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# Non-Markovian effects in strong-field pair creation

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We analyze a quantum kinetic equation describing both boson and fermion pair production and explore analytically and numerically the solution of the non-Markovian kinetic equation. In the low density limit of the kinetic equation we find an analytical solution for the single particle distribution function of bosons and fermions. The numerical investigation for a homogeneous, constant electric field shows an enhancement (bosons) or a suppression (fermions) of the pair creation rate according to the symmetry character of the produced particles. For strong fields non-Markovian effects are important while they disappear for weak fields. Hence it is sufficient to apply the low density limit for weak fields but necessary to take into account memory effects for strong fields. [S0556-2821(99)04607-X]

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## I. INTRODUCTION

A proper description of the preequilibrium evolution of the quark-gluon plasma (QGP), believed to be created in an ultrarelativistic heavy-ion collision can start from a transport equation that incorporates a source term and a collision integral [1]. The source term describes the production of pairs of particles and antiparticles while the collisions lead to thermalization.

The formation of the QGP is assumed to proceed via the creation of a strong chromoelectric field in the region between the two receding nuclei after the collision. The field subsequently decays by emitting quark-antiquark pairs according to the nonperturbative tunneling process of the Schwinger mechanism [2–4]. Within the flux tube model a lot of promising research has been carried out [5–14].

The process of pair production within the Schwinger mechanism has been addressed by many authors [5,6,10,15] in recent years, with the back reaction scenario also considered [16]. A recent application of the Schwinger mechanism of pair creation to the QCD case, solving a transport equation in boost invariant variables with a simple collision and source term, has been performed in [17]. Another interesting calculation was provided by [18] where the polarization of the created quark-antiquark pair in a string fragmentation model for inclusive reactions was discussed.

The pair creation process as a field theoretical problem is solved by analyzing the Dirac equation with an external field coupled to the Maxwell equations, e.g. [16]. In recent studies the open question as to how to link the field theoretical treatments with a kinetic theory has been addressed. Investigations performed for QED have shown that a consistent field theoretical approach leads to a kinetic equation with a modified source term, providing a non-Markovian evolution of the distribution function. This result was first obtained by Rau [19] for the case of fermion pair creation for a constant electric field using a projection method [20].

A generalized treatment allowing for a time dependent field was given in [21,22]. Therein we derived a kinetic equation containing both boson and fermion pair production. For the case of a constant field our results agree with those of [19,23]. The properties of the source term itself such as the momentum dependence and the time structure have been studied [22–24] for a constant and a time dependent field. A detailed analysis of the boson pair creation combined with a systematic numerical study of the time structure of the solution was provided in the approach of [23]. These studies employ an asymptotic expansion of the non-local source term which results in a local source term for particle creation.

In order to give a complete picture of the pair creation process in terms of a kinetic equation alternatively to a field theoretical solution it is necessary to study besides the weak field limit also the strong field behavior where the appearance of non-Markovian aspects of the pair creation process is expected. Therefore the main goal of this article is the study of the time evolution of the system in a regime where memory effects become important. As an illustrative example we consider pair creation in QED. Attention will be paid to the numerical analysis of the evolution of the distribution function for both bosons and fermions. We explore the influence of the different quantum statistical properties of boson and fermion pair creation and how the differences depend on the strength of the external field.

In Sec. II A we discuss the kinetic equation with a new source term for pair production. In Sec. II B we explore the numerical solution of the non-Markovian kinetic equation for weak and strong fields. We also study the influence of the symmetry character of fermions and bosons on the evolution of the distribution function. In Sec. II C we discuss the analytic solution in the Markovian and the low density limits, respectively. We compare the results with the non-Markovian solution and discuss the differences. The results are summarized in Sec. III. Within this study we neglect any influence due to collisions as well as back reactions of the produced charged particles on the initial electric field.

## II. NON-MARKOVIAN KINETIC EQUATION

### A. Source term with non-Markovian character

We consider particle production in a strong external electric field which leads to an unstable vacuum that can decay by creation of electron-positron pairs. Using the field-theoretical model of charged particles in an external, homogeneous, time-dependent field characterized by the vector potential $A_\mu=(0,0,0,A(t))$ with $A(t)=A_3(t)$ and the resulting electric field $E(t)=E_3(t)=-\dot{A}(t)=-dA(t)/dt$, the kinetic equation is derived starting from the Dirac (Klein-Gordon) equation for fermions (bosons). The transition from the in-state to the instantaneous, quasiparticle state at the time $t$ has been achieved by a time-dependent Bogoliubov transformation.

As the final result we obtain the kinetic equation for the single particle distribution function $f(\overline{P},t)=\langle 0|a^\dagger_{\overline{P}}(t)a_{\overline{P}}(t)|0\rangle$ defined as the vacuum expectation value in the time dependent basis of creation and annihilation operators $a^\dagger_{\overline{P}}(t),a_{\overline{P}}(t)$ for electron states at the time $t$ and the 3-momentum $\overline{P}$:

$$\frac{df_\pm(\overline{P},t)}{dt}=\frac{\partial f_\pm(\overline{P},t)}{\partial t}+eE(t)\frac{\partial f_\pm(\overline{P},t)}{\partial P_\parallel(t)}$$

$$=\frac{1}{2}\mathcal{W}_\pm(t)\int_{-\infty}^{t}dt'\mathcal{W}_\pm(t')$$

$$\times[1\pm 2f_\pm(\overline{P},t')]\cos[x(t',t)], \qquad (1)$$

where the upper sign (lower sign) in Eq. (1) corresponds to boson (fermion) pair creation. Details of the derivation are given in [21–23]. The momentum is defined as $\overline{P}=(p_1,p_2,P_\parallel(t))$, with the longitudinal momentum $P_\parallel(t)=p_\parallel-eA(t)$ where $p_\parallel=p_3$. For fermion creation we find, in agreement with [19],

$$\mathcal{W}_-(t)=\frac{eE(t)\varepsilon_\perp}{\omega^2(t)}, \qquad (2)$$

and for boson production our result agrees with [23]

$$\mathcal{W}_+(t)=\frac{eE(t)P_\parallel(t)}{\omega^2(t)}=\frac{P_\parallel(t)}{\varepsilon_\perp}\mathcal{W}_-(t). \qquad (3)$$

We define the total energy $\omega(t)=\sqrt{\varepsilon_\perp^2+P_\parallel^2(t)}$, the transverse energy $\varepsilon_\perp=\sqrt{m^2+\overline{p}_\perp^2}$ and the transverse momentum $\overline{p}_\perp=(p_1,p_2)$. Furthermore $x(t',t)=2[\Theta(t)-\Theta(t')]$ denotes the difference of the dynamical phases which are defined as

$$\Theta(t)=\int_{-\infty}^{t}dt'\omega(t'). \qquad (4)$$

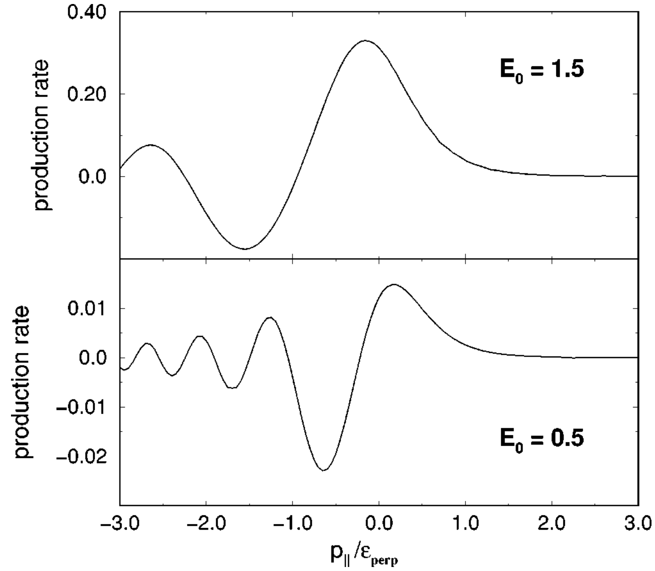


FIG. 1. The dependence of the source term in low density limit on the parallel momentum for fermion pair creation for a strong field (upper panel: $E_0=1.5$) and a weak field (lower panel: $E_0=0.5$).

Equation (1) is characterized by the following properties.

(i) The particles are produced not only at rest $p_\parallel=0$ as assumed in more phenomenological approaches, e.g. [16]. In Fig. 1 the dependence of the source term in low density limit for fermion production on the parallel momentum,

$$S^0_\pm(\overline{P},t)=\frac{1}{2}\mathcal{W}_\pm(t)\int_{-\infty}^{t}dt'\mathcal{W}_\pm(t')\cos[x(t',t)], \qquad (5)$$

is shown. This calculation has been performed for a strong and weak constant electric field, $E(t)=\text{const}$. The production rate is peaked at about zero momentum; for positive momenta it approaches zero, and for negative momenta it is dominated by oscillations due to the choice of a constant electric field. It increases with increasing strength of the external field. Similar results have been obtained recently by different authors [19,21–23].

(ii) The source term and the distribution function have a momentum dependence accounted for by the transverse energy $\varepsilon_\perp$ and by the kinetic momentum $P_\parallel(t)$; i.e., once a solution for $p_\parallel=0$ and $p_\perp=0$ is obtained, the solution for nonvanishing momenta can be generated by simple variable transformations. Therefore, we drop the explicit notation of the dependence on $\overline{P}$ in the distribution function and in the source term.

(iii) Furthermore, the kinetic equation (1) has non-Markovian character. The source term contains a time integration over the statistical factor $[1\pm 2f_\pm(\overline{P},t)]$ and over the non-local cosine function which causes memory effects. This important property will be discussed within this article. Investigating the differences of boson and fermion pair creation for weak and strong fields, we go beyond recent studies, e.g. by [22,23], in the following subsections.

### B. Solution of the non-Markovian kinetic equation

In the previous subsection we have discussed general features of the source term. Now we want to study the time structure of the non-Markovian solution in detail by solving the kinetic equation

$$\frac{d f_{\pm}(t)}{d t}=\frac{1}{2}\mathcal{W}_{\pm}(t)\int_{-\infty}^{t}dt'\mathcal{W}_{\pm}(t')\times[1\pm 2f_{\pm}(t')]\cos[x(t',t)] \tag{6}$$

numerically. The non-Markovian character is obvious and has two different origins. On the one hand coherent phase oscillations are contained, induced by the cosine function with a non-local argument. This non-locality in time leads to non-Markovian effects itself. On the other hand the appearance of a statistical factor under the time integral means that the solution of the differential equation depends on the full time evolution of the distribution function and hence memory effects are included.

The complicated structure requires a self-consistent scheme for the numerical solution. As initial conditions we use $\lim_{t\to-\infty}f_{\pm}(t)=0$. We solve the differential equation within the standard methods and obtain the self-consistent solution by iteration. Another possibility to solve this equation is direct solution by integration. Numerical investigations have shown that this method leads to the same result but more iteration steps are needed to find the solution and thus we decided to solve Eq. (6) in its differential form. In order to demonstrate the numerical solution of the kinetic equation we choose the simple case of a constant electric field $E(t)=E=\text{const}$. On the one hand this is the most simple possible case also for the numerical treatment since this assumption leads to a reduction of the number of integrations. On the other hand this is the standard *Ansatz* for the case that back reactions are not included [19,23] and therefore permits us to compare the results. The vector potential is, in this case,

$$\tilde{A}(\tau)=A(\tau)/\varepsilon_{\perp}=-\tau E_0/e, \tag{7}$$

where the dimensionless variable $E_0=eE/\varepsilon_{\perp}^2$ does not depend on time and the energy is given as

$$\omega_0(\tau)=\sqrt{1+E_0^2(\tau-\Delta\tau)^2}. \tag{8}$$

In our calculations we keep the transverse momentum fixed and hence normalize in units of a constant $\varepsilon_{\perp}$. In Eqs. (7) and (8) we have introduced the dimensionless time variables

$$\tau=t\varepsilon_{\perp} \tag{9}$$

and

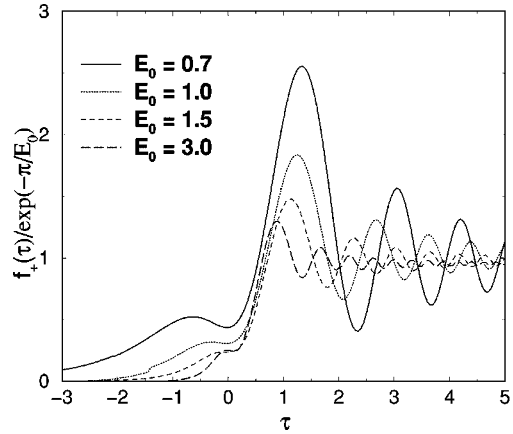


FIG. 2. The boson distribution function as a solution of the non-Markovian equation as function of time for different field strengths at $p_{\parallel}=0$.

$$\Delta\tau=\Delta\tau(p_{\parallel},\overline{p_{\perp}})=\frac{p_{\parallel}\varepsilon_{\perp}(\overline{p_{\perp}})}{eE}. \tag{10}$$

This notation is also convenient to distinguish the weak field $(E_0<1)$ and strong field $(E_0>1)$ limits.

For such a simple model case, we can solve the integral of the dynamical phase (4) and obtain

$$\Theta(\tau)=\frac{1}{2}(\tau-\Delta\tau)\omega_0(\tau)+\frac{1}{2E_0}\ln[E_0(\tau-\Delta\tau)+\omega_0(\tau)]. \tag{11}$$

In the following we discuss the numerical solution of the kinetic equation for bosons and fermions for different field strengths. In Fig. 2 we plot the distribution function for bosons for weak fields and strong fields. The particles are produced at about zero kinetic momentum. Because of the fact that we have no damping mechanism, the distribution oscillates around a constant value. The oscillations to be seen in the figure are due to the choice of a constant electric field. In response to strong fields the frequencies of these oscillations increase while the amplitudes decrease compared with the limit at large times. Other *Ansätze* for the time dependence of the external field such as a Gaussian shape [22] may lead to a damping of these oscillations. But it is important to note that in order to describe a more realistic situation the back reaction of the produced particles on the initial field should be included [16,23]. The curves are normalized to the large time limit

$$f(\tau\to\infty)=\exp\left(\frac{-\pi}{E_0}\right). \tag{12}$$

We observe that for all plotted field strengths the curves converge to this limit. In other words, one can conclude that for large times we obtain the old result given by Schwinger's formula for weak fields as well as for strong fields. The absolute value of the distribution function at $\tau\to\infty$ is larger for strong fields than it is for weak fields.

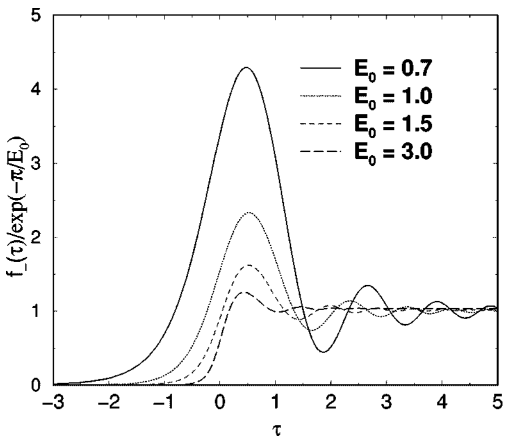


FIG. 3. Same as Fig. 2 for fermions.

We obtain a similar result for fermion pair creation; see Fig. 3. As a result of the different amplitudes for fermion and boson pair production, Eqs. (2) and (3), the shapes of the curves are slightly different, in particular around zero kinetic momenta. We can compare the distribution function for fermions and bosons for a given field strength, Figs. 2 and 3. The onset of particle creation (first maximum) is earlier for fermions than it is for bosons. Both curves reach the same limit for large times and oscillate with the same frequency for a given $E_0$. The results which we obtain for the boson case are in agreement with those of [23] for the case of weak fields, wherein the Markovian limit was employed. But the numerical solution introduced in this section allows us in addition to consider pair creation for strong fields.

### C. Low density and Markovian limit

#### 1. Closed kinetic equation

In the previous subsection we have demonstrated how to solve the kinetic equation in its non-Markovian form and explored the numerical solutions. In this section we want to discuss the solution of the kinetic equation (1) for weak fields where it is possible to apply approximations. The Markovian limit of the non-Markovian equation (1) is defined by the neglect of memory effects in the source term. This requires us to find a local approximation of the non-local argument of the time integral as well as to neglect the dependence of the distribution function on the pre-history of the evolution, $f(\tau')\rightarrow f(\tau)$. Herein we restrict ourselves to the neglect of memory effects in the distribution function and retain the phase oscillation effects. Hence we can compare the results with the full non-Markovian solution of the previous section. Although we will call this the Markovian approximation within this article, it is important to note that the complete Markovian solution was recently explored in [23].

We obtain the kinetic equation

$$\frac{d f_{\pm}^{M}(\tau)}{d\tau}=[1\pm 2 f_{\pm}^{M}(\tau)]S_{\pm}^{0}(\tau)=S_{\pm}^{M}(\tau), \tag{13}$$

where $S_{\pm}^{0}(\tau)$ is the source term in low density limit, Eq. (5).

Taking into account the initial condition $\lim_{\tau\rightarrow-\infty} f_{\pm}^{M}(\tau)=0$, we obtain the following solution of the kinetic equation (13):

$$f_{\pm}^{M}(\tau)=\mp\frac{1}{2}\left(1-\exp\left[\pm 2\int_{-\infty}^{\tau} d\tau' S_{\pm}^{0}(\tau')\right]\right). \tag{14}$$

This result is exact in the Markovian limit and holds for any time-dependent homogeneous electric field. In the lowest order of the expansion of Eq. (14) we obtain

$$f_{\pm}^{0}(\tau)=\int_{-\infty}^{\tau} d\tau' S_{\pm}^{0}(\tau'). \tag{15}$$

This solution is equivalent to the low density limit where effects due to the symmetry character of the created particles are neglected in the source term. Note that both the low density limit and the Markovian approximation are restricted to weak fields, $E_0<1$.

Before we discuss the numerical results for the Markovian approximation and the low density limit, we explore general properties of the distribution function defined in Eq. (14). One requirement of a kinetic theory is that the distribution function for bosons and for fermions must be positive definite for all times and momenta. In order to prove the validity of this important property we rewrite Eq. (15) as

$$f_{\pm}^{0}(\tau)=\frac{1}{2}\int_{-\infty}^{\tau} d\tau' g_{\pm}^{1}(\tau')\int_{-\infty}^{\tau'} d\tau'' g_{\pm}^{1}(\tau'') + \frac{1}{2}\int_{-\infty}^{t} d\tau' g_{\pm}^{2}(\tau')\int_{-\infty}^{\tau'} d\tau'' g_{\pm}^{2}(\tau''), \tag{16}$$

where the functions $g_{\pm}^{1,2}(\tau)$ are given as

$$g_{\pm}^{1,2}(\tau)=\mathcal{W}_{\pm}(\tau)\left\{\begin{matrix}\cos[2\Theta(\tau)]\\ \sin[2\Theta(\tau)]\end{matrix}\right\}. \tag{17}$$

It is easy to transform the integral (16) to the following form:

$$f_{\pm}^{0}(\tau)=\frac{1}{4}\left(\int_{-\infty}^{\tau} d\tau' g_{\pm}^{1}(\tau')\right)^{2}+\frac{1}{4}\left(\int_{-\infty}^{\tau} d\tau' g_{\pm}^{2}(\tau')\right)^{2}. \tag{18}$$

From this quadratic representation of Eq. (15) we can conclude that the distribution function is positive definite as it should be

$$f_{\pm}^{0}(\tau)\geqslant 0. \tag{19}$$

On the other hand this result is very useful for numerical calculations since Eq. (18) just requires us to perform a single integration compared with Eq. (15) in which it was necessary to solve a double integration; hence it makes the numerical treatment easier.

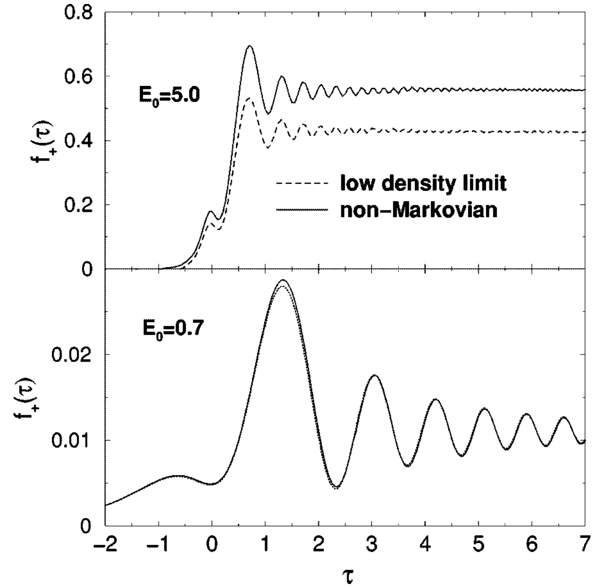


FIG. 4. The time evolution of the distribution functions of bosons ($f_+$) compared with the low density limit $f_+^0$ at $p_\parallel=0$ for a strong field (upper panel: $E_0=5.0$) and a weak field (lower panel: $E_0=0.7$) is shown.

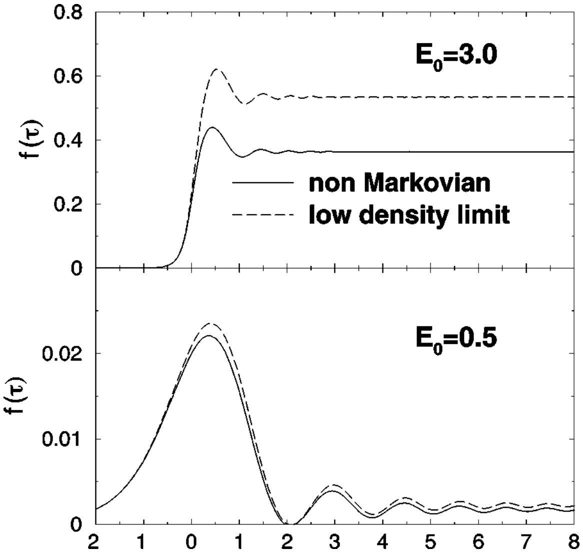


FIG. 5. The time evolution of the distribution functions of fermions ($f_-$) compared with the low density limit $f_-^0$ at $p_\parallel=0$ for a strong field (upper panel: $E_0=3.0$) and a weak field (lower panel: $E_0=0.5$) is shown.

At large times, $\tau\to\infty$, the distribution function is in the asymptotic regime. In the low density limit it is easy to show the feature that $f_0(\infty)\neq 0$. The distribution function evolves from zero at $\tau\to-\infty$ to an asymptotic nonzero value. In the absence of back reaction and collisions (or any other damping mechanism) we observe an accumulation effect. In the next subsection we will elucidate these properties with numerical results.

### *2. Numerical results*

Using the results of Eqs. (18) we can explore the solution of Eq. (14) for the distribution functions in the Markovian limit and in the low density approximation.

In Figs. 4 and 5 we compare the distribution functions of the non-Markovian solutions with the low density limit solutions for fermions and bosons, respectively. In the lower panels we have chosen a weak field and observe that the results agree with each other. This was expected since for weak fields the absolute value of the distribution function is small and consequently the statistical factor $[1\pm 2f(\tau)]$ does not considerably deviate from 1. Hence the low density limit is a good approxmation for relatively small field strengths, $E_0<1$.

In the upper panels we have chosen large field strengths. Both for fermions, Fig. 4, and for bosons, Fig. 5, the low density limit solution shows a different limit for large times. The inclusion of a quantum statistical character in the non-Markovian solution leads for fermion pair creation to a suppression and for boson pair creation to an enhancement compared to the corresponding low density limit. These plots elucidate two important things. On the one hand they demonstrate the influence of the different symmetry character of fermions and bosons. On the other hand we see that for strong fields the low density limit solution provides the wrong results. Technically the reason is clear. The statistical factor $[1\pm 2f(\tau)]$ deviates from 1 and therefore it has to be included in the kinetic equation. Physically this means that the non-Markovian character becomes important.

We have to distinguish different time scales: the memory time and the production time [19,22–24]. While the memory time has quantum mechanical origin and can be considered as the time needed to tunnel the barrier, the production time is the time interval between two creation processes. The memory time is now for strong fields of the same order of magnitude as the production time. A separation of the time scales, which is necessary for the approximation discussed in the previous subsection, is no longer possible. The prehistory affects the evolution of the distribution function and therefore memory effects become important. This is taken into account by the time integration over the statistical factor in the full non-Markovian equation.

In the previous subsection we have also discussed the Markovian approximation understood as the neglect of memory in the distribution function only. In Fig. 6 we compare the three solutions of the kinetic equation for strong fields for bosons (lower panel) and for fermions (upper panel). Besides the already discussed non-Markovian solution and the low density approximation, we also show the Markovian limit. It is needless to point out that for weak fields the Markovian solution also agrees with the non-Markovian solution. Although the Markovian solution is in better agreement with the correct solution than the low density limit, the error is visible and the Markovian limit fails for strong fields. For weak fields the low density limit provides exact results since non-Markovian effects disappear. For strong fields it is unavoidable to solve the non-Markovian equation. Just in a small band of field stengths of about 1 the Markovian limit is a sensible approximation, better than the low density limit, but non-Markovian effects are still very small. Hence for small fields, $E_0<1$, we sug-

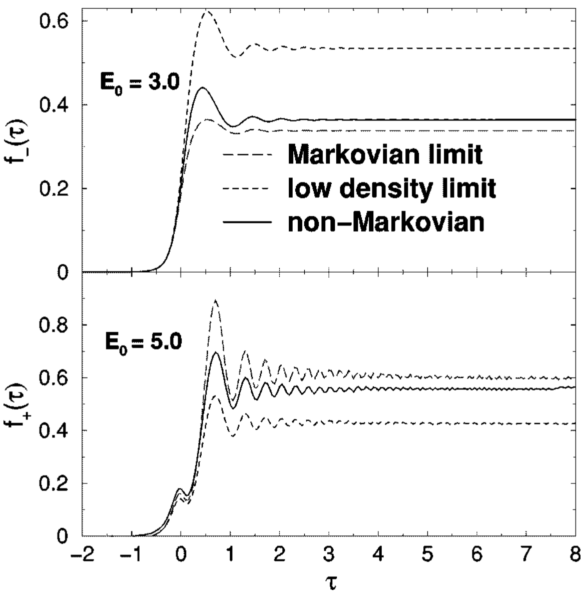


FIG. 6. The time evolution of the distribution functions of fermions (upper panel: $E_0 = 3.0$) and bosons (lower panel: $E_0 = 5.0$) within different approximations for strong fields. Only the full non-Markovian solution provides the correct limit for large times.

gest to use the low density limit, for strong fields, $E_0 > 1$, the non-Markovian solution.

## III. SUMMARY

We have analytically and numerically explored the solution of a quantum kinetic equation describing particle production of boson and fermion pairs. The source term providing the creation of pairs is characterized by its non-Markovian character. The time evolution of the distribution function depends on the entire pre-history of the evolution. We have numerically solved the kinetic equation in its non-Markovian form for both weak and strong fields. We compared these solutions with both the low density limit and the Markovian approximation for which we found an analytic solution in closed form.

In conclusion we can summarize that for weak fields it is appropriate to use the low density limit while for strong fields it is necessary to solve the non-Markovian kinetic equation. In order to give a more complete picture of the physics beyond the Markovian limit it is necessary to study in a next possible step the momentum dependence of the distribution functions.

Although the numerical analysis was performed for a constant field, our results qualitatively hold for any time-dependent field. This will be important when a more realistic scenario is addressed where the electric field is determined self-consistently. In order to incorporate back reactions it is necessary to solve the Maxwell equation that determines the electric field via the conduction current and the polarization current due to the creation of the charged particle pairs [16,23]. Furthermore, it would be of great interest to extend this approach to the QCD case in order to explore the consequences of the new source term for the pre-equilibrium physics in ultrarelativistic heavy-ion collisions.

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