(p)} = D - F$$

$$\alpha_{\Lambda p}^{(p)} = \frac{1}{M_\Lambda - M_N}\frac{2}{\sqrt{6}}(d+3f)(D+F)$$

$$+ \frac{1}{M_\Sigma - M_N}\frac{4}{\sqrt{6}}D\,(d-f)$$

$$\beta_{\Lambda p}^{(p)} = -\frac{1}{\sqrt{6}}(20g_6 + 4g_7 + 12g_8)$$

$$\phi_{\Lambda p}^{(p)} = -\frac{1}{\sqrt{6}}(D+3F)$$

TABLE I. Experimental values of the decay amplitudes including the errors. The numbers have to be multiplied by a factor of 10^{-7}.

$\mathcal{A}_{\Sigma^+ n}^{(s)}$	$\mathcal{A}_{\Sigma^- n}^{(s)}$	$\mathcal{A}_{\Lambda p}^{(s)}$	$\mathcal{A}_{\Xi^- \Lambda}^{(s)}$
0.13 ± 0.02	4.27 ± 0.02	3.25 ± 0.02	-4.51 ± 0.02
$\mathcal{A}_{\Sigma^+ n}^{(p)}$	$\mathcal{A}_{\Sigma^- n}^{(p)}$	$\mathcal{A}_{\Lambda p}^{(p)}$	$\mathcal{A}_{\Xi^- \Lambda}^{(p)}$
-44.4 ± 0.16	1.52 ± 0.16	-23.4 ± 0.56	-14.8 ± 0.55

$$\alpha_{\Xi^- \Lambda}^{(p)} = -\frac{1}{M_\Xi - M_\Lambda}\frac{2}{\sqrt{6}}(d-3f)(D-F)$$

$$- \frac{1}{M_\Xi - M_\Sigma}\frac{4}{\sqrt{6}}D\,(d+f)$$

$$\beta_{\Xi^- \Lambda}^{(p)} = \frac{1}{\sqrt{6}}(20g_6 + 4g_7 - 12g_8)$$

$$\phi_{\Xi^- \Lambda}^{(p)} = -\frac{1}{\sqrt{6}}(D-3F). \quad (40)$$

III. RESULTS AND DISCUSSION

In this section we discuss the numerical values of the LECs and the fit to experiment. There exist eight independent experimental numbers, i.e. s- and p-wave amplitudes for the four decays $\Sigma^+ \to n\,\pi^+$, $\Sigma^- \to n\,\pi^-$, $\Lambda \to p\,\pi^-$ and $\Xi^- \to \Lambda\,\pi^-$, which are not related by isospin. The central values for our parameters are $F_\pi = 92.4$ MeV, $D = 0.75$, $F = 0.50$. The procedure of estimating the counterterms of higher order in the resonance saturation scheme involving the $1/2^-$, $1/2^+$ octets R, B^* introduces eight additional parameters, four of which are determined from the strong resonance decays. Together with the couplings d and f from the weak Lagrangian at lowest order we have then six parameters with which to perform a least-squares fit to the nonleptonic hyperon decays. The experimental values of the decays are shown in Table I, and the fitted chiral expansions of the decay amplitudes read, in units of 10^{-7},

$$\mathcal{A}_{\Sigma^+ n}^{(s)} = 0.00 - 0.04 = -0.04,$$

$$\mathcal{A}_{\Sigma^+ n}^{(p)} = -19.6 - 24.8 = -44.4,$$

$$\mathcal{A}_{\Sigma^- n}^{(s)} = 7.19 - 1.86 = 5.33,$$

$$\mathcal{A}_{\Sigma^- n}^{(p)} = -5.33 + 6.94 = 1.61,$$

$$\mathcal{A}_{\Lambda p}^{(s)} = 3.32 - 1.15 = 2.17,$$

$$\mathcal{A}_{\Lambda p}^{(p)} = -12.4 - 11.0 = -23.4,$$

$$\mathcal{A}_{\Xi^- \Lambda}^{(s)} = -6.06 + 1.92 = -4.14,$$

TABLE II. Numerical values of the LECs obtained from a least-squares fit. The couplings d and f are given in units of 10^{-7} GeV, the g_i in units of 10^{-7}.

d	f	g_1	g_2	g_3	g_4	g_6	g_7	g_8	g_9
0.44	-0.50	0.26	0.14	-0.12	-0.26	0.09	-0.11	0.21	-0.79

$$\mathcal{A}^{(p)}_{\Xi^- - \Lambda} = -10.44 - 4.30 = -14.7, \qquad (41)$$

where the first number is the lowest order piece, involving the weak counterterms d, f, and the second number contains the contributions from contact terms at next order. The contributions from the weak meson decays for the p waves appear also at that order and are, therefore, included in the second number. The results, particularly for the p waves, are in satisfactory agreement with experiment. A fit with *only* the spin-$1/2^-$ resonances, as performed in the quark model in [5], does *not* lead to good agreement between theory and experiment in the framework of heavy baryon chiral perturbation theory, although the $1/2^-$ contribution to the parity-violating decay amplitudes is roughly of the same size as in [5]. The reason for this is that it is not possible to obtain a satisfactory fit for p waves by using only the lowest order couplings d and f. In the usual quark model approach the p-wave amplitudes include explicit $SU(3)$ symmetry breaking corrections of higher chiral order. A much improved fit to the p waves is possible and the contributions from the spin-$1/2^-$ resonances, which contribute only to the s waves, are sufficient to achieve a satisfactory fit for both s and p waves. This indicates that such higher order corrections are essential. By including the spin-$1/2^+$ resonances in the resonance saturation scheme, which contribute to the p waves at next-to-leading order, one is able to account for some of these higher order effects. The contributions from the $1/2^+$ resonances to the parity-conserving decay amplitudes are of comparable size as the the ground state contributions, and apparently, these resonances are crucial in heavy baryon chiral perturbation theory to obtain a satisfactory fit also for the p waves. For completeness, the corresponding values of the couplings g_i from the Lagrangian $\mathcal{L}^{W(1)}_{\phi B}$ are given in Table II.

It should be noted that a very different fit with six parameters was performed in [4]. There loop corrections were included and the exchange of intermediate decuplet states was used in order to estimate the LECs. It turned out that no satisfactory fit was possible and additional counterterms had to be included. Inclusion of the $1/2^-$ and $1/2^+$ resonance states seems then to play an important role for understanding of the nonleptonic hyperon decays. In fact in the case of p waves their contribution is comparable to or even exceeds that from lowest order pieces. In order to make a more definitive statement about their importance one must, of course, go to higher orders and include loops. However, this is beyond the scope of the present work.

IV. SUMMARY

We have in this paper studied the importance of baryon resonances for the nonleptonic hyperon decays at the tree level in chiral perturbation theory. To this end, we included the spin-$1/2^-$ octet from the $(70, 1^-)$ states and the octet of Roper-like $1/2^+$ fields in the effective theory. The most general Lagrangian incorporating these resonances coupled to the ground state baryons introduces eight new parameters, four of which can be determined from the strong decays of the resonances. Integrating out the resonances generates counterterms in the Lagrangian at next-to-leading order. On the other hand, the inclusion of the spin-$3/2^+$ decuplet, as performed in [4], generates terms at the same chiral order as the loop corrections, $\mathcal{O}(p^2)$, which is beyond the accuracy of this calculation and therefore can be neglected. In [5] it was argued that the inclusion of the spin-$1/2^-$ octet is sufficient to obtain a satisfactory fit for both s and p waves. We were able to show that in the framework of chiral perturbation theory the structure of the contributions from these resonances agrees with the results in the quark model to the order we are working. In the quark model the expressions for the p waves include additional explicit $SU(3)$ symmetry breaking corrections of second chiral order, in which case, a much improved fit to the p waves is possible and the contributions from the spin-$1/2^-$ resonances, which contribute only to the s waves, are sufficient to achieve a satisfactory fit for both s and p waves. On the other hand, in chiral perturbation theory the improvement of experimental agreement is brought about by the inclusion of the Roper octet, which is in the same mass range as the $1/2^-$ octet. The reason for this is that the contributions from the lowest order couplings d and f for the p-wave decay amplitudes tend to cancel thus enhancing the contributions from terms of higher chiral order. By including the spin-$1/2^+$ resonances in the resonance saturation scheme, which contribute to the p waves at next-to-leading order, one is able to overcome this problem. By fitting the six parameters of the weak Lagrangian, two from lowest order and four at next-to-leading order, to the eight independent decay amplitudes that are not related by isospin, we obtain satisfactory agreement with experiment. We suggest then that the inclusion of spin-$1/2$ resonances in nonleptonic hyperon decays provides a reasonable estimate of the importance of higher order counterterms. In order to make a more definite statement, one should go to higher orders and include meson loops.

Of course, our fit is not unique. Another satisfactory fit for the decay amplitudes was achieved in [4] by including Goldstone boson loops and spin-3/2 decuplet contributions. The effects of higher resonances like the ones considered in the present work were neglected. By considering only the nonleptonic hyperon decays it is not possible to decide which approach describes nature more appropriately. One must examine other weak processes involving hyperons, e.g. the radiative hyperon decays. This work is under way [11].

APPENDIX: DETERMINATION OF THE $\frac{1}{2}^-$-RESONANCE COUPLINGS s_d AND s_f

The decays listed in the particle data book, which determine the coupling constants s_d and s_f, are $N(1535) \to N\pi$, $N(1535) \to N\eta$ and $\Lambda(1405) \to \Sigma\pi$. The width follows via

$$\Gamma = \frac{1}{8\pi M_R^2} |\mathbf{k}_\phi| |\mathcal{T}|^2 \tag{A1}$$

with

$$|\mathbf{k}_\phi| = \frac{1}{2M_R} [(M_R^2 - (M_B + m_\phi)^2)(M_R^2 - (M_B - m_\phi)^2)]^{1/2} \tag{A2}$$

being the three-momentum of the meson $\phi = \pi, \eta$ in the rest frame of the resonance. The terms M_R and M_B are the masses of the resonance and the ground state baryon, respectively. For the transition matrix one obtains

$$|\mathcal{T}|^2 = \frac{2}{F_\pi^2} (M_R - M_B)^2 [(M_R + M_B)^2 - m_\phi^2] A_{R\phi} \tag{A3}$$

with the coefficients

$$A_{N(1535)\pi} = \frac{3}{2}(s_d + s_f)^2,$$

$$A_{N(1535)\eta} = \frac{1}{6}(s_d - 3s_f)^2, \quad A_{N(1535)\pi} = 2s_d^2. \tag{A4}$$

Using the experimental values for the decay widths we arrive at the central values

$$s_d = 0.17, \quad s_f = -0.12 \tag{A5}$$

where we have chosen the sign of s_d to be positive in accordance with the ground state octet D coupling. We do not present the uncertainties in these parameters here, since for the purpose of our considerations a rough estimate of these constants is sufficient.

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