Deep inelastic structure functions in light-front QCD: Radiative corrections

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Recently, we introduced a unified theory to deal with perturbative and non-perturbative QCD contributions to hadronic structure functions in deep inelastic scattering. This formulation is realized by combining the coordinate space approach based on light-front current algebra techniques and the momentum space approach based on Fock space expansion methods in the Hamiltonian formalism of light-front field theory. In this work we show how a perturbative analysis in the light-front Hamiltonian formalism leads to the factorization scheme we have proposed recently. The analysis also shows that the scaling violations due to perturbative QCD corrections can be rather easily addressed in this framework by simply replacing the hadron target by a dressed parton target and then carrying out a systematic expansion in the coupling constant α_s based on the perturbative QCD expansion of the dressed parton target. The tools employed for this calculation are those available from light-front old-fashioned perturbation theory. We present a complete set of calculations of unpolarized deep inelastic structure functions to order α_s. We extract the relevant splitting functions in all the cases. We explicitly verify all the sum rules to order α_s. We demonstrate the validity of approximations made in the derivation of the new factorization scheme. This is achieved with the help of detailed calculations of the evolution of structure function of a composite system carried out using multi-parton wave functions. [S0556-2821(99)02109-8]

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I. INTRODUCTION

A general cross section in hadron physics contains both short distance and long distance behavior and hence is not accessible to perturbative QCD. Factorization theorems [1] allow one to separate the two behaviors in a systematic fashion. Usually the short distance (perturbative) properties are calculated with Feynman diagrams where the most popular choice of regulator is dimensional regularization. So far there is no non-perturbative implementation of dimensional regularization in field theory. The long distance (non-perturbative) part is given in terms of a set of operator matrix elements which are left for computation, for example, in lattice gauge theory. Since the factorization scale is an artifact reflecting our present inability to do computations in QCD, the two sectors (perturbative and non-perturbative) should merge smoothly. Since currently available formalisms employed to tackle the two sectors use different regulators, different degrees of freedom, etc., this goal is difficult to accomplish in practice. It is desirable to have a method of calculation where same formalism is used to deal with both perturbative and non-perturbative regions of QCD.

Recently we have proposed a new method [2] of calculation of deep inelastic structure functions that combines current algebra techniques and Fock space expansion methods in light-front field theory in a Hamiltonian framework. We have arrived at expressions for various structure functions as the Fourier transform of hadron matrix elements of different components of bilocal vector and axial vector currents on the light-front. By expanding the state of the hadron in terms of multi-parton wave functions, non-perturbative QCD dynamics underlying the structure functions can be explored.

In this work we show that a perturbative analysis in the light-front Hamiltonian formalism leads to the factorization scheme proposed in Ref. [2]. The analysis also shows that the scaling violations due to perturbative QCD corrections can be rather easily addressed in this framework by simply replacing the hadron target by dressed parton target and then carrying out a systematic expansion in the coupling constant α_s based on the perturbative QCD expansion of the dressed parton target. The tools used are those belonging to light-front old fashioned perturbation theory [3–5] which employs transition matrix elements and energy denominators instead of Feynman propagators.

The advantage of using the light-front Hamiltonian formulation is that the matrix elements can be naturally defined in the light-front gauge ($A^+=0$ in light-front coordinates). With this gauge choice, there is no need for the path ordered exponential between fermion field operators in the bilocal current which is mandatory in other popular gauge choices. Since we do not have to deal with four dimensional integrals involving Feynman propagators, we do not encounter some of the problems associated with the use of the non-covariant gauge condition $A^+=0$ in usual covariant perturbative QCD calculations.

Meanwhile, the evaluation of the matrix elements in our approach (see Sec. IV) is straightforward, which also greatly clarifies the physical picture of deep inelastic scattering (DIS). For example, matrix elements of the transverse component of the bilocal vector and axial vector currents are easy to analyze in the present method. In sharp contrast, there are well-known problems [6] associated with γ_5 in dimensional regularization. Also the presence of quark masses poses no problem in our calculations which has been bothersome for a long time in the standard operator product expansion (OPE) or Feynman diagram approach of DIS.

Since our approach deals with probability amplitudes, real and virtual processes are calculated to the same order without any difficulties, in contrast with the Altarelli-Parisi method [7] which deals with probability densities. We first present results for the unpolarized structure functions $F_2(x)$ for a dressed quark (extracted from the plus component of the vector bilocal) and a gluon target to order α_s. We extract the relevant splitting functions. In these cases we also explicitly verify the longitudinal momentum sum rule. Furthermore, interference effects are straightforward to handle. As a result, we explicitly show the invalidity of the popular twist classification [8]. We demonstrate this by analyzing the matrix element of the transverse component of the bilocal vector current (see Sec. IV for details). In the conventional OPE analysis [9], it is customary to ignore the non-trivial structure of the state, and consider the operator structure alone to draw conclusions. A case in point is the transverse component of the bilocal vector current. This operator is not diagonal and in the twist analysis of Ref. [8] will appear to have twist 3. Hence it appears as if this operator matrix element cannot have a parton interpretation. However, our explicit calculations show that this operator matrix *matrix element* is indeed twist 2 and has the familiar parton interpretation. This becomes clear only *after* the evaluation of the matrix elements which includes off-diagonal ones [10].

For a second example, in the case of the transverse polarized structure function, it is popular to ignore quark mass and stress quark-gluon correlations. Explicit calculations [11] employing our methods have shown that the operators that involve γ_5 which do not have an explicit dependence on the quark mass m_q come out proportional to m_q when the matrix element is taken between dressed quark states. In particular all the operators contribute at the same level to the structure function g_T of a dressed quark. This is shown to be essential for the g_2 structure function to obey the Burkhardt-Cottingham sum rule [12] in perturbative QCD.

In this work we also provide a detailed calculation of the evolution of F_2 structure function for a hadronic bound state. An entire analysis is carried out using multi-parton wave functions in momentum space. Both real and virtual processes are accounted for and in the lowest order analysis one begins to see the emergence of the standard evolution equation. The detailed analysis justifies the approximations made in Sec. II which lead to factorization to all orders in perturbation theory. Then what remains unsolved is the non-perturbative contribution that is defined in the same framework. At present, the non-perturbative QCD dynamics and hadronic bound states on the light-front are indeed explored in this same framework. Therefore, we hope that the present work (Ref. [2] and the present paper) can really provide a natural connection of the fully theoretical understanding to the experimental phenomena of DIS. Note that for exclusive processes, factorization has been proved using the light-front formalism by Brodsky and Lepage [3].

The plan of this paper is as follows. In Sec. II we present a perturbative analysis in the light-front Hamiltonian formalism which leads to the factorization and to the concept of the structure function of a dressed parton in DIS. The tools necessary for the evaluation of these functions are discussed in Sec. III. Unpolarized dressed parton distribution functions are discussed in Sec. IV. We also present the explicit verification of the appropriate sum rules in this section. A detailed analysis of the structure function of bound states is carried out in Sec. V which justifies *a posteriori* the approximations made in the study in Sec. II. Finally Sec. VI contains a discussion and conclusions. We leave a detailed discussion of the polarized dressed parton structure function and the associated factorization and evolution equation for polarized hadron structure functions to be presented in a forthcoming paper.

II. FACTORIZATION: A PERTURBATIVE ANALYSIS

In this section we show in detail how the factorization picture discussed in Ref. [2] emerges in a perturbative analysis carried over to all orders in the case where the bilocal operator involved does not change the particle number. The analysis leads to the concept of the structure function of a dressed parton in DIS.

To explicitly demonstrate the factorization picture on the light-front, we consider the F_2 structure function as a specific example in this section. For simplicity we drop reference to the flavor: then,

$$\frac{F_2(x,Q^2)}{x} = \frac{1}{4\pi}\int d\xi^- e^{-i P^+\xi^- x/2} \times\langle PS|[(\psi^+)^\dagger(\xi^-)\psi^+(0)-(\psi^+)^\dagger(0)\psi^+(\xi^-)]|PS\rangle. \tag{2.1}$$

From the discussion in Sec. V B of [2], we have

$$\frac{F_2(x,Q^2)}{x} = q(x,Q^2) = \frac{1}{4\pi}\int d\xi^- e^{-i P^+\xi^- x/2}\sum_{n_1,n_2}\langle PS\mu^2|n_1\rangle\langle n_2|PS\mu^2\rangle\langle n_1|U_h^{-1}[(\psi^+)^\dagger(\xi^-)\psi^+(0) -(\psi^+)^\dagger(0)\psi^+(\xi^-)]U_h|n_2\rangle, \tag{2.2}$$

where $U_h = T^+\exp\{(-i/2)\int_{-\infty}^0 dx^+\tilde{P}^-_{int}(x^+)\}$, and $\tilde{P}^-_{int}\equiv P^{-H}_{int}+P^{-M}_{int}$ is denoted as the hard and mixed light-front interaction Hamiltonian.

In the following, we shall not explicitly evaluate contributions from intermediate states which involve vanishing energy denominators. These contributions are most conveniently included by introducing the wave function renormalization constant associated with the parton active in the high energy process.

Let us consider first few terms in the expression given in Eq. (2.2). The lowest order term yields the function

$$q^{(0)}(x,Q^2)=q(x,\mu^2)=\sum_n |\langle PS,\mu^2_{\rm fact}|n\rangle|^2$$
$$=\sum_s \int^{\mu} d^2k^{\perp}\langle PS\mu^2|b_s^{\dagger}(yP^+,k^{\perp})b_s(yP^+,k^{\perp})|PS\mu^2\rangle. \tag{2.3}$$

Terms linear in the interaction Hamiltonian vanish since the plus component of the bilocal operator conserves particle number on the light-front. Consider the second order contribution

$$q^{(2)}(x,Q^2)=\frac{1}{4}\sum_{nmpk}\langle PS\mu^2|n\rangle\int_{-\infty}^{0}dx_1^+\langle n|\tilde{P}^-_{int}(x_1^+)|m\rangle \times\langle m|\mathcal{O}|p\rangle\left\langle p\left|\int_{-\infty}^{0}dx_2^+\tilde{P}^-_{int}(x_1^+)\right|k\right\rangle \times\langle k|PS\mu^2\rangle. \tag{2.4}$$

Here we have denoted

$$\mathcal{O}=\frac{1}{4\pi}\int d\xi^- e^{-iP^+\xi^-x/2}[(\psi^+)^{\dagger}(\xi^-)\psi^+(0) -(\psi^+)^{\dagger}(0)\psi^+(\xi^-)]. \tag{2.5}$$

Using

$$P^-_{int}(x^+)=e^{iP^-_{free}(x^+)/2}P^-_{int}(0)e^{-iP^-_{free}(x^+)/2}, \tag{2.6}$$

we have

$$q^{(2)}(x,Q^2)=\sum_{nmpk}\frac{1}{P^-_{0n}-P^-_{0m}}\frac{1}{P^-_{0k}-P^-_{0p}}\langle n|\tilde{P}^-_{int}(0)|m\rangle \times\langle m|\mathcal{O}|p\rangle\langle p|\tilde{P}^-_{int}(0)|k\rangle\langle PS\mu^2|n\rangle\langle k|PS\mu^2\rangle. \tag{2.7}$$

The states $|n\rangle$, and $|k\rangle$ are forced to be low energy states with $(k^{\perp})^2<\mu^2$. We can restrict the states $|m\rangle,|p\rangle$ to be high energy states with $(k^{\perp})^2>\mu^2$. The bilocal operator $\mathcal{O}$ picks an active parton in a high energy state whose longitudinal momentum is forced to be xP^+. Further we need to keep only terms in $\tilde{P}^-_{int}$ which cause transitions involving the active parton. (Transitions involving spectators lead to wave function renormalization of spectator states which are canceled by the renormalization process as shown explicitly in Sec. V B.)

To order α_s, a straightforward evaluation (see later) leads to

$$q(x,Q^2)=\mathcal{N}\left\{q(x,\mu^2) +\frac{\alpha_s}{2\pi}C_f\ln\frac{Q^2}{\mu^2}\int_x^1\frac{dy}{y}P(x/y)q(y,\mu^2)\right\}, \tag{2.8}$$

where $\mathcal{N}$ is the wave function renormalization constant of the active parton and P is the splitting function. Including the contribution from the wave function renormalization constant to the same order (α_s), we get

$$q(x,Q^2)=\int dy\ \ \mathcal{P}(x,Q^2;y,\mu^2)q(y,\mu^2), \tag{2.9}$$

where the hard scattering coefficient

$$\mathcal{P}(x,Q^2;y,\mu^2) =\delta(x-y)+\frac{\alpha_s}{2\pi}C_f\ln\frac{Q^2}{\mu^2}\int_0^1 dz\,\delta(zy-x)\tilde{P}(z) \tag{2.10}$$

with $\tilde{P}(x)=P(x)-\delta(1-x)\int_0^1 dyP(y)$.

We note that the above analysis can be carried over to all orders in perturbation theory with the result

$$\mathcal{P}(x,Q^2;y,\mu^2) =\langle yP^+,k^{\perp},s|U_h^{-1}\mathcal{O}U_h|yP^+,k^{\perp},s\rangle =\langle yP^+,k^{\perp},s;(\text{dressed})|\mathcal{O}|yP^+,k^{\perp},s;(\text{dressed})\rangle. \tag{2.11}$$

In evaluating the above expression, only in the interaction Hamiltonians in the extreme left and extreme right of the time ordered product do we need to keep a mixture of soft and hard partons. This is governed by P^{-M}_{int}. They are needed to cause the transition of a soft parton to a hard parton. In the rest of the interaction Hamiltonians occurring in the chain, the partons are restricted to be hard; i.e., they are determined by P^{-H}_{int} only. For the leading logarithmic evolution we are discussing here, they appear ordered in transverse momentum.

III. TOOLS OF CALCULATION

In this section, we outline the basic tools for calculating the perturbative contribution to the structure functions, namely the hard scattering coefficients, $\mathcal{P}(x,Q^2;y,\mu^2)$, given by Eq. (2.11). If we set $k=P$, then $y=1$ and the hard scattering coefficients just become the structure functions of dressed quark and gluon targets in DIS,

$$f_i^p(x,Q^2)=\frac{1}{4\pi}\int d\eta e^{-i\eta x} \times {}_p\langle ks|[\overline{\psi}(\xi^-)\Gamma_i\psi(0)\mp \text{H.c.}]|ks\rangle_p . \quad (3.1)$$

As a matter of fact, we can compute the perturbative QCD correction to the hadronic structure functions by calculating the structure functions of the dressed quarks and gluons. In old-fashioned light-front perturbation theory, the dressed quark or gluon states can be expanded as follows [4]:

$$|Ps\rangle_p=U_h|Ps\rangle=\sqrt{\mathcal{N}}\Bigg\{|Ps\rangle+\sum_n |n\rangle\frac{\langle n|P_{int}^{-M}|Ps\rangle}{(P^- - P_n^-)} + \sum_{mn}|m\rangle\frac{\langle m|P_{int}^{-H}|n\rangle\langle n|P_{int}^{-M}|Ps\rangle}{(P^- - P_m^-)(P^- - P_n^-)}+\cdots\Bigg\}, \quad (3.2)$$

where $|Ps\rangle$, the bare single particle state, and $|n\rangle$, the two-particle state, $|m\rangle$, the three-particle state, etc., are eigenstates of the free Hamiltonian. Introducing the multi-parton amplitudes (wave functions)

$$\Phi_n=\frac{\langle n|P_{int}^{-M}|Ps\rangle}{(P^- - P_n^-)},$$

$$\Phi_m=\sum_n \frac{\langle m|P_{int}^{-H}|n\rangle\langle n|P_{int}^{-M}|Ps\rangle}{(P^- - P_m^-)(P^- - P_n^-)}, \quad (3.3)$$

the expansion in Eq. (3.2) takes the form

$$|Ps\rangle_p=\sqrt{\mathcal{N}}\Bigg\{|Ps\rangle+\sum_n \Phi_n|n\rangle+\sum_m \Phi_m|m\rangle+\cdots\Bigg\}. \quad (3.4)$$

In the above expressions, P_{int}^{-M} and P_{int}^{-H} are the interaction parts of the canonical light-front QCD Hamiltonian, but the former contains the mixed soft and hard partons and the latter only has hard partons. The canonical light-front QCD Hamiltonian is given by the following form in our two-component formalism [4]:

$$P_{int}^-=\int dx^- d^2x^\perp\{\mathcal{H}_{qqg}+\mathcal{H}_{ggg}+\mathcal{H}_{qqgg}+\mathcal{H}_{qqqq}+\mathcal{H}_{gggg}\} \quad (3.5)$$

and

$$\mathcal{H}_{qqg}=g\xi^\dagger\Bigg\{-2\left(\frac{1}{\partial^+}\right)(\partial\cdot A^\perp)+\sigma\cdot A^\perp\left(\frac{1}{\partial^+}\right)(\sigma\cdot\partial^\perp+m) +\left(\frac{1}{\partial^+}\right)(\sigma\cdot\partial^\perp-m)\sigma\cdot A^\perp\Bigg\}\xi, \quad (3.6)$$

$$\mathcal{H}_{ggg}=gf^{abc}\Bigg\{\partial^iA_a^jA_b^iA_c^j+(\partial^iA_a^i)\left(\frac{1}{\partial^+}\right)(A_b^j\partial^+A_c^j)\Bigg\}, \quad (3.7)$$

$$\mathcal{H}_{qqgg}=g^2\Bigg\{\xi^\dagger\sigma\cdot A^\perp\left(\frac{1}{i\partial^+}\right)\sigma\cdot A^\perp\xi+2\left(\frac{1}{\partial^+}\right) \times(f^{abc}A_b^i\partial^+A_c^i)\left(\frac{1}{\partial^+}\right)(\xi^\dagger T^a\xi)\Bigg\} =\mathcal{H}_{qqgg1}+\mathcal{H}_{qqgg2}, \quad (3.8)$$

$$\mathcal{H}_{qqqq}=2g^2\Bigg\{\left(\frac{1}{\partial^+}\right)(\xi^\dagger T^a\xi)\left(\frac{1}{\partial^+}\right)(\xi^\dagger T^a\xi)\Bigg\}, \quad (3.9)$$

$$\mathcal{H}_{gggg}=\frac{g^2}{4}f^{abc}f^{ade}\Bigg\{A_b^iA_c^jA_d^iA_e^j+2\left(\frac{1}{\partial^+}\right)(A_b^i\partial^+A_c^i)\left(\frac{1}{\partial^+}\right) \times(A_d^j\partial^+A_e^j)\Bigg\} =\mathcal{H}_{gggg1}+\mathcal{H}_{gggg2}, \quad (3.10)$$

where the dynamic fermion field operator

$$\psi^+(x)=\begin{bmatrix}\xi(x)\\ 0\end{bmatrix}, \quad (3.11)$$

with

$$\xi(x)=\sum_\lambda \chi_\lambda\int\frac{dk^+d^2k^\perp}{2(2\pi)^3\sqrt{k^+}}[b_\lambda(k)e^{-ikx}+d_{-\lambda}^\dagger(k)e^{ikx}], \quad (3.12)$$

and the transverse gluon field operator

$$A^{i\perp}(x)=\sum_\lambda\int\frac{dk^+d^2k^\perp}{2(2\pi)^3k^+}[\varepsilon^{i\perp}(\lambda)a_\lambda(k)e^{-ikx}+\text{H.c.}], \quad (3.13)$$

with

$$\{b_\lambda(k),b_{\lambda'}^\dagger(k')\}=\{d_\lambda(k),d_{\lambda'}^\dagger,k'\} =2(2\pi)^3k^+\delta(k^+-k'^+)\delta^2(k^\perp-k^{\perp\prime}), \quad (3.14)$$

$$[a_\lambda(k),a^\dagger_{\lambda'}(k')]=2(2\pi)^3k^+\delta(k^+-k'^+)\delta^2(k^\perp-k^{\perp\prime}), \tag{3.15}$$

and χ_λ is the eigenstate of σ_z in the two-component spinor of ψ_+ by the use of the following light-front γ matrix representation [13]:

$$\gamma^0=\begin{bmatrix}0 & -i\\ i & 0\end{bmatrix}, \quad \gamma^3=\begin{bmatrix}0 & i\\ i & 0\end{bmatrix}, \quad \gamma^i=\begin{bmatrix}-i\tilde{\sigma}^i & 0\\ 0 & i\tilde{\sigma}^i\end{bmatrix} \tag{3.16}$$

with $\tilde{\sigma}^1=\sigma^2$, $\tilde{\sigma}^2=-\sigma^1$ and $\varepsilon^i(\lambda)$ the polarization vector of transverse gauge field.

IV. UNPOLARIZED DRESSED PARTON STRUCTURE FUNCTIONS

In this section we present the calculations of the F_2 structure function for dressed quark and gluon targets.

A. Dressed quark structure function from the plus component

The F_2 structure function of a dressed quark is given by

$$\begin{aligned}\frac{F_2(x,Q^2)}{x}&=\frac{1}{4\pi}\int d\eta e^{-i\eta x}\overline{V}_1\\&=\frac{1}{4\pi P^+}\int d\eta e^{-i\eta x}\\&\quad\times{}_p\langle ks|\overline{\psi}(\xi^-)\gamma^+\psi(0)-\overline{\psi}(0)\gamma^+\psi(\xi^-)|ks\rangle_p.\end{aligned} \tag{4.1}$$

The gluon structure function [1] is defined by

$$\begin{aligned}F_2^G(x,Q^2)&=\frac{1}{4\pi P^+}\int d\eta e^{-i\eta x}\\&\quad\times{}_p\langle k\lambda|(-)F^{+\nu a}(\xi^-)F^{+a}_{\ \nu}(0)|k\lambda\rangle_p.\end{aligned} \tag{4.2}$$

The dressed quark or gluon state can be obtained by the perturbative expansion in the old fashioned time-ordered Hamiltonian formulation, as given by Eq. (3.2). But we can also find such states by solving the light-front bound state equation. Let us take the state $|P\rangle$ to be a dressed quark which obeys the eigenvalue equation

$$\left(M^2-\sum_{i=1}^{n}\frac{(\kappa_i^\perp)^2+m_i^2}{x_i}\right)\begin{bmatrix}\Phi_q\\ \Phi_{qg}\\ \vdots\end{bmatrix}=\begin{bmatrix}\langle q|H^H_{int}|q\rangle & \langle q|H^H_{int}|qg\rangle & \cdots\\ \langle qg|H^H_{int}|q\rangle & \cdots & \\ \vdots & \ddots & \end{bmatrix}\begin{bmatrix}\Phi_q\\ \Phi_{qg}\\ \vdots\end{bmatrix}. \tag{4.3}$$

Explicitly, expanding the state in terms of bare states of quark, quark plus gluon, quark plus two gluons, etc., we have

$$\begin{aligned}|P\sigma\rangle_q=\sqrt{\mathcal{N}_q}\Bigg\{&b^\dagger(P,\sigma)|0\rangle+\sum_{\sigma_1,\lambda_2}\int\frac{dk_1^+d^2k_1^\perp}{\sqrt{2(2\pi)^3k_1^+}}\int\frac{dk_2^+d^2k_2^\perp}{\sqrt{2(2\pi)^3k_2^+}}\psi_2(P,\sigma|k_1,\sigma_1;k_2,\lambda_2)\\&\times\sqrt{2(2\pi)^3P^+}\delta^3(P-k_1-k_2)b^\dagger(k_1,\sigma_1)a^\dagger(k_2,\lambda_2)|0\rangle\Bigg\}.\end{aligned} \tag{4.4}$$

The factor $\mathcal{N}_q$ is the wave function renormalization constant for the quark and the function $\psi_2(P,\sigma|k_1\sigma_1,k_2\lambda_2)$ is the probability amplitude to find a bare quark with momentum k_1 and helicity σ_1 and a bare gluon with momentum k_2 and helicity λ_2 in the dressed quark.

Introduce the Jacobi momenta $(x_i,\kappa_i^\perp)$

$$k_i^+=x_iP^+,k_i^\perp=\kappa_i^\perp+x_iP^\perp \tag{4.5}$$

so that

$$\sum_i x_i=1,\sum_i \kappa_i^\perp=0. \tag{4.6}$$

The amplitude ψ_2 is related to the amplitude Φ_2 in Eq. (4.3) by

$$\sqrt{P^+}\psi_2(k_i^+,k_i^\perp)=\Phi_2(x_i,\kappa_i^\perp). \tag{4.7}$$

The two particle amplitude ψ_2 is given by

$$\begin{aligned}&\psi_2(P,\sigma|p_1,s_1;p_2,\rho_2)\\&=\frac{1}{\left[\frac{m^2+(P^\perp)^2}{P^+}-\frac{m^2+(p_1^\perp)^2}{p_1^+}-\frac{(p_2^\perp)^2}{p_2^+}\right]}\\&\quad\times\frac{g}{\sqrt{2(2\pi)^3}}T^a\frac{1}{\sqrt{p_2^+}}\chi^\dagger_{s_1}\left[2\frac{p_2^\perp}{p_2^+}-\frac{\sigma^\perp\cdot p_1^\perp-im}{p_1^+}\sigma^\perp\right.\\&\quad\left.-\sigma^\perp\frac{\sigma^\perp\cdot P^\perp+im}{P^+}\right]\chi_\sigma\cdot(\epsilon^\perp_{\rho_2})^*.\end{aligned} \tag{4.8}$$

We rewrite the above equation in terms of Jacobi momenta ($p_i^+ = x_i P^+, \kappa_i^\perp = p_i^\perp + x_i P^\perp$) and the wave functions Φ_i which are functions of Jacobi momenta. Using the notation $x = x_1$, $\kappa_1 = \kappa$ and using the facts $x_1 + x_2 = 1$, $\kappa_1 + \kappa_2 = 0$, we have

$$\Phi_2^{s_1,\rho_2}(x,\kappa^\perp;1-x,-\kappa^\perp) = \frac{1}{\left[m^2 - \dfrac{m^2+(\kappa^\perp)^2}{x} - \dfrac{(\kappa^\perp)^2}{1-x}\right]} \frac{g}{\sqrt{2(2\pi)^3}} T^a \chi_{s_1}^\dagger \times \left[-2\frac{\kappa^\perp}{1-x} - \frac{\sigma^\perp\cdot\kappa^\perp - im}{x}\sigma^\perp - \sigma^\perp im\right]\chi_\sigma \cdot (\epsilon_{\rho_2}^\perp)^*. \tag{4.9}$$

Evaluating the expression in Eq. (4.1) explicitly, noting that in the present case the contribution from the second term in this expression is zero, we get the quark structure function of the dressed quark

$$\frac{F_{2(q)}^q(x,Q^2)}{x} = \mathcal{N}_q \Bigg\{ \delta(1-x) + \sum_{\sigma_1,\lambda_2} \int dx_2 \int d^2\kappa_1^\perp \int d^2\kappa_2^\perp \,\delta(1-x-x_2) \times \delta^2(\kappa_1^\perp + \kappa_2^\perp) |\Phi_2^{\sigma_1,\lambda_2}(x,\kappa_1^\perp;x_2,\kappa_2^\perp)|^2 \Bigg\}. \tag{4.10}$$

This equation makes manifest the parton interpretation of the quark distribution function; namely, the quark distribution function of a dressed quark is the incoherent sum of probabilities to find a bare parton (quark) with longitudinal momentum fraction x in various multi-particle Fock states of the dressed quark. Since we have computed the distribution function in field theory, there are also significant differences from the traditional parton model [14]. The most important difference is the fact that the partons in field theory have transverse momenta ranging from zero to infinity. Whether the structure function scales or not now depends on the ultraviolet behavior of the multi-parton wave functions. By analyzing various interactions, one easily finds that in super-renormalizable interactions, the transverse momentum integrals converge in the ultraviolet and the structure function scales, whereas in renormalizable interactions, the transverse momentum integrals diverge in the ultraviolet which in turn leads to scaling violations in the structure function.

Taking the bare and dressed quarks to be massless, we arrive at

$$\sum_{\sigma_1,\rho_2} \int d^2\kappa^\perp |\Phi_2^{\sigma_1,\rho_2}(x,\kappa^\perp,1-x,-\kappa^\perp)|^2 = \frac{g^2}{(2\pi)^3} C_f \frac{1+x^2}{1-x} \int d^2\kappa^\perp \frac{1}{(\kappa^\perp)^2} \tag{4.11}$$

where $C_f = (N^2-1)/2N$. Recalling that $|\Phi_2(x,\kappa^\perp)|^2$ is the probability density to find a quark with momentum fraction x and relative transverse momentum $\kappa^\perp$ in a parent quark, we define the probability density to find a quark with momentum fraction x inside a parent quark as the splitting function

$$P_{qq}(x) = C_f \frac{1+x^2}{1-x}. \tag{4.12}$$

Clearly, the probability density to find a gluon with momentum fraction x inside a parent quark is defined as the splitting function

$$P_{Gq}(x) = C_f \frac{1+(1-x)^2}{x}. \tag{4.13}$$

The transverse momentum integral in Eq. (4.11) is divergent at both limits of integration. We regulate the lower limit by μ and the upper limit by Q. Thus we have

$$\frac{F_{2(q)}^q(x,Q^2)}{x} = \mathcal{N}_q\left[\delta(1-x) + \frac{\alpha_s}{2\pi} C_f \frac{1+x^2}{1-x} \ln\frac{Q^2}{\mu^2}\right]. \tag{4.14}$$

The normalization condition reads

$$\mathcal{N}_q\left[1 + \frac{\alpha_s}{2\pi} C_f \int dx \frac{1+x^2}{1-x} \ln\frac{Q^2}{\mu^2}\right] = 1. \tag{4.15}$$

Within the present approximation (valid only up to α_s),

$$\mathcal{N}_q = 1 - \frac{\alpha_s}{2\pi} C_f \int dx \frac{1+x^2}{1-x} \ln\frac{Q^2}{\mu^2}. \tag{4.16}$$

In the second term we recognize the familiar expression of wave function correction of the state n in old fashioned perturbation theory, namely, $\Sigma_m' |\langle m|V|n\rangle|^2/(E_n - E_m)^2$.

Thus, to order α_s,

$$\frac{F_{2(q)}^q(x,Q^2)}{x} = \delta(1-x) + \frac{\alpha_s}{2\pi}\ln\frac{Q^2}{\mu^2} \times C_f \left[\frac{1+x^2}{1-x} - \delta(1-x)\int dy \frac{1+y^2}{1-y}\right]. \tag{4.17}$$

Note that Eq. (4.17) can also be written as

$$\frac{F_{2(q)}^q(x,Q^2)}{x} = \delta(1-x) + \frac{\alpha_s}{2\pi} C_f \ln\frac{Q^2}{\mu^2} \times \left[\frac{1+x^2}{(1-x)_+} + \frac{3}{2}\delta(1-x)\right], \tag{4.18}$$

which is a more familiar expression. By construction, $|\Phi_2(x,\kappa^\perp)|^2$ is a probability density. However, this function is singular as $x \to 1$ (gluon longitudinal momentum fraction approaching zero). To get a finite probability density we have to introduce a cutoff ϵ ($x_{gluon}>\epsilon$), for example. In a physical cross section, this ϵ cannot appear and here we have an explicit example of this cancellation. Note that the function

$$\tilde{P}_{qq}=C_f\frac{1+x^2}{(1-x)_+}+\frac{3}{2}\delta(1-x)$$

does not have a probabilistic interpretation since it includes contributions from virtual gluon emission. This is immediately transparent from the relation

$$\int dx\tilde{P}_{qq}(x)=0. \tag{4.19}$$

We also note that the divergence arising from small transverse momentum (the familiar mass singularity) cannot be handled properly in the present calculation. This is to be contrasted with the calculation of the physical hadron structure function where the mass singularities can be properly absorbed into the non-perturbative part of the structure function.

Let us now explicitly check the longitudinal momentum sum rule for the dressed quark. According to the sum rule,

$$\int_0^1 dx[F^q_{2(q)}(x)+F^q_{2(G)}(x)]=\frac{1}{2(P^+)^2}\ {}_q\langle P|\theta^{++}(0)|P\rangle_q=1. \tag{4.20}$$

Explicit calculations show that the gluon structure function for the dressed quark target $F^q_{2(G)}$ is given by

$$F^q_{2(G)}=\frac{\alpha_s}{2\pi}\ln\frac{Q^2}{\mu^2}C_f x\frac{1+(1-x)^2}{x}. \tag{4.21}$$

From Eqs. (4.18) and (4.21) it follows that

$$\int_0^1 dxF^q_2(x)=\int_0^1 dx[F^q_{2(q)}(x)+F^q_{2(G)}(x)]=1 \tag{4.22}$$

since

$$\int_0^1 dx\ \ x[\tilde{P}_{qq}(x)+P_{Gq}(x)]=0. \tag{4.23}$$

B. Dressed quark structure function from the transverse component

From Bjorken-Johnson-Low (BJL) expansion and light-front current algebra, it also follows that

$$\frac{F_2(x,Q^2)}{x}=\frac{1}{4\pi}\int d\eta e^{-i\eta x}\overline{V}_1=\frac{1}{4\pi P^{i\perp}}\int d\eta e^{-i\eta x}{}_p \times\langle ks|\overline{\psi}(\xi^-)\gamma^i\psi(0)-\overline{\psi}(0)\gamma^i\psi(\xi^-)|ks\rangle_p. \tag{4.24}$$

From Eqs. (4.1) and (4.24), it follows that the structure function F_2 can be expressed not only as a matrix element of the plus component of the bilocal vector current, but also the matrix element of the transverse component of the bilocal current. Next we extract the structure function $F_2(x)$ from the transverse component of the bilocal vector current [Eq. (4.24)]. The operator that appears in this equation is

$$\overline{\psi}(y)\gamma^\perp\psi(0)=(\psi^+)^\dagger(y)\alpha^\perp\psi^-(0)+(\psi^-)^\dagger(y)\alpha^\perp\psi^+(0). \tag{4.25}$$

The constrained fermion field $\psi^-=(1/i\partial^+)[\alpha^\perp\cdot(i\partial^\perp+gA^\perp)+\gamma^0 m]\psi^+$. Hence the operator in the above equation appears to be higher twist (twist 3). Without loss of generality we take the $\perp$ direction along the x axis. The structure function can be explicitly written as

$$\frac{F_2(x,Q^2)}{x}=\frac{1}{8\pi}\frac{P^+}{P^1}\int dy^- e^{-iP^+y^-x/2}\langle P|\xi^\dagger(y) \times[O_m+O_{k^\perp}+O_g]\xi(0)|P\rangle+\text{H.c.}, \tag{4.26}$$

with

$$O_m=im\frac{1}{i\partial^+}\sigma^2,$$

$$O_{k^\perp}=\frac{1}{i\partial^+}[i\partial^1-\sigma^3\partial^2],$$

$$O_g=g\frac{1}{i\partial^+}[A^1+i\sigma^3A^2]. \tag{4.27}$$

First consider contributions from the operator O_m. Only potential non-vanishing contributions are from the diagonal matrix elements for the single quark state and the quark-gluon state. The single quark matrix element vanishes because σ^2 flips helicity. The diagonal contribution from the quark-gluon state also vanishes because of the cancellation between the two terms in Eq. (4.25). Thus the contribution from O_m to F_2 vanishes.

Next consider contributions from the operator $O_{k^\perp}$. Explicit evaluation leads to

$$\frac{F_2(x,Q^2)}{x}\bigg|_{k^\perp}=\mathcal{N}_q\bigg\{\delta(1-x)+\frac{1}{P^1}\sum_{\sigma_1,\lambda_2}\int dx_2\int d^2\kappa_1^\perp\int d^2\kappa_2^\perp\,\delta(1-x-x_2)\times\delta^2(\kappa_1^\perp+\kappa_2^\perp)|\Phi_2^{\sigma_1,\lambda_2}(x,\kappa_1^\perp;x_2,\kappa_2^\perp)|^2\frac{\kappa_1^1+xP^1}{x}\bigg\}$$

$$=\mathcal{N}_q\bigg\{\delta(1-x)+\sum_{\sigma_1,\lambda_2}\int dx_2\int d^2\kappa_1^\perp\int d^2\kappa_2^\perp\,\delta(1-x-x_2)\times\delta^2(\kappa_1^\perp+\kappa_2^\perp)|\Phi_2^{\sigma_1,\lambda_2}(x,\kappa_1^\perp;x_2,\kappa_2^\perp)|^2\bigg\},\tag{4.28}$$

since $\int d^2\kappa_1^\perp\kappa_1^1|\psi_2|^2=0$ as a consequence of rotational invariance. Equation (4.28) gives the same result as Eq. (4.10).

Last, we evaluate the contribution from the quark-gluon correlation operator O_g:

$$\frac{F_2(x,Q^2)}{x}\bigg|_g=\frac{1}{2}\frac{g}{\sqrt{2(2\pi)^3}}\frac{1}{P^1}\sum_{\sigma_1,\lambda_2}\int\frac{dy}{\sqrt{1-y}}d^2\kappa^\perp\chi_\sigma^\dagger[\epsilon_{\lambda_2}^1+i\sigma^3\epsilon_{\lambda_2}^2]\chi_{\sigma_1}\Phi_2^{\sigma_1,\lambda_2}(y,\kappa^\perp;1-y,-\kappa^\perp)+\text{H.c.}=0.\tag{4.29}$$

This is because the quark-gluon amplitude Φ_2 has two types of terms: (a) terms proportional to the quark mass m accompanied by $\sigma^\perp$ which vanish because $\chi_\sigma^\dagger\sigma^\perp\chi_\sigma=0$ and (b) terms proportional to $\kappa^\perp$ which vanish because of rotational symmetry. Thus the contribution from O_g to the structure function vanishes.

From Eq. (4.28) and Eq. (4.10), it follows that the structure function extracted from Eq. (4.24) has the same result given by Eq. (4.17) and hence the same parton interpretation as that extracted from Eq. (4.1). Thus we have explicitly demonstrated the parton interpretation of the transverse component of the bilocal vector matrix element in unpolarized deep inelastic scattering. The classification of twist in DIS or other hadronic collision processes based on the different components of light-front bilocal operators seems unreliable.

C. Dressed gluon structure function

The dressed gluon state can be expanded as

$$|P\lambda\rangle_g=\sqrt{\mathcal{N}_g}\bigg\{a^\dagger(P,\lambda)|0\rangle+\sum_{\sigma_1\sigma_2}\int\frac{dk_1^+d^2k_1^\perp}{\sqrt{2(2\pi)^3k_1^+}}\frac{dk_2^+d^2k_2^\perp}{\sqrt{2(2\pi)^3k_2^+}}\sqrt{2(2\pi)^3P^+}\delta^3(P-k_1-k_2)\psi_{2(q\bar{q})}\times(P,\lambda|k_1\sigma_1,k_2\sigma_2)b^\dagger(k_1\sigma_1)d^\dagger(k_2,\sigma_2)|0\rangle+\frac{1}{2}\sum_{\lambda_1\lambda_2}\int\frac{dk_1^+d^2k_1^\perp}{\sqrt{2(2\pi)^3k_1^+}}\frac{dk_2^+d^2k_2^\perp}{\sqrt{2(2\pi)^3k_2^+}}\sqrt{2(2\pi)^3P^+}\delta^3(P-k_1-k_2)\psi_{2(gg)}\times(P,\lambda|k_1\lambda_1,k_2\lambda_2)a^\dagger(k_1\lambda_1)a^\dagger(k_2,\lambda_2)|0\rangle\bigg\}.\tag{4.30}$$

The factor of $\frac{1}{2}$ is the symmetry factor for identical bosons.

As before we introduce the boost invariant amplitudes

$$\sqrt{P^+}\psi_{2(q\bar{q})}(k_i^+,k_i^\perp)=\Phi_{2(q\bar{q})}(x_i,\kappa_i^\perp),$$

$$\sqrt{P^+}\psi_{2(gg)}(k_i^+,k_i^\perp)=\Phi_{2(gg)}(x_i,\kappa_i^\perp).\tag{4.31}$$

The $q\bar{q}$ wave function of the dressed gluon is given by

$$\Phi_2^{s_1,s_2}(x,\kappa^\perp;1-x,-\kappa^\perp)=\frac{1}{\left[m^2-\frac{m^2+(\kappa^\perp)^2}{x(1-x)}\right]}\frac{g}{\sqrt{2(2\pi)^3}}T^a\chi_{s_1}^\dagger\times\left[\frac{\sigma^\perp\cdot\kappa^\perp}{x}\sigma^\perp-\sigma^\perp\frac{\sigma^\perp\cdot\kappa^\perp}{1-x}-i\frac{m}{x(1-x)}\sigma^\perp\right]\times\chi_{-s_2}(\epsilon_{\rho_2}^\perp)^*.\tag{4.32}$$

The gg wave function of the dressed gluon state given by

$$\Phi_{2(gg)}(x,\kappa^{\perp}) = \frac{g}{\sqrt{2(2\pi)^3}} 2if^{abc} \frac{x(1-x)}{(\kappa^{\perp})^2} \frac{1}{\sqrt{x}} \frac{1}{\sqrt{1-x}} \epsilon^{j}_{\lambda_1} \epsilon^{l}_{\lambda_2} (\epsilon^{i}_{\lambda})^* \times \left[-\kappa^i \delta_{lj} + \frac{\kappa^j}{x} \delta_{il} + \frac{\kappa^l}{1-x} \delta_{ij} \right]. \tag{4.33}$$

The contribution from the first term in Eq. (4.30) to the gluon structure function for the dressed gluon target is given by

$$F^{g(1)}_{2(G)} = \delta(1-x). \tag{4.34}$$

The contribution to the gluon structure function from the $q\bar{q}$ component of the dressed gluon state is a disconnected contribution which we omit. The contribution to the gluon structure function from the gg component of the dressed gluon state is given by

$$F^{g(3)}_{2(G)} = \frac{\alpha_s}{2\pi} \ln\frac{Q^2}{\mu^2} 2N \left[\frac{x}{1-x} + \frac{1-x}{x} + x(1-x) \right] x. \tag{4.35}$$

We define the probability density to a find a gluon with momentum fraction x in the dressed gluon, $P_{GG}(x)$, by

$$P_{GG}(x) = 2N \left[\frac{x}{1-x} + \frac{1-x}{x} + x(1-x) \right]. \tag{4.36}$$

Collecting all three contributions together, we have

$$F^{g}_{2(G)}(x,Q^2) = \mathcal{N}_g \left[\delta(1-x) + \frac{\alpha_s}{2\pi} \ln\frac{Q^2}{\mu^2} 2N \times \left(\frac{x}{1-x} + \frac{1-x}{x} + x(1-x) \right) x \right]. \tag{4.37}$$

The coefficient $\mathcal{N}_g$ is determined from the longitudinal momentum sum rule for the dressed gluon target; namely, we require

$$\int_0^1 dx F^g_2(x) = \int_0^1 dx [F^g_{2(G)}(x) + F^g_{2(q)}(x)] = \frac{1}{2(P^+)^2} \;{}_g\langle P | \theta^{++}(0) | P \rangle_g = 1. \tag{4.38}$$

Thus we need to evaluate

$$\frac{1}{2(P^+)^2} {}_g\langle P | \theta_q^{++}(0) | P \rangle_g . \tag{4.39}$$

Explicit evaluation leads to

$$\frac{1}{2(P^+)^2} {}_g\langle P | \theta_q^{++}(0) | P \rangle_g = \frac{\alpha_s}{2\pi} \ln\frac{Q^2}{\mu^2} \frac{1}{2} \times \int dx [x^2 + (1-x)^2] \mathcal{N}_g . \tag{4.40}$$

We define the probability density to find a quark with momentum fraction x in a dressed gluon as the splitting function $P_{qG}(x)$:

$$P_{qG}(x) = \frac{1}{2} [x + (1-x)^2]. \tag{4.41}$$

From Eq. (4.38) we arrive at

$$\mathcal{N}_g \left[1 + \frac{\alpha_s}{2\pi} \ln\frac{Q^2}{\mu^2} \int dx \left\{ [x^2 + (1-x)^2] + 2N \left(\frac{x}{1-x} + \frac{1-x}{x} + x(1-x) \right) x \right\} \right] = 1. \tag{4.42}$$

Thus to order α_s, we have

$$\mathcal{N}_g = 1 - \frac{\alpha_s}{2\pi} \ln\frac{Q^2}{\mu^2} \int dx \left\{ [x^2 + (1-x)^2] + 2N \left(\frac{x}{1-x} + \frac{1-x}{x} + x(1-x) \right) x \right\}. \tag{4.43}$$

Correspondingly, the complete dressed gluon structure function is given by

$$F^g_{2(G)}(x,Q^2) = \delta(1-x) + \frac{\alpha_s}{2\pi} \ln\frac{Q^2}{\mu^2} \left\{ 2N \left[\left(\frac{x}{(1-x)_+} + \frac{1-x}{x} + x(1-x) \right) x + \frac{11}{12} \delta(1-x) \right] - \frac{1}{3} \delta(1-x) \right\}. \tag{4.44}$$

Including the end point $(x \to 1)$ contributions, we define

$$\widetilde{P}_{GG}(x) = 2N \left\{ \left(\frac{x}{(1-x)_+} + \frac{1-x}{x} + x(1-x) \right) + \frac{11}{12} \delta(1-x) \right\} - \frac{1}{3} \delta(1-x). \tag{4.45}$$

This is indeed the first calculation of the gluon splitting function from the dressed gluon in the literature. Lepage and Brodsky [3] mentioned a Fock space calculation of DIS splitting functions but for gluon splitting function; they just quoted the result from [7]. It is easy to verify that

$$\int_0^1 dx \;\; x [2P_{qG}(x) + \widetilde{P}_{GG}(x)] = 0. \tag{4.46}$$

V. STRUCTURE FUNCTION OF THE HADRON: PARTON PICTURE, SCALE EVOLUTION AND FACTORIZATION

As we have schematically discussed in Sec. II (also see [2]), the non-perturbative contribution to the structure functions and the scaling violations from the perturbative QCD corrections can be unified and treated in the same framework in our formalism. In this section, we shall address the issues associated with scaling violations in the structure function of the "meson-like" bound state and explicitly demonstrate the validity of factorization outlined in Sec. II.

A. Parton picture

Let us first discuss the emergence of the parton picture for the structure function of a composite state. We expand the state $|P\rangle$ for the $q\bar{q}$ bound state in terms of the Fock components $q\bar{q},\ q\bar{q}g,\ \dots$ as follows:

$$|P\rangle=\sum_{\sigma_1,\sigma_2}\int\frac{dk_1^+d^2k_1^\perp}{\sqrt{2(2\pi)^3k_1^+}}\int\frac{dk_2^+d^2k_2^\perp}{\sqrt{2(2\pi)^3k_2^+}}\psi_2(P|k_1,\sigma_1;k_2,\sigma_2)\sqrt{2(2\pi)^3P^+}\,\delta^3(P-k_1-k_2)b^\dagger(k_1,\sigma_1)d^\dagger(k_2,\sigma_2)|0\rangle$$
$$+\sum_{\sigma_1,\sigma_2,\lambda_3}\int\frac{dk_1^+d^2k_1^\perp}{\sqrt{2(2\pi)^3k_1^+}}\int\frac{dk_2^+d^2k_2^\perp}{\sqrt{2(2\pi)^3k_2^+}}\int\frac{dk_3^+d^2k_3^\perp}{\sqrt{2(2\pi)^3k_3^+}}\psi_3$$
$$\times(P|k_1,\sigma_1;k_2,\sigma_2;k_3,\lambda_3)\sqrt{2(2\pi)^3P^+}\,\delta^3(P-k_1-k_2-k_3)b^\dagger(k_1,\sigma_1)d^\dagger(k_2,\sigma_2)a^\dagger(k_3,\lambda_3)|0\rangle+\cdots\ . \tag{5.1}$$

Here ψ_2 is the probability amplitude to find a quark and an antiquark in the meson, ψ_3 is the probability amplitude to find a quark, antiquark and a gluon in the meson, etc.

As in Sec. IV we evaluate the expression in Eq. (4.1) explicitly. The contribution from the first term (from the quark), in terms of the amplitudes

$$\sqrt{P^+}\psi_2(k_i^+,k_i^\perp)=\Phi_2(x_i,\kappa_i^\perp),$$
$$P^+\psi_3(k_i^+,k_i^\perp)=\Phi_3(x_i,\kappa_i^\perp), \tag{5.2}$$

and so on, is

$$\frac{F_2^q(x)}{x}=\sum_{\sigma_1,\sigma_2}\int dx_2\int d^2\kappa_1^\perp\int d^2\kappa_2^\perp\,\delta(1-x-x_2)\,\delta^2(\kappa_1+\kappa_2)|\Phi_2^{\sigma_1,\sigma_2}(x,\kappa_1^\perp;x_2\kappa_2^\perp)|^2$$
$$+\sum_{\sigma_1,\sigma_2,\lambda_3}\int dx_2\int dx_3\int d^2\kappa_1^\perp\int d^2\kappa_2^\perp\int d^2\kappa_3^\perp\,\delta(1-x-x_2-x_3)\,\delta^2(\kappa_1+\kappa_2+\kappa_3)|\Phi_3^{\sigma_1,\sigma_2,\lambda_3}(x,\kappa_1^\perp;x_2,\kappa_2^\perp;x_3,\kappa_3^\perp)|^2$$
$$+\cdots\ . \tag{5.3}$$

Again, the partonic interpretation of the F_2 structure function is manifest in this expression. Using different techniques and approximations, the same result has been also obtained by Brodsky and Lepage [3].

The contribution to the structure function from the second term in Eq. (4.1) is

$$\frac{F_2^{\bar{q}}(x)}{x}=\sum_{\sigma_1,\sigma_2}\int dx_2\int d^2\kappa_1^\perp\int d^2\kappa_2^\perp\,\delta(1-x-x_2)\,\delta^2(\kappa_1+\kappa_2)|\Phi_2^{\sigma_1,\sigma_2}(x_2,\kappa_2^\perp;x,\kappa_1^\perp)|^2$$
$$+\sum_{\sigma_1,\sigma_2,\lambda_3}\int dx_2\int dx_3\int d^2\kappa_1^\perp\int d^2\kappa_2^\perp\int d^2\kappa_3^\perp\,\delta(1-x-x_2-x_3)\,\delta^2(\kappa_1+\kappa_2+\kappa_3)|\Phi_3^{\sigma_1,\sigma_2,\lambda_3}(x_2,\kappa_2^\perp;x,\kappa_1^\perp;x_3,\kappa_3^\perp)|^2$$
$$+\cdots\ . \tag{5.4}$$

The normalization condition guarantees that

$$\int dx\left[\frac{F_2^q(x)}{x}+\frac{F_2^{\bar{q}}(x)}{x}\right]=2 \tag{5.5}$$

which reflects the fact that there are two valence particles in the meson. Since the bilocal current component $\bar{\mathcal{J}}^+$ involves only fermions explicitly, we appear to have missed the contributions from the gluon constituents altogether. The gluonic contribution to the structure function F_2 is most easily calculated by studying the hadron expectation value of the conserved longitudinal momentum operator P^+.

From the normalization condition, it is clear that the valence distribution receives contributions from the amplitudes $\Phi_2, \Phi_3, \ldots$ at any scale μ. This has interesting phenomenological implications. In the model for the meson with only a quark-antiquark pair of equal mass, the valence distribution function will peak at $x=\frac{1}{2}$. If there are more than just the two particles in the system, the resulting valence distribution will no longer be symmetric about $x=\frac{1}{2}$ as a simple consequence of longitudinal momentum conservation.

Equation (5.3) as it stands is useful only when the bound state solution in QCD is known in terms of the multi-parton wave functions. The wave functions, as they stand, span both the perturbative and non-perturbative sectors of the theory. Great progress in the understanding of QCD in the high energy sector has been made in the past by separating the soft (non-perturbative) and hard (perturbative) regions of QCD via the machinery of factorization. It is of interest to see under what circumstances a factorization occurs in the formal result of Eq. (5.3) and a perturbative picture of scaling violations emerges finally. We shall explicitly address this issue in the following section.

B. Perturbative picture of scaling violations in a bound state

To address the issue of scaling violations in the structure function of the ''meson-like'' bound state, it is convenient to separate the momentum space into low-energy and high-energy sectors. Such a separation has been introduced in the past in the study of the renormalization of bound state equations [15] in light-front field theory. The two sectors are formally defined by introducing cutoff factors in the momentum space integrals. How to cut off the momentum integrals in a sensible and convenient way in light-front theory is a subject under active research at the present time. Complications arise because of the possibility of large energy divergences from both small k^+ and large $k^\perp$ regions. In the following we investigate only the effects of logarithmic divergences arising from large transverse momenta, ignoring the subtleties arising from both small x $(x\to 0)$ and large x $(x\to 1)$ regions and subsequently use simple transverse momentum cutoff. For complications arising from $x\to 1$ region see Ref. [3].

1. Scale separation

We define the soft region to be $\kappa^\perp<\mu$ and the hard region to be $\mu<\kappa^\perp<\Lambda$, where μ serves as a factorization scale which separates soft and hard regions. Since it is an intermediate scale introduced artificially purely for convenience, the physical structure function should be independent of μ. The multi-parton amplitude Φ_2 is a function of a single relative transverse momentum $\kappa_1^\perp$ and we define

$$\Phi_2=\begin{cases}\Phi_2^s, & 0<\kappa_1^\perp<\mu,\\ \Phi_2^h, & \mu<\kappa_1^\perp<\Lambda.\end{cases} \tag{5.6}$$

The amplitude Φ_3 is a function of two relative momenta, $\kappa_1^\perp$ and $\kappa_2^\perp$ and we define

$$\Phi_3=\begin{cases}\Phi_3^{ss}, & 0<\kappa_1^\perp,\kappa_2^\perp<\mu,\\ \Phi_3^{sh}, & 0<\kappa_1^\perp<\mu,\mu<\kappa_2^\perp<\Lambda,\\ \Phi_3^{hs}, & \mu<\kappa_1^\perp<\Lambda,0<\kappa_2^\perp<\mu,\\ \Phi_3^{hh}, & \mu<\kappa_1^\perp,\kappa_2^\perp<\Lambda.\end{cases} \tag{5.7}$$

Let us consider the quark distribution function $q(x)=F_2(x)/x$ defined in Eq. (5.3). In presence of the ultraviolet cutoff Λ, $q(x)$ depends on Λ and schematically we have

$$q(x,\Lambda^2)=\sum\int_0^\Lambda\Phi_2^2+\sum\int_0^\Lambda\int_0^\Lambda\Phi_3^2. \tag{5.8}$$

For convenience, we write

$$q(x,\Lambda^2)=q_2(x,\Lambda^2)+q_3(x,\Lambda^2), \tag{5.9}$$

where the subscripts 2 and 3 denote the two-particle and three-particle contributions respectively. Thus, schematically we have

$$q(x,\Lambda^2)=q(x,\mu^2)+\sum\int_\mu^\Lambda|\Phi_2^h|^2+\sum\int_0^\mu\int_\mu^\Lambda|\Phi_3^{sh}|^2 +\sum\int_\mu^\Lambda\int_0^\mu|\Phi_3^{hs}|^2+\sum\int_\mu^\Lambda\int_\mu^\Lambda|\Phi_3^{hh}|^2. \tag{5.10}$$

We investigate the contributions from the amplitudes Φ_3^{sh} and Φ_3^{hs} to order α_s in the following.

2. Dressing with one gluon

We substitute the Fock expansion, Eq. (5.1), in Eq. (4.3) and make projection with a three particle state $b^\dagger(k_1,\sigma_1)d^\dagger(k_2,\sigma_2)a^\dagger(k_3,\sigma_3)|0\rangle$ from the left. In terms of the amplitudes Φ_2,Φ_3, we get

$$\Phi_3^{\sigma_1\sigma_2\lambda_3}(x,\kappa_1;x_2,\kappa_2;1-x-x_2,\kappa_3)=\mathcal{M}_1+\mathcal{M}_2, \tag{5.11}$$

where the amplitudes

$$\mathcal{M}_1=\frac{1}{E}(-)\frac{g}{\sqrt{2(2\pi)^3}}T^a\frac{1}{\sqrt{1-x-x_2}}\;V_1\Phi_2^{\sigma_1'\sigma_2}\times(1-x_2,-\kappa_2^\perp;x_2,\kappa_2^\perp) \tag{5.12}$$

and

$$\mathcal{M}_2=\frac{1}{E}\frac{g}{\sqrt{2(2\pi)^3}}T^a\frac{1}{\sqrt{1-x-x_2}}\;V_2\Phi_2^{\sigma_1\sigma_2'}\times(x,\kappa_1^\perp;1-x,-\kappa_1^\perp) \tag{5.13}$$

with the energy denominator

$$E=\left[M^2-\frac{m^2+(\kappa_1^\perp)^2}{x}-\frac{m^2+(\kappa_2^\perp)^2}{x_2}-\frac{(\kappa_3^\perp)^2}{1-x-x_2}\right] \tag{5.14}$$

and the vertices

$$V_1 = \chi^\dagger_{\sigma_1} \sum_{\sigma_1'} \left[\frac{2\kappa_3^\perp}{1-x-x_2} - \frac{(\sigma^\perp \cdot \kappa_1^\perp - im)}{x} \sigma^\perp + \sigma^\perp \frac{(\sigma^\perp \cdot \kappa_2^\perp - im)}{1-x_2} \right] \chi_{\sigma_1'} \cdot (\epsilon^\perp_{\lambda_1})^* \tag{5.15}$$

and

$$V_2 = \chi^\dagger_{-\sigma_2} \sum_{\sigma_2'} \left[\frac{2\kappa_3^\perp}{1-x-x_2} - \sigma^\perp \frac{(\sigma^\perp \cdot \kappa_2^\perp - im)}{x_2} + \frac{(\sigma^\perp \cdot \kappa_1^\perp - im)}{1-x} \sigma^\perp \right] \chi_{-\sigma_2'} \cdot (\epsilon^\perp_{\lambda_1})^*. \tag{5.16}$$

3. Perturbative analysis

For $\kappa_1^\perp$ hard and $\kappa_2^\perp$ soft, $\kappa_1^\perp + \kappa_2^\perp \approx \kappa_1^\perp$ and the multiple transverse momentum integral over Φ_3 factorizes into two independent integrals and the longitudinal momentum fraction integrals become convolutions. The contribution from $\mathcal{M}_1$ to Φ_3 is

$$\Phi_{3,1}^{\sigma_1,\sigma_2,\Lambda_3}(x,\kappa_1^\perp; x_2,\kappa_2^\perp; 1-x-x_2, -\kappa_2^\perp)$$
$$= -\frac{g}{\sqrt{2(2\pi)^3}} T^a x \sqrt{\frac{1-x-x_2}{1-x_2}} \frac{1}{(\kappa_1^\perp)^2} \chi^\dagger_{\sigma_1} \times \sum_{\sigma_1'} \left[\frac{2\kappa_1^\perp}{1-x-x_2} + \frac{\sigma^\perp \cdot \kappa_1^\perp}{x} \sigma^\perp \right] \chi'_{\sigma_1} \cdot (\epsilon^\perp_{\lambda_1})^* \Phi_2^{\sigma_1',\sigma_2} \times (1-x_2, -\kappa_2^\perp; x_2, \kappa_2^\perp). \tag{5.17}$$

Thus the contribution from $\mathcal{M}_1$ to the structure function is

$$\sum \int |\Phi_{3,1}^{hs}|^2 = \frac{\alpha_s}{2\pi} C_f \ln\frac{\Lambda^2}{\mu^2} \int_x^1 \frac{dy}{y} P_{qq}\left(\frac{x}{y}\right) q_2(y,\mu^2), \tag{5.18}$$

where

$$P_{qq}\left(\frac{x}{y}\right) = \frac{1+\left(\frac{x}{y}\right)^2}{1-\frac{x}{y}}. \tag{5.19}$$

For the configuration $\kappa_1^\perp$ hard, $\kappa_2^\perp$ soft, the contribution from $\mathcal{M}_2$ does not factorize and the asymptotic behavior of the integrand critically depends on the asymptotic behavior of the two-particle wave function Φ_2. To determine this behavior, we have to analyze the bound state equation which shows that for large transverse momentum $\Phi_2(\kappa^\perp) \approx 1/(\kappa^\perp)^2$. Thus the contribution from $\mathcal{M}_2$ for scale evolution is suppressed by the bound state wave function. Analysis of the interference terms (between $\mathcal{M}_1$ and $\mathcal{M}_2$) shows that their contribution also is suppressed by the bound state wave function.

For the configuration $\kappa_1^\perp$ soft, $\kappa_2^\perp$ hard, contributions from $\mathcal{M}_1$ and the interference terms are suppressed by the wave function. The contribution from $\mathcal{M}_2$ factorizes both in transverse and longitudinal space and generates a pure wave function renormalization contribution

$$\sum \int |\Phi_{3,2}^{sh}|^2 = \frac{\alpha_s}{2\pi} C_f \ln\frac{\Lambda^2}{\mu^2} \int_0^1 dy \frac{1+y^2}{1-y} q_2(x,\mu^2). \tag{5.20}$$

We have seen that even though the multi-parton contributions to the structure function involve both coherent and incoherent phenomena, in the hard region coherent effects are suppressed by the wave function.

4. Corrections from the normalization condition

In the dressed quark calculation, we have seen that the singularity that arises as $x \to 1$ from real gluon emission is canceled by the correction from the normalization of the state (virtual gluon emission contribution from wave function renormalization). In the meson bound state calculation, so far we have studied the effects of a hard real gluon emission. In this section we study the corrections arising from the normalization condition of the quark distribution in the composite bound state.

Collecting all the terms arising from the hard gluon emission contributing to the quark distribution function, we have

$$q(x,\Lambda^2) = q_2(x,\mu^2) + q_3(x,\mu^2) + \frac{\alpha_s}{2\pi} C_f \ln\frac{\Lambda^2}{\mu^2} \int_x^1 \frac{dy}{y} P_{qq}\left(\frac{x}{y}\right) q_2(y,\mu^2) + \frac{\alpha_s}{2\pi} C_f \ln\frac{\Lambda^2}{\mu^2} q_2(x,\mu^2) \int dy P(y). \tag{5.21}$$

We have a similar expression for the antiquark distribution function.

The normalization condition on the quark distribution function should be such that there is one valence quark in the bound state at any scale Q. We choose the factorization scale $\mu = Q_0$. Let us first set the scale $\Lambda = Q_0$. Then we have (in the truncated Fock space)

$$\int_0^1 dx \;\; q_2(x,Q_0^2) + \int_0^1 dx \;\; q_3(x,Q_0^2) = 1. \tag{5.22}$$

Next set the scale $\Lambda = Q$. We still require

$$\int_0^1 dx \;\; q_2(x,Q^2) + \int_0^1 dx \;\; q_3(x,Q^2) = 1. \tag{5.23}$$

We note that the evolution of q_3 requires an extra hard gluon which is not available in the truncated Fock space. Thus in the present approximation $q_3(x,Q^2) = q_3(x,Q_0^2)$.

Carrying out the integration explicitly, we arrive at

$$\int_0^1 dx \;\; q_2(x,Q_0^2)\left[1+\frac{2\alpha_s}{2\pi}C_f \ln\frac{Q^2}{Q_0^2}\int dy P(y)\right] + \int_0^1 dx \;\; q_3(x,Q^2)=1. \tag{5.24}$$

Thus we face the necessity to ''renormalize'' our quark distribution function. Let us define a renormalized quark distribution function

$$q_2^R(x,Q_0^2)=q_2(x,Q_0^2)\left[1+2\frac{\alpha_s}{2\pi}C_f \ln\frac{Q^2}{Q_0^2}\int_0^1 dy P(y)\right] \tag{5.25}$$

so that, to order α_s,

$$\int_0^1 dx \;\; q_2^R(x,Q_0^2)+\int_0^1 dx \;\; q_3(x,Q_0^2)=1. \tag{5.26}$$

We have

$$q_2(x,Q_0^2)=q_2^R(x,Q_0^2)\left[1-2\frac{\alpha_s}{2\pi}C_f \ln\frac{Q^2}{Q_0^2}\int_0^1 dy P(y)\right]. \tag{5.27}$$

Collecting all the terms, to order α_s, we have the normalized quark distribution function

$$q(x,Q^2)=q_2^R(x,Q_0^2)+\frac{\alpha_s}{2\pi}C_f \ln\frac{Q^2}{Q_0^2}\int_0^1 dy \;\; q_2^R(y,Q_0^2) \times\int_0^1 dz \;\; \delta(zy-x)\tilde{P}(z)+q_3(x,Q^2) \tag{5.28}$$

with $\tilde{P}(z)=P(z)-\delta(z-1)\int_0^1 dy P(y)$.

We see that just as in the dressed quark case, the singularity arising as $x\to 1$ from real gluon emission is canceled in the quark distribution function once the normalization condition is properly taken into account. From this derivation we begin to recognize the emergence of the Altarelli-Parisi evolution equation.

C. Summary

In this section we have carried out an analysis of the scale evolution of structure functions of a meson-like composite system. We have separated the parton transverse momenta into soft and hard parts. The three body wave function which is a function of two relative momenta has soft, hard and mixed components. The mixed components of the three body wave function which are functions of soft and hard momenta are responsible for the scale evolution of the soft part of the structure function in perturbation theory.

In the analysis with wave functions, there are two contributions to the three body wave function: one where the gluon is absorbed by the quark and second where the gluon is absorbed by the anti-quark (spectator). There appears a non-vanishing contribution when the hard gluon is absorbed by the anti-quark. This corresponds to the transition caused by the interaction Hamiltonian when the active parton remains soft, while a hard spectator makes transition to a soft spectator state. This leads to wave function renormalization of the spectator anti-quark but this is eventually canceled by the normalization condition as discussed in detail in Sec. V B 4. This justifies *a posteriori* the prescription given in Sec. II that we need to keep only those terms in $P^{-(H)}$ which cause transitions involving the active parton.

In the wave function analysis, there are also contributions that are omitted *a priori* in the calculational scheme which lead to factorization in Sec. II. All of these contributions are suppressed by the asymptotic behavior of the bound state wave function as we have explicitly shown. In summary, the detailed analysis carried out with the help of multi-parton wave functions in Sec. V B justifies the approximations made in Sec. II which lead to the emergence of factorization to all orders in perturbation theory and to the simple scale evolution picture.

VI. CONCLUSION

We have shown that a perturbative analysis in the light-front Hamiltonian formalism leads to the factorization scheme proposed in Ref. [2]. It is shown that the scaling violations due to perturbative QCD corrections can be rather easily addressed in this framework by simply replacing the hadron target by a dressed parton target and then carrying out a systematic expansion in the coupling constant α_s based on the perturbative QCD expansion of the dressed parton target. The calculational procedure utilizes techniques of old-fashioned perturbation theory, the main ingredients of which are transition matrix elements and energy denominators.

The main advantage of the present method can be summarized as follows. The bilocal currents are defined in the light-front gauge $A^+=0$, and since the bilocality is only in the light-front longitudinal (x^-) direction, the path-ordered exponential between fermion field operators in the bilocal current is replaced by unity (*irrespective* of which component of the current is considered). This results in an extremely simplified operator structure and a straightforward parton picture. Further, the calculations do not employ Feynman propagators and as a result we encounter neither the usual problems associated with using a non-covariant gauge in a covariant calculation nor the problems associated with the unphysical pole of the propagator. The calculations are straightforward and γ_5 or the presence of quark masses poses no special problem. The physical picture is very clear at every stage of the calculation. Also the regularization scheme used in this framework for perturbative contributions can be directly applied to the construction of hadronic bound states which is the major topic of current research on light-front field theory [16,17]. Thus, once the light-front bound state structures are found, a complete theoretical understanding of structure functions can become possible.

In addition, the approach uses probability amplitudes rather than probability densities and hence interference effects are easy to handle. Exploiting this feature, we have clarified the parton interpretation of the matrix element of the transverse component of the bilocal vector current. We have presented real and virtual corrections to the structure function F_2 for a dressed quark and gluon in a transparent manner. The splitting functions are extracted and the longitudinal momentum sum rule is verified explicitly to order α_s.

We have carried out, with the help of multi-parton wave functions, a detailed analysis of the scale evolution of the structure function of a composite system which justifies the approximations made in Sec. II which lead to the emergence of factorization to all orders in perturbation theory and to a simple scale evolution picture. A complete fourth order calculation is necessary to establish the viability of the new approach for the perturbative domain. Such a calculation is presently under way. The investigation of main contribution to DIS structure functions, nonperturbative QCD dynamics, is also in progress. We shall leave the discussion of these topics for future publications.

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