Mass spectra and decay widths of hadrons in the relativistic string model

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(Received 6 July 1998; published 8 April 1999)

A relativistic string model for hadrons is presented. The center-of-mass motion is separated correctly. The vibrational and rotational motions of the string are solved by using the WKB approximation. Moreover, their decay widths are calculated by taking into account the pair creation of quarks inside the string. Both the mass spectra and the decay widths are reproduced fairly well in comparison with experimental data. [S0556-2821(99)00311-2]

PACS number(s): 12.39.Ki, 12.40.Yx, 13.25.−k, 13.30.Eg

I. INTRODUCTION

It is well known that hadrons are composed of quarks and antiquarks which are governed by quantum chromodynamics (QCD). In the hadron structure, gluons also play an important role which is shown in deep inelastic lepton-nucleon scattering. For example the total momentum of hadrons cannot be explained without the degrees of freedom of gluons. The string model proposed by Johnson and Thorn [1] is one of the models in which the gluon degrees of freedom are incorporated explicitly into the model. According to the string picture, mesons are made up of a quark and an antiquark which are connected by the classical gluon field. The string model could reproduce rotational excited states called Regge trajectories [2]. Then Migdal *et al.* [3] extended the model to baryons which are viewed as a quark and a diquark connected by the same gluon field as in mesons. The diquark picture was suggested from many phenomena [4]. However the treatment in [3] is unsatisfactory because the separation of the center-of-mass motion is unclear and the nonrelativistic approximation adopted in the paper is not justified.

It is the purpose of this paper to present a relativistic string model for hadrons and calculate the mass spectra and the decay widths of both the vibrational and rotational modes. Our improved model is fully relativistic and the center-of-mass motion is eliminated by choosing correctly the center of mass frame. Moreover, the decay process can be incorporated by taking into account the pair production inside the gluon flux. The rate of pair production is calculated by analogy with the Schwinger mechanism in QED [5,6].

This paper is organized as follows. In the next section, a classical relativistic string model is described and conditions for choosing the center of mass frame are presented. In Sec. III, the dynamical system is transformed into Hamiltonian form and canonical quantization is applied to the system. The mass spectra of hadrons are obtained with the use of the WKB approximation and the decay widths with the use of the Schwinger mechanism. The numerical results are presented in Sec. IV. The concluding remarks are given in the final section.

II. CLASSICAL THEORY OF THE STRING MOTION

Let us start to present our model of hadron strings. It is assumed that a meson is described by a gluon string whose two ends are occupied by a quark and an antiquark. For a baryon, the antiquark is replaced with a diquark. As a result, all the hadrons have universal stringlike structure. The geometry is shown in Fig. 1. The Lagrangian of this system is given by [3]

$$L(r_i,\dot{r}_i,\omega)=-\sum_{i=1}^{2} m_i\sqrt{1-\dot{r}_i^2-\omega^2}-V, \tag{2.1}$$

where the first terms represent the kinetic energy of the quark and antiquark (diquark) and the last one the energy of the gluon string. We have assumed that the whole system rotates with angular velocity $\omega=\dot{\theta}$. Each end of the string has mass m_i and the distance r_i which is measured from the center of gravity ($i=1,2$). The contribution to the Lagrangian from the gluon flux is given by

$$V(r_i,\omega)=\sum_{i=1}^{2}\int_0^{r_i} a\sqrt{1-r^2\omega^2}dr, \tag{2.2}$$

where a is the energy density of the string (per unit length) and the factor $\sqrt{1-r^2\omega^2}$ in the right-hand side comes from the Lorentz contraction along the transverse direction of the string. The canonical momenta are defined by

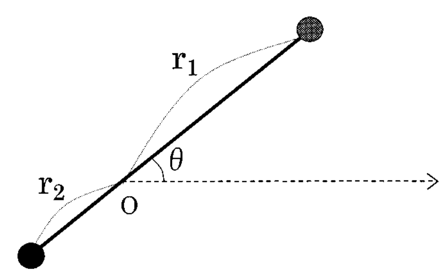

FIG. 1. The schematic picture of our hadron strings.

$$p_i \equiv \frac{\partial L}{\partial \dot{r}_i} = \frac{m_i \dot{r}_i}{\sqrt{1-v_i^2}} \quad (i=1,2), \tag{2.3}$$

$$l \equiv \frac{\partial L}{\partial \dot{\theta}} = \sum_{i=1}^{2} \left(\frac{m_i r_i^2 \omega}{\sqrt{1-v_i^2}} + \int_0^{r_i} \frac{a \omega r^2}{\sqrt{1-\omega^2 r^2}} dr \right), \tag{2.4}$$

where $v_i^2 \equiv \dot{r}_i^2 + \omega^2 r_i^2$. These quantities denote the radial momenta of each quark and the total orbital angular momentum respectively.

Now let us discuss the center of mass of our system. Provided that the origin of our coordinate system coincides with the center of mass, it is proved that the following subsidiary conditions hold:

$$p_1 - p_2 = 0, \tag{2.5}$$

$$\dot{p}_1 - \dot{p}_2 = 0. \tag{2.6}$$

We will show that these conditions are equivalent to the definition of the center of mass; the total momentum of our system vanishes. Evidently the first condition Eq. (2.5) means that the radial component of the total momentum vanishes because the gluon field does not have the radial component of the momentum. On the other hand the transverse component is given by

$$\begin{aligned} p_\theta &= r_1 p_1 - r_2 p_2 + \int_0^{r_1} \frac{a \omega r}{\sqrt{1-\omega^2 r^2}} dr - \int_0^{r_2} \frac{a \omega r}{\sqrt{1-\omega^2 r^2}} dr \\ &= \frac{1}{\omega} \left(\frac{m_1 \omega^2 r_1}{\sqrt{1-v_1^2}} - \frac{m_2 \omega^2 r_2}{\sqrt{1-v_2^2}} - a\sqrt{1-\omega^2 r_1^2} + a\sqrt{1-\omega^2 r_2^2} \right) \\ &= \frac{1}{\omega} \left(\frac{\partial L}{\partial r_1} - \frac{\partial L}{\partial r_2} \right) \end{aligned} \tag{2.7}$$

with the help of Eq. (2.1). By using the Lagrange equation for r_i, the second condition Eq. (2.6) is rewritten as

$$\frac{\partial L}{\partial r_1} - \frac{\partial L}{\partial r_2} = 0. \tag{2.8}$$

Thus it is shown that the our new conditions Eqs. (2.5) and (2.6) is equivalent to the definition of the center of mass.

It should be noted that the center of mass motion has been completely eliminated due to the conditions (2.5) and (2.6). In order to show this sfact, we will show that our Lagrangian is described by only the relative coordinate $r \equiv r_1 + r_2$. In fact let us consider any variation, $r_i \to r_i + \delta r_i$ with using the two conditions. Then the variation of L is written as

$$\delta L = \sum_i \left(\frac{\partial L}{\partial \dot{r}_i} \delta \dot{r}_i + \frac{\partial L}{\partial r_i} \delta r_i \right) = p\, \delta(\dot{r}_1 + \dot{r}_2) + \dot{p}\, \delta(r_1 + r_2), \tag{2.9}$$

where the canonical equations of motion and the condition of relative momentum ($p \equiv p_1 = p_2$) were used. Noting that $p = (\partial L)/(\partial \dot{r})$, one can see that the Lagrangian contains only the relative coordinates, $L(r, \dot{r}, \dot{\theta})$. Therefore the center-of-mass motion has been eliminated completely from our Lagrangian. Thus the Hamiltonian is given by

$$H(p,l,r) \equiv \dot{r} p + \omega - L = \sum_i \left(\frac{m_i}{\sqrt{1-v_i^2}} + \int_0^{r_i} \frac{a}{\sqrt{1-\omega^2 r^2}} dr \right). \tag{2.10}$$

It is evident that the first term means the kinetic energy of a quark or an antiquark (diquark) at each end and the second the energy of the string.

III. QUANTUM THEORY OF THE STRING MOTION

Now we will go over the quantum theory of our system. The excited states of hadrons are described by the Schrödinger equation,

$$H(\hat{p}, \hat{l}, r) \psi(r,\theta) = E \psi(r,\theta). \tag{3.1}$$

By using the separation of variables $\psi = \phi(r) \exp il\theta$, the angular momentum is quantized as follows: $l = 0,1,2,3, \ldots$. Introducing a new function by $\phi(r) \equiv \chi(r)/r$, the Schrödinger equation is rewritten as

$$H\left(-i\hbar \frac{\partial}{\partial r}, l, r \right) \chi(r) = E \chi(r). \tag{3.2}$$

[Note that $r\hat{p}(1/r) = -i\partial/\partial r$.] Let us use the WKB approximation following Sec. 31 in [7]; Substituting the expression, $\chi(r) = A(r)\exp(iS(r)/\hbar)$ into Eq. (3.2), we obtain (up to the second order of $\hbar$)

$$H\left(\frac{\partial S}{\partial r}, l, r \right) = E, \tag{3.3}$$

$$\frac{\partial}{\partial r}\left(\frac{\partial H}{\partial p} A \right) + \frac{\partial H}{\partial p} \frac{\partial A}{\partial r} = 0, \tag{3.4}$$

where $A(r)$ and $S(r)$ are real unknown functions. The first equation with the following quantization condition is devoted to the determination of the energy E and the phase function $S(r)$, that is,

$$\oint p\, dr = 2 \int_a^b p(r) dr = \left(n + \frac{1}{2} \right) h, \quad n = 0,1,2, \ldots, \tag{3.5}$$

where a and b are the turning points and n means the quantum number for the vibrational mode. This condition, however, does not hold in the case of $l=0$ where the particle can come to the origin ($a=0$). In this case one should take the

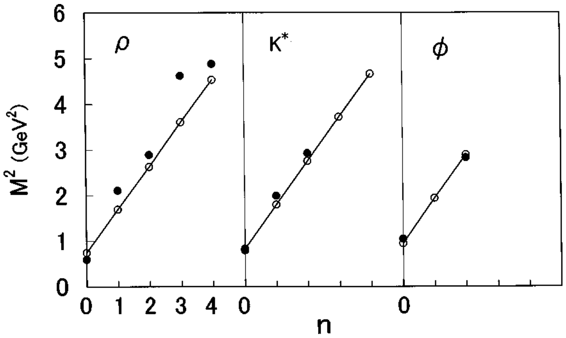

FIG. 2. The vibrational excited states of mesons. The quantum number of the vibrational motion is denoted by *n*. The filled and open circles represent the experimental and theoretical values, respectively.

boundary condition, $\chi(0)=0$ from the definition $\phi(r)=\chi(r)/r$. Thus we have two solutions, $\chi(r)\sim\cos(\int_0^r pdr-\pi/2)$ and $\chi(r)\sim\cos(\int_b^r pdr-\pi/4)$. Since these two solutions coincide between $a<r<b$, the right-hand side of Eq. (3.5) should be replaced by $(n+\frac{3}{4})h$ [8]. Equation (3.4) is rewritten as

$$\frac{\partial}{\partial r}\left(\frac{\partial H}{\partial p}A^2\right)=0\rightarrow A^2=C\left(\frac{\partial H}{\partial p}\right)^{-1} \tag{3.6}$$

and gives the wave function $A(r)$. The factor C on the right hand side is the normalization constant which is determined by

$$\int_0^\infty A(r)^2 dr=1. \tag{3.7}$$

Thus we obtain a relativistic hadron string model in which the center-of-mass motion is completely eliminated.

Finally we discuss the decay process in the framework of our model. The decay is viewed as a pair production of $\overline{q}q$ inside the flux tube which is known as Schwinger mechanism. According to [6], the probability of the pair production in a unit space-time volume in the tube is given by

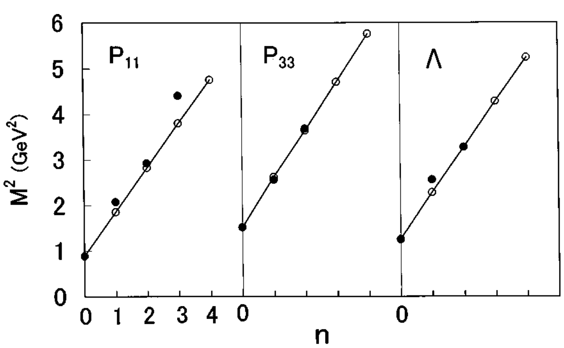

FIG. 3. The vibrational excited states of baryons. The quantum number of the vibrational motion is denoted by *n*. The filled and open circles represent the experimental and theoretical values, respectively.

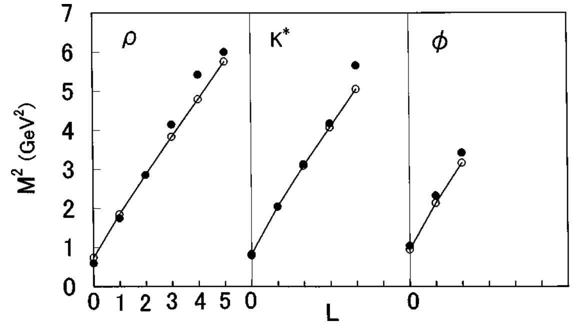

FIG. 4. The rotational excited states of mesons. The orbital angular momentum is denoted by *L*. The filled and open circles represent the experimental and theoretical values, respectively.

$$w=\frac{a^2}{4\pi^3}\sum_q\sum_{n=1}^\infty\frac{1}{n^2}\exp\left(-\frac{n\pi m_q^2}{a}\right). \tag{3.8}$$

Here Σ_q indicates a summation over all quark flavors with mass m_q (q=u,d,s). The probability of the decay (pair production) in the time interval dt is given by

$$dW=\int|\phi(r)|^2 wV(r)r^2 drdt, \tag{3.9}$$

where $V(r)$ denotes the volume of the string when its length is r in the center of mass system and it is given by

$$V(r)=\sum_i\int_0^{r_i}\sqrt{1-\omega^2r^2}S_0 dr. \tag{3.10}$$

In the right-hand side, S_0 denotes the area of the cross section of the tube which is not moving along the transverse direction. The decay width of the hadron is given by

$$\Gamma=\frac{dW}{dt}=\int A(r)^2 wV(r)dr. \tag{3.11}$$

This formula gives the decay widths of the excited hadrons and is one of main results of this paper.

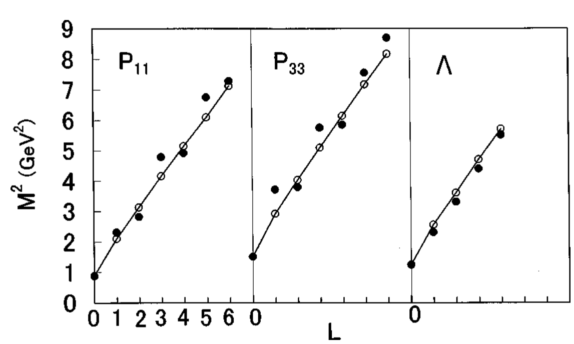

FIG. 5. The rotational excited states of baryons. The orbital angular momentum is denoted by *L*. The filled and open circles represent the experimental and theoretical values, respectively.

TABLE I. Decay widths of vibrational excited states of hadrons. The experimental and theoretical values are denoted by Γ_{exp} and Γ_{theor}, respectively.

Meson	Γ_{exp} (MeV)	Γ_{theor} (MeV)	Baryon	Γ_{exp} (MeV)	Γ_{theor} (MeV)
$\rho(770)$	151	151	$P_{11}(939)$		
$\rho(1450)$	310 ± 60	236	$P_{11}(1440)$	250–450	227
$\rho(1700)$	235 ± 50	296	$P_{11}(1710)$	50–250	288
$\rho(2150)$	363 ± 50	345	$P_{11}(2100)$	160–360	338
$\rho(2212)$	286 ± 80	389			
$K^*(892)$	50	149	$P_{33}(1232)$	115–125	136
$K^*(1412)$	227 ± 22	232	$P_{33}(1600)$	250–450	215
$K^*(1714)$	323 ± 110	294	$P_{33}(1920)$	150–300	269

IV. NUMERICAL RESULTS

We have calculated the mass spectra and the decay widths of hadrons by using our string model. Since the string is made up of the gluon field, the gluon degrees of freedom are incorporated into our model explicitly. The current quark masses are determined as $m_u=m_d=10$ MeV and $m_s=150$ MeV by fitting to the masses of the ground state hadrons. Thus there are only two parameters a and S_0. The former determines the mass spectra (the Regge slope) and the latter the decay width. We take $a=0.15$ GeV^2 in order to reproduce the observed Regge slope.

Now let us calculate the mass spectra by using Eq. (3.5). The excited states are classified into two categories; the vibrational mode (the yo-yo mode [9]) and the rotational one (the leading Regge trajectories). Consider first the vibrational excited states. The mass spectra are shown in Fig. 2 for ρ, K^* and ϕ mesons. In the same way, the mass spectra of P_{11}, P_{33}, and Λ baryons are shown in Fig. 3. The diquark mass is taken so as to reproduce the lowest mass for each case: $m_d=210$ MeV for P_{11} resonances, $m_d=550$ MeV for P_{33} resonances and $m_d=350$ MeV for Λ resonances. Moreover the masses of the excited states that lie on the corresponding Regge trajectories are calculated and the results are shown in Figs. 4 and 5 for mesons and baryons respectively. The L means the orbital angular momentum. The total angular momentum is given by the additive sum of the orbital and spin angular momenta: $J=L+S$. Our model reproduces the observed mass spectra [10] fairly well.

Next let us calculate the decay width by using Eq. (3.11). There remains one parameter S_0. Its value should be the same for all the hadrons because the color charge of each end is the same for all the hadrons considered in this paper. For convenience, we take the value so as to reproduce the decay width of $\rho(770)$:151 MeV. The calculated decay widths are shown in Tables I and II. It is found that the predictions by our model are consistent with the experimental values [10] although the agreement is not always satisfactory. The enhancement of the decay widths at higher excited states is explained by the elongation of the string. The smallness of the width of $K^*(890)$ may be due to the small phase space

TABLE II. Decay widths of rotational excited states of hadrons. The experimental and theoretical values are denoted by Γ_{exp} and Γ_{theor}, respectively.

Meson	Γ_{exp} (MeV)	Γ_{theor} (MeV)	Baryon	Γ_{exp} (MeV)	Γ_{theor} (MeV)
$\rho(770)$	151	151	$P_{11}(939)$		
$\rho(1318)$	106	247	$P_{11}(1520)$	110–135	258
$\rho(1691)$	160 ± 10	318	$P_{11}(1680)$	120–140	287
$\rho(2037)$	427 ± 120	358	$P_{11}(2190)$	350–550	343
$\rho(2330)$	400 ± 100	401	$P_{11}(2220)$	320–550	395
$\rho(2450)$	400 ± 250	436	$P_{11}(2600)$	500–800	450
			$P_{11}(2700)$	350–900	459
$K^*(892)$	50	148	$P_{33}(1232)$	115–125	133
$K^*(1430)$	98–109	237	$P_{33}(1930)$	250–450	216
$K^*(1770)$	164 ± 17	304	$P_{33}(1950)$	290–350	307
$K^*(2045)$	198 ± 30	371	$P_{33}(2400)$	330–480	331
$K^*(2380)$	178 ± 37	400	$P_{33}(2420)$	300–500	348
			$P_{33}(2750)$	350–500	412
			$P_{33}(2950)$	330–700	443

volume which is not taken into account in our treatment: The threshold energy of $K^*\rightarrow K+\pi$ is near the mass of K^*. There is a tendency that the observed decay widths of the second excited resonances are larger than the predicted ones in general. This deviation seems to come from other decay modes besides the one pair production of $\overline{q}q$.

Lastly some comments are in order. The velocity of the quark is near the light velocity in the vibrational motion as well as in the rotational one. This fact means that the relativistic treatment is necessary for the description of hadrons. The value of S_0 was determined from the width of $\rho(770)$. If our string is described by the elongated MIT bag, the string tension a is equal to $2BS_0$ where B is the bag constant. Consequently we get $S_0=3\ \text{fm}^2$ and $B^{1/4}=178$ MeV, the latter is consistent with the usual value.

V. SUMMARY

In conclusion, we have developed a relativistic string model for hadrons. It is characteristic of our model that the center-of-mass motion is eliminated completely in fully relativistic manner. Moreover the decay process is incorporated by taking into account the pair production of $\overline{q}q$. Generally the decay widths are enhanced by the elongation of the string but in the rotational cases they are suppressed by the Lorentz contraction along the transverse direction. Although our model is so simple, the mass spectra and the decay widths have been reproduced fairly well. We would like to note that the string model is also useful to describe the gluon component of the structure functions of hadrons [11,12] and multiple production in hadron or nuclear collisions at high energies.

[1] K. Johnson and C.B. Thorn, Phys. Rev. D **13**, 1934 (1975); K. Johnson, Acta Phys. Pol. B **6**, 865 (1975).
[2] M. Iwasaki, N. Tanokami, and T. Nakai, Phys. Lett. B **314**, 391 (1993).
[3] A.B. Migdal, S.B. Khokhlachev, and V.Yu. Borue, Phys. Lett. B **228**, 167 (1989); A.B. Migdal, Nucl. Phys. **A518**, 358 (1990).
[4] M. Anselmino, E. Predazzi, S. Ekelin, S. Fredriksson, and D.B. Lichtenberg, Rev. Mod. Phys. **65**, 1199 (1993).
[5] J. Schwinger, Phys. Rev. **82**, 664 (1951).
[6] E.G. Gurvich, Phys. Lett. **87B**, 386 (1979); A. Casher, H. Neuberger, and S. Nussinov, Phys. Rev. D **20**, 179 (1979); N.K. Glendenning and T. Matsui, *ibid.* **28**, 2890 (1983).
[7] P.A.M. Dirac, *The Principles of Quantum Mechanics* (Oxford University Press, Oxford, 1958), Sec. 31.
[8] A.B. Migdal and V.P. Krainov, *Approximation Methods in Quantum Mechanics* (W.A. Benjamin, Inc., New York, 1969), p. 120.
[9] F. Takagi, Prog. Theor. Phys. **94**, 385 (1995).
[10] Particle Data Group, R.H. Barnett *et al.*, Phys. Rev. D **54**, 1 (1996).
[11] A. Kokubo, M. Maruyama, and F. Takagi, Prog. Theor. Phys. Suppl. **120**, 263 (1995).
[12] A. Kokubo, Ph.D. thesis, Tohoku University, 1996.