

Radiative corrections to the decays $K_L^0 \rightarrow e^+ e^-$ and $K_L^0 \rightarrow \mu^+ \mu^-$

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(Received 8 January 1999; published 5 April 1999)

We calculate the rates and lepton (l) invariant mass distributions for decays of the form $0^{-+} \rightarrow l^+ l^- \gamma$, which are important radiative corrections to the purely leptonic decays $0^{-+} \rightarrow l^+ l^-$. Our approach uses the loop diagrams which arise by including the two photon intermediate state and we retain the imaginary parts of the loops—a radiative extension of the “unitarity bound” for the process. These results are compared with those obtained using a model in which the meson couples directly to the leptons. [S0556-2821(99)06709-0]

PACS number(s): 13.20.Eb, 13.40.Ks, 14.40.Aq

I. INTRODUCTION

Recently, a few electron events in the channel $K_L^0 \rightarrow e^+ e^-$, where K_L^0 is the long lived neutral K meson, have been observed [1]. The same experiment sees thousands of muon events, $K_L^0 \rightarrow \mu^+ \mu^-$. One of the experimental acceptance conditions is that the invariant mass of the two leptons be within a few MeV of the K mass. This means events are lost if a photon of sufficient energy is also emitted, i.e. $K_L^0 \rightarrow l^+ l^- \gamma$, with $l = e$ or μ .

Our purpose is to estimate the size of this radiative correction by calculating

$$\frac{1}{\Gamma_0} \int_{4m^2}^{(M-\Delta)^2} ds \frac{d\Gamma}{ds}. \quad (1)$$

Here Γ is the rate for $K_L^0 \rightarrow l^+ l^- \gamma$, Γ_0 is the rate for $K_L^0 \rightarrow l^+ l^-$ and s is the square of the lepton invariant mass, $s = (p + p')^2$, where p and p' are the momenta of the lepton and antilepton. The K mass is denoted by M , the lepton mass by m and Δ is an experimental parameter. In the next section, we examine a tree level model which treats the meson-lepton interaction as a point coupling. Section III contains results for a model in which the meson-lepton point interaction is replaced by a loop diagram with two photons in the intermediate state. We conclude with a comparison of the two approaches. Corrections to $K_L^0 \rightarrow \pi^+ \pi^- \gamma$ are presented in an appendix.

II. RADIATIVE CORRECTIONS IN THE TREE APPROXIMATION

The differential decay width $d\Gamma/ds$ has been calculated in a simple model [2] where the meson-lepton coupling is taken to be a pseudoscalar interaction with an effective coupling constant g . This leads to a transition amplitude T of the form

$$T = g e \left(\bar{u}(p) \not{\epsilon} \frac{(m + \not{p} + \not{k})}{2p \cdot k} \gamma_5 v(p') + \bar{u}(p) \gamma_5 \frac{(m - \not{p}' - \not{k})}{2p' \cdot k} \not{\epsilon} v(p') \right), \quad (2)$$

as illustrated in Fig. 1. Here, ϵ is the photon polarization vector and k is the photon momentum. Unfortunately the result is given only in the limit $M \gg m$, a condition that is clearly not satisfied for K decay into muons. The extension to include terms of all orders in m^2/M^2 is straightforward and we find that Eq. (2) gives a differential decay width for $K_L^0 \rightarrow l^+ l^- \gamma$ of the form

$$\frac{1}{\Gamma_0} \frac{d\Gamma}{ds} = \frac{\alpha}{\pi M^4} \frac{1}{\sqrt{1 - 4m^2/M^2}} \frac{1}{M^2 - s} \times \left[(M^4 + s^2 - 4M^2 m^2) \ln \left(\frac{1+v}{1-v} \right) - 2M^2 s v \right] \quad (3)$$

where $v = \sqrt{1 - 4m^2/s}$. This is a slight extension of Bergström's expression [2] by the terms proportional to m in the numerator and denominator.

In an attempt to estimate the model dependence of our corrections, we will compare our results with “model independent” corrections given by keeping only the universal soft bremsstrahlung correction terms of the form

$$T \rightarrow g e \bar{u}(p) \gamma_5 v(p') \left(\frac{p \cdot \epsilon}{p \cdot k} - \frac{p' \cdot \epsilon}{p' \cdot k} \right). \quad (4)$$

In this case the contents of the square brackets in Eq. (3) are replaced by

$$(2s^2 - 4sm^2) \ln \left(\frac{1+v}{1-v} \right) - 2vs^2. \quad (5)$$



FIG. 1. Diagrams for the radiative corrections to the tree model are shown. A dashed line denotes a meson, a wavy line denotes a photon and a solid line denotes a lepton.

For the expressions (3) and (5), the integral in Eq. (1) can be evaluated analytically to determine the fraction of lepton pairs missed. Using Eq. (3) we get

$$\frac{1}{\Gamma_0} \int_{4m^2}^{(M-\Delta)^2} ds \frac{d\Gamma}{ds} = \frac{\alpha}{\pi} \frac{1}{\sqrt{1-\varepsilon^2}} F(\delta, \varepsilon) \quad (6)$$

where

$$\delta = \frac{2\Delta}{M} - \frac{\Delta^2}{M^2}, \quad \varepsilon = \frac{2m}{M}$$

and

$$\begin{aligned} F(\delta, \varepsilon) = & \{-2(2-\varepsilon^2)[\ln \delta + 2 \ln(1+\beta_-^2)] \\ & -2[1-\delta + \frac{1}{2}(1-\delta)^2] + 2(1-\sqrt{1-\varepsilon^2})^2 \\ & + \varepsilon^2(1 + \frac{3}{8}\varepsilon^2)\} \ln \alpha_+ + (2-\varepsilon^2) \left[-\text{Li}_2\left(\frac{\alpha_+^2}{\beta_+^2}\right) \right. \\ & \left. + \text{Li}_2\left(\frac{\beta_-^2}{\alpha_+^2}\right) \right] - 2\sqrt{1-\varepsilon^2} \ln\left(\frac{1-\beta_-^2/\alpha_+^2}{1-\alpha_+^2/\beta_+^2}\right) \\ & + (\frac{13}{4} - \frac{1}{4}\delta + \frac{3}{8}\varepsilon^2) \sqrt{1-\delta} \sqrt{1-\delta-\varepsilon^2}, \end{aligned} \quad (7)$$

with

$$\alpha_+ = \frac{\sqrt{1-\delta}}{\varepsilon} + \frac{\sqrt{1-\delta-\varepsilon^2}}{\varepsilon}, \quad (8)$$

$$\beta_{\pm} = \frac{1}{\varepsilon} \pm \frac{\sqrt{1-\varepsilon^2}}{\varepsilon}. \quad (9)$$

$\text{Li}_2(x)$ is a Spence function or dilogarithm defined as

$$\text{Li}_2(x) = - \int_0^x \frac{dt}{t} \ln(1-t).$$

Numerical values of $\text{Li}_2(x)$ can easily be obtained using MAPLE or MATHEMATICA.

For expression (5) we again have Eq. (6) where now

$$\begin{aligned} F_{\text{pole}}(\delta, \varepsilon) = & \{-2(2-\varepsilon^2)[\ln \delta + 2 \ln(1+\beta_-^2)] \\ & -2[(2-\varepsilon^2)(1-\delta) + (1-\delta)^2] \\ & + 2(1-\sqrt{1-\varepsilon^2})^2 + \varepsilon^2(2-\frac{3}{4}\varepsilon^2)\} \ln \alpha_+ \\ & + (2-\varepsilon^2) \left[-\text{Li}_2\left(\frac{\alpha_+^2}{\beta_+^2}\right) + \text{Li}_2\left(\frac{\beta_-^2}{\alpha_+^2}\right) \right] \\ & - 2\sqrt{1-\varepsilon^2} \ln\left(\frac{1-\beta_-^2/\alpha_+^2}{1-\alpha_+^2/\beta_+^2}\right) \\ & + (\frac{11}{2} - \frac{3}{2}\delta - \frac{3}{4}\varepsilon^2) \sqrt{1-\delta} \sqrt{1-\delta-\varepsilon^2}. \end{aligned} \quad (10)$$

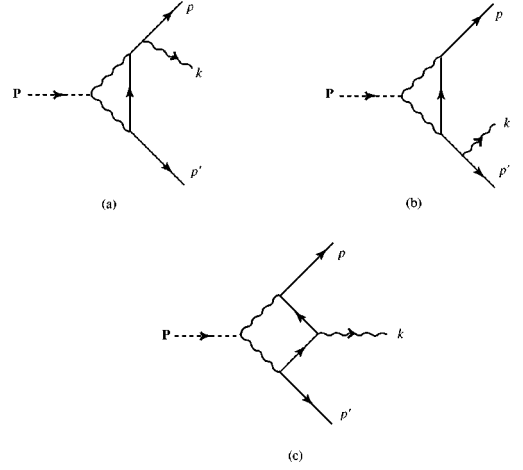


FIG. 2. Diagrams for the radiative corrections to the one-loop model are shown. A dashed line denotes a meson, a wavy line denotes a photon and a solid line denotes a lepton.

III. RADIATIVE CORRECTIONS USING A ONE-LOOP MODEL

All the experimental results announced so far [3] find the rate for $K_L^0 \rightarrow \mu^+ \mu^-$ to be near the theoretical lower limit given by multiplying the rate for $K_L^0 \rightarrow \gamma \gamma$ by the rate for $\gamma \gamma \rightarrow \mu^+ \mu^-$:

$$\Gamma = \Gamma(K_L^0 \rightarrow \gamma \gamma) \frac{\alpha^2}{2\beta} \left[\frac{m}{M} \ln\left(\frac{1+\beta}{1-\beta}\right) \right]^2, \quad (11)$$

where

$$\beta = \sqrt{1-4m^2/M^2}.$$

The rate for $K_L^0 \rightarrow e^+ e^-$ as given in [1] is larger than the unitarity bound given by Eq. (11) and consistent with predictions from chiral perturbation theory [4]. Nevertheless, it seems reasonable, in attempting to extend the calculation of Eq. (1) beyond the result obtained using Eq. (2), to calculate the absorptive part of $K_L^0 \rightarrow l^+ l^- \gamma$ diagrams shown in Fig. 2. In particular the box diagram takes us beyond simple bremsstrahlung off external legs. Like Eq. (11), the bremsstrahlung pole terms of Fig. 2, which vary as $1/\omega$, where ω is the photon energy, contain a factor of the lepton mass. It has been suggested that the terms which vary as ω to a positive power might not include a lepton mass factor and could thus be anomalously large [5].

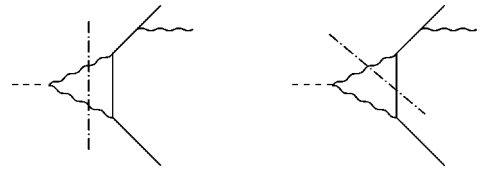


FIG. 3. Typical cut diagrams for the contributions to the absorptive part of the one-loop model are shown. The dot-dashed lines indicate the propagators which are put on mass shell. The sum of these two diagrams determines the imaginary part of the diagram in Fig. 2(a). Figures 2(b) and 2(c) have similar cuts.

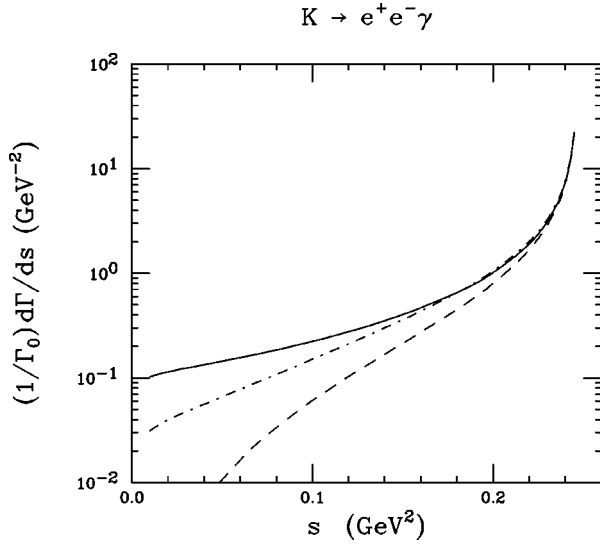


FIG. 4. The e^+e^- invariant mass (s) distribution for the decay $K_L^0 \rightarrow e^+e^- \gamma$ is shown normalized to the decay width $\Gamma_0(K_L^0 \rightarrow e^+e^-)$. The solid line is the result of Ref. [2], the dashed line is the contribution of the $1/\omega$ poles, Eq. (5), and the dot-dashed line is the result for the loop model of Sec. III.

To evaluate the diagrams of Fig. 2, we take the K -photon-photon effective Lagrangian to be

$$A_{\gamma\gamma} \phi F^{\mu\nu} \tilde{F}_{\mu\nu}, \quad (12)$$

where $F^{\mu\nu}$ is the photon field tensor, $\tilde{F}_{\mu\nu}$ is its dual, ϕ is the K_L^0 field and $A_{\gamma\gamma}$ is a constant. This leads to the vertex function

$$\Gamma_{\mu\nu}(k, k') = 2A_{\gamma\gamma} \epsilon_{\mu\nu\alpha\beta} k^\alpha k'^\beta, \quad (13)$$

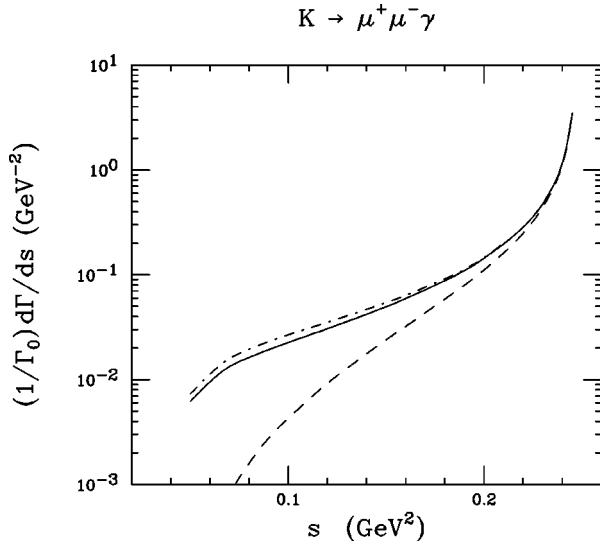


FIG. 5. Same as Fig. 3 for $K \rightarrow \mu^+ \mu^- \gamma$.

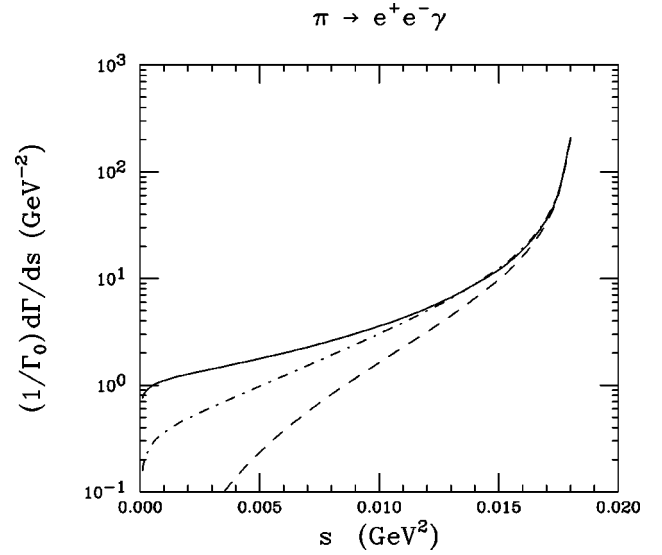


FIG. 6. Same as Fig. 1 for $\pi \rightarrow e^+e^- \gamma$.

where k and k' are the photon momenta. In general, this expression can also include a form factor which depends on k^2 and k'^2 . The implications of including this additional factor are discussed below.

In this case, the expression for $d\Gamma/ds$ is very complicated and we will not attempt to write it out. The box graph involves integrals of the form

$$\int d^4q \frac{q^\mu q^\nu q^\alpha, q^\mu q^\nu, q^\mu}{q^2 [(q+p)^2 - m^2] [(q+p+k)^2 - m^2] (q+P)^2}. \quad (14)$$

The triangle graphs involve similar integrals with one or two factors of q in the numerator and either the second or third factor in the denominator of Eq. (14) omitted. These integrals can be expanded in terms of the external momenta as outlined in the Appendix of Passarino and Veltman [6]. The momentum expansion and its scalar coefficients are given by a computer code called LOOP [7,8] which is a slight modification of a code written by Veltman called FORMFACTOR. Within this code the integrals are evaluated in terms of Spence functions as defined above and these functions are then evaluated numerically.

Once the amplitudes are determined they are squared and summed over spin, including the photon polarization, in the

TABLE I. The fractional radiative correction for $K_L^0 \rightarrow e^+e^-$, as defined by Eq. (1), for several values of the cutoff Δ is shown. The K mass is taken to be 497.67 MeV. The second column is given by Eq. (7), the third column given by the model of Sec. III and the fourth column by Eq. (10).

Δ (MeV)	Modified model [2]	Loop	Model indep.
7.64	0.161	0.154	0.120
5.67	0.178	0.172	0.137
3.67	0.203	0.197	0.161
1.67	0.249	0.244	0.207

TABLE II. Same as Table I for $K_L^0 \rightarrow \mu^+ \mu^-$.

Δ (MeV)	Modified model [2]	Loop	Model indep.
7.67	0.0217	0.0224	0.0171
5.67	0.0244	0.0251	0.0198
3.67	0.0284	0.0290	0.0236
1.67	0.0357	0.0363	0.0309

usual way. To check for errors we replace the photon polarization by its momentum and look for gauge invariance. This is our only real check but it is a very powerful one since gauge invariance requires a delicate cancellation among the three diagrams [9].

The real part of the amplitudes diverges because the effective coupling, Eq. (12), has dimension 5. The absorptive part has several contributions: $K \rightarrow \gamma\gamma$ followed by $\gamma\gamma \rightarrow l^+ l^- \gamma$ as well as $K \rightarrow \gamma l^+ l^-$ followed by $\gamma l \rightarrow \gamma l$. This is illustrated by the cut diagrams of Fig. 3. In the first of these diagrams, the intermediate photons are on shell ($k^2 = k'^2 = 0$), which is equivalent to our assumption that $A_{\gamma\gamma}$ is constant. In the second, one of the photons is virtual, and the effective coupling has the general form $A_{\gamma\gamma^*} = A_{\gamma\gamma} f(k^2)$, where $f(k^2)$ is a form factor normalized to $f(0) = 1$ [10]. Our numerical calculation of the complete absorptive part cannot separate these contributions; so we have effectively assumed $A_{\gamma\gamma^*} = A_{\gamma\gamma}$ throughout. This assumption is justified in the case of electrons, since the form factor correction to the width of the Dalitz decay $K_L^0 \rightarrow e^+ e^- \gamma$ is only a few percent due to the preference for low $e^+ e^-$ invariant mass. For the muon case, the form factor correction to the Dalitz decay width is 20–25% [10], and there could be a discernible effect in the one loop contributions to $d\Gamma/ds$.

IV. RESULTS AND CONCLUSIONS

In Figs. 4, 5 and 6, we show the differential width obtained from the absorptive part of the one-loop calculation as a function of the lepton invariant mass, s , for $K_L^0 \rightarrow e^+ e^- \gamma$, $K_L^0 \rightarrow \mu^+ \mu^- \gamma$, and $\pi^0 \rightarrow e^+ e^- \gamma$. For comparison we also plot Eqs. (3) and (5). The result of the loop calculation is almost the same as Bergström's differential width, as modified by us to include the lepton mass, and both differ substantially from the ‘‘model independent’’ width where only the $1/\omega$ terms are kept.

In Tables I, II, and III we give the integrated width, Eq. 1, for several values of Δ . Again the result from the loop calculation is very similar to that given by Eq. (7) and quite different from that given by Eq. (10). For electrons the correction is large, but not anomalously so, and there is no indication that the diagrams of Fig. 2 are not proportional to the lepton mass.

The moral would seem to be that, except for very small

TABLE III. Same as Table I for $\pi^0 \rightarrow e^+ e^-$. We use $m_\pi = 135$ MeV.

Δ (MeV)	Modified model [2]	Loop	Model indep.
4	0.098	0.098	6.71×10^{-2}
3	0.111	0.111	7.91×10^{-2}
2	0.129	0.129	9.67×10^{-2}
1	0.161	0.161	0.128

invariant masses, the more complicated calculation of the loop model of Sec. III is unnecessary and Eqs. (3) and (6) are sufficient. We have made no attempt to calculate the radiative corrections within the acceptance bin, $(M - \Delta)^2 < s < M^2$. For the model of Sec. II Bergström [2] has given a complete expression for the correction from virtual photons. This, together with Eq. (3), is all that is needed. To calculate the virtual corrections for the model of Sec. III is beyond the scope of this work.

ACKNOWLEDGMENTS

It is a pleasure to thank Jack Ritchie for drawing our attention to this problem, for many discussions, and for preliminary versions of Tables I and II and Figs. 4 and 5. We also thank Jack Smith for providing us with many references, in particular Ref. [4]. This work was supported in part by the U.S. Department of Energy under Contract No. DE-FG013-93ER40757 and in part by the National Science Foundation under Grant No. PHY-98-02439.

APPENDIX

In a model for the $K^0 \rightarrow \pi^+ \pi^-$ vertex similar to Eq. (2), where the $K\pi\pi$ vertex is taken as a constant, the differential rate for $K^0 \rightarrow \pi^+ \pi^- \gamma$ has only the $1/\omega$ terms and is therefore given by Eq. (5) multiplied by M^2/s to remove the spinor factor [11]. The integrated rate is given by Eq. (6) with

$$\begin{aligned}
 F_{\text{pole}}(\delta, \varepsilon) = & \{-2(2 - \varepsilon^2)[\ln \delta + 2 \ln(1 + \beta_-^2)] - 4(1 - \delta) \\
 & + 2(1 - \sqrt{1 - \varepsilon^2})^2 + 2\varepsilon^2\} \ln \alpha_+ + (2 - \varepsilon^2) \\
 & \times \left[-\text{Li}_2\left(\frac{\alpha_+^2}{\beta_+^2}\right) + \text{Li}_2\left(\frac{\beta_-^2}{\alpha_+^2}\right) \right] \\
 & - 2\sqrt{1 - \varepsilon^2} \ln\left(\frac{1 - \beta_-^2/\alpha_+^2}{1 - \alpha_+^2/\beta_+^2}\right) \\
 & + 4\sqrt{1 - \delta}\sqrt{1 - \delta - \varepsilon^2}. \tag{A1}
 \end{aligned}$$

For the four values of Δ used in $K_L^0 \rightarrow l^+ l^- \gamma$, $\Delta = 7.67, 5.67, 3.67,$ and 1.67 MeV, the fractional radiative corrections are 0.0127, 0.0145, 0.0172 and 0.0223.

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