Multiplicity distributions at high energies as a sum of Poissonian-like distributions

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It is shown that at collider energies experimental distributions in the multiplicity n of negatively charged particles in inelastic and nonsingle diffractive $\bar{p}p$ collisions are well parametrized by a sum of so-called Gupta-Sarma distributions having the Poisson distribution as a particular case. This extends the earlier description of the multiplicity distributions in hadron-hadron collisions at c.m. energies below 65 GeV by the two parameter sum of Poissonian distributions. Implications of the proposed parametrization for the CERN LHC energy are discussed. [S0556-2821(99)00507-X]

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I. INTRODUCTION

Charged particle multiplicity distributions in inelastic hadron-hadron collisions at high energies are usually described by the negative binomial (NB); for an experimental review see Ref. [1]. However the appearance of the shoulder structure observed for the first time by the UA5 Collaboration [2] has led to the use of the weighted sum of two NB's with five free parameters [3], where the first NB describes the contribution of soft events (events without minijets) and the second one describes the contribution of semihard events (events with minijets). The aim of this paper is to extend to the collider energies another phenomenological parametrization giving better agreement with lower energy data than NB both for $p(\bar{p})p$ [4,5] and meson-proton [6,5] collisions. In the Refs. [4,5] the multiplicity distribution P_n of negatively charged particles produced in inelastic $p(\bar{p})p$ collisions at the center of mass energies $\sqrt{s}$ below 63 GeV have been fairly well described by a two parameter sum of Poissonian distributions. This approach is based on a simple minded quark-parton model in which quarks q interact pairwise independently of one another with the same conditional probability ε and each qq interaction leads to the same multiplicity distribution in the final state. The probabilities for events with 0, 1, 2 or 3 qq interactions are equal respectively to $(1-\varepsilon)^3$, $3\varepsilon(1-\varepsilon)^2$, $3\varepsilon^2(1-\varepsilon)$ and ε^3 and in terms of a probability generating function PGF, (for a mathematical formalism see Refs. [7,8]) it leads to the relation

$$G(z)=\sum P_n z^n=(1-\varepsilon+\varepsilon\varphi(z))^3, \tag{1}$$

where $G(z)$ is the PGF for the final distributon and $\varphi(z)$ is the PGF for events with one parton-parton collision. The PGF's for events with two or three parton-parton collisions are simply convolutions $\varphi^2(z)$ and $\varphi^3(z)$. Good description of the experimental data has been obtained with $\varphi(z)=\exp(S(z-1))$, the PGF for the Poisson distribution. This parametrization had the strong energy dependence of the parameter ε [4], more smooth energy dependence of the ε was observed [5] when the Poissonian was replaced by the Poisson distribution truncated at zero multiplicity with the PGF $\varphi'(z)=(\varphi(z)-\varphi(0))/(1-\varphi(0))$. The need for the truncation at zero is explained for pp collisions by nonzero electric charge of the initial uu, ud and dd pairs leading to the reduced probability to have zero charged particles in the final state. This is not the case for $\bar{p}p$ collisions where $\bar{u}u$ and $\bar{d}d$ pairs (5 out of 9 combinations) are neutral.

The model [4–6] is modified in the present paper in the two aspects:

(a) The Poisson distribution is replaced by the so-called Gupta-Sarma (GS) distribution [9,10] with the parton distribution function (PDF)

$$g(z)=\exp\left(\frac{-S(1-z)}{1+r(1-z)}\right). \tag{2}$$

The GS distribution having the Poissonian as a particular case at $r=0$ and known in mathematical statistics under the name of the Pólya-Aeppli distribution [11] has physical interpretation in the framework of different models [12–14], see discussion in Refs. [15,16]. In the approach advocated by Biyajima *et al.* [12] and Finkelstein [13] the multiplicity distribution originates from the Poisson distribution of some clusters, each cluster obeys Furry-Yule (or truncated at zero Bose-Einstein) distribution, finally it leads to the PGF (2). One can note that the same form (2) is valid when the PDF for cluster decay distribution is a linear fraction $(1+\Delta(1-z))/(1+r(1-z))$, usual for the theory of branching processes, Ref. [17]. In the Gupta-Sarma approach [9,10] the system after collision is viewed as one highly excited hadron emitting entity obeying the branching process with the probability per infinitesimal time Δt to produce k new particles proportional to $\lambda^k\Delta t$, where λ is positive constant. The solution of the corresponding evolution equation for initial condition with zero particles leads to the PDF of the form (2). In the Chau-Huang approach [14] the GS distribution is obtained from the statistical Ising model.

(b) As suggested in the Ref. [5] events with zero parton-parton collisions can represent the diffractivelike processes, with the fraction of diffractive-like events given by the $(1-\varepsilon)^3$. It has been established that the multiplicity distribution for diffractive system with effective mass M looks like the multiplicity distribution in pp collisions at the c.m. energy $\sqrt{s}=M$, see Ref. [18] for a review on diffraction. In this paper we approximate the diffractive contribution by the form (1) with $\varphi(z)$ equal to the PGF for the Poissonian un-

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TABLE I. Results of the fits to the negatively charged particle multiplicity distributions in inelastic pp collisions [22–39].

Ref.	$\sqrt{s}$ (GeV)	ε	S	S_d	χ^2/NDF
[22]	6.84	0.336 ± 0.002	0.203 ± 0.006	0.032 ± 0.013	21.1/5
[23]	7.87	0.381 ± 0.004	0.325 ± 0.011	0	3.9/5
[24]	9.78	0.441 ± 0.015	0.521 ± 0.040	0	7.5/6
[25]	10.69	0.470 ± 0.014	0.500 ± 0.044	0	2.8/5
[26]	11.46	0.470 ± 0.008	0.671 ± 0.019	0.013 ± 0.040	18.6/7
[27]	13.76	0.472 ± 0.009	0.940 ± 0.028	0	19.1/7
[28]	13.90	0.483 ± 0.010	0.855 ± 0.032	0.003 ± 0.041	7.3/7
[29]	16.66	0.491 ± 0.006	1.170 ± 0.021	0.012 ± 0.021	18.3/10
[30]	18.17	0.523 ± 0.031	1.224 ± 0.094	0	2.4/7
[31]	19.42	0.551 ± 0.016	1.197 ± 0.054	0.074 ± 0.074	9.0/8
[32]	19.66	0.538 ± 0.011	1.243 ± 0.034	0.070 ± 0.071	7.3/10
[33]	21.7	0.512 ± 0.011	1.460 ± 0.040	0.135 ± 0.063	14.3/11
[34]	23.76	0.563 ± 0.012	1.486 ± 0.039	0.112 ± 0.066	10.4/11
[35]	23.88	0.561 ± 0.019	1.601 ± 0.074	0.246 ± 0.148	11.2/10
[36]	26.0	0.577 ± 0.010	1.638 ± 0.034	0.102 ± 0.054	8.4/10
[28]	27.6	0.542 ± 0.013	1.720 ± 0.059	0.096 ± 0.057	17.8/13
[37]	27.6	0.555 ± 0.016	1.565 ± 0.077	0.027 ± 0.068	4.3/9
[38]	30.4	0.528 ± 0.021	2.000 ± 0.077	0.300 ± 0.157	3.1/14
[39]	38.8	0.576 ± 0.008	2.059 ± 0.031	0.273 ± 0.085	8.3/13
[38]	44.5	0.538 ± 0.021	2.441 ± 0.086	0.438 ± 0.167	4.9/16
[38]	52.6	0.549 ± 0.016	2.647 ± 0.068	0.381 ± 0.142	12.2/18
[38]	62.2	0.552 ± 0.017	2.887 ± 0.070	0.426 ± 0.168	16.7/17

der the crude assumption that the integrated over M distribution is similar to the distribution at some effective mass $\overline{M}$ and the final PGF is given by

$$G(z)=(1-\varepsilon)^3\varphi_d+3\varepsilon(1-\varepsilon)^2 g(z)+3\varepsilon^2(1-\varepsilon)\, g^2(z) +\varepsilon^3 g^3(z) \qquad (3)$$

with

$$\varphi_d(z)=[1-\varepsilon+\varepsilon\exp(S_d(z-1))]^3. \qquad (4)$$

More careful description of the diffractivelike events with integration over M is given in Refs. [19,20].

In the Sec. II the main characteristics of the Gupta-Sarma distribution are summarized. In Sec. III we present results of fits to the available pp data and to the $\bar{p}p$ data obtained at the $S\bar{p}pS$ collider. In Sec. IV the discussion and the conclusions are given.

II. CHARACTERISTICS OF THE GUPTA-SARMA DISTRIBUTION

The mean multiplicity $\langle n\rangle$ and its dispersion $D=(\langle n^2\rangle-\langle n\rangle^2)^{1/2}$ are easily obtained from the PDF (2) using formulas $\langle n\rangle=dg/dz|_{z=1}$ and $D^2=\langle n\rangle+d^2\ln g(z)/dz^2|_{z=1}$

TABLE II. Results of the fits to the negatively charged particle multiplicity distributions in $\bar{p}p$ interactions [2,40].

Events	$\sqrt{s}$ (GeV)	ε	S	S_d	r	χ^2/NDF
NSD	200	0.456	5.965 ± 0.076		0.170 ± 0.056	19.7/29
		0.264 ± 0.038	7.441 ± 0.304		0.297 ± 0.084	9.2/28
NSD	546	0.456	8.429 ± 0.053		0.468 ± 0.030	61.3/45
		0.352 ± 0.018	9.453 ± 0.187		0.558 ± 0.038	32.6/44
inel.	546	0.456	7.410 ± 0.091	11.071 ± 0.458	0.832 ± 0.075	39.4/44
		0.536 ± 0.027	6.743 ± 0.067	11.811 ± 0.326	0.794 ± 0.067	29.9/43
NSD	900	0.456	10.400 ± 0.100		0.703 ± 0.065	77.0/52
		0.304 ± 0.029	12.213 ± 0.394		0.823 ± 0.095	20.9/51

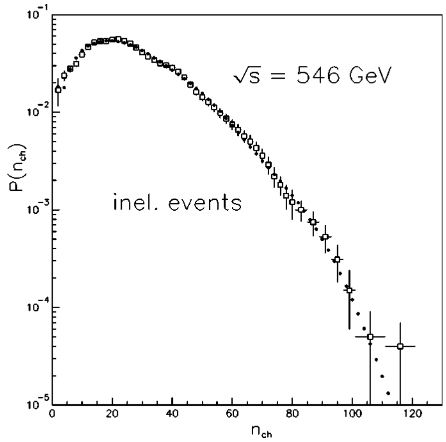

FIG. 1. Multiplicity distribution for inelastic antiproton-proton collisions at the c.m. energy 546 GeV [40] (squares) compared with results of the fit (full dots).

$$\langle n\rangle = S, \quad D^2 = \langle n\rangle(1+2r). \tag{5}$$

To obtain expressions for probabilites g_n one can use the method proposed by Finkelstein [13]. The PGF is expressed as a sum of powers $[z/(1+r(1-z))]^k$, where the denominator represents the well known NB. Then, the contribution to g_n with $n\neq 0$ from the kth term is equal to the NB probability to have $n-k$ particles and finally it gives

$$g_n = g_0\sum_{k=1}^{n}\frac{(n-1)!}{(n-k)!(k-1)!k!}a^k b^{n-k} \tag{6}$$

with

$$a=\frac{S}{(1+r)^2}, \quad b=\frac{r}{1+r}, \tag{7}$$

and

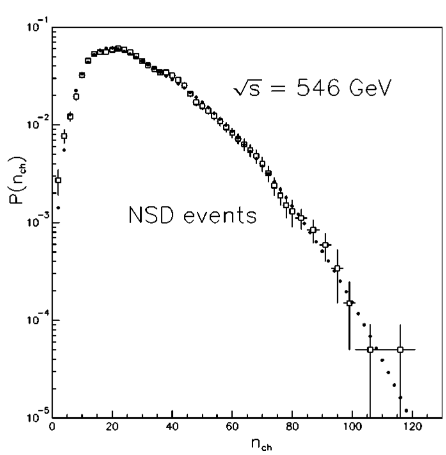

FIG. 2. Multiplicity distribution for NSD collisions at the c.m. energy 546 GeV [40] (squares) compared with results of the fit (full dots).

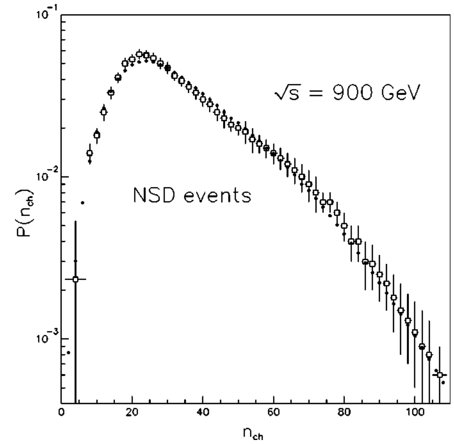

FIG. 3. Charge particle multiplicity distribution for NSD collisions at the c.m. energy 900 GeV [2] (squares) compared with results of the fit (full dots).

$$g_0 = g(0) = \exp\left(\frac{-S}{1+r}\right). \tag{8}$$

One can note [12,15] that the GS distribution is a particular case of the partially coherent laser distribution (PCLD) (see the review of the PCLD in [7]). Indeed the PGF for the PCLD is the product of the PDF (2) and $(1+r(1-z))^{-k}$, i.e., the convolution of the GS and NB distributions. It gives the expressions for g_n in terms of the Laguerre polynomials [21]

$$g_n = \left(\frac{r}{1+r}\right)^n \exp\left(\frac{-S}{1+r}\right) L_n^{-1}\left(\frac{-S}{r(1+r)}\right). \tag{9}$$

Using iteration relations for the Laguerre polynomials [21] one can obtain next iteration relations for g_n

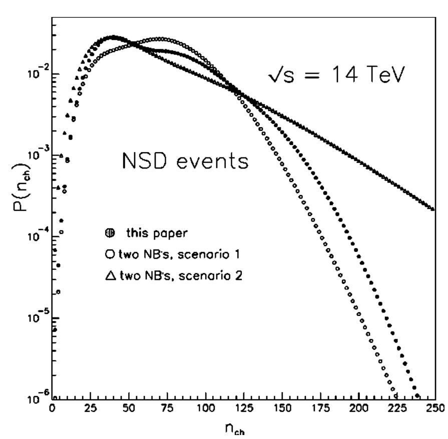

FIG. 4. NSD multiplicity distribution at the LHC energy 14 TeV, predicted by the parametrization, given in the text, compared to the two predictions from the paper [45], based on the parametrization by the weighted sum of the two NB's.

$$(n+1)g_{n+1}=(a+2nb)g_n-(n-1)b^2g_{n-1} \qquad (10)$$

at $n>1$ and

$$g_1=a\,g_0. \qquad (11)$$

These iteration relations can be useful for calculations at large n values.

III. RESULTS OF FITS

Both for pp and $\overline{p}p$ data we calculate the number of negatively charged particles as $n=(n_{ch}-2)/2$, i.e., we count the number of produced pairs of charged particles. As mentioned above, we truncate the distribution for parton-parton collision in the fits to the pp data and do not truncate it for $\overline{p}p$ data at the collider energies.

The pp data used [22–39] are the same as in Refs. [4,5] with additional measurement from Ref. [31]. In Table I the results of fits to the distribution with the PGF (3) are given for the case $r=0$. The agreement with experimental data is good, this is expected since even the two-parameter parametrization with the diffractive contribution concentrated at zero multiplicity was successful [4,5]. One can note also that the mean multiplicity for diffractive contribution, proportional to S_d increases slowly with energy.

Nonsingle diffractive (NSD) multiplicity distributions, measured by the UA5 Collaboration [2,40] have been parameterized by the distribution (3) without diffractive component, i.e., by the distribution with three other ''parton-parton collision'' components normalized by the factor $(1-(1-\varepsilon)^3)^{-1}$. Inelastic multiplicity distribution at $\sqrt{s}=546$ GeV [40] has been parametrized by the full distribution (3). The results of the fits are given in Table II both for $\varepsilon=0.456$ and for free ε. The fixed value of the ε was chosen on the assumption that the fraction of diffractive like events is equal to 16%, the fraction of the single diffractive events measured by the CDF Collaboration at 546 GeV [41]. The fraction measured by the UA5 Collaboration is equal to 11% [40], corresponding conditional probability ε is equal to 0.52.

The results of the fits with free ε are illustrated in Figs. 1–3 respectively for inelastic and NSD data at 546 GeV [40] and for NSD data at 900 GeV [2]. The quality of the fits is quite qood, the fluctuations in the parameter ε are explained by the our crude treatment of the diffractive component and possible bias in the experimental data. We have ignored also the nonnegligible contribution of the double diffraction processes in the NSD data. The influence of the high multiplicity tail on the fit parameters has been observed also, the fit in the region $n_{ch}<80$ of the inelastic data at 546 GeV gives more reasonable value of S_d near 3, significantly smaller than the values S_d in the Table II.

IV. DISCUSSION AND CONCLUSIONS

The possibility that multiplicity distributions at high energies can split into several structures has been predicted more the 25 years ago [42]. For example Nielsen and Olesen made the statement [43]: ''if we go to high enough energy one should see a separation of the multiplicity spectrum in a series of equidistant peaks at $n\sim n_1,\ 2n_1,3n_1,\ \ldots$.'' Kaidalov and Ter-Martirosyan in the framework of the quark-gluon string model have predicted three peaks in the multiplicity distribution at $\sqrt{s}=100$ TeV [44]. These Regge-type models [42,44] in principle predict more than three peaks (structures) in contrast to our approach with maximum three nondiffractive structures. The prediction for NSD multiplicity distribution at the CERN Large Hadron Collider (LHC) energy $\sqrt{s}=14$ TeV, calculated with parameters $\varepsilon=0.456$, $r=0.8$ and S fixed by the expected mean multiplicity $\langle n_{ch}\rangle=67.2$ is given in the Fig. 4 in comparison with the two predictions, given in the papers [45,46], based on the parametrization by the weighted sum of two NB's.[1] Our prediction seems to be intermediate between these two negative binomial scenarios. All three predictions, given in the Fig. 4, do not show peak structures expected in the Regge-type models.

It is of interest to compare the GS parametrization with other parametrizations, applied to the e^+e^- annihilation processes and lepton-nucleon interactions. From our point of view the use of the GS distribution in $pp(\overline{p}p)$ collisions to describe multiplicity distribution for one parton-parton collision has no direct connection with more elementary $\overline{q}q$ pair production in e^+e^- annihilation. Indeed, the effective c.m. energy of the parton-parton pair in hadron collisions spans a wide energy range, whereas in e^+e^- annihilation it is fixed by the initial energy. Good description of the e^+e^- and lepton hadron data has been obtained using phenomenological modified negative binomial distribution (MNBD) [48–54] or QCD motivated generalized negative binomial distribution (GNBD) [55–58]. Nevertheless it is worth to note that GS distribution is the limit of the NB (or MNBD) distribution when k goes to infinity and therefore is quite similar to the NB (or MNBD) distribution with high k values.

Our last comment concerns the behavior of the parameters S and r of the GS distribution. Their positiveness (nonnegativeness) at high energies indicates that the cumulants and combitants of this distribution are nonnegative. This allows the interpretation of the GS distribution in the framework of the hierarchical cluster models with Poisson superposition [59].

In conclusion, the multiplicity distributions at collider energies have been fairly well parameterized by the sum of Poisson-like GS distributions, with one GS distribution describing the multiplicity distribution for one ''parton-parton'' collision. An attempt has been made to connect the fractions of events with 1, 2 or 3 ''parton-parton'' collisions with the fraction of diffractivelike events in a framework of the simple minded parton model.

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