B_c mesons in a Bethe-Salpeter model

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We apply our Bethe-Salpeter model for mesons to the B_c family with parameters fixed in our previous investigation. We evaluate the mass of the pseudoscalar B_c meson as 6.356 and 6.380 GeV/c^2 and the lifetime as 0.47 and 0.46 ps, respectively, in two reductions of the Bethe-Salpeter equation, in good agreement with the recently reported mass of 6.40 ± 0.39 (stat) ± 0.13 (syst) GeV/c^2 and lifetime of $0.46^{+0.18}_{-0.16}$ (stat) ± 0.03 (syst) ps by the CDF Collaboration. We evaluate the decay constant of the B_c meson and compare different contributions to its decay width. [S0556-2821(99)02203-1]

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Recently the Collider Detector at Fermilab (CDF) Collaboration reported the observation of the bottom-charmed mesons B_c in 1.8 TeV $p\bar{p}$ collisions using the CDF detector at the Fermilab Tevatron [1]. This pseudoscalar state is the lowest energy state of the family of mesons composed of a $\bar{b}$ antiquark and a c quark. Since this state lies below the (BD) threshold and has nonvanishing flavor quantum numbers, it decays only through weak interactions. This eliminates uncertainties encountered in strong decays and renders the decay width of B_c more theoretically tractable.

Different approaches have been used to evaluate the spectrum of B_c mesons. Non-relativistic potential models have been used by Eichten and Quigg [2] where they discussed four potentials and gave mass values for the B_c meson in the range 6.248–6.266 GeV/c^2. Gershtein *et al.* [3] used two potentials and reported predictions of 6.253 and 6.264. QCD sum rules have been used by Chabab [4] where he predicted a prediction of 6.25 GeV/c^2.

In this paper, we extend our model [5,6] based on the Bethe-Salpeter equation (BSE) to include the bottom-charmed mesons. The BSE provides an appealing starting point to describe hadrons as relativistic bound states of quarks, just as the Dirac equation provides a relativistic description of a fermion in an external field. The BSE for a bound state may be written in momentum space in the form

$$G^{-1}(P,p)\psi(P,p)=\int \frac{1}{(2\pi)^4}V(P,p-p')\psi(P,p')d^4p', \tag{1}$$

where P is the four-momentum of the bound state and p is the relative four-momentum of the constituents. The BSE has three elements: the interaction kernel (V) and the propagator (G) which we provide as input, and the amplitude (ψ) obtained by solving the equation. We also solve for the energy, which is contained in the propagator.

Different approaches have been developed to make the four dimensional problem BSE more tractable and physically appealing. These include the instantaneous approximation (IA) and quasi-potential equations (QPE) [7–9]. In the IA, the interaction kernel is taken to be independent of the relative energy. In QPE, the two particle propagator is modified in a way which keeps covariance and reduces the four-dimensional BSE to a three-dimensional equation. Of course, there is considerable freedom in carrying out this reduction.

We have used two reductions of the QPE to study the meson spectrum [5,6]. These reductions correspond to different choices of the two particle propagator used to reduce the problem into three dimensions. We refer to these reductions as A and B. Reduction A corresponds to a spinor form of the Thompson equation [10] and reduction B corresponds to a new QPE introduced in Ref. [11]. These two reductions are chosen because they are shown to give good fits to the meson spectrum.

We assume the interaction kernel to consists of a one gluon exchange interaction V_{OGE} in the ladder approximation, and a phenomenological, long range scalar confinement potential V_{CON} given in the form

$$V_{\mathrm{OGE}}+V_{\mathrm{CON}}=-\frac{4}{3}\alpha_s\frac{\gamma_\mu\otimes\gamma_\mu}{(p-p')^2} +\sigma\lim_{\mu\to 0}\frac{\partial^2}{\partial\mu^2}\frac{\mathbf{1}\otimes\mathbf{1}}{-(p-p')^2+\mu^2}. \tag{2}$$

TABLE I. Values of the parameters used in reductions A, B.

	Reduction A	Reduction B
m_b (GeV)	4.65	4.68
m_c (GeV)	1.37	1.39
m_s (GeV)	0.397	0.405
m_u (GeV)	0.339	0.346
σ (GeV^2)	0.233	0.211
γ	0.616	0.444
β (GeV)	0.198	0.187

TABLE II. Spectrum of B_c mesons in different channels (GeV/c^2).

State	This work reduction A	This work reduction B	Eichten and Quigg Ref. [2]	Gershtein *et al.* [3] Martin potential	Gershtein *et al.* [3] BT potential
1^1S_0	6.356	6.380	6.264	6.253	6.246
1^3S_1	6.397	6.415	6.337	6.317	6.337
1^3P_0	6.673	6.692	6.700	6.683	6.700
1^3P_2	6.751	6.773	6.747	6.743	6.747
1^1P_1	6.752	6.777		6.729	6.736
2^1S_0	6.888	6.874	6.856	6.867	6.856
2^3S_1	6.910	6.891	6.899	6.902	6.899
1^3D_1	6.984	6.955	7.012	7.008	7.012

Here, α_s is the strong coupling, which is weighted by the meson color factor of $\frac{4}{3}$, and the string tension σ is the strength of the confining part of the interaction. We adopt a scalar Lorentz structure V_{CON} as discussed in Refs. [5,6].

In our model the strong coupling is assumed to run as in the leading log expression for α_s

$$\alpha_s(Q^2)=\frac{4\pi\alpha_s(\mu^2)}{4\pi+\beta_1\alpha_s(\mu^2)\ln(Q^2/\mu^2)}, \tag{3}$$

where $\beta_1=11-2n_f/3$ and $n_f=4$ is the number of quark flavors taken to be fixed. At the scale of the Z boson, $\alpha_s(\mu^2=M_Z^2)\simeq 0.12$ and Q^2 is related to the meson mass scale through

$$Q^2=\gamma^2 M_{\mathrm{meson}}^2+\beta^2, \tag{4}$$

where γ and β are parameters determined by a fit to the meson spectrum. In our formulation of BSE there are therefore seven parameters: four masses $m_u=m_d, m_s, m_c, m_b$, the string tension σ, and the parameters γ and β used to govern the running of the coupling constant.

In Refs. [5,6] we fitted the meson spectrum using these seven parameters. However, we did not include the B_c mesons in our fit. Table I shows the values of the parameters obtained by the fits in reductions A and B.

In this paper we extend our model to evaluate the properties of the B_c mesons using these same values of the parameters. In Table II we compare the spectrum of B_c mesons obtained in reduction A and B with the work of Eichten and Quigg [2], Gershtein *et al.* [3] using the Martin potential, and Gershtein *et al.* [3] using the Buchmuller-Tye (BT) potential. The first row compares with the experimental results [1] of 6.40 ± 0.39 (stat) ± 0.13 (syst) GeV/c^2. Both reduction A and B compare reasonably well with the experimental results though the experimental uncertainties are large.

In our formalism the mesons are taken as bound states of a quark and an antiquark. The wave functions for the mesons are calculated by solving reductions of Bethe-Salpeter equation [5,6]. We construct the meson states as [12]

$$\begin{aligned}|M(\mathbf{P_M},J,m_J)\rangle &= \sqrt{2M}\int d^3\mathbf{p}\langle Lm_L Sm_S|Jm_J\rangle\langle sm_s\bar{s}m_{\bar{s}}|Sm_S\rangle\Phi_{Lm_L}(\mathbf{p})\\ &\times\left|\bar{q}\left(\frac{m_{\bar{q}}}{M_{q\bar{q}}}\mathbf{P_M}-\mathbf{p},m_{\bar{s}}\right)\right\rangle\left|q\left(\frac{m_q}{M_{q\bar{q}}}\mathbf{P_M}+\mathbf{p},m_s\right)\right\rangle,\end{aligned} \tag{5}$$

where the quark states are given by

$$|q(\mathbf{p},m_s)\rangle=\sqrt{\frac{(E_q+m_q)}{2m_q}}\begin{pmatrix}\chi^{m_s}\\ \dfrac{\sigma\cdot\mathbf{p}}{(E_q+m_q)}\chi^{m_s}\end{pmatrix},$$

$$M_{q\bar{q}}=m_q+m_{\bar{q}},$$

$$E_q=\sqrt{m_q^2+\mathbf{p}^2}. \tag{6}$$

In Eq. (5) M is the meson mass. The meson and the constituent quark states satisfy the normalization condition

$$\langle M(\mathbf{P'_M},J',m'_J)|M(\mathbf{P_M},J,m_J)\rangle = 2E\,\delta^3(\mathbf{P'_M}-\mathbf{P_M})\,\delta_{J',J}\delta_{m'_J,m_J}, \tag{7}$$

$$\langle q(\mathbf{p}',m'_s)|q(\mathbf{p},m_s)\rangle=\frac{E_q}{m_q}\delta^3(\mathbf{p}'-\mathbf{p})\,\delta_{m'_s,m_s}. \tag{8}$$

TABLE III. Leptonic decay constant of $B_c(f_{B_c})$ in MeV.

This work A	This work B	Eichten and Quigg Ref. [2]	Gershtein *et al.* [3] Martin potential	Gershtein *et al.* [3] BT potential
578	490	500	512	500

TABLE IV. Various contributions to the decay width of B_c in 10^{-12} GeV.

	$\Gamma(b\to \mathrm{X})$	$\Gamma(c\to \mathrm{X})$	Γ (ann)
Reduction A	0.75	0.51	0.14
Reduction B	0.78	0.55	0.11

In previous works [13–15], we have used the wavefunctions of our model to evaluate the semileptonic form factors for B to D and D^* mesons, and the leptonic decay constants. Here, we are interested in the leptonic decay constant. The weak decay constants for the pseudoscalar and vector mesons are defined by

$$\langle 0|J_\mu|P(p)\rangle = i f_P p_\mu ,$$

$$\langle 0|J_\mu|V(p)\rangle = M_V f_V \varepsilon_\mu ,$$

$$J_\mu = V_\mu - A_\mu , \tag{9}$$

where P and V are pseudoscalar and vector states and V_μ and A_μ are the vector and axial vector currents.

Taking into account the relativistic effects, the expressions of the decay constants in terms of the wave functions are given by [16]

$$f_i = \sqrt{\frac{12}{M}}\int_0^\infty \frac{p^2 dp}{2\pi^3}\sqrt{\frac{(m_q+E_q)(m_{\bar{q}}+E_{\bar{q}})}{4E_q E_{\bar{q}}}}F_i(p), \tag{10}$$

(11)

where the subscript i represents P or V and

$$F_P(p) = \left[1 - \frac{p^2}{(m_q+E_q)(m_{\bar{q}}+E_{\bar{q}})}\right]\psi_P(p), \tag{12}$$

$$F_V(p) = \left[1 - \frac{p^2}{3(m_q+E_q)(m_{\bar{q}}+E_{\bar{q}})}\right]\psi_V(p), \tag{13}$$

where ψ_P , ψ_V are the wave functions of the pseudoscalar and vector states respectively. The nonrelativistic limit of these expressions yields a relation between f_i and the wave function at the origin in coordinate space $R(0)$,

$$f_i = \sqrt{\frac{3}{\pi M}}R(0). \tag{14}$$

The leptonic decay constant (f_{B_c}) is relevant for the annihilation channel of the B_c pseudoscalar meson. In Table III we compare different predictions for this quantity.

The lifetime of B_c is a very important quantity which may help us understand the basic properties of the weak interaction at a fundamental level especially since the strong interaction effects can be estimated reliably. The total width can be approximated by the sum of the widths of $\bar{b}$-quark decay with the spectator c quark, the c-quark decay with the spectator $\bar{b}$ quark, and the annihilation channel $B_c^+ \to l^+\nu_l(c\bar{s},u\bar{s})$, $l=e,\ \mu,\ \tau$. Since all these decays lead to different final states, we have no interference between different amplitudes. The total width is then given by

$$\Gamma(B_c\to X) = \Gamma(b\to X) + \Gamma(c\to X) + \Gamma(\mathrm{ann}). \tag{15}$$

Neglecting the quark binding effects, we obtain for the b and c inclusive widths in the spectator approximation,

$$\Gamma(b\to X) = \frac{G_F^2|V_{cb}|^2 m_b^5}{192\pi^3}9,$$

$$\Gamma(c\to X) = \frac{G_F^2|V_{cs}|^2 m_c^5}{192\pi^3}5. \tag{16}$$

The width of the annihilation channel is given by

$$\Gamma(\mathrm{ann}) = \sum_i \frac{G_F^2}{8\pi}|V_{bc}|^2 f_{B_c}^2 M_{bc} m_i^2\left(1-\frac{m_i^2}{m_{Bc}^2}\right)^2 \cdot C_i , \tag{17}$$

where $C_i=1$ for the $\tau\nu_\tau$ channel and $C_i=3|V_{cs}|^2$ for the $c\bar{s}$ channel, and m_i is the mass of the heaviest fermion (τ or c). Table IV shows various contributions to the width of B_c in our model.

We have used $V_{cb}=0.041$, and $V_{cs}=0.96$. From Table IV we see that both reductions predict that the b decay dominates c decay in B_c meson.

In Table V, we compare the lifetime of B_c in different models with the CDF experimental result. The experimental result indicates that the binding effects may not be very important as suggested by Quigg [17].

In conclusion, we have evaluated the meson spectrum of the B_c mesons in two reductions of BSE. We used parameters fixed from our previous fits and our results for properties of B_c agree with the recent measurement of the CDF Collaboration of B_c mass. We also predicted the leptonic

TABLE V. Comparison of the lifetime of B_c meson (in ps) in different models.

Experiment [1]	Reduction A	Reduction B	Quigg [17]	Gershtein *et al.* [3]
$0.46^{+0.18}_{-0.16}$(stat)± 0.03(syst)	0.47	0.46	$1.1-1.4$	0.55 ± 0.15

decay constant and evaluated various contributions to the decay width of B_c. The partial width of B_c due to b-quark decay dominates that due to the c-quark decay. Our result for the B_c lifetime is in good agreement with the CDF measurement.

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